Aerodynamic Design of Axial Flow Compressor & Fans
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Lecture 19
Design Concepts (Contd.)

Hello, and welcome to lecture-19. We are discussing our third module for Design Concepts.

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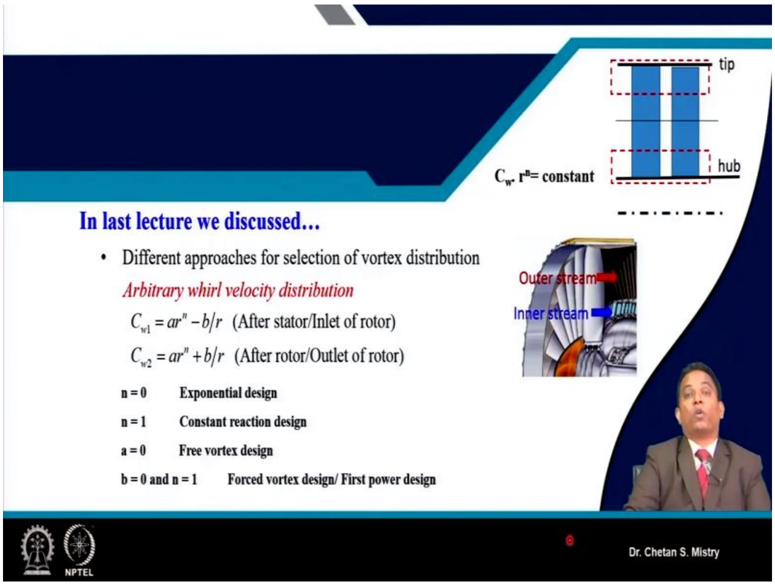

In last lecture, we were discussing about our free vortex concept. And we realized, when we are having say free vortex kind of designed, where you will be having blades to be highly twisted. And in order to address that high twist, we are having different configuration, it says we can take $C_w \cdot r^n = constant$, where we have seen that's what will be varying between some range. The 'n', that's what is varying from $-1\ to\ 2$.

We can say, when we are discussing about the design of axial flow compressor, in order to take care of what is happening near my hub and near my tip region, this exponent we are taking to be a higher number. And in order to compensate what losses we are getting, in sense of loading, in particular mid-section, these exponents, we are taking that to be a lower value maybe in the range of 0.8.

So, this is what is one of the philosophy what we have discussed. Then we were discussing about the application of our vortex equation, what it says, in order to have that to be applicable, it needs to satisfied what assumptions we have made. So, if we consider, say for high bypass ratio fan, say exit of my flow coming out from fan, which is entering inside my core engine, that is where my pressure rise requirement is comparatively low.

And if we are considering on upper portion of my blade, where my radius it is changing, my speed is changing. That is where we are considering our loading to be higher, my pressure rise expected in that particular region is higher. If that's what is your case, then you need to modify your vortex energy equation in such a way that, it need to have consideration of variation of stagnation enthalpy with the radius.

Then, we started discussing about different approaches for the vortex selection. And the recent approach we have assumed in which we are considering our total enthalpy that's what is remains constant along the span, we can say, my h_0 that is not varying along my hub to shroud or we can say, our work input that's what is remains constant. And we are considering arbitrary whirl velocity component distribution.

So, we have seen, at the entry of our rotor or say at the exit of the stage, previous stage or previous stator, we can say, my C_{w1}, that is given by this formula, say $ar^n - b/r$, we can say at the exit of our rotor, that's what is given by $ar^n + b/r$. Now, here also, we are taking different numbers, say this exponent what we are taking, 'n', we discussed in last lecture, when we are considering our $n = 0$ that design approach it is called exponential design.

When we are considering $n = 1$, that is what we are referring as say constant reaction design. When we are considering our constant $a = 0$, that is what we are defining as a free vortex design. And if we consider my $b = 0$ and $n = 1$, that's what is defined as a forced vortex design, sometimes people they are considering that as say first power design.

So, today we will be discussing about how do we use this exponent for our design calculation and what all will be the impact on our calculation for flow angle for the calculation of our degree of reaction.

