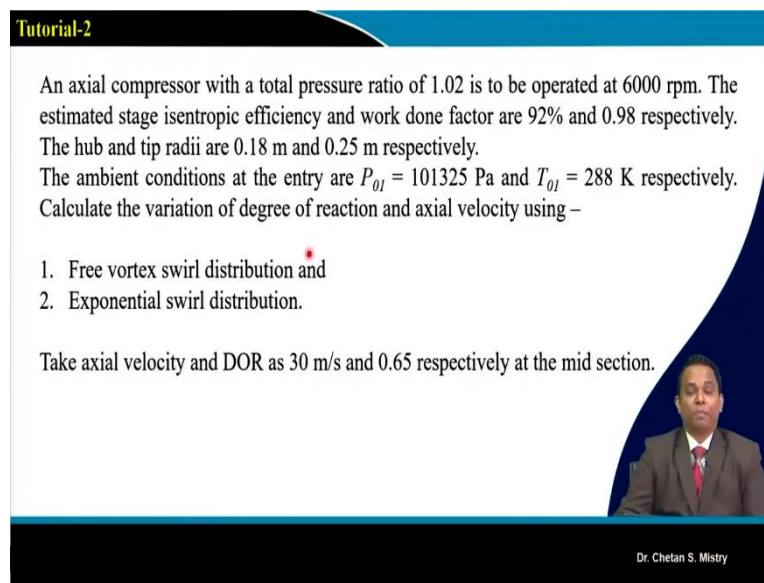


**Aerodynamic Design of Axial Flow Compressor & Fans**  
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**Lecture 22**  
**Design Concepts (Contd.)**

Hello, and welcome to lecture-22. In last lecture, we were solving a numerical, that's what was to calculate what is the variation of my flow angle by considering free vortex configuration and by considering, say constant reaction configuration.

Now, today, let us take one more numerical, that's what will be based on the free vortex and exponential kind of swirl distribution. So, that's what we'll be giving you some other kind of feeling in sense of understanding the methods or design concepts what all we have discussed in past few lectures.

(Refer Slide Time: 01:15)




**Tutorial-2**

An axial compressor with a total pressure ratio of 1.02 is to be operated at 6000 rpm. The estimated stage isentropic efficiency and work done factor are 92% and 0.98 respectively. The hub and tip radii are 0.18 m and 0.25 m respectively. The ambient conditions at the entry are  $P_{01} = 101325$  Pa and  $T_{01} = 288$  K respectively. Calculate the variation of degree of reaction and axial velocity using –

1. Free vortex swirl distribution and
2. Exponential swirl distribution.

Take axial velocity and DOR as 30 m/s and 0.65 respectively at the mid section.

  
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So, we have our axial flow compressor with a total pressure ratio of 1.02 and it is operating at 6000 rpm. The estimated stage isentropic efficiency and work done (factor) are 92% and 0.98 respectively. The hub and tip radius are 0.18 m and 0.25 m, respectively. Consider the ambient conditions at the entry as  $p_{01} = 101325$  Pa and kelvin  $T_{01} = 288$  K, respectively. Calculate the variation of degree of reaction and axial velocity using free vortex swirl distribution and exponentials swirl distribution. It says take axial velocity and degree of reaction as 30 m/s and 0.65 at the mid station.

So, here in this case it says we are looking for calculation what is happening with our degree of reaction and what is happening with the change of axial velocity. That means, we need to have some understanding of these methods.

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**Tutorial contd.**

Given stage data,

$P_{01} = 101325 \text{ Pa}$	$r_t = 0.25 \text{ m}$
$T_{01} = 288 \text{ K}$	$r_h = 0.18 \text{ m}$
$\pi_c = 1.02$	$r_m = \frac{r_t + r_h}{2} = 0.215 \text{ m}$
$R_m = 0.65$	$N = 6000 \text{ rpm}, \omega = \frac{2\pi N}{60} = 628.32 \text{ rad/s}$
	$\eta = 92\%$
	$\lambda = 0.98$

**Hint**

Calculate required work ( $\Delta T_0$ ) at mean. Degree of reaction at mean is known

↓

Use general constant work whirl distribution

↓

Use vortex energy equation to find axial velocity distribution and variation of reaction



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So, let us see how do we proceed with. It says, we can have this data, so, pressure, temperature pressure ratio, my tip radius, hub radius, rotational speed, efficiency and blockage factor; they are known to us. We would like to calculate the degree of reaction. So, in order to have your degree of reaction to be known, we need to have the calculation of whirl velocity component, okay. Now, when I say, we are looking for whirl velocity component, we need to go with arbitrary distributed whirl velocity distribution at the entry and at the exit. So, let us move with.

*Given stage data,*

$$P_{01} = 101325 \text{ Pa}$$

$$T_{01} = 288 \text{ K}$$

$$\pi_c = 1.02$$

$$R_m = 0.65$$

$$r_t = 0.25 \text{ m}$$

$$r_h = 0.18 \text{ m}$$

$$r_m = \frac{r_t + r_h}{2} = 0.215 \text{ m}$$

$$N = 6000 \text{ rpm}, \omega = \frac{2\pi N}{60} = 628.32 \frac{\text{rad}}{\text{s}}$$

$$\eta = 92\% \text{ and } \lambda = 0.98$$

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**Tutorial contd.**

The stage total temperature ratio

$$\tau_c = 1 + \frac{\pi_c^{\frac{\gamma-1}{\gamma}} - 1}{\eta}$$

$$= 1 + \frac{1.02^{1.4} - 1}{0.92}$$

$$\tau_c = 1.0061$$

Hence stage exit total temperature  $T_{02} = \tau_c T_{01} = 1.0061 \times 288$   
 $\Rightarrow T_{02} = 289.77 K$

Stage total temperature rise is given by  $\Rightarrow \Delta T_0 = T_{02} - T_{01} = 1.77 K$


Let's assume generalized constant work swirl distribution as,

$$C_{w1} = ar^n - \frac{b}{r} \text{ (at rotor inlet)}$$

$$C_{w2} = ar^n + \frac{b}{r} \text{ (at rotor exit)}$$

Calculate required work ( $\Delta T_0$ ) at mean station.  
 Degree of reaction at mean is known

Given stage data,  
 $T_{01} = 288 K$   
 $\pi_c = 1.02$



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So, what we know, is we are looking for say  $\Delta T_0$  for calculation in a later stage, we are looking for a degree of reaction, that is what needs to be calculated at mid station, okay. So, if we write down we have our temperature ratio, that's what is given by

$$\tau_c = 1 + \frac{\pi_c^{\frac{\gamma-1}{\gamma}} - 1}{\eta}$$

If you look at our pressure ratio is known to us efficiency is also known to us that's what will be giving me my temperature ratio as 1.0061.

