Aerodynamics Design of Axial Flow Compressors & Fans
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Lecture 17
Design Concepts (Contd.)

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Hello, and welcome to lecture-17 for module-3. In last lecture we were discussing about the three-dimensional flow in axial flow compressor. We were discussing about the effect of boundary layer growth, we were discussing about the reason for change of axial velocity, work done factor, blockage factor and some of the reasons which are responsible for the three-dimensionality in the flow.

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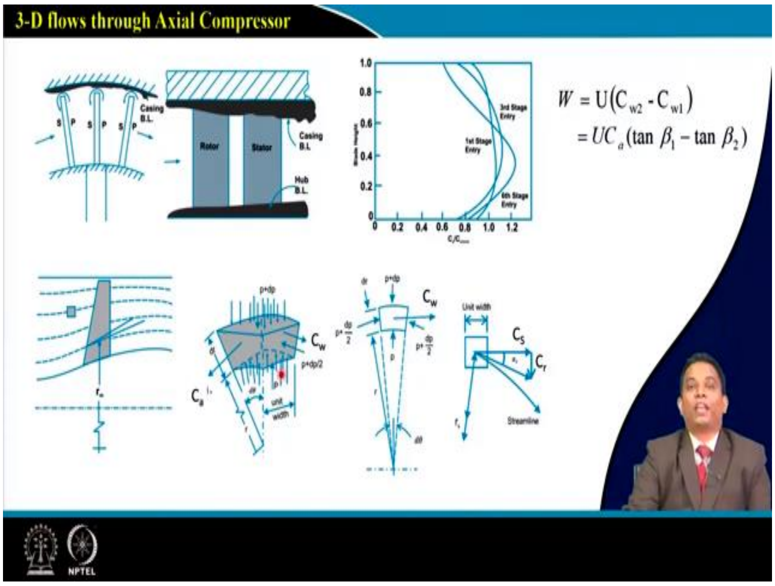

If you look at here, this is what we have started discussing. So, we can understand, the axial flow compressor, that's what is working under adverse pressure gradient. So, we are imposing our flow and adverse pressure gradient and that is the reason why there are more chances for my flow to get separated through this axial flow compressor stage.

One more thing is because of presence of solid body near the casing as well as near the hub region, we will see there is a growth of boundary layer from both the walls. If we consider, say this is what is one of the stage; my axial flow compressor it is made up of number of stages, it may be 8, it may be 10, it may be 12.

So, if I am increasing my number of stages, that's what will lead to growth of boundary layer till end of my stages. It may be possible that my last stage which will be having shorter blade, whole my blade that will be covered with this growth of boundary layer. So, we have our flow to be three-dimensional in that sense. If we consider the growth of boundary layer from the both the walls, that's what is making our passage, because of presence of low momentum fluid within the boundary layer, it will be making our passage to be converging passage.

So, we can understand, we are forcing our flow to work under adverse pressure gradient at the same time because of growth of this boundary layer, we are having our passage to be convergent passage and that's what will be giving us a chocking kind of situation. Now, we have also understood in order to have free rotation of our wheel, we need to provide certain amount of clearance between casing and our blade.

Now, this is what is one of the area for the research for many. Here if you look at, on my pressure side we will be having say higher pressure on our suction side we will be having lower pressure and we can understand, our rotation of wheel that's what is from suction side to the pressure side. Now, my flow that will try to move from pressure side to the suction side because of presence of clearance and this clearance, that's what will be giving me a flow, that's what is called tip leakage flow.

Now, the situation here near the tip region, that will be getting worse, because we can understand we are having growth of boundary layer itself with our walls near the casing and along with that we will be having flow complexity because of presence of this tip leakage flow. So, that is the reason why, what we are considering in sense of distribution of our velocity mainly axial velocity that's what will not remain constant.

So, here if you look at, we have discussed for our first stage; if we look at, we are having the growth of boundary layer from both the walls and if you consider there is a deformation of my velocity profile near the hub region as well as near the tip region. And we are assuming our axial velocity will be constant; so, that's what is contradicting.

Now, at the same time if I am moving further downside, say maybe for the sixth stage we are considering, we will be having great deformation of velocity profile and this deformation of velocity profile, you will understand, that's what is not in line to what we are assuming. We are assuming our axial velocity to be constant when we are doing our design.