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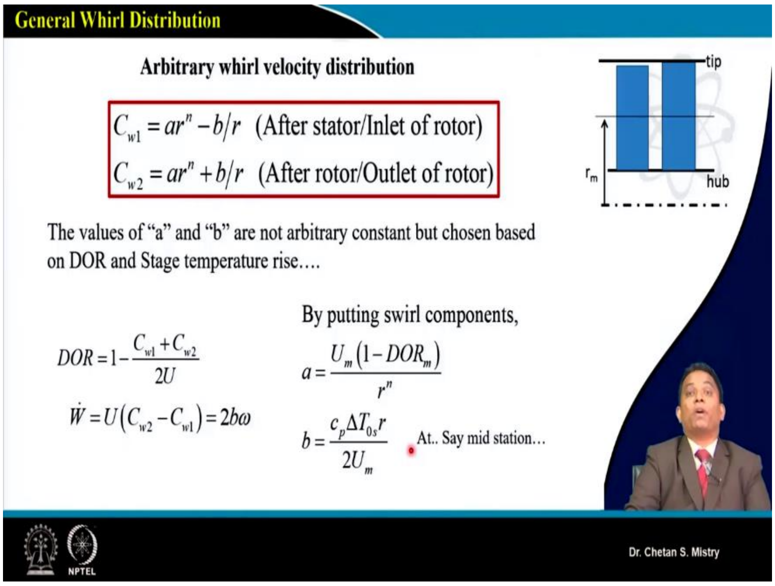

So, let us consider, before going into detail, what we have discussed in last lecture, we are having unknowns like constant a and b. These constants, we are calculating based on our formula for degree of reaction and my stage total pressure rise. If that's what is your case, and if I will be simplifying the equation for my degree of reaction and work done, we will be getting the value of constant 'a' and constant 'b'.

And we have discussed in last lecture, say that's what is a function of my radius; my a, that's what is varying with my radius, my b also is varying with my radius, and it is also a function of my peripheral speed at the mid station. So, conventionally what all books we are referring, if we go through, they people, they are doing the calculation at the mid-section. So, calculation of mid-section, that's what is very important.

So, what all calculation we also will be doing; initially, we will be doing those calculation at the mid-section, and then after we will apply the logic for the variation of our whirl velocity component from hub to mid-section, and from say, mid-section to tip. So, we must have the details what is coming in sense of my flow angles, in sense of my velocity components at the mid-section that need to be known to us. If not, then we need to assume.

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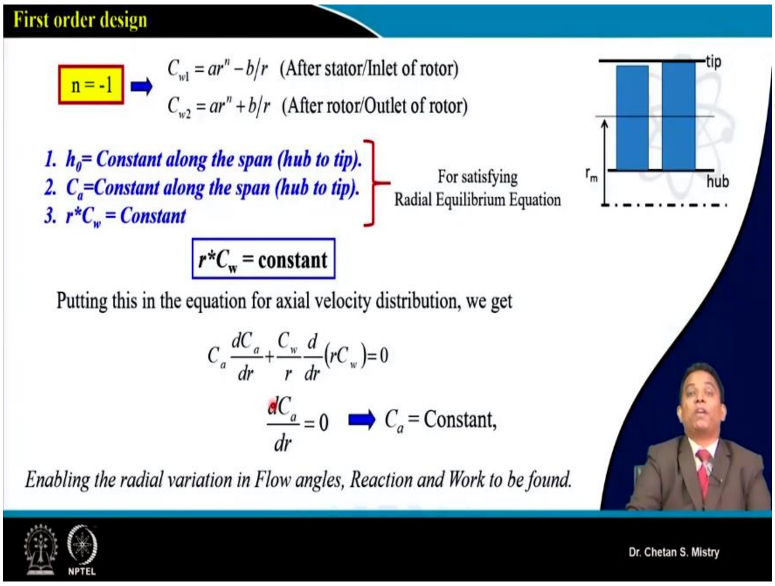

So, let us move, say… we consider first, say $n = -1$, we can say that as say… first order design, okay. Now my whirl component, as we have discussed, my C_{w1} and C_{w2}, that's what is given by this equation.

$$C_{w1} = ar^n - \frac{b}{r}$$

$$C_{w2} = ar^n + \frac{b}{r}$$

And from our fundamentals, what we know, in order to satisfy the radial equilibrium equation, we need to have our work input or my h_0, that needs to be constant along my span.

My axial velocity also remains constant from hub to tip. And, we know my $C_w \cdot r = constant$, that is giving me free vortex concept. So, we can say, if I will be satisfying all these equations, all these three questions, or three conditions, I say, that's what is satisfying my radial equilibrium equation. And that's what is compulsory, okay. So, if we consider, say here, if I will be putting my $n = -1$ in this formula, that's what is giving me, my $C_w \cdot r = constant$.

So, you can say, that's what is in line to what we are assuming as free vortex concept. And if you made the assumption, say, you know, like my h_0, that's what is constant, then in my equation, I can write down this is what is my formulation, it says

$$C_a \frac{dC_a}{dr} + \frac{C_w}{r}\frac{d}{dr}(rC_w) = 0$$

In that equation, if you will be putting our $C_w \cdot r = constant$, it says my $\frac{dC_a}{dr} = 0$. The meaning is my axial velocity remains constant. Now, you can understand, if I am taking my exponent as say $n = -1$, that's what is giving me my free vortex design concept. And that's what is satisfying all the three condition what we are looking for, for satisfying of radial equilibrium equation.