$$\tau_c = 1 + \frac{1.02^{\frac{0.4}{1.4}} - 1}{0.92} = 1.0061$$

Since this is what is low speed compressor, you can see my pressure ratio that is also say lower, it is say 1.02, that's what will be giving me my temperature ratio in this range. We know what is our entry temperature, so, based on the temperature ratio, we can calculate, what will be my outlet temperature.

$$T_{02} = \tau_c T_{01} = 1.0061 \times 288 = 289.77 K$$

Now, once the outlet temperature is known to me, we can calculate what is total temperature rise in the stage, that's what we are writing as,

$$\text{Stage total temperature rise is given by } \Rightarrow \Delta T_0 = T_{02} - T_{01} = 1.77 K$$

Now, at this moment, we are not having idea how the whirl velocity component it is being distributed. So, we can safely assume, say generalized constant work distribution we can say  $C_{w1}$ , that's what is  $ar^n - b/r$ , that's what we are considering at the entry or at the rotor inlet; my  $C_{w2}$  we can say  $ar^n + b/r$  at exit, okay.

$$C_{w1} = ar^n - \frac{b}{r} \quad (\text{at rotor inlet})$$

$$C_{w2} = ar^n + \frac{b}{r} \quad (\text{at rotor exit})$$

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**Tutorial contd.**

Introducing a non-dimensional radius,  $R = r/r_m$

The swirl-distribution in terms of non-dimensional radius can be expressed as

$$C_w = a(r_m R)^n \pm \frac{b}{r_m R} = [ar_m^n]R^n \pm \left[\frac{b}{r_m}\right]\frac{1}{R}$$

$$C_w = AR^n \pm \frac{B}{R}$$

Where  $A = ar_m^n$  and  $B = \frac{b}{r_m}$


We know, Vortex Energy Eqn.

$$C_a \frac{dC_a}{dr} + \frac{C_w}{r} \frac{d}{dr}(rC_w) = 0$$

The simplified Vortex Energy Equation with constant spanwise work as

$$\frac{1}{2} \frac{dC_a^2}{dR} + \frac{C_w}{R} \frac{d}{dR}(RC_w) = 0$$

The above equation can be solved for  $C_a$  for given swirl distribution



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Now, in order to simplify this solution, let me assume, let me introduce one non-dimensional parameter, say... 'R', that is nothing but my radius (r) or my location at particular station divided by my mean radius ( $r_m$ ). So, if this is what is your case, in place of 'R' in this equation, what we have assumed as,  $C_{w1}$  and  $C_{w2}$ , I can replace that with say  $r_m R$ .

*The swirl distribution in terms of non – dimensional radius can be expressed as*

$$C_w = a(r_m R)^n \pm \frac{b}{r_m R} = [ar_m^n]R^n \pm \left[\frac{b}{r_m}\right]\frac{1}{R}$$

So, if I will be putting this formula, it says my whirl component distribution that can be written as

$$C_w = AR^n \pm \frac{B}{R}$$

So, where we are putting our constant as  $A = ar_m^n$  and B that is nothing but  $\frac{b}{r_m}$ . Now, from our vortex energy equation, so this is what is my vortex energy equation, what we have written. Since, we have modified our equation in terms of 'R'; so, in place of writing 'r', I can put my 'R'. And if I am simplifying this equation, this is what is a formulation for say my distribution of axial velocity or variation of axial velocity with radius and variation of my whirl velocity component with the radius.

*The simplified Vortex Energy Equation with constant spanwise work as*

$$\frac{1}{2} \frac{dC_a^2}{dR} + \frac{C_w}{R} \frac{d}{dR}(RC_w) = 0$$

So, this is the equation which we will be using for calculation of variation of axial velocity for both the method methods.

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**Tutorial contd.**

**1. Free-vortex swirl distribution (n=-1)**

In the general whirl distribution equation,

$$C_w = AR^n \pm \frac{B}{R}$$

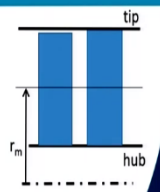

The free-vortex swirl distribution is obtained by putting  $n = -1$

$$C_w = AR^{-1} \pm \frac{B}{R}$$

$$C_w = \frac{A}{R} \pm \frac{B}{R}$$

Thus, the swirl components at inlet and exit to rotor are given by

$$C_{w1} = \frac{A}{R} - \frac{B}{R} \text{ (Before rotor)}$$

$$C_{w2} = \frac{A}{R} + \frac{B}{R} \text{ (After rotor)}$$



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Now, let us take first method. What it says? It says free vortex distribution. So, when I say free vortex, we can say, my exponent n we are putting as -1. So, let me put  $n = -1$ . So, my  $C_w$  formula, that's what will be changed to

$$C_w = AR^{-1} \pm \frac{B}{R} = \frac{A}{R} \pm \frac{B}{R}$$

So, you can say, we can safely write down my

$$C_{w1} = \frac{A}{R} - \frac{B}{R} \text{ (Before Rotor)}$$

and  $C_{w2}$ , that is

$$C_{w2} = \frac{A}{R} + \frac{B}{R} \text{ (After rotor)}$$

So, now we know what is our whirl component at the entry and what is our whirl component at the exit.

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**Tutorial contd.**

Using this swirl distribution in Simplified Vortex Energy Equation

$$\frac{1}{2} \frac{dC_a^2}{dR} + \frac{C_w}{R} \frac{d}{dR} (RC_w) = 0$$

We get

$$\frac{1}{2} \frac{dC_a^2}{dR} + \frac{\left(\frac{A \pm B}{R \pm R}\right)}{R} \frac{d}{dR} \left( R \left( \frac{A \pm B}{R \pm R} \right) \right) = 0$$

$$\frac{1}{2} \frac{dC_a^2}{dR} + \frac{A \pm B}{R^2} \frac{d}{dR} (A \pm B) = 0$$

$$\frac{1}{2} \frac{dC_a^2}{dR} = 0$$

$$C_a = \text{Constant}$$

The axial velocity remains constant throughout the span

*(A small video inset of Dr. Chetan S. Mistry is visible in the bottom right corner of the slide.)*

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Now, what we learned? We are having our axial velocity variation, that's what we have derived with simplified vortex energy equation. So, in this equation in place of  $C_w$ , we can write down it is  $\frac{A}{R} \pm \frac{B}{R}$ , if we are putting this in this formula, we will be getting say

*Simplified Vortex Energy Equation*

$$\frac{1}{2} \frac{dC_a^2}{dR} + \frac{C_w}{R} \frac{d}{dR} (RC_w) = 0$$

We get,

$$\frac{1}{2} \frac{dC_a^2}{dR} + \frac{\frac{A}{R} \pm \frac{B}{R}}{R} \frac{d}{dR} \left( R \left( \frac{A}{R} \pm \frac{B}{R} \right) \right) = 0$$

$$\frac{1}{2} \frac{dC_a^2}{dR} + \frac{A \pm B}{R^2} \frac{d}{dR} (A \pm B) = 0$$

$$\frac{1}{2} \frac{dC_a^2}{dR} = 0$$

$$C_a = \text{Constant}$$

If that's what is your case, you can say my axial velocity is constant.

