Aerodynamic Design of Axial Flow Compressor & Fans
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Lecture 21
Design Concepts (Contd.)

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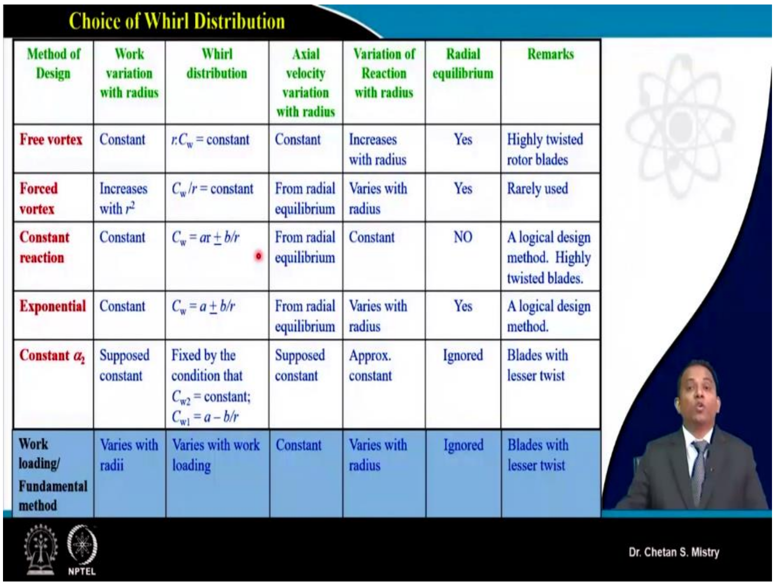

Hello, and welcome to lecture-21st. In last few lectures, we were discussing about different design approaches for axial flow compressors and fans. So, we were discussing about design methods name as say… free vortex design, forced vortex design, constant reaction design, exponential design, constant α_2 and work loading or our fundamental method.

Now, this is what is a compilation of all design methods what we have discussed in past few lectures. That's what is in sense of the variation of work with the radius, whirl distribution, axial velocity variation with the radius, variation of degree of reaction. Whether this method is opting for or is it satisfying a radial equilibrium and we will be putting some remarks that's what is related to the final blades what we will be getting.

So, if you look at, very first method, what we have discussed it is free vortex method. And as we have discussed, during 50s and 60s, people, they were using this method for the development of gas turbine engines application to Aero engines, okay. Now, in that, what we are looking for is by work variation, that's what will remains constant. I will be having my whirl distribution as say $rC_w = constant$.

And, if we look at, this is what it says axial velocity also we are assuming to be constant. And, we have derived with it says my degree of reaction, that's what is varying all the way from hub to tip. And, this is what is satisfying our radial equilibrium equation. And if you look at, this method, that's what is giving highly twisted blades.

And we have realized, there may be chances for my degree of reaction to go 0 or maybe negative value when we are looking for free-vortex design concept, okay. In order to overcome what limitations we are having with the free vortex design, we have opt for different approaches; one of them, that's what is a forced vortex design approach. We can say my work variation, that's what is varying with the function of r^2.

Here, whirl distribution, we are assuming $\frac{C_w}{r} = constant$. And, if we look at our axial velocity, we are calculating based on radial equilibrium, and it says my degree of reaction, that's what is varying from hub to tip. And yes, this is what is satisfying our radial equilibrium equation.

So, if you recall from our radial equilibrium requirement, we say, we need to have constant work input all the way from hub to shroud, we need to have our axial velocity to be constant, and $C_w \cdot r = constant$. If we are satisfying these three requirements, then we can say our radial equilibrium, it is satisfied. Then we have discussed, if you are able to have two parameters, that's what is known to us, we can calculate our third parameter. So, that's what we have done in our forced vortex design.

Then we have started discussing about the constant reaction design. So, if we look at, it says my work variation along the radius, that's what is constant; my whirl distribution, that's what we have arbitrarily chosen. Whirl velocity component, it says my C_{w1} and C_{w2}, that's what is varying in the form of $ar \pm \frac{b}{r}$. So, at the entry we are assuming my whirl distribution, at the exit we are assuming different whirl distribution; my axial velocity, that's what we are calculating based on radial equilibrium equation.

And as we have decided with, say constant reaction; so, it says my variation of degree of reaction, that's what is constant. And the limitation with this method, it is not satisfying radial equilibrium, okay. But, it says this is what is more logical design and it may be used for design of special kind of compressors, okay.

Then we are having our exponential design method; if you are looking at my work variation, that's what is constant and my whirl velocity distribution, we can say that's what is based on my selection of arbitrary whirl distribution. We can calculate our axial velocity based on radial equilibrium equation and it says my degree of reaction, that's what is varying along my span or with radius from hub to shroud. And, we can say, this is what is satisfying my radial equilibrium equation. This, also people used to say as a more logical design.

So, many fans, many compressors in actual engines; we are finding, they are having of this kind of design configuration. We have discussed about the variation of my exponent from ranging of say maybe 0.8 to 2, that's what is of special category and there we are using this kind of design concepts. These days people started talking about constant α_2 design, in which at the exit of my rotor, α_2 we are assuming to be constant, okay.

So, there are special requirements downstream of my rotor, we are having stator and for more challenging designs, people they are opting for constant α_2 approach. It says, my work done supposed to be constant; we can say, my whirl distribution, you can say, my C_{w2}, that's what is constant. We are assuming our whirl component at the exit of rotor to be constant.

And at the entry, we can assume that to be say $a - b/r$. We are having this axial velocity, we can say, it is supposed to be constant; we can say my degree of reaction also is around constant, you can say, about constant, approximately constant. Here, we are ignoring our radial equilibrium. The beauty of this design is we will be having less twisted blades, okay!

So, recent compressors what we are looking for, say… for LP compressor for HP compressor, even for high bypass ratio fans or for low bypass ratio fans, people, they are looking for the special kind of requirements. And, these designs, that's what will be catering those. Then we were discussing about say work loading or the fundamental method, that's what it says my work done or work, that's what is varying along my radius; we are having variation of my C_w component.

Because at all stations, we are not assuming our C_w at entry and C_w at the exit. Basically, that's what we are calculating based on our fundamental understanding of aerodynamic work and thermodynamic work. We can say, our axial velocity we are assuming to be constant, we will be having degree of reaction, that's what is varying all the way from hub to tip. And, in this case also, our radial equilibrium we are ignoring. Now, the case is what blades we are getting, that's what is having less twisted blade.

So, it is all the choice of designer to meet special requirements and based on that he or she will be doing the design or group of people, they are doing their design and finally based on their expectation, if the design is meeting with, they will be going for finalizing that design approach. So, there is no unique method in sense; you are having multiple choice here when you are doing your design, okay. So, with this background, let us try to solve a numerical, that's what will be giving you idea how do we approach with say different design concepts.

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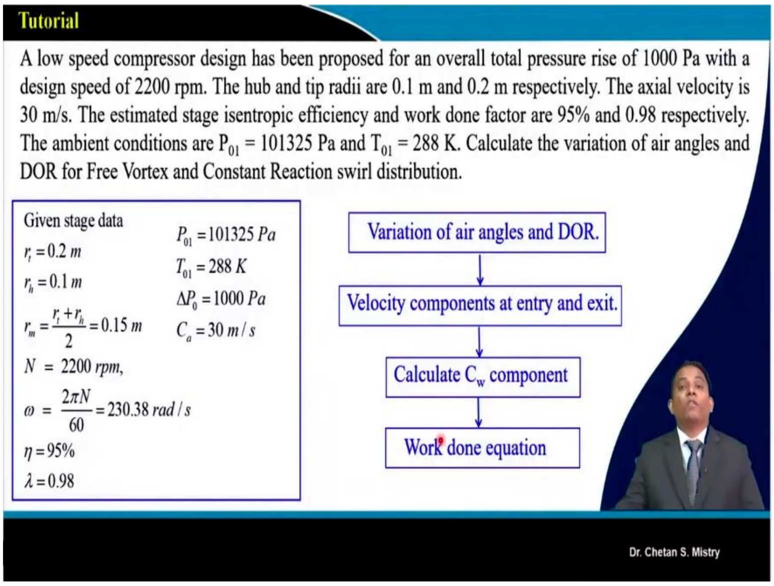

So, let us take a numerical. it says a low speed compressor need to be designed to have overall pressure rise of 1000 pascal with design speed of 2200 rpm. The hub and tip radius are 0.1 m and 0.2 m respectively. The axial velocity is 30 m/s, the estimated stage isentropic efficiency and work done factor, they are 95% and 0.98 respectively. The ambient conditions are $p_{01} = 101325\ Pa$ and temperature as 288 K. Calculate the variation of air angles and degree of reaction for free vortex and constant reaction whirl distribution.

