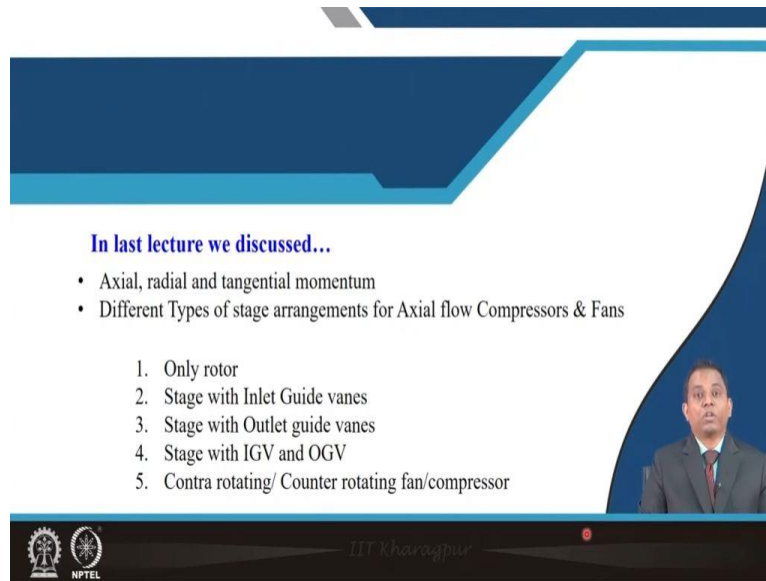


Aerodynamic Design of Axial Flow Compressors & Fans
Professor Chetankumar Sureshbhai Mistry
Department of Aerospace Engineering
Indian Institute of Technology, Kharagpur
Lecture – 10
Stage Configuration and parameters (Contd.)

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In last lecture we discussed...

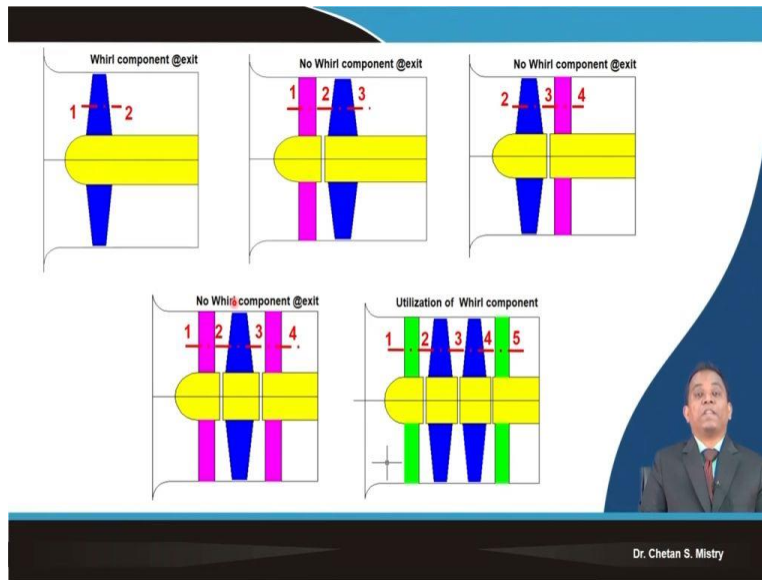
- Axial, radial and tangential momentum
- Different Types of stage arrangements for Axial flow Compressors & Fans
 1. Only rotor
 2. Stage with Inlet Guide vanes
 3. Stage with Outlet guide vanes
 4. Stage with IGV and OGV
 5. Contra rotating/ Counter rotating fan/compressor

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Hello, and welcome to lecture-10. In our last session, we were discussing about axial, radial and tangential momentum. And we have realized this is what is direct in relation with your mechanical part; but still, the change of velocity components names say, axial velocity, tangential velocity and a radial velocity; they are having their own importance. If you look at next we have discussed about the different types of stage arrangement for axial flow compressor and fans. What all configurations we have discussed is with only rotor configuration; say stage with inlet guide vanes.