So, you know, this is what will give you idea, this is what will give you, say… method, in order to calculate how my flow angles, degree of reaction and work, that's what is been varying along my radius. So, we can say $n = -1$, that's what is giving us my first order design. And, we can say, that's what is say my free vortex kind of design.

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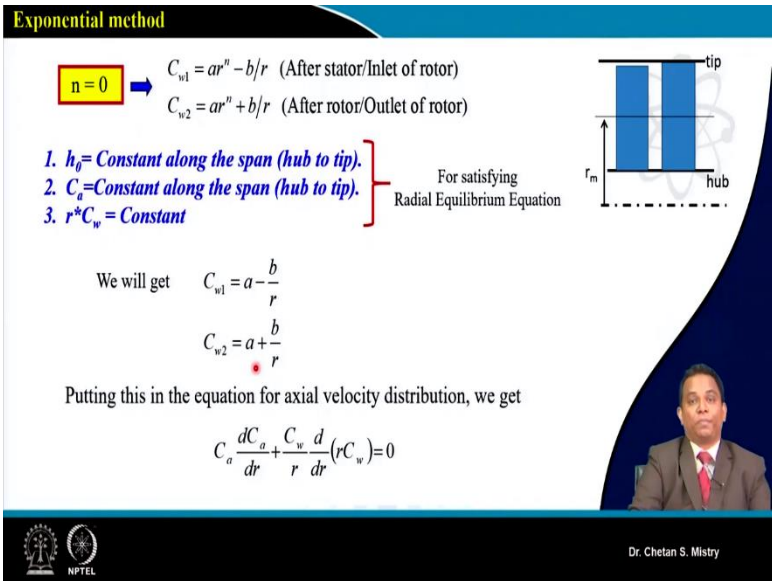

Now, let me move to the next. Suppose, if I consider my $n = 0$, so in this equation, if you look at, that's what is say my C_{w1} and my C_{w2}, and as we have discussed, what is our requirement? I need to satisfy my radial equilibrium equation. And in order to satisfy my radial equilibrium equation, I need to have satisfaction of my $h_0 = constant$, I need to have my axial velocity to be constant.

And I need to have my $C_w \cdot r = constant$. If these three are meeting, then we can say, we are satisfied with radial equilibrium. If you recall, I was discussing, say, two have the parameter if we know, then it will be possible for us to do the calculation for third parameter, that is also one of the way to use our radial equilibrium equation.

So, here in this case, if you recall, say, we are assuming our whirl component, that's what is varying here by certain proportion, we can say by this equation. If that's what is your case, and if I will be putting my $n = 0$, my C_{w1} and C_{w2}, that's what I can write down as say,

$$C_{w1} = a - \frac{b}{r}$$

$$C_{w2} = a + \frac{b}{r}$$

Now, you can understand this is what is not giving me the satisfaction of say my free vortex, say $C_w \cdot r = constant$. But as we have discussed, we can do the calculation of other parameters. So, let me write down the equation. This is what is our equation. So, we can say my variation in axial velocity, that's what can be calculated by using this formula. Just understand one thing, this is what is representing our situation where my C_w is changing, that means my axial velocity may change, okay.

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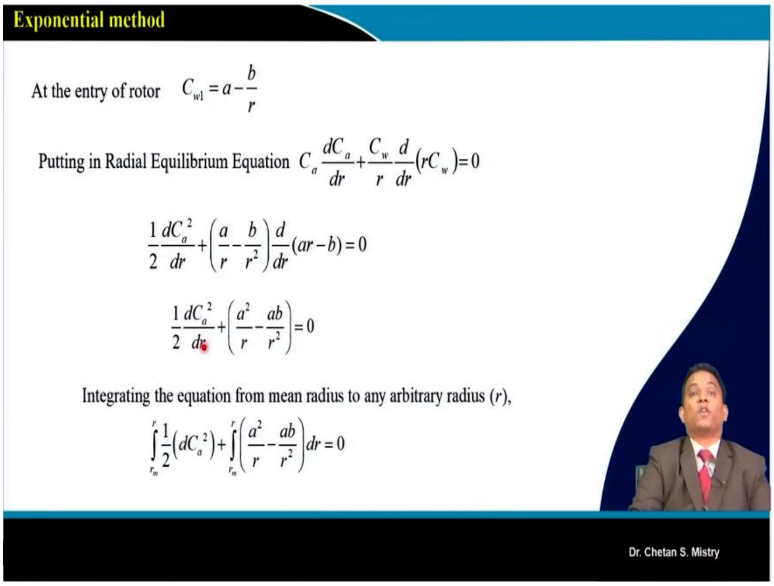