So, this is what is a numerical. We can say what all data that's what is given. We know what is my tip radius, it is 0.2 m, hub radius is 0.1 m. You can say, you can calculate your mid radius,

$$Given\ data,$$

$$r_t = 0.2\ m$$

$$r_h = 0.1\ m$$

$$r_m = \frac{r_t + r_h}{2} = 0.15\ m$$

that's what is 0.15 m.

We have our rotational speed as say 2200 rpm, you can say, this speed is say low speed compressor, we will be having our efficiency to be 95% and you can see work done factor it is given 98%.

$$N = 2200\,rpm$$
$$\omega = \frac{2\pi N}{60} = 230.38\frac{rad}{s}$$
$$\eta = 95\%$$
$$\lambda = 0.98$$

So, you can say, this is what may be, the first stage. We are having our entry pressure and entry temperature and we are expecting our total pressure rise of 1000 Pa and axial velocity also is known to us.

$$P_{01} = 101325\,Pa$$
$$T_{01} = 288\,K$$
$$\Delta P_0 = 1000\,Pa$$
$$C_a = 30\,m/s$$

Now, what all we are looking for is, we are looking for variation of my air angles and degree of reaction.

So, you can understand if we are looking for air angles, we must know what all are the velocities at the entry of my rotor, what are the velocities at the exit of my rotor. When I say velocities, we can say what are my absolute velocities, my relative velocities. Based on that we will be calculating our C_w, that is nothing but my whirl component, because that's what is important in order to calculate your degree of reaction, okay. And, this C_w, that's what you can calculate based on what all work done is given to you, okay.

So, in order to meet these requirements, let us move from bottom to top approach. Say, initially we will be implementing our work done equation for the calculation of parameters.

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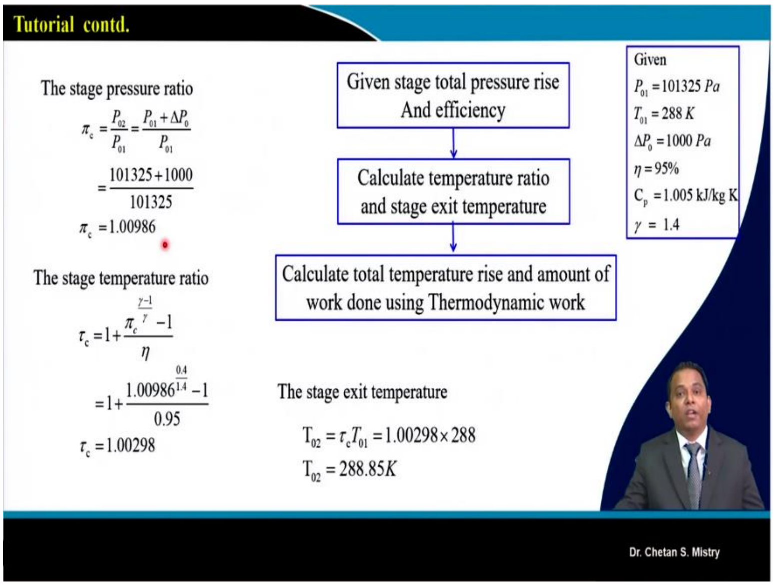

So here, what we know? We know what is my total pressure rise required, we know what is our efficiency and based on that we can calculate what will be my temperature ratio. Once we will be calculating our temperature ratio, we are able to calculate our thermodynamic work. So, let us move step by step.

Say very first point it says my stage pressure rise, you can say P_{02} by P_{01}. You can say, what pressure rise we are expecting is say 1000 Pa, that's what will be giving me my pressure ratio of 1.00986 pressure ratio.

$$\textit{The stage pressure ratio,}$$

$$\pi_c = \frac{P_{02}}{P_{01}} = \frac{P_{01} + \Delta P_0}{P_{01}} = \frac{101325 + 1000}{101325} = 1.00986$$

Now, based on this pressure ratio, we can calculate what will be our temperature ratio. So here, this temperature ratio we are correlating in sense of my pressure ratio and efficiency.

$$\textit{The stage temperature ratio,}$$

$$\tau_c = 1 + \frac{\pi_c^{\frac{\gamma-1}{\gamma}} - 1}{\eta}$$

So, these two are known to me, if I will be putting, that's what we will be giving me my temperature ratio as say 1.00298.

$$\tau_c = 1 + \frac{1.00986^{\frac{0.4}{1.4}} - 1}{0.95}$$

$$\therefore \tau_c = 1.00298$$

Now, once this temperature ratio is known to us, we can calculate what will be my outlet temperature. So, we can say we are able to calculate what will be my ΔT_0. So here, if you look at, since this is what is low speed application, we are expecting our pressure rise in the range of 1000 Pa, that is why my pressure ratio is lower. And that is the reason why my T_{02} is also coming to be lower, okay.

$$The\ stage\ exit\ temperature,$$

$$T_{02} = \tau_c T_{01} = 1.00298 \times 288 = 288.85\ K$$

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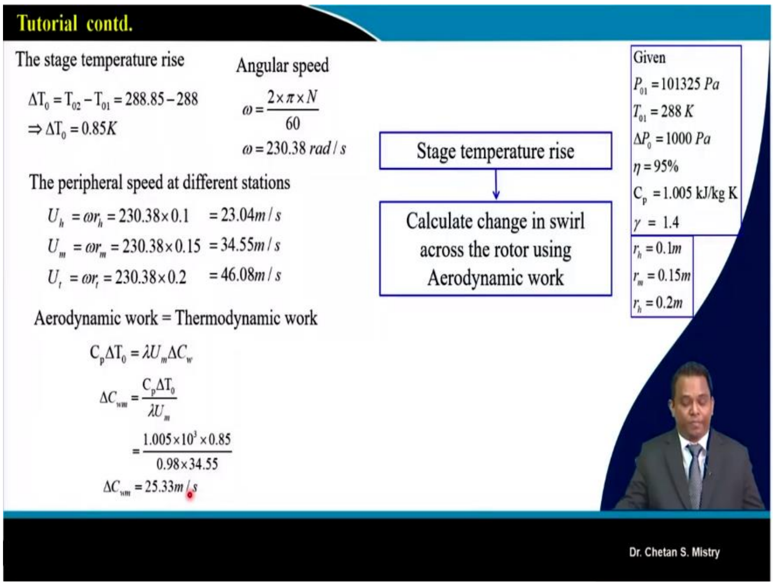

Now, once we are calculating this T_{02}, we are able to calculate what will be my ΔT_0, okay. Now, let us try to calculate what all will be my peripheral speed at different stations. So, in order to calculate, we can say my peripheral speed at the hub. It is my, you know, angular speed into hub radius, my angular speed into say mid radius, my angular speed into tip radius. So, this radius at hub, tip and mid-section, that's what is given to us. So, if you will be putting this, it will be giving me my peripheral speed at different stations.

$$The\ peripheral\ speed\ at\ different\ stations$$

$$U_h = \omega r_h = 230.38 \times 0.1 = 23.04\ m/s$$

$$U_m = \omega r_m = 230.38 \times 0.15 = 34.55\, m/s$$

$$U_t = \omega r_t = 230.38 \times 0.2 = 46.08\, m/s$$

Just understand, we are interested in calculation of what is happening at different stations, mainly at three stations, mid-section, hub-section and tip-section.