So, we will be having say inlet guide vanes that will be followed with rotor. We have stage with outlet guide vanes; means rotor will be followed by the stator. We have inlet guide vane as well as outlet guide vane. And we have discussed important aspects for the contra rotating or counter rotating fan or compressor configuration.

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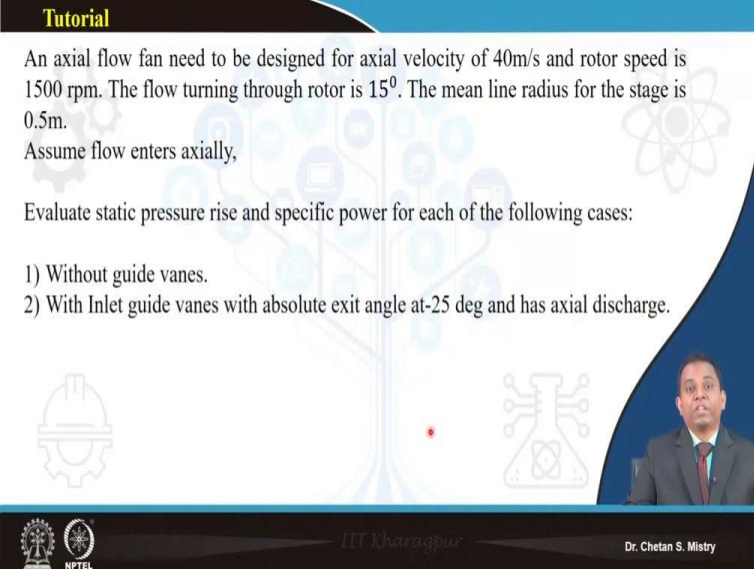
So, in our configuration for our understanding, if you look at here, say when we are using only rotor configuration, we realize we will be having whirl component; that is what will be coming out from the exit. And we realize based on our requirement, we need to design our fan. So, when we are not expecting much pressure rise and our whole purpose it is for the handling the mass to rate this kind of configuration, that's what is preferred. We have inlet guide vanes that what will be followed by your rotor; under that configuration we tried to avoid the whirl component that's what is coming out from my rotor.

And if that's what is your case, you need to have your rotor inlet condition at certain angle. So, my inlet guide vane, that is what will be providing my flow to the rotor at certain angle and my exit we are considering to be axial. Here, we are, you know, try to avoiding the whirl component. Same way here if we consider we are having a rotor that will be followed with the stator. So, here also we are trying to remove the whirl component coming out from the rotor and we are trying to give the axial exit, it is based on the requirement.

In line to that we can have configuration in which we are having say... inlet guide vane that will be followed by rotor; and that will be followed with my stator configuration or say outlet guide vane. Then, we have discussed very important aspect, that's what is say our contra rotating fan or counter rotating fan, or compressor. In which we have utilized the whirl component that is coming out from my rotor-1 that will go inside rotor-2. That is what will be sucked by rotor-2, which will

be increasing our relative velocity at the entry of my rotor-2; and we will be having comparatively high pressure rise compared to all these configurations. And we have discussed this configuration that's what is having now more attractions for future engine developments.

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Tutorial

An axial flow fan need to be designed for axial velocity of 40m/s and rotor speed is 1500 rpm. The flow turning through rotor is 15° . The mean line radius for the stage is 0.5m.
Assume flow enters axially,

Evaluate static pressure rise and specific power for each of the following cases:

- 1) Without guide vanes.
- 2) With Inlet guide vanes with absolute exit angle at-25 deg and has axial discharge.

The slide features a blue header with the word 'Tutorial' in yellow. The background is white with faint technical icons like a gear, a hard hat, and a molecular structure. A small video inset of a man in a suit is visible in the bottom right corner. The footer contains the IIT Kharagpur logo, the NPTEL logo, and the name 'Dr. Chetan S. Mistry'.

Now, if you want to have the idea, let us take one of the tutorial which will give you idea what all we are discussing in sense of stage configuration. How do we use that for our purpose? So, take one numerical, say, we have an axial flow fan that need to be designed for axial velocity of 40 m/s , rotor speed is 1500 rpm. My flow turning angle is 15° for the rotor, mean line diameter or mean line radius for the stage is 0.5 meter.