So, we can say for free vortex, we have also seen, say... this is what is our required condition, we need to have satisfaction of three conditions. One, that's what is say... constant total enthalpy; second, that's what is my constant axial velocity; and third, that's what is  $C_w r = \text{Constant}$ , okay. So, that is what it says my axial velocity is coming to be constant. And that's what will remain 30 m/s at the entry of my rotor, as well as at the exit of my rotor.

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**Tutorial contd.**

Knowing constant axial velocity through the blade row,  
The degree of reaction can be expressed as

$$DOR = 1 - \frac{C_{w2} + C_{w1}}{2U}$$

Putting values of  $C_{w2}$  and  $C_{w1}$  as per free vortex distribution

$$DOR = 1 - \frac{C_{w2} + C_{w1}}{2U} = 1 - \frac{\frac{A}{R} + \frac{B}{R} + \frac{A}{R} - \frac{B}{R}}{2U}$$

$$DOR = 1 - \frac{A}{UR}$$

The rotor speed  $U$  can be expressed in terms of mean rotor speed  $U_m$  and radii


We get

$$U = U_m \left( \frac{r}{r_m} \right) = U_m R$$

since  $R = r/r_m$

We know,

$$C_{w1} = \frac{A}{R} - \frac{B}{R} \text{ (Before rotor)}$$

$$C_{w2} = \frac{A}{R} + \frac{B}{R} \text{ (After rotor)}$$


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Now, next, that's what is to calculate your degree of reaction. So, degree of reaction, that's what we can rewrite down as

$$DOR = 1 - \frac{C_{w2} + C_{w1}}{2U}$$

In this equation, we can write down my  $C_{w1}$  and  $C_{w2}$ , that's what is  $\frac{A}{R} - \frac{B}{R}$ , and second, that's what is  $\frac{A}{R} + \frac{B}{R}$ .

So, if you are putting this together,

$$DOR = 1 - \frac{\frac{A}{R} + \frac{B}{R} + \frac{A}{R} - \frac{B}{R}}{2U}$$

It says my degree of reaction, it is nothing but

$$DOR = 1 - \frac{A}{UR}$$

Now, what we have assumed? We have taken our  $R = \frac{r}{r_m}$ . So, we can write down my peripherals speed at particular station, that's what we can write down,

$$U = U_m \left( \frac{r}{r_m} \right) = U_m R$$

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**Tutorial contd.**

The degree of reaction thus becomes

$$DOR = 1 - \frac{A}{U_m R^2}$$

So at mean section,

$$DOR_m = 1 - \frac{A}{U_m}$$

Constant  $A = U_m(1 - DOR_m)$

We know,

$$U_m = \omega \times r_m = 628.32 \times 0.215$$

$$U_m = 135 \text{ m/s}$$

So, prescribing a mean reaction of 0.65

$$A = 135 \times (1 - 0.65)$$

$$A = 47.25$$

The constant 'B' can be evaluated,

$$C_p AT_0 = \lambda U A C_w$$

$$C_p AT_0 = \lambda U (C_{w2} - C_{w1})$$

$$C_p AT_0 = \lambda U \left( \frac{A}{R} + \frac{B}{R} - \frac{A}{R} + \frac{B}{R} \right)$$

$$C_p AT_0 = \frac{2\lambda U B}{R} = 2\lambda U_m B$$

$$B = \frac{C_p AT_0}{2\lambda U_m} = \frac{1.005 \times 10^3 \times 1.77}{2 \times 0.98 \times 135}$$

$$B = 6.72$$

Given


$$r_m = \frac{r_1 + r_2}{2} = 0.215 \text{ m}$$

$$\omega = \frac{2\pi N}{60} = 628.32 \text{ rad/s}$$

$$\Delta T_0 = 1.77 \text{ K}$$

$$U_m = 135 \text{ m/s}$$

$$\lambda = 0.98$$



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So, this is what is your case, you can write down my degree of reaction, that's what is given by

$$DOR = 1 - \frac{A}{U_m R^2}$$

Now, what we know for our case at the mid station, so, we can say  $r/r_m$ , so,  $r_m/r_m$  that's what we will be getting cancel. So, at mid-station we can write down, it is given by

$$DOR_m = 1 - \frac{A}{U_m}$$



So, you can calculate what will be the constant A. So, for that, we are looking for U mean, so, U mean we are writing as

$$U_m = \omega \times r_m$$

Since my hub diameter - hub radius is given, my tip radius is given, I can calculate what will be my mean radius. So, this is what is my mean radius and omega we already have calculated based on  $\frac{2\pi N}{60}$ . It is 628.32 rad/s.

$$U_m = 628.32 \times 0.215 = 135 \text{ m/s}$$

So, we will get our main peripheral speed as 135 m/s. Since my Um is known to me, I can calculate what will be my constant A. So, my constant, that's what is coming 47.25, this is what is my 'A'.

*So, prescribing a mean reaction of 0.65*

$$A = 135 \times (1 - 0.65) = 47.25$$

Now, what we know from our fundamental understanding of energy balance, my aerodynamic work and thermodynamic work, they both will be same.

So, if we are putting in this form, it says my B will be coming in sense of  $\Delta T_0$ .

$$C_p \Delta T_0 = \lambda U \Delta C_w$$

$$C_p \Delta T_0 = \lambda U (C_{w2} - C_{w1})$$

$$C_p \Delta T_0 = \lambda U \left( \frac{A}{R} + \frac{B}{R} - \frac{A}{R} + \frac{B}{R} \right)$$

$$C_p \Delta T_0 = \frac{2\lambda U B}{R} = 2\lambda U_m B$$

So, you can say B, that is given by

$$B = \frac{C_p \Delta T_0}{2\lambda U_m} = \frac{1.005 \times 10^3 \times 1.77}{2 \times 0.98 \times 135} = 6.72$$

Now, we already have calculated our  $\Delta T_0$ , it is 1.77.  $C_p$  I can write down 1.005 into 10 to power 3 divided by 2. Lambda is known to us, that's what is 0.98 and this ( $U_m$ ) is what is 135.