Now, here in this case, if you look at, we can say, we can correlate our aerodynamic work and thermodynamic work. And, based on that, if we are putting this equation, it says, my change in whirl component, that's what is a function of $C_p\Delta T_0$, my work done factor and my peripheral speed at the mid-station. So, we will be doing our calculation what is happening at the mid-section first. So, it says my delta C_w ΔC_w, that's what is coming as say 25.33 meter per second m/s.

$$Aerodynamic\ work = Thermodynamic\ work$$

$$C_p\Delta T_0 = \lambda U_m \Delta C_w$$

$$\therefore \Delta C_{wm} = \frac{C_p\Delta T_0}{\lambda U_m} = \frac{1.005 \times 10^3 \times 0.85}{0.98 \times 34.55}$$

$$\therefore \Delta C_{wm} = 25.33\, m/s$$

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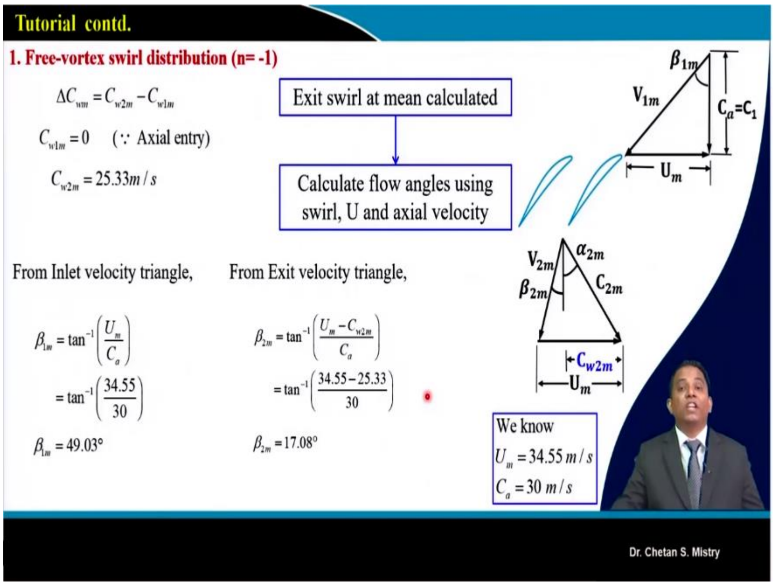

Now, what we are asked for? We are asked for implementing two different design approaches; first, it is free vortex design in which we can say my $n = -1$, okay. So, let us try to approach with say first design methodology, that's what is say free vortex method. So, here, if you look at, my ΔC_w, that is nothing but a change in my tangential velocity component, we can write down here it is

$$\Delta C_{wm} = C_{w2m} - C_{w1m}$$

Since, we are considering our entry to be axial one, that is the reason why my C_{w1}, that is coming 0. So, if you are putting these numbers, it says we are able to calculate the whirl component at the exit and at the mid-station, okay.

$$C_{w1m} = 0(\because Axial\ entry)$$

$$\therefore C_{w2m} = 25.33\ m/s$$

Now, if you are considering this is what will be my velocity triangle, do not forget to plot velocity triangle in order to understand, and in order to calculate various angles and various velocity components.

So, here if you look at, this is what is representing my inlet velocity triangle, this is what is representing my outlet velocity triangle, for the rotor, okay. So, from my velocity triangle, I can say my $\tan\beta_1$, that's what we can say, it is U_m/C_a . So, based on that this U_m we have calculated. C_a, that's what is given, it is 30 m/s. So, based on that we can calculate what will be my entry air angle, β_1. In line to that, based on my exit velocity triangle, we can calculate what will be my β_2

$$From\ inlet\ velocity\ triangle,$$

$$\beta_{1m} = \tan^{-1}\left(\frac{U_m}{C_a}\right)$$

$$= \tan^{-1}\left(\frac{34.55}{30}\right)$$

$$\therefore \beta_{1m} = 49.03°$$

So, here this $\tan\beta_2$, we can write down. That is nothing but $\frac{U_m - C_{w2m}}{C_a}$. This U_m is known to us, C_{w2} is known to us, my axial velocity, that's what is known to us. We can calculate, what is my β_2 at a mid-section, okay. So, you can understand, we have calculated what is our whirl component and what will be the change in these angles.

$$From\ exit\ velocity\ triangle,$$

$$\beta_{2m} = \tan^{-1}\left(\frac{U_m - C_{w2m}}{C_a}\right)$$
$$= \tan^{-1}\left(\frac{34.55 - 25.33}{30}\right)$$
$$\therefore \beta_{2m} = 17.08°$$

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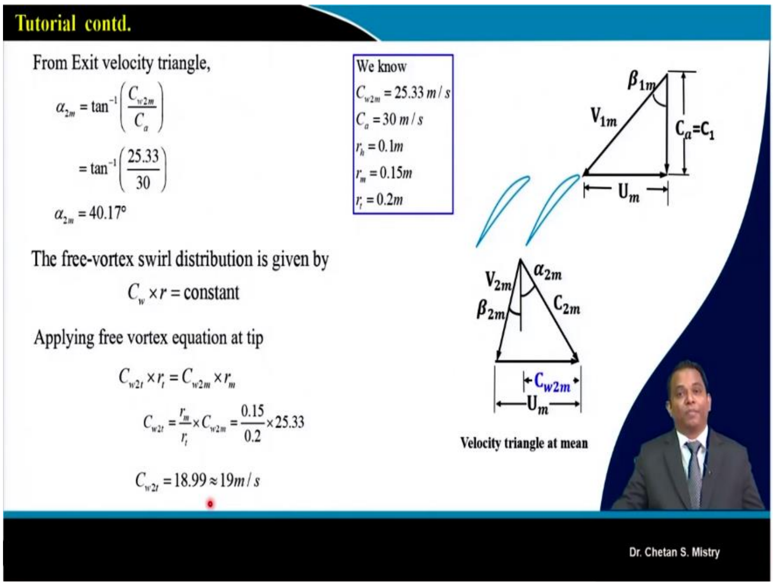

Now, at the exit; similarly, we can calculate what is our absolute flow angle. So, if you are putting this, say it says $\tan\alpha_2$, that's what is given by C_{w2}/C_a, C_a is nothing but this is what is my axial velocity component. So, it says I will be having my α_2, that is 40.17°. Now, what all we learn, for free vortex design, we are considering my $C_w \cdot r = constant$.

$$From\ Exit\ velocity\ triangle,$$
$$\alpha_2 = \tan^{-1}\left(\frac{C_{w2m}}{C_a}\right)$$

$$= \tan^{-1}\left(\frac{25.33}{30}\right)$$

$$\therefore \alpha_2 = 40.17°$$

So, here in this case, since my C_w at the mid station is known to me and my radius is known to me. So, I will be writing say we can write down,

$$C_{w2t} \times r_t = C_{w2m} \times r_m$$

So, you can understand, we are basically calculating the constant. And, what we have assumed? Throughout my span, my $C_w \cdot r = constant$. So, that's what will be giving us the idea to calculate what is happening in sense of change of C_{w2} at mid station, at tip and at the hub. So, let us calculate what is happening at the tip. It says my C_{w2} at the tip, it is 19 m/s, okay.

$$\therefore C_{w2t} = \frac{r_m}{r_t} \times C_{w2m} = \frac{0.15}{0.2} \times 25.33 = 18.99 \approx 19\ m/s$$