So, let us have a look at. Suppose if I consider at the entry, my C_{w1}, that's what I am writing as say $a - b/r$. So, in this equation, if you look at since my C_{w1}, or C_w, that's what is changing, then I need to do calculation, what is happening with my axial velocity, okay. So, if I will be simplifying this equation, that's what will be coming as

$$\frac{1}{2}\frac{dC_a^2}{dr} + \left(\frac{a}{r} - \frac{b}{r^2}\right)\frac{d}{dr}(ar - b) = 0$$

If I am simplifying this equation, this is what will be giving me how my variation of axial velocity, that's what is happening.

$$\frac{1}{2}\frac{dC_a^2}{dr} + \left(\frac{a^2}{r} - \frac{ab}{r^2}\right) = 0$$

Now, in order to do the calculation, how my axial velocity is varying, I will be putting my limit from say my mid-section to any station r. So, I can say this is what is my integral form of my axial velocity and this is what is my integral form for my variation in radius.

$$\textit{Integrating the equation from mean radius to any arbitrary radius } (r),$$

$$\int_{r_m}^{r} \frac{1}{2}\,(dC_a^2) + \int_{r_m}^{r} \left(\frac{a^2}{r} - \frac{ab}{r^2}\right) dr = 0$$

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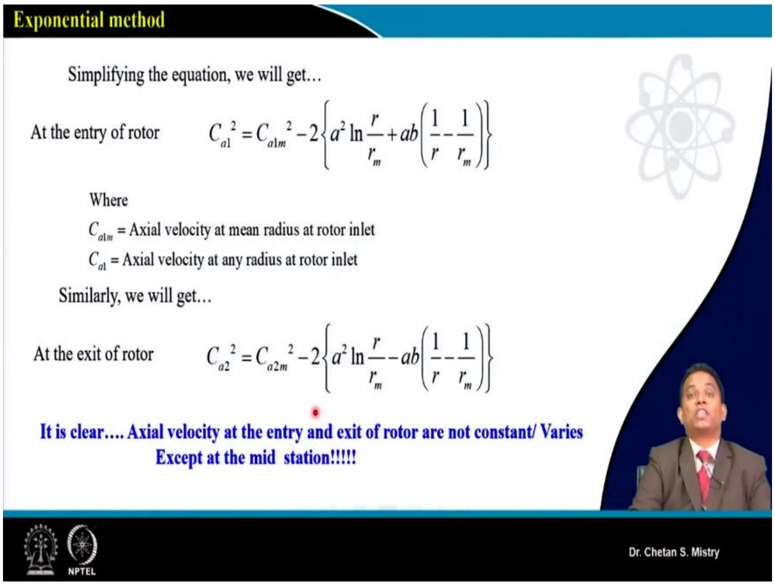

If we are simplifying this equation, we will be getting, at the entry of my rotor the axial velocity that's what is very it is in sense of my say axial velocity variation at the mid-section or my magnitude of axial velocity at the mid-section and this is what is a function of my radius, okay. We will not go into detail in sense of derivation, because this is what is my design course; and in design course, maybe with using your pen and paper try to calculate this C_{a1} what you are getting, okay.

$$C_{a1}^2 = C_{a1m}^2 - 2\left\{a^2 \ln\frac{r}{r_m} + ab\left(\frac{1}{r} - \frac{1}{r_m}\right)\right\}$$

So, it says C_{a1m} that is nothing but my axial velocity at the mid-station at the entry, okay, and my C_{a1} that is nothing but my axial velocity at any station, okay. So, you can understand, at mid station, I know my axial velocity, suppose; then I can calculate how my axial velocity is varying along the span at the entry of my rotor by using this formula, okay.

Now, if we are considering what is happening at the exit of my rotor, so, in line to that we can write down the equation for say exit velocity or exit axial velocity, that is also in sense of C_{a2} at the mid-station and this is what is in the form of say my radius.

$$C_{a2}^2 = C_{a2m}^2 - 2\left\{a^2 \ln\frac{r}{r_m} - ab\left(\frac{1}{r} - \frac{1}{r_m}\right)\right\}$$

So, if we try to look at here, just understand, like this is what is giving a different kind of feeling, what it says? We are having, not satisfaction of our radial equilibrium, what is the reason?

Because my axial velocity at the entry and exit, that’s what is not remains constant, it is varying, okay. And that’s what is not satisfying my radial equilibrium equation. Now, the question must arise in your mind, say if it is not satisfying the radial equilibrium then what needs to be done, okay? So, that is what we will be discussing very soon.

But you can understand, when I am considering my exponential design, in which I will be having my axial velocity at the entry of my rotor and my axial velocity at the exit of my rotor, that’s what is not same, it is varying. Now, immediate question will come, what is happening with my degree of reaction?

