Introduction to CFD
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Lecture - 59
Basics of Turbulence Modeling (continued)

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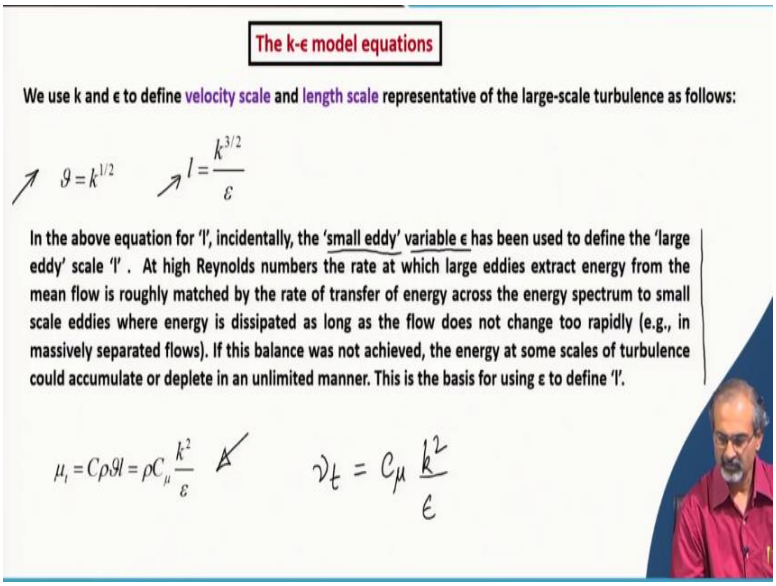

In this lecture we continue our discussion on the k epsilon model. We had discussed about the conservation equations for turbulent kinetic energy for the mean flow as well as for the fluctuations. In a similar manner, we could derive the exact equation for the dissipation rate which we are not going to do very elaborately here because of paucity of time. But nevertheless it can be done.

But it is more interesting for us now to get into the k epsilon model itself instead of pursuing the exact equations. So, we would look at k and epsilon to define velocity scale and length scale, which are representative of the large scale turbulence. These are the two equations we are proposing on more of dimensional basis. And we would like to note the point here that in this equation for l.

Remember that l is the length scale associated with the larger eddies. However, in this equation for l we have the dissipation which is associated with the small eddies. We have talked about this fact that dissipation occurs at the smallest scales. So, what is the basis on which we are introducing that when we are defining the length scale for a larger eddy.

So, the point is that at high Reynolds numbers, the rate at which the larger eddies extract energy from the mean flow, it is roughly matched by the rate of transfer of energy across the energy spectrum to small scale eddies where energy is dissipated as long as the flow does not change too rapidly. That means there is some kind of an equilibrium. If this balance was not achieved, the energy at some scales of turbulence could accumulate or deplete in an unlimited manner.

So, this is the basis for using epsilon when we define the length scale for the larger entities. And then we have the eddy viscosity defined in terms of k and epsilon. So, alternatively you can also write it as C mu k square by epsilon.

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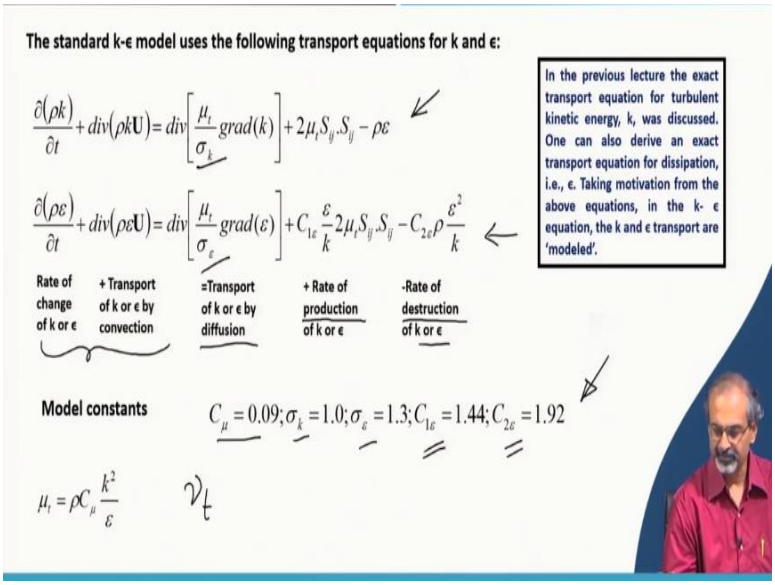

Now, let us have a look at the standard k epsilon model which is going to be used for simulating the transport of k and epsilon. So, you may recall that the equation for turbulent kinetic energy in its exact form was containing many more terms than what you have here in the model equation here. So, some of the effects have been simplified and modeled. We are not going to do it in an exact manner.