So, the thing is, here what we are assuming, that's what is not coming. So, that means in order to understand what all need to be address, we need to have certain parameter, okay. And in order to address this issue, we have introduced one parameter, that's what is called work done factor in last lecture. We have discussed about how we are calculating our actual work done. We must realize, our actual work done, that's what will be lower than that of what we are expecting.

Now, many researchers, many companies, those who are working in an area of axial flow compressor, they are having their own database over the year, that's what they have generated based on their experience may be based on their theoretical study based on their computational study and that is how they have come up with some parameter.

Mainly, for Americans, they used to say that as a blockage factor. So, that blockage factor they are calculating based on growth of boundary layer, based on say displacement thickness and they are coming up with some numbers and those numbers they are using when they are doing their actual design. For our case, we have given what plot, that's what was discussed by Cohen and Rogers; that is what we say with increase of number of stages our work done factor, that's what is going to reduce.

Now, when we move further, we know, we are having our axial flow compressor that may be working under say adverse pressure gradient. It may be possible that at the exit we will be having pressure to be high and when I say my pressure to be high my density also will be higher and my continuity says my density into area into velocity that need to be constant at entry as well as at the exit.

Now, suppose if we are assuming, we are having our axial velocity to be constant and if my density is increasing at the exit, we can say our flow passage area will be having lower area at the exit and having higher area at the entry means it will be having converging kind of passage. So, you can see here, this is what is kind of passage we will be getting at the exit of my compressor stage.

Now, what all we are assuming up till now? We have done two dimensional analysis and for those two dimensional analysis we have assumed, say… our flow is moving parallel to my casing and it is moving parallel to my hub, but in actual scenario if you consider, what is happening, now my upper stream lines that will try to follow the path, that's what was given by my casing; in line to that near hub my stream lines will try to follow the part of the curvature of my hub.

And, that's what will be giving us some kind of inclination to our flow, that means, here we will be having some other component of velocity that's what is coming into the picture. So, up till now what wall we are assuming is say we are having axial velocity to be constant and we are not having any component in a radial direction.

Now, because of this kind of curvature, because of high pressure ratio, we will be having a new component that will be coming into the picture that's what we say radial velocity component. And, we have taken say one of the particle here and for that particle we try to write down the equation for balancing the inertia force and pressure force.

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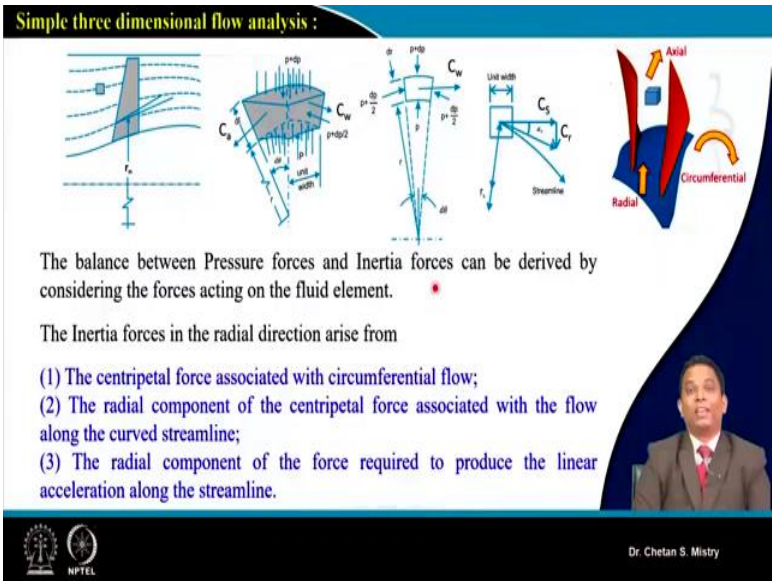

Let us move ahead, say, this is what is our scenario. If we consider for this case, say… we can say, we are having this direction as my axial direction, I will be having my circumferential direction and I will be having my radial direction that will be along the span. If I will be selecting one of the elements that's what is placed between these two blades, suppose it may be say a rotor blade or it may be a straighter blade.

So, if this is what is my element; I will consider the control volume approach here. The same element we can represent here that's what will be having some radius r and elemental part we can say it is having say radius dr or distance dr. This inclination we are considering as $d\theta$. Now, we know, because of the movement of this particle or within the blade passage, we will be having the rise of pressure that's what is happening, that's what is my diffusive action.