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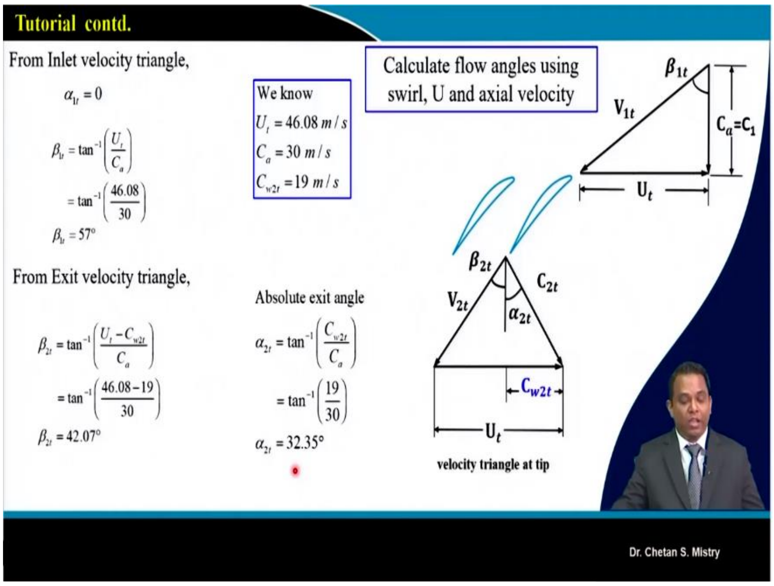

Now, once we are able to calculate our whirl component at the tip, we can calculate what all are the flow angles. So, what we know? my α_1, that's what is 0 because we are considering axial entry. So, based on my velocity triangle, we can calculate my $tan\,\beta_1$, that is $tan\,\beta_1 = U/C_a$, do not forget this U is at tip, okay, and my axial velocity we are as assuming to be constant. So, U_{tip} what we have calculated earlier, it is 46.08 and C_a it is known to us, that's what is 30 m/s.

$$From\ inlet\ velocity\ triangle,$$

$$\alpha_{1t} = 0$$

$$\beta_{1t} = \tan^{-1}\left(\frac{U_t}{C_a}\right)$$

$$= tan^{-1}\left(\frac{46.08}{30}\right)$$

$$\therefore \beta_{1t} = 57°$$

So, based on that, we can calculate our β_{1t} it is 57°. Same way, we can calculate what is happening with my β_{2t}. So, if we are looking at, it says my from velocity triangle, we can write down my

$$\tan \beta_{2t} = \frac{U_{tip} - C_{w2t}}{C_a}$$

$$\therefore \beta_{2t} = \tan^{-1}\left(\frac{U_{tip} - C_{w2t}}{C_a}\right)$$

$$= \tan^{-1}\left(\frac{46.08 - 19}{30}\right)$$

$$\therefore \beta_{2t} = 42.07°$$

So, that's what is giving my β_{2t} as 42.07°, okay. Now, same way we can calculate what is happening with my absolute triangle. So, that's what is nothing but tan inverse C_{w2} by C_a and that's what is giving alpha to as 32.35°.

$$absolute\ exit\ angle$$

$$\alpha_{2t} = \tan^{-1}\left(\frac{C_{w2t}}{C_a}\right)$$

$$= \tan^{-1}\left(\frac{19}{30}\right)$$

$$\therefore \alpha_{2t} = 32.35°$$

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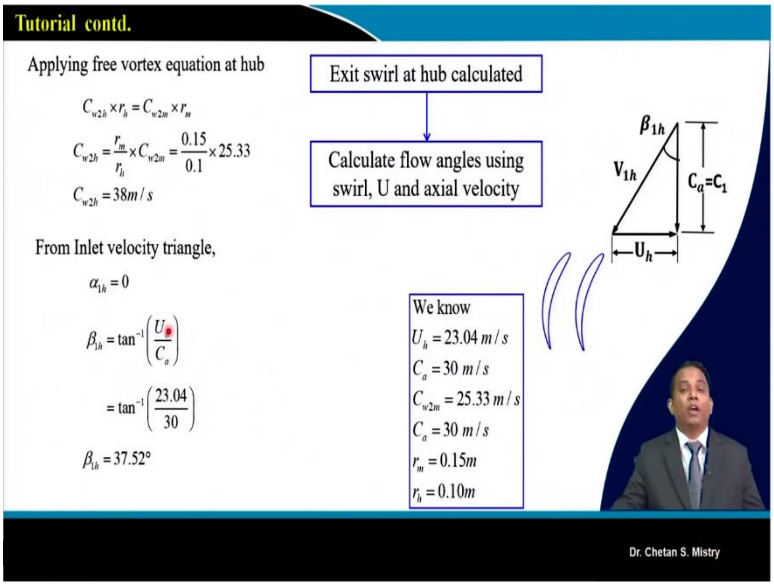

Now, you know, based on what understanding we have, we can do our calculation at hub station also, what we know from the free vortex concept, my $C_w \cdot r = constant$. Since, this constant at mid station it is known to me, I can do calculation what is happening at the hub. So, I can write down this equation as say

$$C_{w2h} \times r_h = C_{w2m} \times r_m$$

$$\therefore C_{w2h} = \frac{r_m}{r_h} \times C_{w2m} = \frac{0.15}{0.1} \times 25.33$$

$$\therefore C_{w2h} = 38 \, m/s$$

So, if you are putting these numbers, it says my C_{w2} at the hub is coming 38 m/s. In line to what we have discussed, we can do our calculation for β_{1h}. Here also, remember this β_1 we are calculating at the hub; so, you will be having change in your velocity triangle, do not forget, okay. So, make a habit to plot the velocity triangle, that's what we will be giving you indication say you are putting your U at hub, okay. And, that's what is giving me β_1 at the hub.

$$From\ Inlet\ velocity\ triangle,$$

$$\alpha_{1h} = 0$$

$$\beta_{1h} = \tan^{-1}\left(\frac{U_h}{C_a}\right)$$

$$= \tan^{-1}\left(\frac{23.04}{30}\right)$$

$$\therefore \beta_{1h} = 37.52°$$

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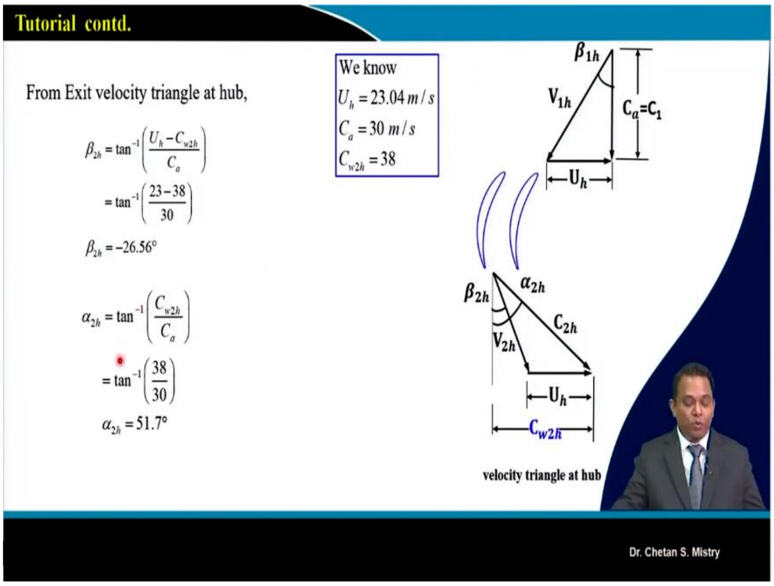

Same way, we can calculate based on our exit velocity triangle β_{2h}, okay. And this is what is similar to what we have done calculation at the mid-section inline to what we have done calculation at the tip-section, okay. And this is what will be giving me, what is happening in sense of my α_{2h} and what is happening in sense of my β_{2h}.