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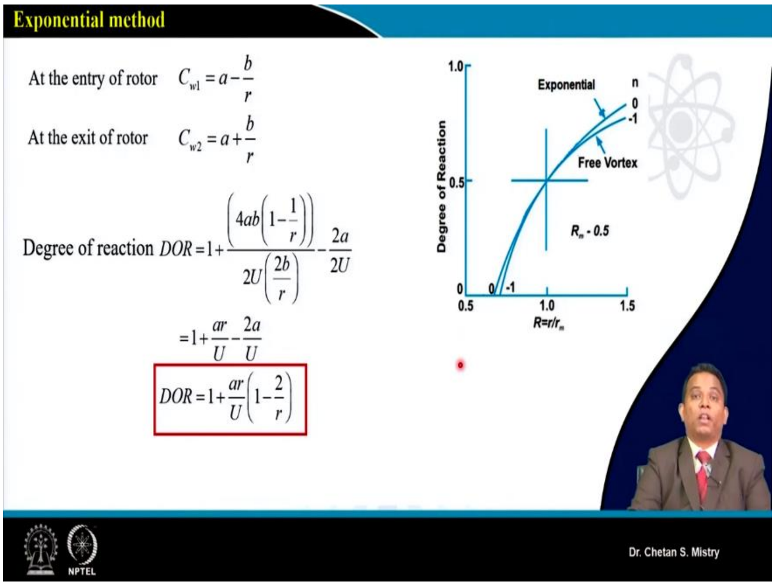

So, if we consider here, say, from my fundamental equation, I can write down my C_{w1} and C_{w2}, and the degree of reaction equation, if I will be putting this C_{w1} and C_{w2}, that's what is saying, my degree of reaction, that's what is varying with my radius, okay. So, my degree of reaction, that's what is varying with my radius.

$$At\ the\ entry\ of\ rotor, C_{w1} = a - \frac{b}{r}$$

$$At\ the\ exit\ of\ rotor, C_{w2} = a + \frac{b}{r}$$

$$Degree\ of\ Reaction,$$

$$DOR = 1 + \frac{4ab\left(a - \frac{1}{r}\right)}{2U\left(\frac{2b}{r}\right)} - \frac{2a}{2U}$$

$$= 1 + \frac{ar}{U} - \frac{2a}{U}$$

$$\therefore DOR = 1 + \frac{ar}{U}\left(1 - \frac{2}{r}\right)$$

So, let us try to look at here, what is happening? What all numbers we are discussing? What we have discussed? We are having two approaches, where my $n = -1$ and when 'n' we are considering to be 0. So, if you look at here, this is what is representing along my span what is happening and this is what is representing the variation of degree of reaction.

So, you can say somewhere and downside you will be having your hub and somewhere here you will be having your tip, just do not get confused, this is not hub and this... So, this is what is normalized with respect to my mid-section, okay. So, if we look at considerably here, so here, if you try to look at, suppose say… if I am taking my $n = -1$ condition, so you can say, this is what is giving me my degree of reaction to be 0 at 1 station.

Suppose if I will be extending that line further on my y axis, it says it is giving me my degree of reaction to be negative. Same way, if you are considering my $n = 0$, it may be possible further extension, that's what will be giving me my degree of reaction to be negative. So, we can say that degree of reaction, that's what is a very important parameter. And as we have discussed, that's what is a thermodynamic parameter in order to check the diffusion, that's what is happening in my axial flow compressor, okay.

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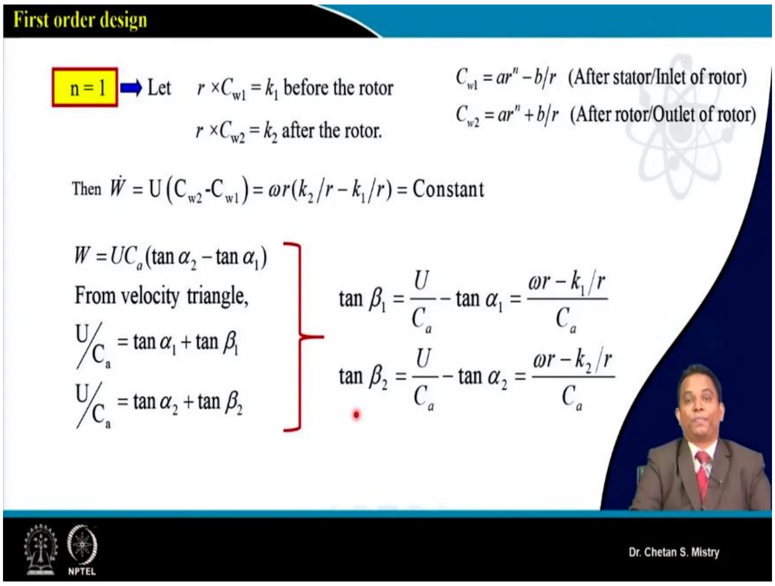

Now, with this, the next step, that's what will be coming is let say, let us consider my $n = 1$ condition. So, when I am taking my $n = 1$ condition, say in this formula, if I will be putting, say I am simplifying this form, as say I am assuming my $r \times C_{w1} = k_1$ and I am assuming my $r \times C_{w2} = k_2$, okay.