So, you can say, I will be representing my pressure downside as say 'p' and on upper side, I will put my pressure as $p + dp$. Now, as we have seen, say this is what is representing my axial velocity component that's what is in my axial direction and we will be having one more component in my circumferential direction that's what is my whirl component C_w component.

Now, if I will be considering say unit width kind of configuration, say for my tangential velocity component or whirl component that can be represented here. In line to that, if you will be putting that in large view, we will see, I will be having number of streamlines they are present and because of the streamline, we will be having one of the component of velocity that will remain present that's what is a C_r, it will be having magnitude comparatively low but that's what will be there.

So, today we will be discussing more in sense of what is happening in radial direction. So, before going into detail, we will see, my pressure force and inertia force on this element need to be balanced and we can say this inertia force in a radial direction that's what is arising because of your centripetal action associated with the circumferential flow. So, you can say this is what is representing say what is happening in sense of my centripetal action.

The radial component of centripetal force associated with the flow along the curved stream line. So, this is what we can say it is a curved stream line along which we will be having one of the component for centripetal force. We also will be having say radial component of force associated to produce a linear acceleration along the stream line. So, that's what we say, this is what is representing my linear acceleration component.

Now, we are moving more towards analyzing what is happening in sense of my flow or the action on my fluid particle in order to understand the flow three-dimensionality. Now, in order to understand that part, let us move into more detail.

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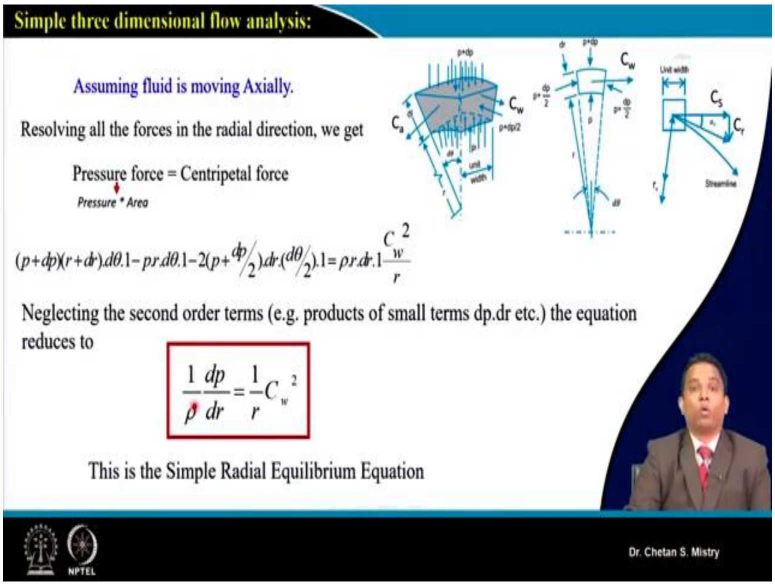

Let us see, say this is what I say simple three-dimensional analysis. Before going into detail analysis, we will be making certain assumptions to simplify the problem. Our first assumption, it is the radial movement of your flow is governed by radial equilibrium of the forces. So, we are more interested in what is happening with our radial movement. So, we are considering say balance of my radial forces.

The radial movement, that's what will be occurring within the passage only not outside; be careful, we are not interested in analyzing our flow what is happening between stator and rotor or what is happening in the diffusing passage. We are at this moment interested what is happening in our blade passage. Third, we will see, the flow analysis that's what is involved balancing the radial force, that's exerted by the blade rotation.

And gravitational force, we can neglect because our working fluid we are assuming as say air. Now, for this elemental part, as we have discussed, we will be having say this angle that's what is say $d\theta$, my pressure force acting is say 'p', on upper part my pressure force acting is $p + dp$. Now, we will try to balance the force in radial direction.

So, assuming, if we assume our flow to be moving in axial direction and if we will be resolving our force component in radial direction, we can say, my pressure force that need to be balanced by my centripetal force.

So, if we consider our pressure force, what we know? Pressure force, it is given by $Pressure \times Area$. So, that's what we are writing here for this upper part it is $(p + dp)(r + dr) \cdot d\theta \cdot 1$, we can say, our width to be say unity or 1. Same way from lower side we can write down, this is what is $p \cdot r \cdot d\theta \cdot 1$.