Broadly both the equations on the left hand side contain the rate of change and the transport through convection. And then on the right hand side, you have the source terms where you are talking about transport by diffusion. And then you have the production of k or epsilon and the destruction of k or epsilon. And there are certain model constants which are used which are stated here.

So, C mu is used in the mu t or nu t expression. Sigma k here figures here in the k epsilon sigma epsilon here and the other two coefficients figure in the epsilon equation as source term coefficients. So, let us try to explore a little further on these equations. As we do that let us use a slightly simplified nomenclature.

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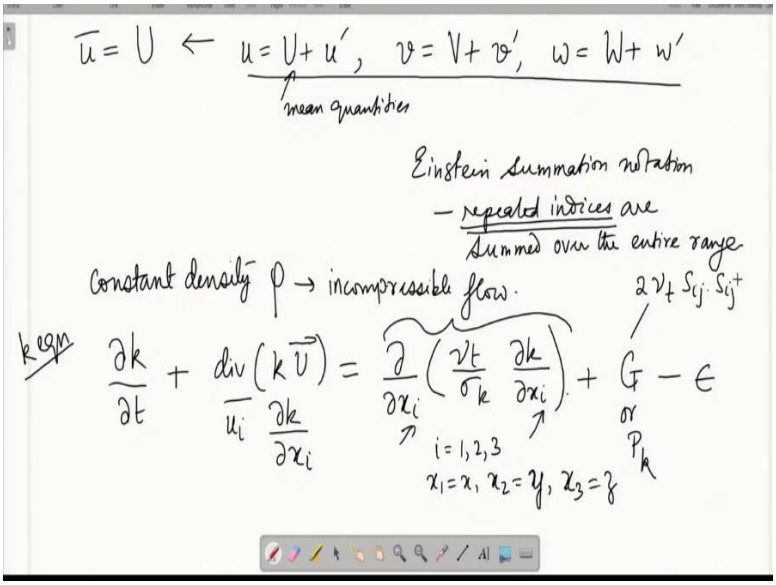

Because it will be more convenient for us to express it this way that we just put u bar to represent capital U remember that u is capital U plus u dash, v is capital V plus v dash, w is capital W plus w dash that was the nomenclature we were using now. And we are just using the overbar to represent the mean quantities. That is one. The other thing is that we are going to use the index notation which makes the representation of the equations simpler.

So, in the index notation we often use the Einstein summation notation where you will see that repeated indices wherever they occur are summed over the entire range. So, we understand this aspect further as we write the equations. So, if we follow these steps and we also consider that we are talking about a constant density case since, we are talking about incompressible flow then the representations become simpler.

And you can then write the k equation of the k epsilon model either this way or you can prefer to write it this way. So, we are just showing alternative forms. So, we said repeated indices. So, you can understand that here you have a repeat of index i. So, basically i in a three dimensional flow will go from 1 to 3 that means x 1 will mean x, x 2 will mean y, x 3 will mean z and so on.

So, you will have del del x del del y and del del z all summed up through a common expression like this. So, that is the convenience of the index notation. Then you have the production of k which is often represented as G or in some books also as P k, production of k and then minus epsilon. So, this is your k equation. And then we recall that G is 2 nu t S ij scalar product of the rate of deformation tensor.