So, that's what is giving me what is my constant B and that is coming 6.72, okay. So, based on this, we can calculate what is our constant A and B. Now, what is our target? We are looking for calculation of our degree of reaction.

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**Tutorial contd.**

From given swirl distribution

$$C_{w2} + C_{w1} = \frac{2A}{R}$$

$$C_{w2} - C_{w1} = \frac{2B}{R}$$

Values of swirl velocity at hub, mean and tip can be obtained by solving the set of resulting equations at each radii

At hub,  $R_h = 0.837$

$$C_{w2h} + C_{w1h} = \frac{2A}{R_h} = \frac{2 \times 47.25}{0.837} = 112.9$$

$$C_{w2h} - C_{w1h} = \frac{2B}{R_h} = \frac{2 \times 6.72}{0.837} = 16.05$$

Swirl velocity component at hub -rotor exit  $C_{w2h} = 64.48 \text{ m/s}$

Swirl velocity component at hub- rotor entry  $C_{w1h} = 48.42 \text{ m/s}$

Since  $R = \frac{r}{r_m}$


$R_h = \frac{0.18}{0.215} = 0.837$

$R_m = \frac{0.215}{0.215} = 1$

$R_t = \frac{0.25}{0.215} = 1.16$

$A = 47.25$

$B = 6.72$



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So, you know, for that purpose, we can write down, here in this case, my  $C_{w1} + C_{w2}$ , that's what is given by

$$C_{w1} + C_{w2} = \frac{2A}{R}$$

and we can write down  $C_{w2} - C_{w1}$ , that's what is given by

$$C_{w2} - C_{w1} = \frac{2B}{R}$$

So, at a hub, this is what it says; it is given at this station is 0.387, we can have this  $R_h$ , it is 0.837.

So, we can put this equation. It says, this is what will be giving me, what will be the variation of my whirl component at the entry and exit at the hub, okay. So, this is what will be giving me how my whirl velocity component, that's what is varying at the entry and exit.

At hub,  $R_h = 0.837$

$$C_{w2h} + C_{w1h} = \frac{2A}{R_h} = \frac{2 \times 47.25}{0.837} = 112.9$$

$$C_{w2h} - C_{w1h} = \frac{2B}{R_h} = \frac{2 \times 6.72}{0.837} = 16.05$$

*Swirl velocity component at hub – rotor exit,  $C_{w2h} = 64.48 \text{ m/s}$*

*Swirl velocity component at hub – rotor entry,  $C_{w1h} = 48.42 \text{ m/s}$*

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**Tutorial contd.**

Similarly, The swirl components at Mean and tip section can be calculated,

Swirl velocity component at mean –rotor exit  $\Rightarrow C_{w2m} = 53.97 \text{ m/s}$

Swirl velocity component at mean- rotor entry  $\Rightarrow C_{w1m} = 40.53 \text{ m/s}$

Swirl velocity component at tip –rotor exit  $\Rightarrow C_{w2t} = 46.52 \text{ m/s}$

Swirl velocity component at tip- rotor entry  $\Rightarrow C_{w1t} = 34.94 \text{ m/s}$

The degree of reaction is given by,

$$DOR = 1 - \frac{A}{U_m R^2}$$

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Now, inline to that, we can do our calculation at the mean-section, inline to that we can do our calculation at the tip-section. So, we can say, our swirl velocity component at the mid station it is coming  $53.97 \text{ m/s}$  and at the entry it is  $40.53 \text{ m/s}$ . Same way, we can do our calculation at the tip, that's what is coming say  $46.52$  and at the entry it is  $34.94$ .

*Similarly, the swirl components at Mean and Tip section can be calculated,*

*Swirl velocity component at mean – rotor exit,  $C_{w2m} = 53.97 \text{ m/s}$*

*Swirl velocity component at mean – rotor entry,  $C_{w1m} = 40.53 \text{ m/s}$*

*Swirl velocity component at tip – rotor exit,  $C_{w2t} = 46.52 \text{ m/s}$*

*Swirl velocity component at tip – rotor entry,  $C_{w1t} = 34.94 \text{ m/s}$*

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**Tutorial contd.**

$$DOR = 1 - \frac{A}{U_m R^2}$$

At mean,  $DOR_m = 1 - \frac{47.25}{135} = 0.65$  (Given)

At hub,  $DOR_h = 1 - \frac{47.25}{135 \times 0.83^2} = 0.49$


At tip,  $DOR_t = 1 - \frac{47.25}{135 \times 1.16^2} = 0.73$

Since  $R = r/r_m$ , from given values of radii

$R_h = 0.18/0.215 = 0.837$

$R_m = 0.215/0.215 = 1$

$R_t = 0.25/0.215 = 1.16$



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Now, based on this understanding, we can calculate our degree of reaction, that's what we have derived as

$$DOR = 1 - \frac{A}{U_m R^2}$$

So, different 'R' value we can put. Suppose, if I will be putting at say mid-station, that's what is  $U_m$ , because 'R' is 1.

$$\text{At mean, } DOR_m = 1 - \frac{47.25}{135} = 0.65 \text{ (given)}$$

At hub, we can calculate because my ratio of  $r/r_m$ , you can calculate, that's what is coming 0.83, okay. This is giving me my degree of reaction at the hub as 0.49.

$$\text{At hub, } DOR_h = 1 - \frac{47.25}{135 \times 0.83^2} = 0.49$$

At tip, we are getting our degree of reaction to be 0.73.

$$\text{At tip, } DOR_t = 1 - \frac{47.25}{135 \times 1.16^2} = 0.73$$

So, this is what is all calculation when we are considering our free vortex configuration, in which our axial velocity, that's what is constant; and, my degree of reaction, that's what is going from 0.49 at the hub to 0.73 at the tip.