Same way on both the side we can write down $2(p + {}^{dp}/_{2})$ into this equation $(dr \cdot \left({}^{d\theta}/_{2}\right) \cdot 1)$ and that's what will be balanced by my centripetal force and that centripetal force we are writing as say $(m \cdot \frac{\omega^2}{r})$, or here in this case, we are writing in sense of $\frac{mC_w^2}{r} = \rho \cdot r \cdot dr \cdot 1\ C_w^2/r$ because my whirl component, that's what is responsible for that centripetal action.

$$(p + dp)(r + dr) \cdot d\theta \cdot 1 - p \cdot r \cdot d\theta \cdot 1 - 2\left(p + {}^{dp}/_{2}\right) \cdot dr \cdot \left({}^{d\theta}/_{2}\right) \cdot 1 = \rho \cdot r \cdot dr \cdot 1\ C_w^2/r$$

Now, if we will be rearranging all these terms and if we will be neglecting the lower order terms, we will be coming up with the equation it says $\frac{1}{\rho}\frac{dp}{dr}$ that's what is equal to $\frac{1}{r}\mathrm{C}_{\mathrm{w}}^2$.

$$\frac{1}{\rho}\frac{dp}{dr} = \frac{1}{r}\ C_w^2$$

Now, you can understand, this is what is representing what is happening or how my pressure is changing along with the radius or we can say how my pressure is changing along the span and that's what is known as simple radial equilibrium equation.

So, this is what is very important equation when we are analyzing our flow through any turbo machinery. So, this is what we can say, based on our assumptions we are coming with the solution, that's what is say… we will be having

$$\frac{1}{\rho}\frac{dp}{dr} = \frac{1}{r}\ C_w^2$$

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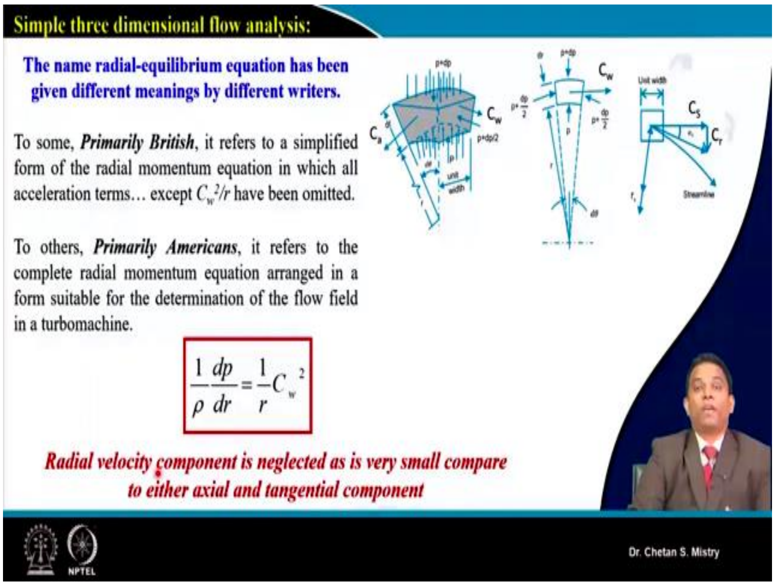

Now, this equation if we consider, this is what is named as say radial equilibrium equation that has different meanings. So, as I told on both the sides of Pacific, people they having some different thought process in sense of designing, the say your axial flow compressor. So, what it says? Primarily for British, it refers as a simplified form of radial momentum equation in which all the acceleration term except this $\frac{C_w^2}{r}$ have been omitted.

So, we are having only action because of centripetal force. For Americans, they say this is what is referred as more complete radial equilibrium equation, that's arranged in a suitable form in order to determine the flow field within the turbo machinery. So, now here if you look at, this is what is my fundamental equation, I say, that we say as a radial equilibrium equation.

Now, you will get surprise we have started talking about our radial velocity component, we are interested in analyzing our flow for say three dimensionality and here if you look at, in equation nowhere we are getting C_r. So, the thing is, in radial velocity component what we are considering, that's what is having very small magnitude and that's what if you are comparing with your axial velocity and tangential velocity that will becoming as a lower magnitude and that's the reason why in our simplified radial equilibrium equation, we are not having any term that's what is correlating my dr.