Consider, say flow is entering axially. Evaluate the static pressure rise and specific power for each of the following cases. In which we are having without guide vane configuration and we have inlet guide vane configuration, which has absolute entry angle at absolute exit angle of -25° , and my discharge is axial one.

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Tutorial Contd.

1) Without guide vanes

Station - 1

Station - 2

Rotor

Peripheral velocity $U_m = \frac{2\pi r_m N}{60}$

$U_m = \frac{2\pi * 0.5 * 1500}{60}$

$U_m = 78.54 \text{ m/s}$

Static pressure rise $\Delta P_{\text{rotor}} = \frac{1}{2} \rho (V_1^2 - V_2^2)$

Specific Power = UC_{w2}

U, V_1, V_2, C_{w2}

Given :
 $C_a = 40 \text{ m/s}$,
 $N = 1500 \text{ rpm}$,
 For rotor
 $\Delta\beta = \beta_1 - \beta_2 = 15^\circ$

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Now, for this configuration what all data we know is we are having our axial velocity; it is 40 m/s . We have our peripheral speed, we can count by using my rotational speed of 1500 rpm . For rotor, my $\Delta\beta$ that is my flow deflection angle; it is given 15° . Now, let us see suppose if I am taking my case-1, which is say without guide vanes. So, in order to understand that we can have this as a line diagram, we can have our station-1 and station-2. For that station-1 and 2, we can make our velocity triangle. It says my entry to the rotor is axial; so, we can say this is what is my axial entry.

So, this represents my inlet velocity triangle and this is what is representing my outlet velocity triangle from the rotor. Now, what we want to calculate? We are looking for what is my static pressure rise in the rotor; that's what is given by $\frac{1}{2} \rho (V_1^2 - V_2^2)$. Second, we are looking for specific power; that's what is UC_{w2} . It means with given data, we need to calculate our peripheral speed; we need to calculate what will be my relative velocity at the entry, what will be my relative velocity at the exit. We are looking for what will be my swirl component or my whirl component coming out from the rotor.

With this data, let us start with. So, very first point we can say we are looking for calculation of peripheral speed. So, my mean radius is given to me, it is say 0.5 meter ; so, from that we can calculate what is our peripheral speed. So, this peripheral speed I can write down

$$\text{peripheral Velocity, } U_m = \frac{2\pi RN}{60}$$

$$\therefore U_m = \frac{2\pi \times 0.5 \times 1500}{60}$$

$$\therefore U_m = 78.54 \text{ m/s}$$

So, this rotational speed is given, it is 1500; and my radius is given 0.5. So, we can calculate what is my peripheral speed. So, it says this is coming 78.54 m/s.

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Tutorial Contd.

From entry velocity triangle,

$$\tan \beta_1 = \frac{U}{C_a} = \frac{78.54}{40} = 1.963$$

$$\beta_1 = \tan^{-1}(1.963) = 63^\circ$$

We know $\Delta\beta = \beta_1 - \beta_2 = 15^\circ$

$$\beta_2 = 63^\circ - 15^\circ = 48^\circ$$

Relative velocity at inlet

$$V_1 = U / \sin \beta_1$$

$$= 78.54 / \sin(63^\circ)$$

$$V_1 = 88.14 \text{ m/s}$$

From geometry,

$$\frac{U}{C_a} = \tan \alpha_2 + \tan \beta_2$$

$$\alpha_2 = \tan^{-1}\left(\frac{U}{C_a} - \tan \beta_2\right)$$

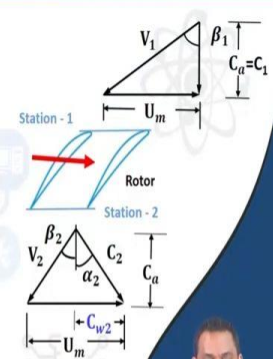
$$\alpha_2 = \tan^{-1}\left(\frac{78.54}{40} - 1.963\right)$$