So, if you look at, my β_{2h}, just careful, that's what is coming -26.56°. Just look it, say on one side, we are putting this β_2, okay.

$$From\ exit\ velocity\ triangle\ at\ hub,$$

$$\beta_{2h} = \tan^{-1}\left(\frac{U_h - C_{w2h}}{C_a}\right)$$

$$= \tan^{-1}\left(\frac{23 - 38}{30}\right)$$

$$\therefore \beta_{2h} = -26.56°$$

So, do not get confused that angle is coming negative just you need to put that or you need to the represent in such a way that it will be on the other side of my velocity triangle, okay. Now, if you look at my blade or say my aerofoil shape also will be changing accordingly, okay. Now α_2 you can calculate that's what is coming 51.7°.

$$\alpha_{2h} = \tan^{-1}\left(\frac{C_{w2h}}{C_a}\right)$$

$$= \tan^{-1}\left(\frac{38}{30}\right)$$

$$\therefore \beta_{2h} = 51.7°$$

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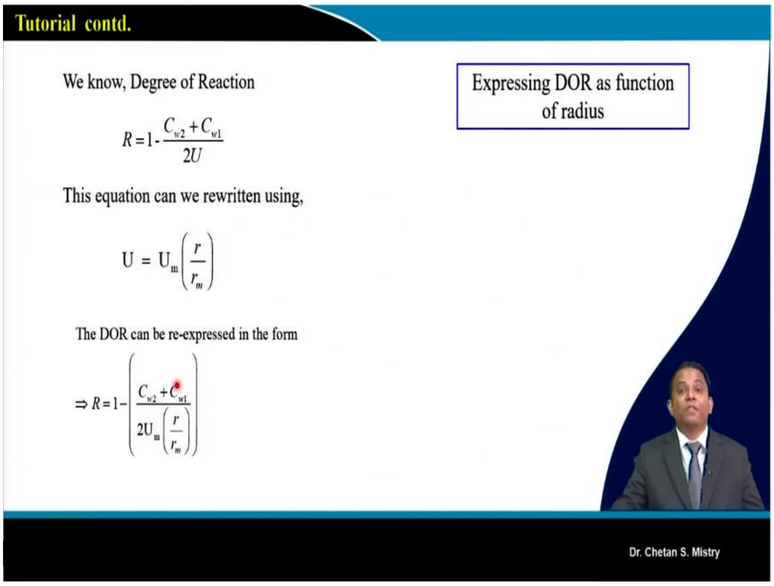

Now, here if you look at, we have done all our calculation for C_{w1} and C_{w2} at different stations, we have calculated our flow angles. Now, let us move with the next step, that's what is for the calculation of degree of reaction. So, my degree of reaction we can write down it is

$$DOR = 1 - \frac{C_{w2} + C_{w1}}{2U}$$

Now, let us try to put that or rewrite. We can say, my peripheral speed at any station, that's what is we are writing as say my peripheral speed at the mid-station into this radius ratio.

$$U = U_m\left(\frac{r}{r_m}\right)$$

This r is nothing but that's what is representing particular station and r_m, that's what is representing my mid station, okay. So, if you are putting this in the form of formula, it says my degree of reaction we can represent by this form.

$$The\ DOR\ can\ be\ re-expressed,$$

$$R = 1 - \left(\frac{C_{w2} + C_{w1}}{2U_m \left(\frac{r}{r_m} \right)} \right)$$

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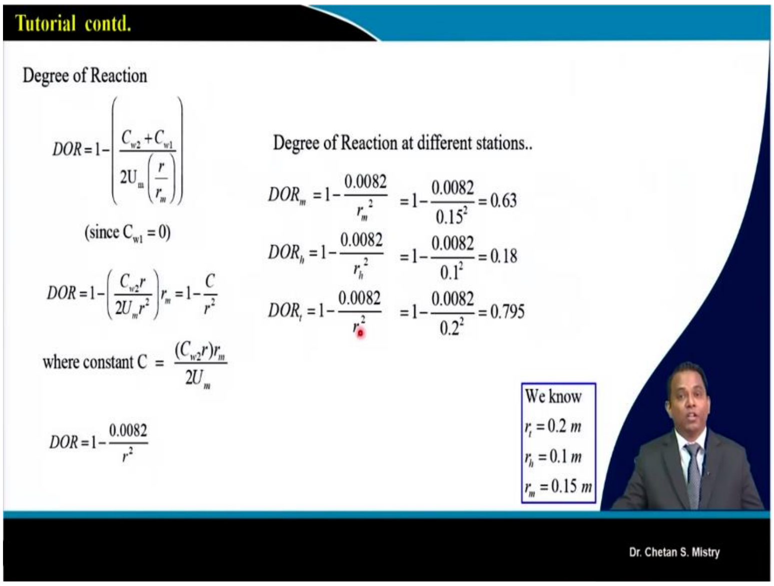

Let us try to say re-formulate that. So, this is what will be my degree of reaction. Now, what we are looking for is, say variation of degree of reaction at different stations. So, you can say, we can write down this degree of reaction in a more simplified way, it says degree of reaction it is given by

$$DOR = 1 - \left(\frac{C_{w2} + C_{w1}}{2U_m \left(\frac{r}{r_m} \right)} \right)$$

$$since, C_{w1} = 0$$

$$DOR = 1 - \left(\frac{C_{w2} r}{(2U_m r^2)} \right) r_m = 1 - \frac{C}{r^2}$$

There is a reason behind calculating, you can straightaway do calculation by putting some calculation of say C_{w2} at hub, C_{w1} at say hub; you can put C_{w1} at tip, C_{w2} at tip that way also you can do the calculation. But, let me simplify this. So, it says this constant we are writing as

$$Constant = \frac{(C_{w2} r) r_m}{2U_m}$$

and this is what will be giving me my degree of reaction, that's what is given by

$$DOR = 1 - \frac{0.0082}{r^2}$$

So, this is nothing but some constant divided by r^2. Now, these r as we have discussed, they are different locations. We know what are our radius at tip, we know what are our radius at hub and radius at the mid station. So, let me write down degree of reaction at mid station, degree of reaction at hub and my degree of reaction at tip.

$$Degree\ of\ Reaction\ at\ different\ stations..$$

$$DOR_m = 1 - \frac{0.0082}{r_m^2} = 1 - \frac{0.0082}{0.15^2} = 0.63$$

$$DOR_h = 1 - \frac{0.0082}{r_h^2} = 1 - \frac{0.0082}{0.1^2} = 0.18$$

$$DOR_t = 1 - \frac{0.0082}{r_t^2} = 1 - \frac{0.0082}{0.2^2} = 0.795$$

Let me put all these radius values at mid station, at hub and at the tip, that's what is giving me the variation of my degree of reaction. So, if you look at carefully, it says my degree of reaction at the hub is coming 0.18, at mid station it is coming 0.63 and at tip it is coming around 0.79. So, you can say, for free vortex design, we will be having the variation of degree of reaction from hub to shroud, okay.

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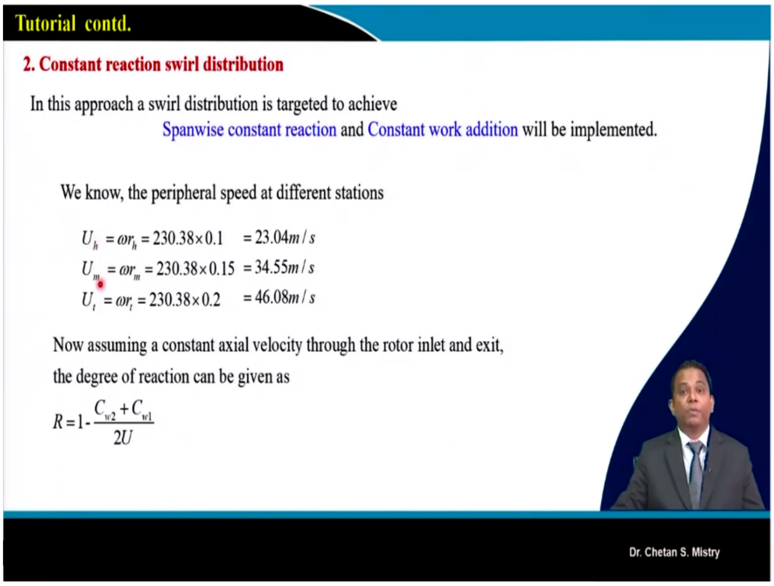