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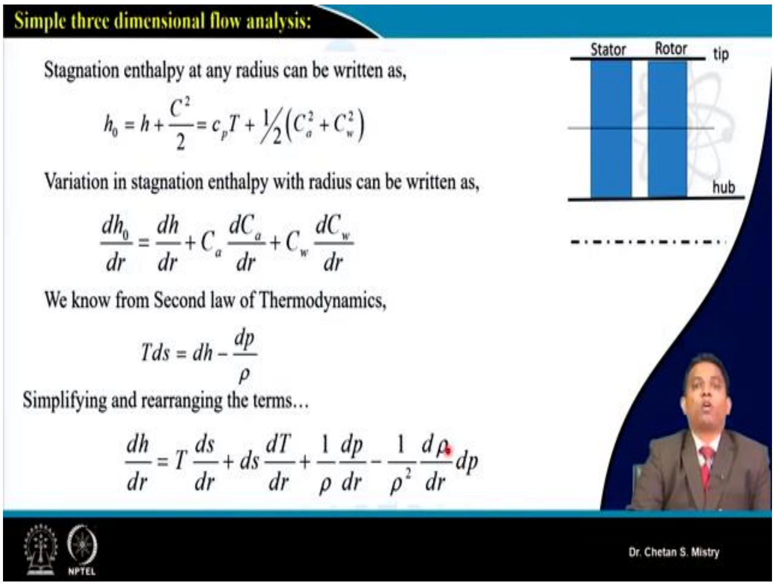

Now, let us try to understand what is happening with this particle when we are considering the analysis of flow. So, what we know? My stagnation enthalpy at any radius that can be written as say $h_0 = h + \frac{C^2}{2}$, where C is my absolute velocity and from our understanding of velocity triangle I can write down that in terms of axial velocity and tangential velocity, okay.

$$h_0 = h + \frac{C^2}{2} = c_p T + \frac{1}{2}(C_a^2 + C_w^2)$$

So, if we are considering there is a variation of this stagnation enthalpy along the radius, we can write down, $\frac{dh_o}{dr}$ that is nothing but it is $\frac{dh}{dr} + C_a \frac{dC_a}{dr} + C_w \frac{dC_w}{dr}$, so this is what is representing my variation of stagnation enthalpy along the radius and that's what is a function of my static enthalpy, it is a function of my axial velocity and that's what is a function of my tangential velocity, or whirl component of my velocity.

$$\frac{dh_o}{dr} = \frac{dh}{dr} + C_a\frac{dC_a}{dr} + C_w\frac{dC_w}{dr}$$

Now, we have our fundamental equation from second law of thermodynamics, that's what is say, my Tds that's what is given by

$$Tds = dh - \frac{dp}{\rho},$$

If this is what is known to us and if we are writing this equation in terms of variation along the radius, this equation can be reformed in this equation form, that's what is representing my change of say entropy, that's what is representing change in my static temperature, that's what is representing in sense of my change of temperature and in sense of change of my density.

So, all these parameters they are of importance to us, so slowly we are moving towards say analyzing more realistic picture what is happening.

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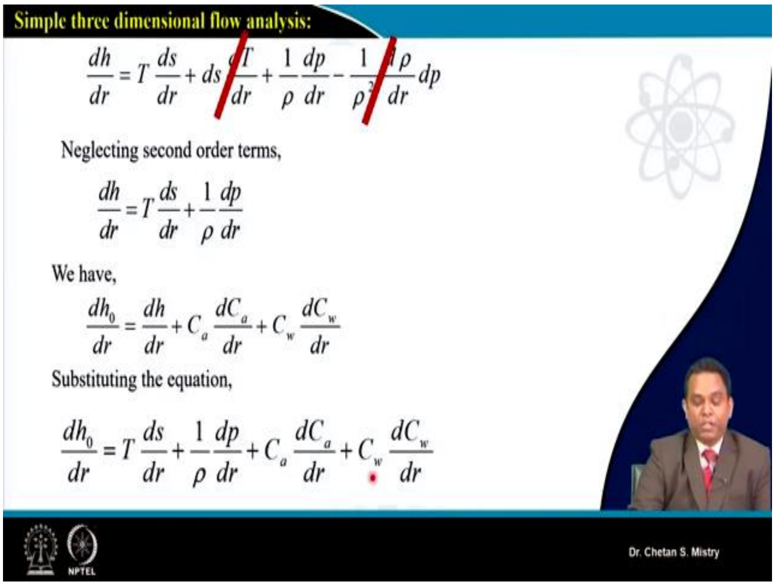