So, if we consider here, and if I will be putting that in my work done equation, that's what is $U(C_{w2} - C_{w1})$, that's what is coming to be constant, okay.

$$\dot{W} = U(C_{w2} - C_{w1}) = \omega r(k_2/r - k_1/r) = constant$$

So, we can say, this is what is my situation where my h_0 is constant along my span, okay. What we know from our velocity triangle, we must know, like what will be my variation of β_1 and what will be the variation of my β_2, because we are interested in calculation what is happening with our degree of reaction.

$$W = UC_a(\tan\alpha_2 - \tan\alpha_1)$$

$$From\ velocity\ triangle,$$

$$\frac{U}{C_a} = \tan\alpha_1 + \tan\beta_1$$

$$\frac{U}{C_a} = \tan\alpha_2 + \tan\beta_2$$

$$\therefore \tan\beta_1 = \frac{U}{C_a} - \tan\alpha_1 = \frac{\omega r - k_1/r}{C_a}$$

$$and\ \tan\beta_2 = \frac{U}{C_a} - \tan\alpha_2 = \frac{\omega r - k_2/r}{C_a}$$

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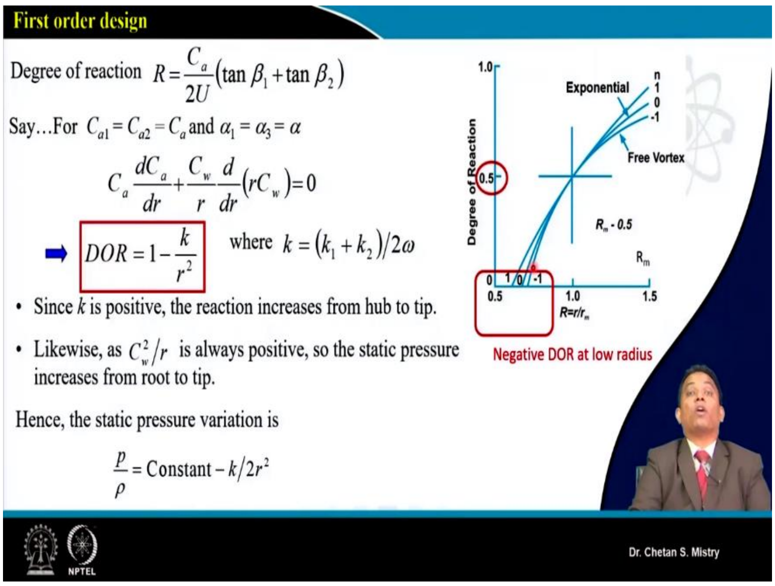

So, under this condition, when we are taking our $n = 1$, we will say the degree of reaction simply will be kept in terms of my $\Delta\beta$ or β_1 and β_2. Suppose, say if I am assuming my axial velocity to be constant, okay; if this is what is your case, just realize, one of the conditions we are putting $C_w \cdot r = constant$, that's what is my one condition, and what we are assuming? We are assuming our axial velocity to be constant, so that's what is my second condition.

If that's what is your case, you can understand, you are able to calculate what is happening with my stagnation enthalpy or my work done. So, as we have seen, out of three constants, if two constants are known to me, my third constant we can calculate, okay. So here, my degree of reaction we can say, that's what is coming in sense of

$$Degree\ of\ reaction, R = \frac{C_a}{2U}(\tan\beta_1 + \tan\beta_2)$$

$$For\ C_{a1} = C_{a2} = C_a\ \ and\ \alpha_1 = \alpha_3 = \alpha$$

$$C_a \frac{dC_a}{dr} + \frac{C_w}{r}\frac{d}{dr}(rC_w) = 0$$

$$\therefore DOR = 1 - \frac{k}{r^2}$$

$$where\ k = \frac{k_1 + k_2}{2\omega}$$

The value of this k is always positive, and that's what is increasing from hub to tip, okay. Now here, in this case, likewise, say my $\frac{C_w^2}{r}$ is always positive, that's what is giving me my static pressure rise it is happening from say root to tip or may be from hub to shroud, okay.

So, we can say my static pressure rise, that's what is given by

$$Static\ pressure\ variation, \frac{p}{\rho} = constant - {}^{k}\!/_{2r^2}$$

So, here if we try to look at, say this is what is my situation. We are talking about different values of 'n', we are talking about the exponent value of 'n'. We say, we are considering $n = -1$, that is what we are discussing as or we are selecting that as a free-vortex design.