After doing all this calculation, let us move with the second approach, that's what is say constant reaction approach and for this constant reaction approach, we are assuming our degree of reaction to be constant and constant work addition, okay. This is what all we know from our previous calculation for say peripheral speed at hub, peripheral speed at mid station and my peripheral speed at tip station. What we know, our degree of reaction, that's what is given by

$$R = 1 - \frac{C_{w2} + C_{w1}}{2U}$$

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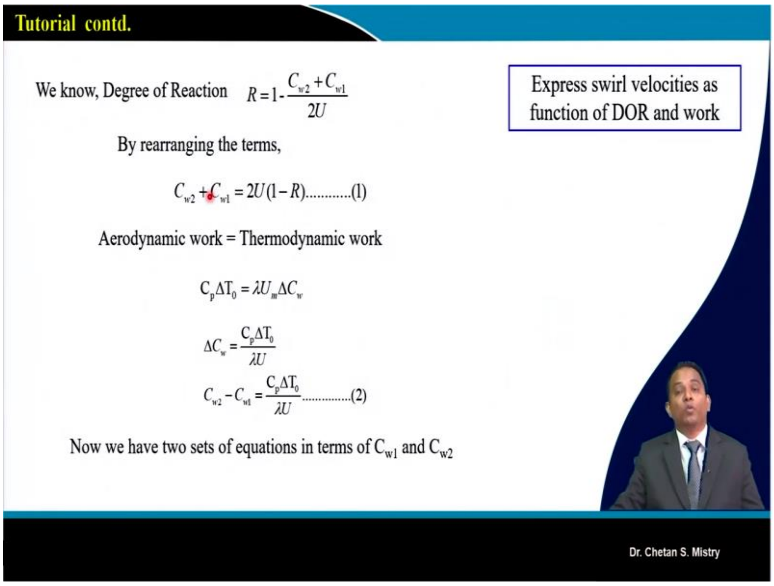

Now, if you are arranging this term, it says, my $C_{w2} + C_{w1}$, that's what is given by $2U(1 - R)$, okay.

$$C_{w2} + C_{w1} = 2U(1 - R)$$

What we know? We are having this ΔC_w term or change of C_w and C_{w2} and C_{w1}, that's what is also coming in my work equation. So, let me put my aerodynamic work and thermodynamic work to be same.

If we are putting that, it says… my $C_{w2} - C_{w1}$, that's what we are representing in the form of $\frac{C_p \Delta T_0}{\lambda U}$. I am sure, this is what all we have discussed in our class, but this is just for your understanding what all we are working at this moment, okay. So, our requirement, it is to calculate what is my C_{w1} and what is my C_{w2}, at entry and at the exit of my rotor.

$$C_p \Delta T_0 = \lambda U_m \Delta C_w$$

$$\Delta C_w = \frac{C_p \Delta T_0}{\lambda U}$$

$$C_{w2} - C_{w1} = \frac{C_p \Delta T_0}{\lambda U}$$

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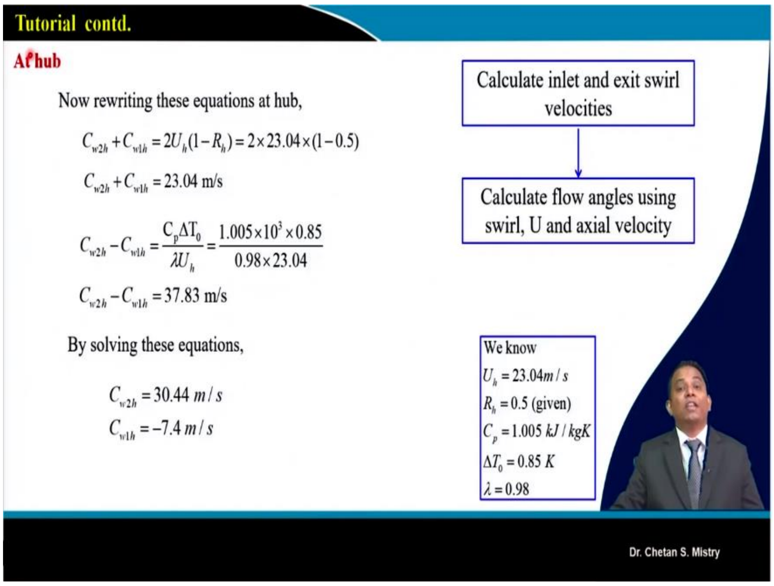

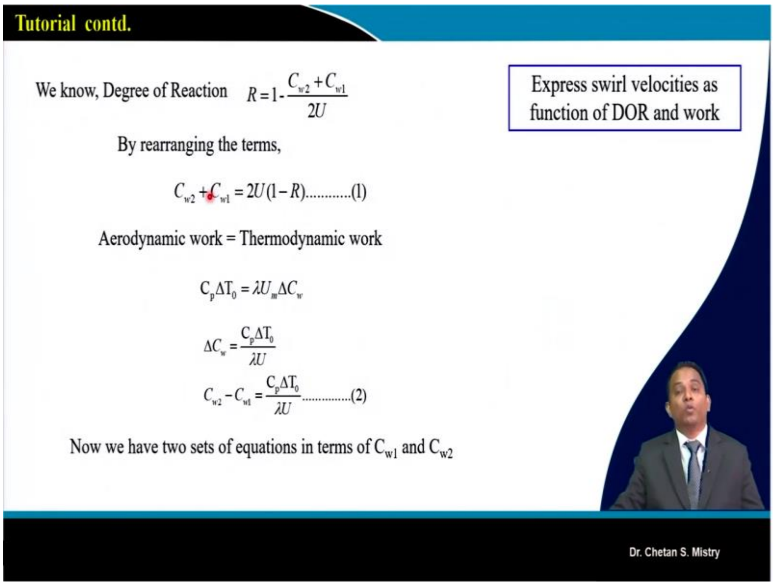

So, if we will be putting, say… we are looking for our calculation at the hub station. So, we can write down, we are having two equations what we have seen; one, that's what is in sense of degree of reaction, second, that's what is instance of my ΔT_0. So, ΔT_0, that's what is known to me and this degree of reaction, that's what we are looking for, okay.

So, it says, if we are assuming, say… our degree of reaction to be 0.5, that's what is given to you, say at mid station or say this is what is at the hub station it is given say it is 0.5. If we are considering that because it is a constant reaction design. It says my degree of reaction to be 0.5, so you can understand at hub, my degree of reaction is 0.5.

So, if I will be putting this in this equation for degree of reaction, and my work done equation, where I know all these numbers. If we will be putting this into case, it says my C_{w1} and C_{w2} at the hub station we can calculate, and that's what is coming my C_{w1} at the hub is -7.4 m/s. And my C_{w2} at the hub is 30.44 m/s.

$$C_{w2h} + C_{w1h} = 2U_h(1 - R_h) = 2 \times 23.04 \times (1 - 0.5)$$

$$C_{w2h} + C_{w1h} = 23.04\ m/s$$

$$C_{w2h} - C_{w1h} = \frac{C_p \Delta T_0}{\lambda U_h} = \frac{1.005 \times 10^3 \times 0.85}{0.98 \times 23.04}$$

$$C_{w2h} - C_{w1h} = 37.83\frac{m}{s}$$

By solving these equations,

$$C_{w2h} = 30.44\ m/s\ \&\ C_{w1h} = -7.4\ m/s$$

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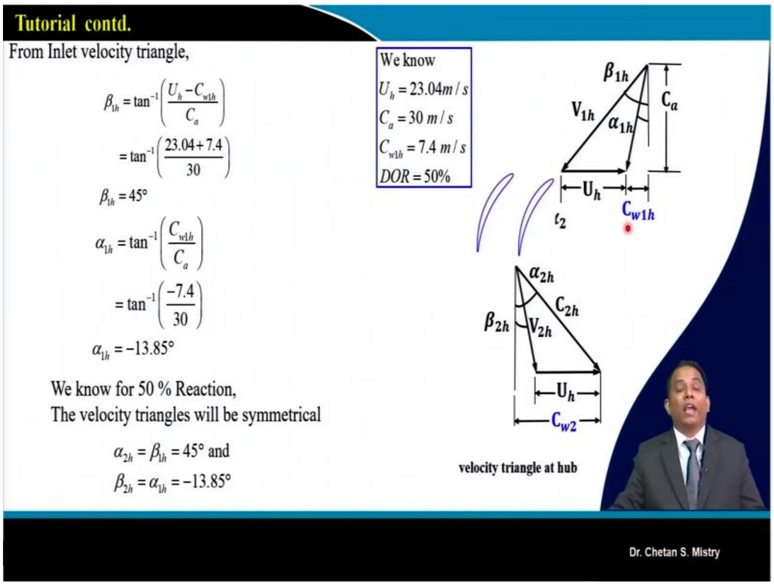

Now, once this is what is known to you, we can put our velocity triangle, be careful, we have done our calculation for C_{w1}. And if you look at this C_{w1}, that's what is coming to be negative. Now, you are expert enough to make your velocity triangle, okay. So, you can say, I am having negative swirl, that's what is coming at the entry.