So, if we are putting here, say this is what is a same form what we are putting. Now, in the case if we consider this $ds\frac{dT}{dr}$, so, we can say this is what is representing say change of entropy and change of my static temperature along the radius. If you are considering, these terms, that's what is a smaller term. So, we can omit that part and we are omitting the second order formulation. And if we are neglecting this part, that's what is say my $\frac{dh}{dr}$, it is given by $T\frac{ds}{dr} + \frac{1}{\rho}\frac{dp}{dr}$.

$$\frac{dh}{dr} = T\frac{ds}{dr} + \frac{1}{\rho}\frac{dp}{dr}$$

Now, from our fundamental equation based on my definition for stagnation enthalpy, we have written $\frac{dh_o}{dr}$ that is given by dh by dr plus change of my axial velocity component and change of my tangential velocity component.

$$\frac{dh_0}{dr} = \frac{dh}{dr} + C_a\frac{dC_a}{dr} + C_w\frac{dC_w}{dr}$$

If we will be rearranging this term in sense of say my entropy and this form, I can rewrite. So, in place of $\frac{dh}{dr}$, I will be writing as say $\frac{dh_0}{dr}$ as

$$\frac{dh_0}{dr} = T\frac{ds}{dr} + \frac{1}{\rho}\frac{dp}{dr} + C_a\frac{dC_a}{dr} + C_w\frac{dC_w}{dr}$$

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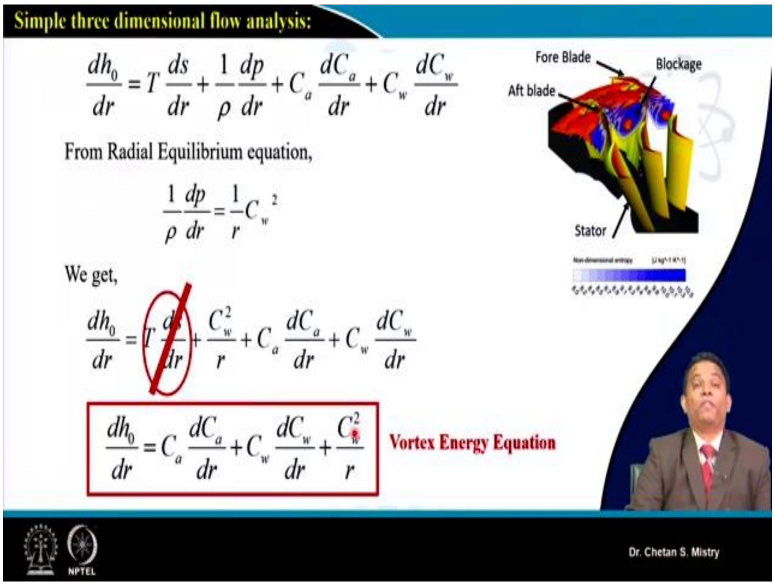

Now, here if you look at, this is what is a term, we can say, that can be represented in other form, because what we know this $\frac{dp}{dr}$, from our radial equilibrium equation, we can write down that as a $\frac{C_w^2}{r}$, okay. So, let me replace this equation here it says $\frac{dh_o}{dr}$, that is given by this entropy change that will be in sense of my centripetal force, that will be in sense of my axial velocity component and change in axial velocity along with the radius, my tangential velocity component and change in tangential velocity component along my radius.

$$\frac{dh_0}{dr} = T\frac{ds}{dr} + \frac{C_w^2}{r} + C_a\frac{dC_a}{dr} + C_w\frac{dC_w}{dr}$$

Now, the term what we are putting here, this is what is say $T\frac{ds}{dr}$, this is what is representing, we can say, change of entropy; that is nothing but it is representing the irreversibility associated with the process. Now, when I say irreversibility, that is nothing for our axial flow compressor, in our fundamental lecture we have discussed, that's what is defined as a losses.

So, many times, say… if we are considering say our transonic flow, we know because of presence of my stock formation, we will be having losses to be more and under that condition this terminology, that is what people they are considering when they are doing the calculation. For the sake of brevity, let me show you, people generally they realize this entropy term, that's what is related with the thermodynamics in it, it has nothing to do with axial flow compressor.

But just understand one thing, our fundamental equation what we are deriving at this moment that's what is giving you idea this change of entropy can not be neglected. Let me show you, say here in this case, say… this is what we have discussed from our own work, so this blockage of our flow, that is what we have represented in sense of say entropy, okay.