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**Tutorial contd.**

**2. Exponential swirl distribution (n=0)**


In the general whirl distribution equation,

$$C_w = AR^n \pm \frac{B}{R}$$

The exponential swirl distribution is obtained by putting  $n = 0$

$$C_w = AR^0 \pm \frac{B}{R}$$
$$C_w = A \pm \frac{B}{R}$$

The swirl components at inlet and exit to rotor are given by

$$C_{w1} = A - \frac{B}{R} \text{ (Before rotor)}$$
$$C_{w2} = A + \frac{B}{R} \text{ (After rotor)}$$


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Now, what next is asked to us, it is to consider exponential swirl velocity distribution. So, if that's what we are considering, we say my 'n', that's what is equal to 0. So, in equation, if I will be writing my  $n = 0$ ,

$$C_w = AR^n \pm \frac{B}{R}$$

$$C_w = AR^0 \pm \frac{B}{R}$$

$$C_w = A \pm \frac{B}{R}$$

It says my  $C_{w1}$ , that is given by

$$C_{w1} = A - \frac{B}{R} \text{ (Before rotor)}$$

and my  $C_{w2}$ , that's what is given by

$$C_{w2} = A + \frac{B}{R} \text{ (After rotor)}$$

So, at the entry, we are considering  $A - \frac{B}{R}$ , and at the exit of my rotor, it is  $A + \frac{B}{R}$ . So, these are the two whirl components, that's what we have come up with.

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**Tutorial contd.**

Using this swirl distribution in Simplified Vortex Energy Equation

$$\frac{1}{2} \frac{dC_a^2}{dR} + \frac{C_w}{R} \frac{d}{dR} (RC_w) = 0$$


We will solve the above for rotor inlet, i.e.

$$C_w = A - \frac{B}{R}$$

The equation will be  $\frac{1}{2} \frac{dC_a^2}{dR} + \left( \frac{A}{R} - \frac{B}{R^2} \right) \frac{d}{dR} (AR - B) = 0$

$$\frac{1}{2} \frac{dC_a^2}{dR} + \left( \frac{A^2}{R} - \frac{AB}{R^2} \right) = 0$$

Integrating the equation from mean radius ( $R=1$ ) to any arbitrary radius ( $R$ ),

$$\int_{R=1}^R \frac{1}{2} (dC_a^2) + \int_{R=1}^R \left( \frac{A^2}{R} - \frac{AB}{R^2} \right) dR = 0$$


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Now, based on our vortex energy equation,

$$\frac{1}{2} \frac{dC_a^2}{dR} + \frac{C_w}{R} \frac{d}{dR} (RC_w) = 0$$

If we will be putting this equation, where my  $C_w$ , I am writing as  $A - \frac{B}{R}$ . So, if we are putting this together, at the entry what it says? At entry, we can say  $A - \frac{B}{R}$ . So, this is what will be giving me the variation of axial velocity. So, you can understand this is what is not coming to be 0.

$$\text{At rotor inlet, } C_w = A - \frac{B}{R}$$

$$\text{The equation will be } \frac{1}{2} \frac{dC_a^2}{dR} + \left( \frac{A}{R} - \frac{B}{R^2} \right) \frac{d}{dR} (AR - B) = 0$$

$$\frac{1}{2} \frac{dC_a^2}{dR} + \left( \frac{A^2}{R} - \frac{AB}{R} \right) = 0$$

That means we need to literally calculate what will be the variation of axial velocity at particular station. So, for the sake of simplicity, what we are putting, we are considering mean radius; we know, that's what is  $r/r_m$  is say 1. So, this I am putting as 1 limit and arbitrary radius, I say it is  $R$ . So, putting my limit and putting for integration, say  $R$  equal to 1 to  $R$  integrating this equation and this formula.

$$\int_{R=1}^R \frac{1}{2} (dC_a^2) + \int_{R=1}^R \left( \frac{A^2}{R} - \frac{AB}{R^2} \right) dR = 0$$

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**Tutorial contd.**

Integrating the equation from mean radius ( $R=1$ ) to any arbitrary radius ( $R$ ),

$$\int_{R=1}^R \frac{1}{2} (dC_a^2) + \int_{R=1}^R \left( \frac{A^2}{R} - \frac{AB}{R^2} \right) dR = 0$$

The axial velocity variation at rotor inlet is given by

$$C_{a1}^2 = C_{a1m}^2 - 2 \left\{ A^2 \ln R + AB \left( \frac{1}{R} - 1 \right) \right\}$$


Where  
 $C_{a1m}$  = Axial velocity at mean radius at rotor inlet  
 $C_{a1}$  = Axial velocity at any radius at rotor inlet

The axial velocity profile at the rotor exit can be deduced by replacing  $B$  with  $-B$

Thus at exit,

$$C_{a2}^2 = C_{a2m}^2 - 2 \left\{ A^2 \ln R - AB \left( \frac{1}{R} - 1 \right) \right\}$$

It can be observed that the axial velocity is not constant through the blade span



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When we are simplifying this term, my axial velocity at the entry, that's what we are getting as

$$C_{a1}^2 = C_{a1m}^2 - 2 \left\{ A^2 \ln R + AB \left( \frac{1}{R} - 1 \right) \right\}$$

Now, here in this case, just remember  $C_{a1m}$  is nothing but my axial velocity at the mean radius, and  $C_{a1}$  is nothing but my axial velocity at particular station.

So, in line to that, if you will try to solve the equation by putting say,  $A + \frac{B}{R}$ , you can get my  $C_{a2}$ , okay. That is nothing but my axial velocity at the exit of my rotor, that's what will be coming in this formula.

$$C_{a2}^2 = C_{a2m}^2 - 2 \left\{ A^2 \ln R - AB \left( \frac{1}{R} - 1 \right) \right\}$$

So, it says my axial velocity is varying at entry, my axial velocity is varying at the exit, it is not remaining constant, okay.

(Refer Slide Time: 17:56)

**Tutorial contd.**


Hence the degree of reaction needs to be evaluated from the first principle

We know,  $DOR = 1 + \frac{C_{a1}^2 - C_{a2}^2}{2U(C_{w2} - C_{w1})} - \frac{C_{w2} + C_{w1}}{2U}$

from the expressions of axial velocity distribution

$$C_{a1}^2 - C_{a2}^2 = C_{a1m}^2 - 2 \left\{ A^2 \ln R + AB \left( \frac{1}{R} - 1 \right) \right\} - C_{a2m}^2 + 2 \left\{ A^2 \ln R - AB \left( \frac{1}{R} - 1 \right) \right\}$$

Assuming that axial velocity at mean radius remains constant before and after rotor  
i.e.  $C_{a1m} = C_{a2m}$  (A common design approach)

$$C_{a1}^2 - C_{a2}^2 = 4AB \left( 1 - \frac{1}{R} \right)$$


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So, if we are looking for calculating our degree of reaction, so this is what will be the formula,

$$DOR = 1 + \frac{C_{a1}^2 - C_{a2}^2}{2U(C_{w2} - C_{w1})} - \frac{C_{w2} + C_{w1}}{2U}$$