We are taking our $n = 0$, that is what we have considered as an exponential design and we have taken $n = 1$ condition, that is what we are considering as a first design approach or first order design. If that's what is your case, you can say this is what is representing how my degree of reaction that's what is varying.

Now, interestingly as we have discussed for earlier 2 cases, if you are comparing all the three cases, we will see our degree of reaction will be having more chances to go negative near the hub region. But if you consider a variation of my degree of reaction for say my free vortex design, you can say, that's what is say less in that sense and here this is what is varying in more way, okay. And, this is what will give you idea how do we use this exponent for the calculation of our degree of reaction and how this is what will be impacting on our design criteria, okay.

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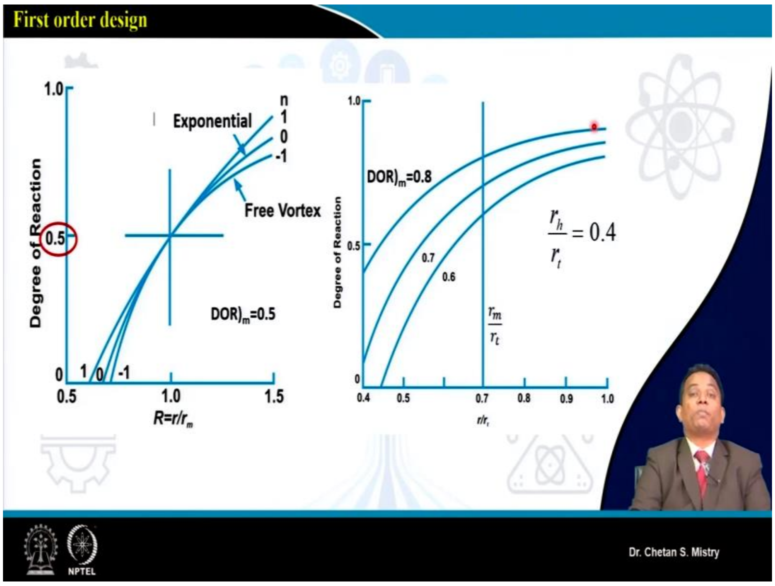

Now, if this is what is known to us, if we will be putting, the idea will come, we will say, say… this design what we are discussing where our degree of reaction at the mid-section we have assumed to be 0.5, okay and since we are assuming this to be 0.5, for all exponent design what we have realized, I will be having my degree of reaction to be going to negative near the hub region. Now the question will arise what needs to be done?

If I am looking for my degree of reaction to be positive. So, this is what it says, if I will be assuming my $r_h/r_t = 0.4$. So, you can understand my hub radius, that's what is say less, where we are discussing that's what is say hub to tip radius it is coming say lower value. Under that condition, if you are considering, say… all these three design concepts if you will be implementing, it says for say… your free vortex concept, if I will be considering my degree of reaction to be of different values, you can see, if I am assuming degree of reaction to be 0.6; you can say, still I am having chances for my degree of reaction to go negative.

So, it says, at mid-section, I need to go with say high degree of reaction, okay. It says, I need to go with say more degree of reaction near the mid station and that's what it says you can assume safely your degree of reaction to be 0.8. Remember one thing, if you are considering that to be 0.8, at tip you will be having my degree of reaction to be coming very high. So, you can understand, we are having or we are targeting our degree of reaction parameter for the design configurations, okay.

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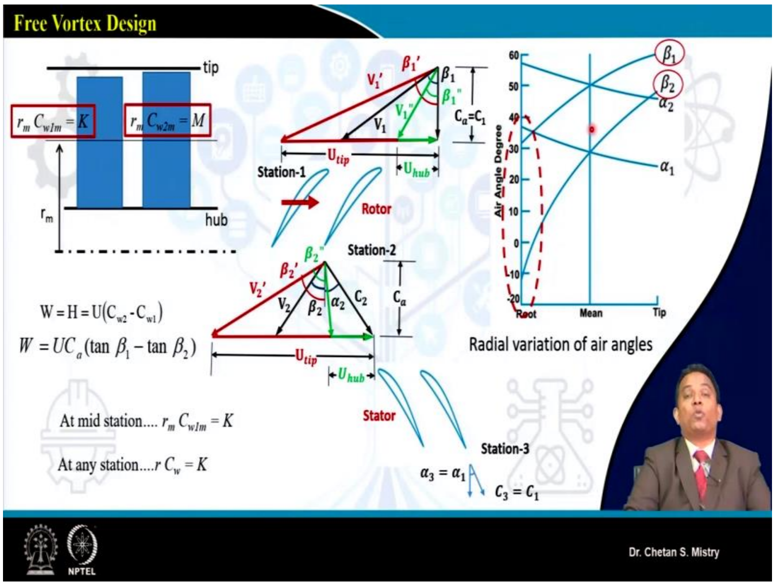

Now, here if we consider, we have discussed about our free vortex concept, let us come here again. So, if we consider, at say entry station, I am writing my $C_w \cdot r = constant$, I am saying that as say k and at the exit of my rotor, I will say this is what is given by my $C_w \cdot r = constant$, okay. If this is what is the kind of configuration what we are having, okay, and at end what we know? At mid station, we can have our calculation.