$$\alpha_2 = 40.44^\circ$$

Tangential velocity at exit

$$C_{w2} = C_a \tan \alpha_2$$

$$= 40 \tan(40.44^\circ)$$

$$C_{w2} = 34.1 \text{ m/s}$$


We know:
 $C_a = 40 \text{ m/s}$
 $U = 78.54 \text{ m/s}$

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Now, we have our next parameter that need to be calculated. That's what is say we are looking for my V_1 ; that's what is my relative velocity at the entry of my rotor. So, based on my velocity triangle I can write down my;

$$\tan \beta_1 = \frac{U}{C_a}$$

Since my entry is axial, so we can write down $\frac{U}{C_a}$. We know what is my peripheral speed now; we know what is my axial speed or axial velocity.

$$\therefore \tan \beta_1 = \frac{78.54}{40} = 1.963$$

$$\beta_1 = \tan^{-1}(1.963) = 63^\circ$$

So, if I am putting that number, it says my blade angle at the entry, it is coming 63° . Now, what we know? We have our $\Delta\beta$ that's what is known to us. So, let me put $\Delta\beta$ that is $\beta_1 - \beta_2$. It is given that angle or the difference is 15° ; so, I can calculate what will be my β_2 .

$$\begin{aligned}\Delta\beta &= \beta_1 - \beta_2 \\ \therefore \beta_2 &= \beta_1 - \Delta\beta = 63^\circ - 15^\circ = 48^\circ\end{aligned}$$

Now, once my β_2 is known; my β_1 , it is known to me. I can calculate what will be my speed; that's what is say my relative velocity at the entry. It is...

$$\begin{aligned}V_1 &= \frac{U}{\sin \beta_1} \\ &= \frac{78.54}{\sin(63^\circ)} \\ &= 88.14 \text{ m/s}\end{aligned}$$

you can say by using velocity triangle. So that is giving me my relative velocity to be 88.14 m/s . Now, we are looking for our exit condition because we want to calculate what will be my relative velocity at the exit and what will be my whirl component. So, in order to do that calculation, from our velocity triangle and from geometry, we can write down,

$$\begin{aligned}\frac{U}{C_a} &= \tan \alpha_2 + \tan \beta_2 \\ \therefore \alpha_2 &= \tan^{-1} \left(\frac{U}{C_a} - \tan \beta_2 \right)\end{aligned}$$

Since, by β_2 is known to me, I can calculate what is by α_2 . So, if I am putting these numbers, it says my α_2 is coming 40.44° .

$$\begin{aligned}\therefore \alpha_2 &= \tan^{-1} \left(\frac{78.54}{40} - 1.963 \right) \\ \therefore \alpha_2 &= 40.44^\circ\end{aligned}$$

Once my α_2 is known to me, I can calculate what is my tangential speed or tangential velocity. So, C_{w2} that is given by,

$$C_{w2} = C_a \tan \alpha_2$$

So, if I am putting those numbers, my axial velocity is known to me; it is 40 m/s . And α_2 we have calculated 40.44° ; this is giving me my whirl component or say whirl velocity at the exit of rotor as 34.1 m/s .

$$\therefore C_{w2} = 40 \tan(40.44^\circ)$$

$$\therefore C_{w2} = 34.1\text{ m/s}$$

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Tutorial Contd.

From Exit velocity triangle,

Relative velocity at exit
 $V_2 = (U - C_{w2}) / \sin \beta_2 = (78.54 - 34.1) / \sin(48^\circ)$
 $V_2 = 59.8\text{ m/s}$

Static pressure rise $= (\Delta P)_r = \frac{1}{2} \rho (V_1^2 - V_2^2)$

Assuming density $\rho = 1.225\text{ kg/m}^3$

$$\Delta P_{\text{rotor}} = \frac{1}{2} \times 1.225 \times (88.14^2 - 59.8^2)$$

$$\Delta P_{\text{rotor}} = 2568\text{ Pa}$$

Specific Power $= UC_{w2} = 78.54 \times 40.44$
 Specific Power $= 3176.16\text{ J/kg}$

We know:
 $C_a = 40\text{ m/s}$
 $U = 78.54\text{ m/s}$
 $V_1 = 88.14\text{ m/s}$
 $C_{w2} = 34.1\text{ m/s}$
 $\beta_2 = 48^\circ$