So, when you are doing your post processing, in order to discuss the losses, people they are conventionally they are using the measurement or say they are using this term as say entropy change. The measurement of entropy, that's what is always a challenging, but we are converting that in form of some pressure and in sense of temperature, okay. For the sake of simplicity, for our case, we will be considering this as say negligible or we can, we are omitting at this moment, because it will be giving unnecessarily complexity.

So, let us try to simplify that we are omitting this part. So, it says my $\frac{dh_0}{dr}$, now, it will be

$$\frac{dh_0}{dr} = C_a\frac{dC_a}{dr} + C_w\frac{dC_w}{dr} + \frac{C_w^2}{r}$$

Now, this equation, that's what is known as vortex energy equation, we will be using this equation again and again when we are discussing the design part.

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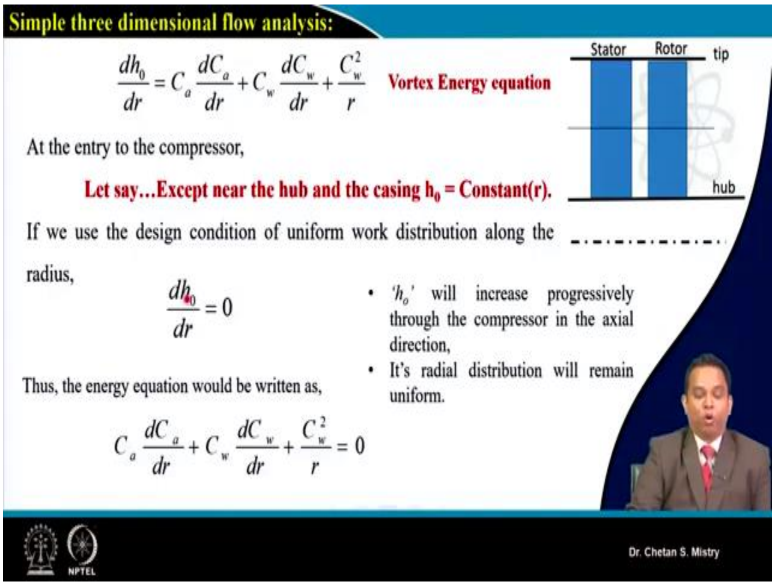

So, this is what is my formula for say vortex energy equation. Now, at the entry of my compressor, say if we are considering near the hub and near the casing along that if we are considering, my total enthalpy along my radius, let us assume to be constant. So, if that's what is your case, say for uniform work distribution, you can straightaway write down $\frac{dh_0}{dr} = 0$, okay.

Now, if this is what is your case then my energy equation that will be reduced in the form of

$$C_a \frac{dC_a}{dr} + C_w \frac{dC_w}{dr} + \frac{C_w^2}{r} = 0$$

Now, fundamentally let us try to understand what we mean by assuming constant work distribution. So, it says my h_0 that will be increased progressively through my compressor in axial direction, we have realized that when we have discussed about the T-S diagram, we understand that part, say along my axial direction I will be having the rise of h_0, that's what is happening.

It is radial distribution, that's what is remains uniform. So, what assumption we are making? Say, we are assuming, say our constant h_0 along the span, we can say, that's what is valid at this moment, okay.

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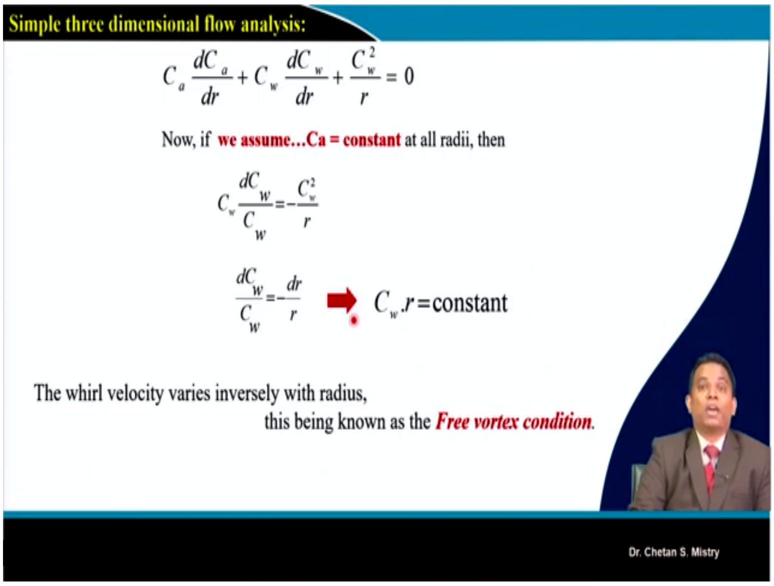