If you are simplifying all these things together, that's what will be giving me my

$$C_{a1}^2 - C_{a2}^2 = 4AB \left( 1 - \frac{1}{R} \right)$$

(Refer Slide Time: 18:20)

**Tutorial contd.**


From the exponential swirl distribution,

$C_{w1} = A - \frac{B}{R}$  (Before rotor)  $\rightarrow$   $C_{w2} - C_{w1} = \frac{2B}{R}$ ,  
 $C_{w2} = A + \frac{B}{R}$  (After rotor)  $C_{w2} + C_{w1} = 2A$

The expression for degree of reaction can be obtained by substituting the derived values

$$DOR = 1 + \frac{4AB \left( 1 - \frac{1}{R} \right)}{2U \left( \frac{2B}{R} \right)} - \frac{2A}{2U} = 1 + \frac{AR}{U} - \frac{2A}{U}$$

$$\Rightarrow DOR = 1 + \frac{AR}{U} \left( 1 - \frac{2}{R} \right)$$

$$\Rightarrow DOR = 1 + \frac{A}{U_m} \left( 1 - \frac{2}{R} \right) \quad \text{Since } U = U_m R$$


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Now, here in this case, what all we know, we are putting our degree of reaction. Since we know what is our  $C_{w1}$ ,  $C_{w2}$ , we have our equation in the formation of  $C_{w2} - C_{w1}$ , that's what is correlating, just remember, that's what is correlating my B and this plus, that's what is correlating my A. So, A and B, that's what is a constant.

*From the exponential swirl distribution,*

$$C_{w1} = A - \frac{B}{R} \quad (\text{Before rotor})$$

$$C_{w2} = A + \frac{B}{R} \quad (\text{After rotor})$$

$$\therefore C_{w2} - C_{w1} = \frac{2B}{R}$$

$$\therefore C_{w2} + C_{w1} = \frac{2A}{R}$$

So, this is what we already seen in our past calculation, so will not be repeating the same thing.

So, if you are putting these together, my degree of reaction, that's what is coming as

$$DOR = 1 + \frac{\left(4AB \left(1 - \frac{1}{R}\right)\right)}{2U \left(\frac{2B}{R}\right)} - \frac{2A}{2U} = 1 + \frac{AR}{U} - \frac{2A}{U}$$

$$\Rightarrow DOR = 1 + \frac{AR}{U} \left(1 - \frac{2}{R}\right)$$

$$\Rightarrow DOR = 1 + \frac{A}{U_m} \left(1 - \frac{2}{R}\right) \quad \text{since } U = U_m R$$

(Refer Slide Time: 19:07)

**Tutorial contd.**

As before, A can be evaluated at the mean radius ( $R = 1$ )  
 $A = U_m(1 - DOR_m)$


with  $DOR_m = 0.65$ , the value of 'A' is  
 $A = 135 \times (1 - 0.65)$   
 $\Rightarrow A = 47.25$

As calculated  
 $U_m = 135 \text{ m/s}$

The constant 'B' can be evaluated by expression for work done

$$B = \frac{C_p \Delta T_0}{2\lambda U_m}$$
$$= \frac{1.005 \times 10^3 \times 1.77}{2 \times 0.98 \times 135}$$
$$B = 6.72$$

From the given swirl distribution

$$C_{w2} + C_{w1} = 2A$$
$$C_{w2} - C_{w1} = \frac{2B}{R}$$


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Now, you know, we want to calculate what is our constant 'A'. So, constant 'A', we are writing as say

$$A = U_m(1 - DOR_m)$$

Since we already know my degree of reaction at mid station, that is given 0.65. If I am putting into this equation, that's what will be giving me my constant A and that is 47.25.

$$DOR = 0.65 \text{ (given)}$$

$$\therefore A = 135 \times (1 - 0.65) = 47.25$$

Same way, we can calculate our B. The B is nothing but that's what is based on our energy balance, say my thermodynamic work and aerodynamic work both are same. Based on that we can say, my B is

$$B = \frac{C_p \Delta T_0}{2\lambda U_m} = \frac{1.005 \times 10^3 \times 1.77}{2 \times 0.98 \times 135} = 6.72$$

That's what is giving me B as 6.72, okay.

(Refer Slide Time: 19:59)

**Tutorial contd.**

at hub,  $R_h = 0.837$

$$\Rightarrow C_{w2h} + C_{w1h} = 2A = 2 \times 47.25 = 94.5$$

$$\Rightarrow C_{w2h} - C_{w1h} = \frac{2B}{R_h} = \frac{2 \times 6.72}{0.837} = 16.05$$

Swirl velocity component at hub – rotor exit  $\Rightarrow C_{w2h} = 55.27 \text{ m/s}$   
 Swirl velocity component at hub- rotor entry  $\Rightarrow C_{w1h} = 39.22 \text{ m/s}$

Swirl velocity component at mid – rotor exit  $\Rightarrow C_{w2m} = 53.97 \text{ m/s}$   
 Swirl velocity component at mid- rotor entry  $\Rightarrow C_{w1m} = 40.53 \text{ m/s}$

Swirl velocity component at tip – rotor exit  $\Rightarrow C_{w2t} = 53.04 \text{ m/s}$   
 Swirl velocity component at tip- rotor entry  $\Rightarrow C_{w1t} = 41.46 \text{ m/s}$

Since  $R = r/r_m$


$$R_h = 0.18/0.215 = 0.837$$

$$R_m = 0.215/0.215 = 1$$

$$R_t = 0.25/0.215 = 1.16$$

$$A = 47.25$$

$$B = 6.72$$



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Now, once we have calculated these constants, we can calculate what will be my  $C_{w1}$  at the entry and at the exit, at three different stations; at hub, mean and tip station. So, this is what is a formulation, that's what we have already discussed earlier. So, at hub, we can say, we are getting  $55.27 \text{ m/s}$  at the entry, we are getting our whirl component as  $39.22 \text{ m/s}$ . For mid station at the exit, we are getting that as a  $53.97 \text{ m/s}$  and at the entry we are getting  $40.53 \text{ m/s}$ .

In line to that, at the tip also we can calculate. It says my  $C_{w2}$  at the tip, it is  $53.04$  and at the tip -at the entry, we can say, it is  $41.46 \text{ m/s}$ . So, because of repetitive stage, we are not showing all calculations here, maybe you can do pen paper work and you can get with this number and verify these numbers.