Calculate:
 $V_2 = ?$
 $\Delta P_{\text{rotor}} = ?$

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Now, what we are looking for is at the exit, we are looking for our relative velocity at the exit. So, based on my velocity triangle, I can write down my V_2 , that's what is given by U minus C_{w2} divided by $\sin \beta_2$.

$$V_2 = \frac{U - C_{w2}}{\sin \beta_2}$$

$$\therefore V_2 = \frac{78.54 - 34.1}{\sin(48^\circ)}$$

$$\therefore V_2 = 59.8\text{ m/s}$$

If I am putting these numbers, it says my relative velocity that is coming 59.8 m/s . So, now in order to calculate what is my static pressure rise in the rotor, I will say my ΔP is given by

$$(\Delta P)_r = \frac{1}{2} \rho (V_1^2 - V_2^2)$$

So, if I am putting this number, I am looking for my density. Let me assume since density is not given, I can safely assume density as 1.225 kg/m^3 .

$$\therefore (\Delta P)_{\text{rotor}} = \frac{1}{2} \times 1.225 \times (88.14^2 - 59.8^2)$$

So, this is what will be giving me my ΔP across the rotor; means static pressure rise across the rotor. So, this it is coming 2568 Pa or 2.56 kPa.

$$\therefore (\Delta P)_{\text{rotor}} = 2568 \text{ Pa}$$

We are looking for specific power; this specific power it is given by

$$\text{Specific Power} = UC_{w2} = 78.54 \times 40.4$$

$$\therefore \text{Specific Power} = 3176.16 \text{ J/kg}$$

So, my peripheral speed is known to me, my C_{w2} is known to me. If I will be putting this number, it says my specific power that is coming 3176.16 J/kg ; be careful about the units.

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Tutorial Contd.

2) With inlet guide vanes and axial flow discharge

Static pressure rise (rotor) $(\Delta P)_r = \frac{1}{2} \rho (V_2^2 - V_3^2)$

Static pressure rise (stator) $(\Delta P)_s = \frac{1}{2} \rho (C_1^2 - C_2^2)$

Specific Power $= UC_{w2}$

From exit velocity triangle,

$$\tan \beta_3 = \frac{U}{C_a}$$

$$= \frac{78.54}{40} = 1.963$$

$$\beta_3 = 63^\circ$$

Given:
 $C_a = 40 \text{ m/s}$,
 $N = 1500 \text{ rpm}$,
 For rotor,
 $\Delta\beta = 15^\circ$,
 $\alpha_2 = -25^\circ$,
 $U = 78.54 \text{ m/s}$

Calculate:
 $\beta_3 = ?$

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Now, we have our second case in which we have our inlet guide vanes, in which my flow is entering inside the rotor at some angle α_2 . That's what is given, say it is -25° . And here if we are looking for we want to calculate our static pressure rise in rotor, we are looking for our static pressure rise in the stator or say inlet guide vane. That's what is given by for my rotor;

$$\text{Static Pressure Rise (Rotor)} = (\Delta P)_r = \frac{1}{2} \rho (V_2^2 - V_3^2)$$

and for stator, it is

$$\text{Static Pressure Rise (Stator)} = (\Delta P)_s = \frac{1}{2} \rho (C_1^2 - C_2^2)$$

And my specific power if I am looking for, it is

$$\text{Specific Power} = UC_{w2}$$

So, here in this case, we are looking for calculation of my relative velocity components V_2 , V_3 . We are interested in what is my absolute velocity C_1 , C_2 and we are also interested in what is my C_{w2} . Now, let us start from the exit velocity triangle. We can straightway write down my β_3 , that's what is a function of U and C_a . So, since my exit it is axial one, we can say

$$\tan \beta_3 = \frac{U}{C_a}$$

If I am putting this numbers, it says my

$$\beta_3 = 63^\circ$$

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Tutorial Contd.