Now, let us try to simplify more in other sense. So, this is what is my equation it says

$$C_a \frac{dC_a}{dr} + C_w \frac{dC_w}{dr} + \frac{C_w^2}{r} = 0$$

Let me assume, say… we assume, we know our axial velocity for our analysis, that’s what we are assuming to be constant. If that’s what is your case, then we can say our equation, that will be simplify in this form ($C_w \frac{dC_w}{C_w} = -\frac{C_w^2}{r}$), and if I am simplifying further it says $\frac{dC_w}{C_w} = -\frac{dr}{r}$.

And, if I will be writing in a simplified form, it says, my $C_w \cdot r = constant$ and if you recall in your fundamentals of fluid mechanics or in aerodynamics, you have realized when we are writing $C_w \cdot r = constant$, that’s what is defined as a free vortex flow, okay. So, we are coming to a stage, where I say, this condition as a free vortex condition. So, this is what is say my free vortex condition, okay.

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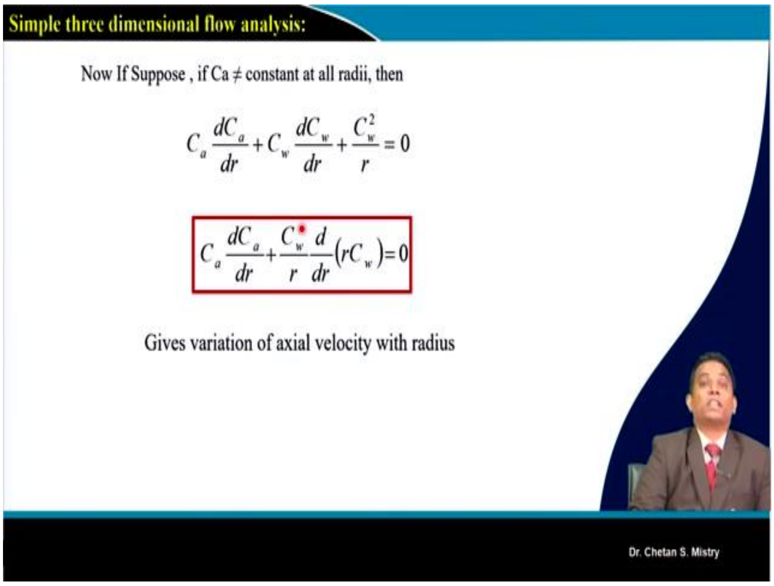

Now, suppose say if I will be saying, say… as we have discussed because of presence of three dimensionality of my flow, we are considering our axial velocity is not remains constant. Suppose if I consider my axial velocity is not constant, then my equation formulation that will be coming as say

$$C_a \frac{dC_a}{dr} + \frac{C_w}{r}\frac{d}{dr}(rC_w) = 0$$

And this equation, that's what will be helping us in order to calculate the variation of my axial velocity along the span, this is what is very important equation, we should not forget this equation, okay.

So, initially we have started with certain assumptions then we have come up with some formulation, where we have assumed our inertia force and pressure force that's what was coming to be say same. We have equalized that and based on that we have come up with the equation that's what is called say your radial equilibrium equation. Then with our understanding of fundamental equations, say… energy equation, based on that we have correlated our enthalpy variation with radius.

Then based on our understanding for the second law of thermodynamics we have incorporated the change of my enthalpy and we have our radial equilibrium equation and based on that simplified form we have come up with some formulation that's what is representing the change of total enthalpy or stagnation enthalpy along my radius in the form of my axial velocity, in the form of my tangential velocity and that is what we are using for our calculation or for further design part.

And then we have seen, if we are assuming our $\frac{dh}{dr}$, that's what is equal to 0 that means my constant work along the span, that's what is giving me say formulation to be $\frac{dh_0}{dr} = 0$. If we assume further say my axial velocity to be constant, then we have come up with the relation, that's what is called my free vortex equation and this is what is a formula, that's what is talking about say change or say variation of my axial velocity along my radius.

So, this is what is all, we say, today we have learned with. Now, we will be moving further as this session, this module is specifically for say design concepts. So, in next lecture we will be discussing about what is happening in sense of change of our say different design approaches. Thank you, thank you very much!