So, this is what is my velocity triangle at the entry of my rotor and this is what will be my velocity triangle at the exit of my rotor. Now, what we are interested in is say what is happening near the tip region. So, I will be putting my tip $U_{tip} = \frac{\pi D_{tip} N}{60}$ since my tip diameter is larger, I will be having this as a configuration, U_{tip}, okay, that's what is giving me my relative velocity component to be V_1'.

And if you look at, that is giving me my blade angle, or we can say my airflow angle, at the entry of rotor as β_1'. In line to that at the exit of rotor, I will be having the variation of my angle, that's what is given by β_2', okay. Now, we know, near the hub region, again, and again, we know the radius near the hub, that's what is lower, that is the reason why my U_{tip}, that's what will be coming to be lower.

And this is what will be the change of my angle β_1'', okay. And here at the exit, if I will be putting this is what will be giving me β_2''. Now, we need to be very careful here, you can say, all the way from my hub to shroud, my β_1 and β_2, that's what is changing. The change in this β_2 and β_1 is because of my change of peripheral speed, and if you are considering here, this is what is representing what is happening with my β_1 and β_2.

So, in this particular region, if you are looking at, it says this is what is the station at hub, where my $\Delta\beta$ is coming to be large, that may be in the range of may be 35° to 42°, okay. And if you are looking near a tip region, that is where my angle $\Delta\beta$, that's what is coming to be lower, okay. When I say my $\Delta\beta$, that's what is to be larger, you can understand, that is where you need to have more amount of work that needs to be done.

And if I am considering this $\Delta\beta$ variation, that's what is giving us idea about what is happening in sense of my blade twist, okay. And that is the reason, we must realize my blade for free vortex design, that's what will be highly twisted blade. So, you can understand here, like what all we started discussing is, we have taken 2 approaches, where we have assumed our stagnation enthalpy to be constant throughout my span, and my arbitrary variation of say whirl component.

And that's what we have defined in the form of, say, some constant, that's what is say C_{w1} and C_{w2}. Then, the exponent what we have in the equation for C_{w1} and C_{w2}, that is what we have assume to be $n = 0$, we have assumed that to be $n = 1$, and then we have assumed, it is $n = -1$.

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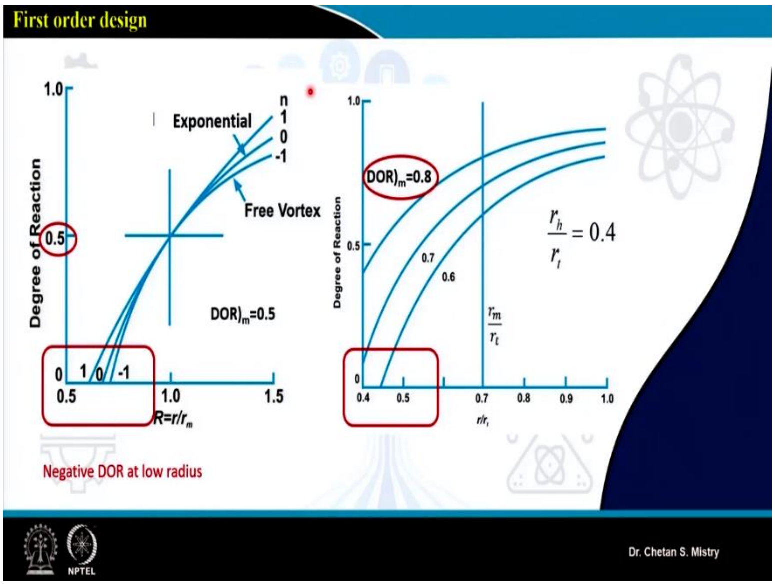

And if we go back, if we try to look at what is happening, say we have realized, say we will be having the variation of degree of reaction. We can say, when I am having my $n = 1$, that's what is given free vortex concept. When I am taking my $n = 0$, that's what is giving me my exponential design concept, and my $n = -1$, that's what is given first order design concept.

With this background, we will be moving ahead in order to understand what is happening in sense of this whirl distribution. So, these are three approaches what we are discussing, okay. And in next lecture, we will be discussing about what all are the other approaches which are possible. Thank you very much for your kind attention. See you in the next lecture. Thank you! Thank you very much!