So, that's what it says, my β_1 at the hub, we can calculate it is U_h minus C_{w1h}. Since, this is what is in negative direction, you can say, that's what is getting added up here. This is what is you need to take care of. That's what is giving me my β_1 at the hub as 45°, same way we can do a calculation for α_1, that is coming -13.85°. Now, what design approach we are discussing? That's what is constant reaction design approach.

So, what it says for 50-person reaction? We will be having our blades to be symmetrical blade. So, for symmetrical blade what it says? my α_2 and β_1, that's what is same, and my β_2 and α_1, that's what is coming to me same. So, you can say my α_2 and β_1 that will be 45°, and my β_2 and α_1, that's what is -13.85°.

$$\beta_{1h} = \tan^{-1}\left(\frac{U_h - C_{w1h}}{C_a}\right)$$

$$= \tan^{-1}\left(\frac{23.04 + 7.4}{30}\right)$$

$$\therefore \beta_{1h} = 45°$$

$$\alpha_{1h} = \tan^{-1}\left(\frac{C_{w1h}}{C_a}\right)$$

$$= \tan^{-1}\left(\frac{-7.4}{30}\right)$$
$$\therefore \alpha_{1h} = -13.85°$$

$$We\ know\ for\ 50\%\ Reaction,$$
$$The\ velocity\ triangle\ will\ be\ symmetrical$$
$$\alpha_{2h} = \beta_{1h} = 45° \quad and$$

$$\beta_{2h} = \alpha_{1h} = -13.85°$$

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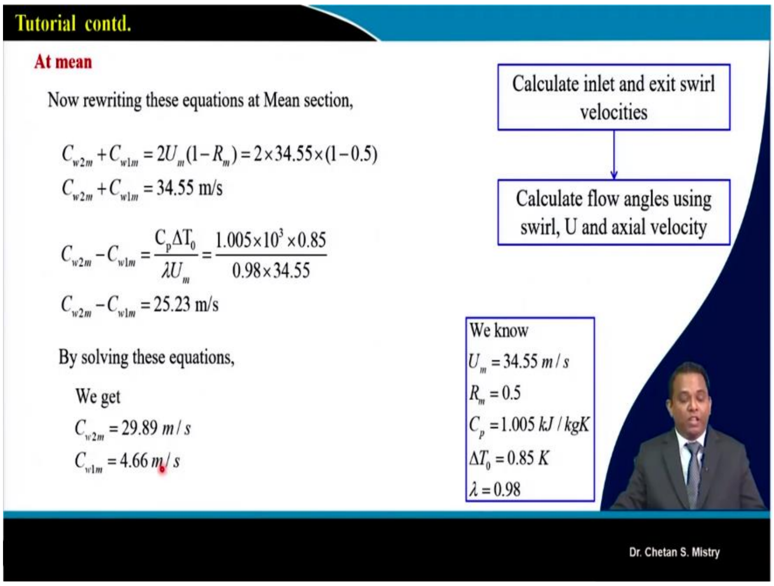

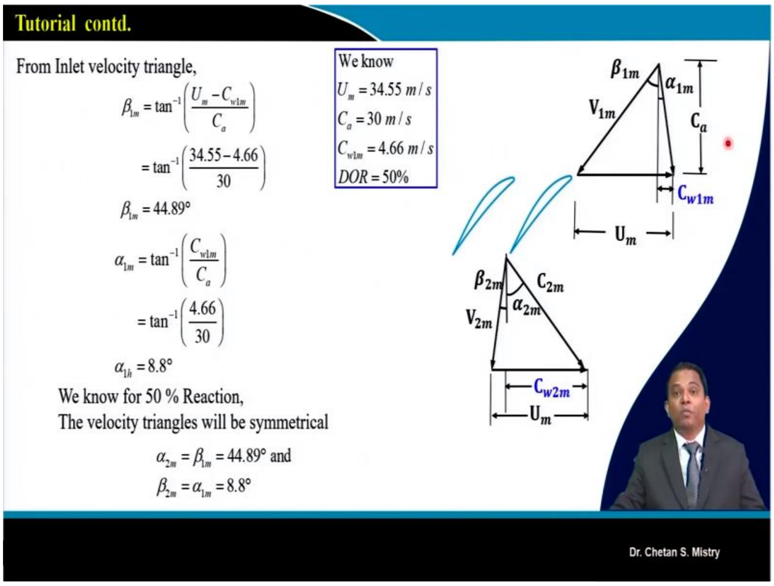

Inline to that we can do our calculation at the mid station. So, we can rewrite the equation and we will be doing our calculation for two different approaches; one, that's what is constant reaction and by work done equation, we will be getting our C_{w2} variation at the mid station, we can calculate our C_{w1} calculation at the mid station. Once this whirl components, that's what we are calculating with, that's what will be giving us idea how to do calculation for variation of our flow angles.

$$C_{w2m} + C_{w1m} = 2U_m(1 - R_m) = 2 \times 34.55 \times (1 - 0.5)$$

$$C_{w2m} + C_{w1m} = 34.55\ m/s$$

$$C_{w2m} - C_{w1m} = \frac{C_p \Delta T_0}{\lambda U_m} = \frac{1.005 \times 10^3 \times 0.85}{0.98 \times 34.55}$$

$$C_{w2m} - C_{w1m} = 25.23\ m/s$$

$$By\ solving\ these\ equations,$$

$$C_{w2m} = 29.89\ m/s\ \&\ C_{w1m} = 4.66\ m/s$$

So here, if you look at this is what is by β_1. So, β_1, that's what we are calculating, just look at here at the mid station, my C_{w1}, that's what is coming positive. So, that's what is on other side, you can see my α_1, that's what is coming to be positive, it is coming 8.8°. Same way, as we are having our degree of reaction to be constant, and that to it is 0.5, we can say, we will be having symmetrical blading.

So, it says my α_2 at the mid station and my β_1 at the mid station, that's what is same and it is 44.89°. We have our α_{1m} and β_{2m}, that's what is coming to be same, and that's what is 8.8°. So, just look at how you will be changing your velocity triangle. So, this is what is my velocity triangle at the mid station, okay. Always, repeatedly I am saying, make a habit of plotting the velocity triangle when you are doing your calculations, okay.