*Swirl velocity component at hub – rotor exit  $\Rightarrow C_{w2h} = 55.27 \text{ m/s}$*

*Swirl velocity component at hub – rotor entry  $\Rightarrow C_{w1h} = 39.22 \text{ m/s}$*

*Swirl velocity component at mid – rotor exit  $\Rightarrow C_{w2m} = 53.97 \text{ m/s}$*

*Swirl velocity component at mid – rotor entry  $\Rightarrow C_{w1m} = 40.53 \text{ m/s}$*

*Swirl velocity component at tip – rotor exit,  $C_{w2t} = 53.04 \text{ m/s}$*

*Swirl velocity component at tip – rotor entry,  $C_{w1t} = 41.46 \text{ m/s}$*

(Refer Slide Time: 21:10)

**Tutorial contd.**

**Variation of axial velocity**

The axial velocity variation at rotor inlet is given by

$$\Rightarrow C_{a1}^2 = C_{a1m}^2 - 2 \left\{ A^2 \ln R + AB \left( \frac{1}{R} - 1 \right) \right\}$$

At hub (with given  $C_{a1m} = 30 \text{ m/s}$ )

$$C_{a1h}^2 = 30^2 - 2 \left\{ 47.25^2 \ln 0.837 + (47.25 \times 6.72) \left( \frac{1}{0.837} - 1 \right) \right\}$$

$$\Rightarrow C_{a1h} = 39.63 \text{ m/s}$$

Similarly at tip (with given  $C_{a1m} = 30 \text{ m/s}$ )

$$C_{a1t}^2 = 30^2 - 2 \left\{ 47.25^2 \ln 1.16 + (47.25 \times 6.72) \left( \frac{1}{1.16} - 1 \right) \right\}$$

$$\Rightarrow C_{a1t} = 18.02 \text{ m/s}$$

Since,  $R = r/r_m$

$R_h = 0.18/0.215 = 0.837$


$R_t = 0.215/0.215 = 1$

$R_t = 0.25/0.215 = 1.16$

$C_{a1m} = 30 \text{ m/s}$

$A = 47.25$

$B = 6.72$



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Now, we are looking for what is happening with our axial velocity. So, if you recall, we have derived  $C_{a1}$  that is nothing but my axial velocity at the entry. That's what we have correlated in sense of our axial velocity at the mid station and constant A, B and R, R is nothing but it is at particular location.

$$C_{a1}^2 = C_{a1m}^2 - 2 \left\{ A^2 \ln R + AB \left( \frac{1}{R} - 1 \right) \right\}$$

So, if I am looking for my axial velocity at the entry near the hub, I will be putting these numbers. It says my  $C_{a1}$  at the mid station is  $30 \text{ m/s}$ , this A we have calculate is 47.25, B we have calculated as say 6.72 and R we have already calculated for hub, it is nothing but  $r_h/r_m$ , it is 0.837, okay. And, that's what will be giving me my axial velocity at the entry near the hub.

$$C_{a1h}^2 = 30^2 - 2 \left\{ 47.25^2 \ln 0.837 + (47.25 \times 6.72) \left( \frac{1}{0.837} - 1 \right) \right\}$$

$$\Rightarrow C_{a1h} = 39.63 \text{ m/s}$$

Similarly, we can calculate our axial velocity at the entry, near the tip region, it is coming 18.02.

$$C_{a1t}^2 = 30^2 - 2 \left\{ 47.25^2 \ln 1.16 + (47.25 \times 6.72) \left( \frac{1}{1.16} - 1 \right) \right\}$$

$$\Rightarrow C_{a1t} = 18.02 \text{ m/s}$$

Just imagine, at the mid station, we are having our axial velocity to be 30 m/s, okay, and at the hub, we are having 39.63, near the tip, that's what is 18.02 m/s. So, it is a great variation of my axial velocity.

(Refer Slide Time: 22:38)

**Tutorial contd.**

The axial velocity variation at rotor exit is given by

$$\Rightarrow C_{a2}^2 = C_{a2m}^2 - 2 \left\{ A^2 \ln R - AB \left( \frac{1}{R} - 1 \right) \right\}$$

At hub (with given  $C_{a1m} = 30 \text{ m/s}$ )

$$C_{a2h}^2 = 30^2 - 2 \left\{ 47.25^2 \ln 0.837 - (47.25 \times 6.72) \left( \frac{1}{0.837} - 1 \right) \right\}$$


$$\Rightarrow C_{a2h} = 42.63 \text{ m/s}$$

At tip (with given  $C_{a1m} = 30 \text{ m/s}$ )

$$C_{a2t}^2 = 30^2 - 2 \left\{ 47.25^2 \ln 1.16 - (47.25 \times 6.72) \left( \frac{1}{1.16} - 1 \right) \right\}$$

$$\Rightarrow C_{a2t} = 12.23 \text{ m/s}$$

Since,  $R = r/r_m$   
 $R_h = 0.18/0.215 = 0.837$   
 $R_m = 0.215/0.215 = 1$   
 $R_t = 0.25/0.215 = 1.16$   
 $C_{a1m} = 30 \text{ m/s}$   
 $A = 47.25$   
 $B = 6.72$



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Similarly, we can do over calculation at the exit.

$$C_{a2}^2 = C_{a2m}^2 - 2 \left\{ A^2 \ln R - AB \left( \frac{1}{R} - 1 \right) \right\}$$

So, if we are putting our exit at mid station, we have our axial velocity as 30 m/s. And this constant A and B we can put, that's what is giving me my axial velocity at the exit near the hub is 42.63.

$$C_{a2h}^2 = 30^2 - 2 \left\{ 47.25^2 \ln 0.837 - (47.25 \times 6.72) \left( \frac{1}{0.837} - 1 \right) \right\}$$

$$\Rightarrow C_{a2h} = 42.63 \text{ m/s}$$

Just look at these numbers, this is what is showing some great variation compared to 30 m/s at the mid station.

Same way at the tip station, we can calculate, that's what is coming 12.23 m/s, okay.

$$C_{a2t}^2 = 30^2 - 2 \left\{ 47.25^2 \ln 1.16 - (47.25 \times 6.72) \left( \frac{1}{1.16} - 1 \right) \right\}$$

$$\Rightarrow C_{a2t} = 12.23 \text{ m/s}$$

So, this is what is indication, when we are considering our exponential design, we are having say variation of our axial velocity at the entry and our variation of axial velocity at the exit of my rotor.