We know $\Delta\beta = \beta_2 - \beta_3 = 15^\circ$
 $\beta_2 = 63^\circ + 15^\circ = 78^\circ$

Tangential velocity at rotor entry
 $C_{w2} = C_a \tan \alpha_2 = 40 \tan(-25^\circ)$
 $C_{w2} = -18.65 \text{ m/s}$

Relative velocity at rotor entry
 $V_2 = (U - C_{w2}) / \sin \beta_2$
 $= (78.54 - (-18.65)) / \sin(78^\circ)$
 $V_2 = 99.36 \text{ m/s}$

Relative velocity at rotor exit $\Rightarrow V_3 = U / \sin \beta_3$
 $= 78.54 / \sin(63^\circ)$
 $V_3 = 88.14 \text{ m/s}$

Given:
 $C_a = 40 \text{ m/s}$,
 $N = 1500 \text{ RPM}$
 For rotor,
 $\Delta\beta_{\text{rotor}} = 15^\circ$
 $\alpha_2 = -25^\circ$
 $U = 78.54 \text{ m/s}$

Now we know:
 $\beta_3 = 63^\circ$

Calculate:
 $\beta_2 = ?$
 $V_2 = ?$
 $V_3 = ?$

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Now, what is known to us? We know my $\Delta\beta$; it is given say 15° . Since, we have calculated our β_3 , so, we can calculate what is our β_2 angle and that β_2 angle is coming 78° .

$$\Delta\beta = \beta_2 - \beta_3 = 15^\circ$$

$$\therefore \beta_2 = 63^\circ + 15^\circ = 78^\circ$$

Now, once this is what is known to me, based on my velocity triangle, we will try to calculate what will be my C_{w2} . So, this C_{w2} we can write down its

$$C_{w2} = C_a \tan \alpha_2 = 40 \tan(-25^\circ)$$

this α_2 value, that's what is given to us, it is -25° . So, we can calculate our whirl component, it is -18.65 .

$$\therefore C_{w2} = -18.65 \text{ m/s}$$

Remember the thing where we are putting our inlet guide vanes; that is what will be deciding the magnitude of my whirl component. We can calculate our relative velocity based on the velocity triangle at the entry of my rotor. It says relative velocity V_2 ; it is

$$V_2 = \frac{U - C_{w2}}{\sin \beta_2}$$

I know what is my U , I know what is C_{w2} and what is my β_2 ; so, based on that, we can do our calculation for relative velocity.

$$\therefore V_2 = \frac{78.54 - (-18.65)}{\sin(78^\circ)}$$

$$\therefore V_2 = 99.36 \text{ m/s}$$

This relative velocity is coming 99.36 m/s . If you compare with the earlier number when we are having only rotor, then this relative velocity coming to be lower.

Now, since we have provided with our inlet guide vane, this relative velocity is going to increase. Now, we are looking for what will be our exit relative velocity from this rotor; that's what is given by

$$\begin{aligned} \text{Relative velocity at rotor exit} \Rightarrow V_3 &= \frac{U}{\sin \beta_3} \\ &= \frac{78.54}{\sin(63^\circ)} \\ &= 88.14 \text{ m/s} \end{aligned}$$

So, if I am putting that numbers, that's what is known to us; we will be getting our relative velocity to be 88.14 m/s.

(Refer Slide Time: 16:55)

Tutorial Contd.

Static Pressure rise in rotor

$$\Delta P_{rotor} = \frac{1}{2} \rho (V_2^2 - V_3^2)$$

$$\Delta P_{rotor} = \frac{1}{2} \times 1.225 \times (99.36^2 - 88.14^2)$$

$$\Delta P_{rotor} = 1289 Pa$$

Absolute velocity at rotor entry

$$C_2 = C_a / \cos \alpha_2 = 40 / \cos(-25^\circ)$$

$$\Rightarrow C_2 = 44.14 m/s$$

Static Pressure rise in stator

$$\Delta P_{stator} = \frac{1}{2} \rho (C_1^2 - C_2^2)$$

$$\Delta P_{stator} = \frac{1}{2} \times 1.225 \times (40^2 - 44.14^2)$$

$$\Delta P_{stator} = -213 Pa$$

We know:

$C_a = 40 m/s$
 $\alpha_2 = -25^\circ$
 $U = 78.54 m/s$
 $V_1 = 88.14 m/s$
 $C_{w2} = 34.1 m/s$
 $\beta_2 = 48^\circ$
 $C_1 = C_a = 40 m/s$

Calculate:

$\Delta P_{rotor} = ?$
 $C_2 = ?$
 $\Delta P_{stator} = ?$

So, once this is what is known to us in sense of relative velocity at the entry of my rotor, and at the exit of my rotor, we can calculate what will be the static pressure rise in the say rotor. So, that is given by

Static Pressure rise in rotor

$$\Delta P_{rotor} = \frac{1}{2} \rho (V_2^2 - V_3^2)$$

Since, my V_2 is known to me, we have already calculated; and my V_3 is also known to me, we can calculate what is my ΔP_0 ; that's what is coming 1289 pascal.

$$\therefore \Delta P_{rotor} = \frac{1}{2} \times 1.225 \times (99.36^2 - 88.14^2)$$

$$\therefore \Delta P_{rotor} = 1289 Pa$$

Now, what we know? We know what is our absolute velocity at the rotor entry; that's what is given by

Absolute Velocity at rotor entry,

$$C_2 = \frac{C_a}{\cos \alpha_2} = \frac{40}{\cos(-25^\circ)}$$

$$\Rightarrow C_2 = 44.14 \text{ m/s}$$

That's what will be giving me my absolute velocity as 44.14 m/s. We are interested in calculating what will be the static pressure rise in our stator. If I am putting this number, it says

Static Pressure rise in Stator

$$\Delta P_{stator} = \frac{1}{2} \rho (C_1^2 - C_2^2)$$

$$\Delta P_{stator} = \frac{1}{2} \times 1.225 \times (40^2 - 44.14^2)$$

$$\therefore \Delta P_{stator} = -213 \text{ Pa}$$

So, if we are putting our C_1 and C_2 , it says my static pressure rise is coming -213 Pa.

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Tutorial Contd.

Net pressure rise = $\Delta P_{rotor} + \Delta P_{stator}$
 = 1289 - 213
 = 1076 Pa

Specific Power = $U(C_{w3} - C_{w2})$
 = $78.54 \times (0 - (-18.65))$
 = 1464.77 J/kg

We know:
 $\Delta P_{rotor} = 1289 \text{ Pa}$
 $\Delta P_{stator} = -213 \text{ Pa}$
 $C_{w2} = -18.65 \text{ m/s}$
 $C_{w3} = 0 \text{ m/s}$

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Now, since we have our total pressure rise in rotor; we have pressure rise in stator. We can calculate what will be my net pressure rise. So, this is what is coming 1076 Pa.

$$\text{Net Pressure Rise} = \Delta P_{rotor} + \Delta P_{stator}$$

$$= 1289 - 213$$

$$= 1076 \text{ Pa}$$

We are interested what is our specific power; that's what is

$$\begin{aligned}\text{Specific Power} &= U(C_{w3} - C_{w2}) \\ &= 78.54 \times (0 - (-18.65)) \\ &= 1464.77 \text{ J/kg}\end{aligned}$$

So, if we are putting these numbers, it says my specific power is coming 1464.77 J/kg. So, basically for solving this kind of numericals, you need to make first the sketch - rough diagram, what instructions is given. Then, after you have your understanding of velocity triangle; look at what angles are given; based on that you construct your velocity triangle.

Once this velocity triangles are known to you, you just put what all velocities are known to you. If we have those velocities known based on our trigonometry, we can calculate the velocity component or say angle components. Once we are calculating our velocity components, if we are calculating relative velocity components; that is what will be helpful for calculating my static pressure rise in rotor. If we are calculating absolute velocity component that is what will be helpful to us for calculating static pressure rise in my stator.

And whirl component, that's what is helpful to us for calculation of our specific power. So, I am sure, this is what will be giving you idea in sense of how to do this calculation for different configurations. We have five different configurations which are possible. You can do practice based on your understanding, and build your confidence. Thank you! Thank you very much for this session! See you in the next lecture.