$$\beta_{1m} = \tan^{-1}\left(\frac{U_m - C_{w1m}}{C_a}\right)$$

$$= \tan^{-1}\left(\frac{34.55 - 4.66}{30}\right)$$

$$\therefore \beta_{1m} = 44.89°$$

$$\alpha_{1m} = \tan^{-1}\left(\frac{C_{w1m}}{C_a}\right)$$

$$= \tan^{-1}\left(\frac{4.66}{30}\right)$$

$$\therefore \alpha_{1m} = 8.8°$$

$$We\ know\ for\ 50\%\ Reaction,$$
$$The\ velocity\ triangle\ will\ be\ symmetrical$$

$$\alpha_{2m} = \beta_{1m} = 44.89° \quad and$$

$$\beta_{2m} = \alpha_{1m} = 8.8°$$

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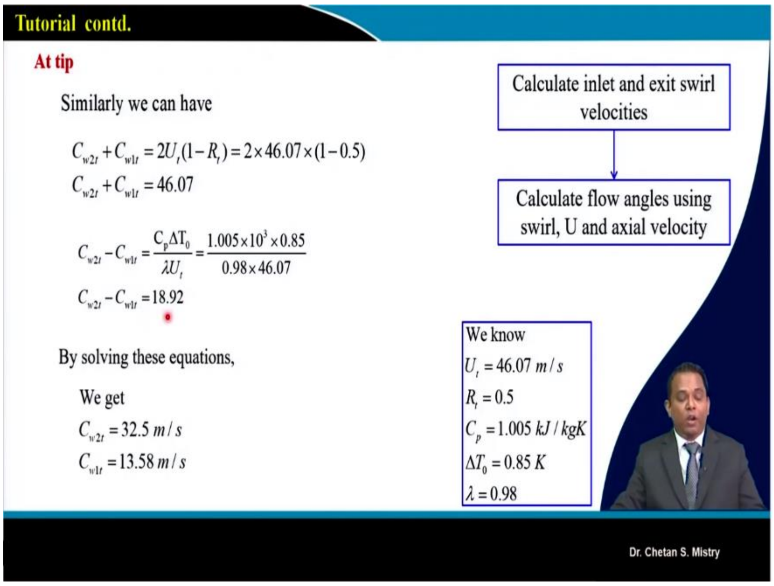

Now, at the tip station also, in line to what all we have discussed, we can do our calculation for C_{w1} and C_{w2} at tip station. So, this is what you can say my degree of reaction is 0.5, that's what is giving my $C_{w2t} + C_{w1t}$ at tip as some number. Same way based on our work balance, we are getting $C_{w2t} - C_{w1t} = 18.92$. Now based on that, we are able to calculate our C_{w2} and C_{w1} at tip station.

$$C_{w2t} + C_{w1t} = 2U_t(1 - R_t) = 2 \times 46.07 \times (1 - 0.5)$$
$$C_{w2t} + C_{w1t} = 46.07$$

$$C_{w2t} - C_{w1t} = \frac{C_p \Delta T_0}{\lambda U_t} = \frac{1.005 \times 10^3 \times 0.85}{0.98 \times 46.07}$$

$$C_{w2t} - C_{w1t} = 18.92 \frac{m}{s}$$

By solving these equations,

$$C_{w2t} = 32.5\ m/s \ \&\ C_{w1t} = 13.58\ m/s$$

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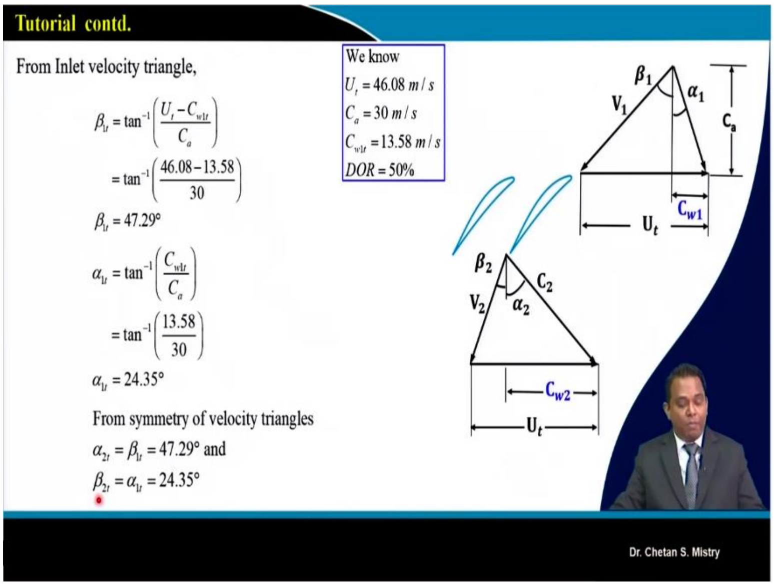

Now, as we have discussed, since my tip diameter, that's what is larger. So, U value, that's what will be coming to be larger. We can do our calculation for β_1, that's what we can have as say 47.29°. We have our α_1 as say 24.35° and because of degree of reaction to be 50%, we will be having symmetrical blading. And, for those symmetrical blading my α_2 and β_1, that's what is coming 47.29° and my β_2 and α_1, they are coming say 24.35°, okay.

$$\beta_{1t} = \tan^{-1}\left(\frac{U_t - C_{w1t}}{C_a}\right)$$

$$= \tan^{-1}\left(\frac{46.08 - 13.58}{30}\right)$$

$$\therefore \beta_{1t} = 47.29°$$

$$\alpha_{1t} = \tan^{-1}\left(\frac{C_{w1t}}{C_a}\right)$$

$$= \tan^{-1}\left(\frac{13.58}{30}\right)$$

$$\therefore \alpha_{1t} = 24.35°$$

$$We\ know\ for\ 50\%\ Reaction,$$

$$The\ velocity\ triangle\ will\ be\ symmetrical$$

$$\alpha_{2t} = \beta_{1t} = 47.29° \quad and$$

$$\beta_{2t} = \alpha_{1t} = 24.35°$$

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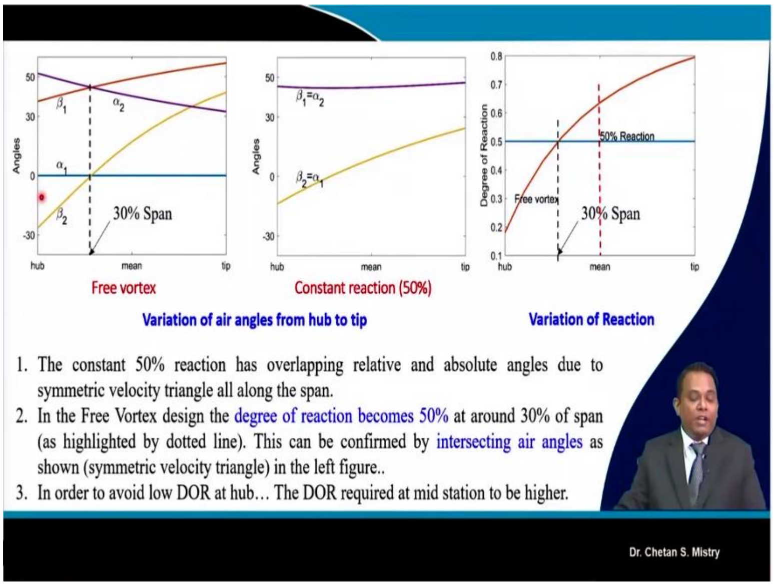

Now, after doing all this calculation, here, this is what is very important, that's what we need to observe. So, this is what is representing the variation of my angle from hub, mid and tip station using pre-vortex concept and this is what is with constant reaction concept.

So, here if you look at, say my $\Delta\beta$, if you are looking at for free vortex concept, we have discussed, that's what will be coming to be large, okay. And, if we are looking at, say… variation of my $\Delta\beta$, that's what is from hub to tip is coming larger, that's what is giving highly twisted blade.

Remember this is what is interpolation at particular three stations; hub, mid and tip-section. Now, here for constant reaction. If you are comparing these angles, these angles are coming to be lower, okay. And this is what is representing how my degree of reaction, that's what is varying. So, this line, that's what is representing my free vortex concept. And this line, that's what is representing my 50% reaction concept. So, if you try to compare these two, it says at mid-section for free vortex, my degree of reaction is coming to be large, maybe around 0.65, okay.

And that is the reason why if you look at, near the hub region, our degree of reaction is coming to be larger, maybe around 0.18, okay. And if you compare both the design approaches, it says, for free vortex design, approximately at 30% span, your degree of reaction is coming 0.5. So, I am sure, this is what will be giving you idea, when we are taking two different approaches for our design.

Say… we have discussed about this design approach, that’s what is called free-vortex design concept. And we have taken the approach, that’s what is constant reaction design approach, okay. So, it is advisable that you do your pen paper calculation, literally sit down and do the calculation, that’s what will give you more confidence in sense of doing the calculation for variation of my whirl velocity components, variation of my degree of reaction, variation of flow angles. Thank you! Thank you very much for your attention!