(Refer Slide Time: 23:35)

**Tutorial contd.**

The degree of reaction at various span locations can be evaluated

$$DOR = 1 + \frac{A}{U_m} \left( 1 - \frac{2}{R} \right)$$

$$DOR_h = 1 + \frac{47.25}{135} \left( 1 - \frac{2}{0.837} \right) = 0.51$$

$$DOR_m = 1 + \frac{47.25}{135} \left( 1 - \frac{2}{1} \right) = 0.65 \text{ (as prescribed)}$$

$$DOR_t = 1 + \frac{47.25}{135} \left( 1 - \frac{2}{1.162} \right) = 0.74$$

Since,  $R = \frac{r}{r_m}$ ,


$R_h = 0.18 / 0.215 = 0.837$

$R_m = 0.215 / 0.215 = 1$

$R_t = 0.25 / 0.215 = 1.16$

$U_m = 135 \text{ m/s}$

$A = 47.25$



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Now, if you are looking for calculation of degree of reaction, we can say, that's what is given by

$$DOR = 1 + \frac{A}{U_m} \left( 1 - \frac{2}{R} \right)$$

this is what we derived, okay. So, whenever you are taking any exponent, maybe you can come up with some other exponent, make a habit of deriving this thing, that's what will be helping you in order to calculate what all parameters, that's what is going to change. Mainly degree of reaction, my axial velocity, they are of great interest at the initial stage of design.

So, if we consider my degree of reaction at the hub, that's what is coming 0.51. If we consider at the mid station, that's what is given 0.65 and at the tip, that's what is coming 0.74.

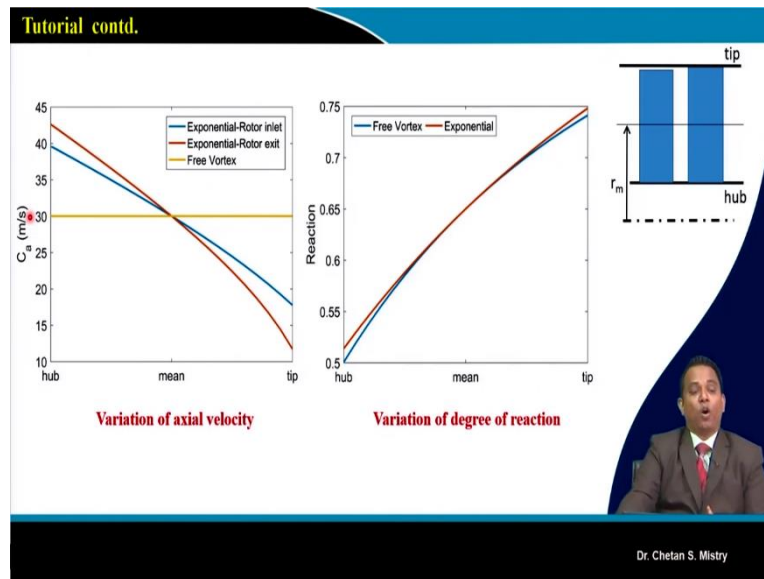
$$DOR_h = 1 + \frac{47.25}{135} \left( 1 - \frac{2}{0.837} \right) = 0.51$$

$$DOR_m = 1 + \frac{47.25}{135} \left( 1 - \frac{2}{1} \right) = 0.65$$

$$DOR_t = 1 + \frac{47.25}{135} \left( 1 - \frac{2}{1.162} \right) = 0.74$$

So, this is what is giving you idea, how my degree of reaction, that's what is changing when we are considering our exponential whirl distribution.

(Refer Slide Time: 24:42)



So, let us have comparison of these 2. So, if you are looking at, say... this is what is representing along the span, how axial velocity it is varying. When we are considering we are having free vortex design, my axial velocity is 30 m/s. And that's what is constant throughout the span. Now, here in this case, if you are considering say, exponential method, we are having variation of axial velocity at the entry, as well as at the exit.

So, if you look at, this is what is representing the variation of axial velocity at the entry. So, you can see from hub to mid-section, my axial velocity, that's what is going to decrease, and on later stage from a mid-section to tip, my axial velocity is going to increase. But at the same time for the exit, if you look at, I am having larger velocity in near the hub, that's what is going to decrease here. In line to that, near the tip region, our axial velocity at the exit is coming to be lower, okay.


And if you try to compare the degree of reaction, we can have, look at these numbers, if we look at, there is not much variation in sense of degree of reaction, that's what is happening both in your hub and tip region. So, we can say, this is what is giving some different kind of feeling.

(Refer Slide Time: 26:15)

**Tutorial contd.**

	Free vortex loading					Exponential loading					
	$C_{w1}$ (m/s)	$C_{w2}$ (m/s)	$\Delta C_w$ (m/s)	$C_a$ (m/s)	DOR	$C_{w1}$ (m/s)	$C_{w2}$ (m/s)	$\Delta C_w$ (m/s)	$C_a$ (Inlet) (m/s)	$C_a$ (Exit) (m/s)	DOR
Tip	34.97	46.52	11.55	30	0.73	41.46	53.04	11.56	39.63	42.63	0.74
Mean	40.53	53.97	13.44	30	0.65	40.53	53.97	13.44	30	30	0.65
Hub	48.42	64.48	16.05	30	0.49	39.22	55.27	16.05	18.02	12.23	0.51

- The change in swirl velocity is same for both distributions ( a consequence of same blade loading ) at different span locations.
- The inlet and outlet swirl are different for both distributions (except at mean radius).



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Now, let us see what is happening in sense of my whirl distribution. So, here if you look at, this is what is representing the comparison of free vortex and exponential loading at tip, mean and hub station. So, if you compare, my  $\Delta C_w$ , for both the methods, they are coming same, okay, they are coming same. If you are looking at, the entry  $C_{w1}$ , exit  $C_{w1}$ , entry  $C_{w1}$  and exits  $C_{w1}$ , for both the methods at the mid station, they are same. But, when we are comparing near the hub region, you can say, we are having say, great reduction for exponential method, okay. And, when we are comparing near the tip region, my whirl component is coming to be higher.

So, this is what all is giving us idea of what all we need to do in sense of systematic calculation. So, by solving this numerical, we are initiating some design idea to build the confidence that you can do your design based on what all approaches are given to you. And at the same time, this is also giving you hint, what all parameters they are getting changed when I am assuming particular design.

So, free vortex design, that's what we discuss as, that's what is giving highly twisted blade. When we are looking at constant reaction design, we have discussed, that's what is not satisfying your radial equilibrium. When you are going with say exponential method; again, you will be having variation of axial velocity. We have discussed about our fundamental methods also where we are neglecting our radial equilibrium. So, with this, we are stopping with. Thank you very much! See you in the next class.