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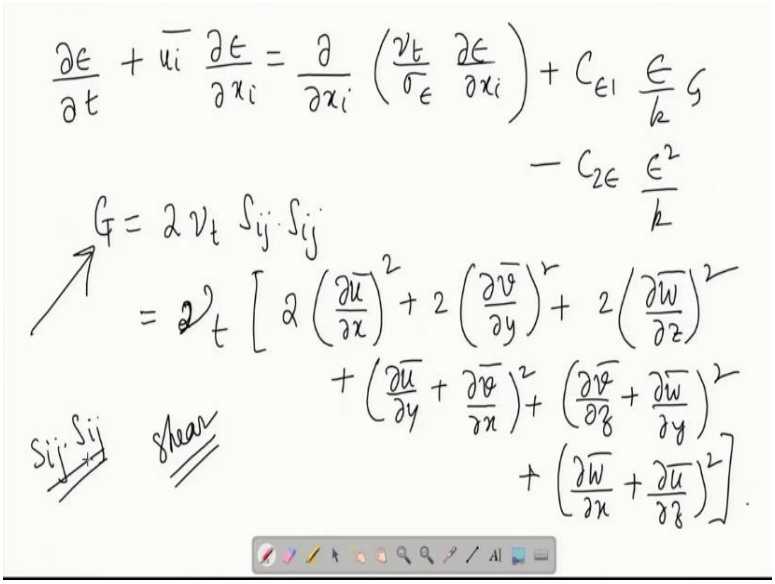

On a similar approach, you can write the epsilon equation. This is how the equations look like. And then the G term is a very important term for us. So, as we said that it is 2 nu t S ij scalar product and then that will turn out to be nu t times. So, as you can see that the production of turbulent kinetic energy involves a whole lot of velocity gradients related to the mean flow.

These overbar quantities are all related to the mean flow which means as long as you have large gradients available in the mean flow there will be a large production of turbulent kinetic energy. And that happens when you have shear existing in the flow. That is why we had emphasized that in order for turbulence to exist and thrive you need to have shear in the flow. So, this expression of course, we had done briefly earlier.

So, you can also do it as a homework problem doing the scalar product of these two tensors. So, now, what we have is we have the Reynolds averaged Navier Stokes equations. And additionally we have continuity equation and the two transport equations k and epsilon. So, these equations have to be solved as a family in order to compute the turbulent flow.

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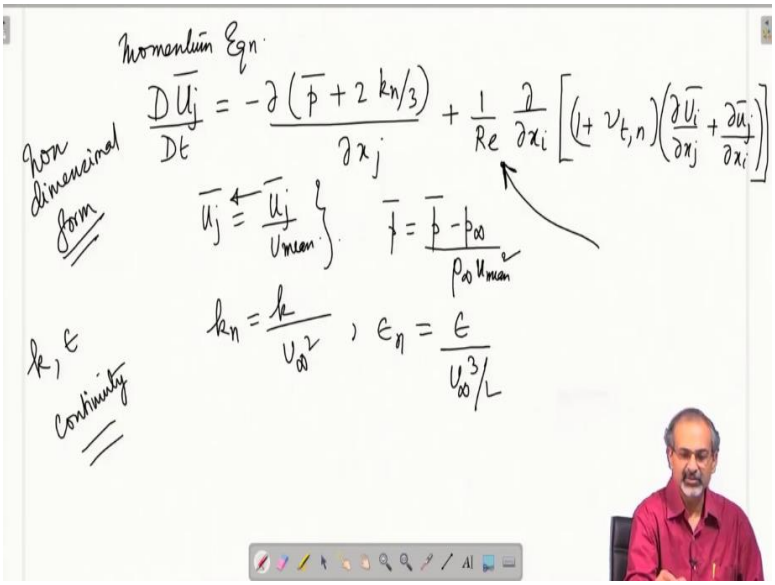

So, if you look at the momentum equation it will look like this. So, let us use this. We are going to use normalized coordinates or other variables here so that we can come up with non dimensional form of the equations. So, what we have basically done over here is that here we are calling u j as basically u j by the reference velocity or mean velocity which is existing. So, the u j bar is now replaced by a normalization or non dimensionalization.

Similarly for v and w, when it comes to p bar the earlier value of p bar has been replaced by a non dimensionalization which works like this. Similarly, for turbulent kinetic energy and epsilon you can do these non dimensionalizations and express the equations in non dimensional form. The advantages that then you could simulate for different Reynolds numbers in order to show the effect of Reynolds number on the turbulent properties.

So, we can do this non dimensionalization for the k equation, the epsilon equation and the continuity equation. And then we solve for the system. But then of course, before you solve you have to also set up the boundary conditions. So, let us briefly discuss about boundary conditions. So, let us set a simple problem for us ourselves say it is flow through a channel.

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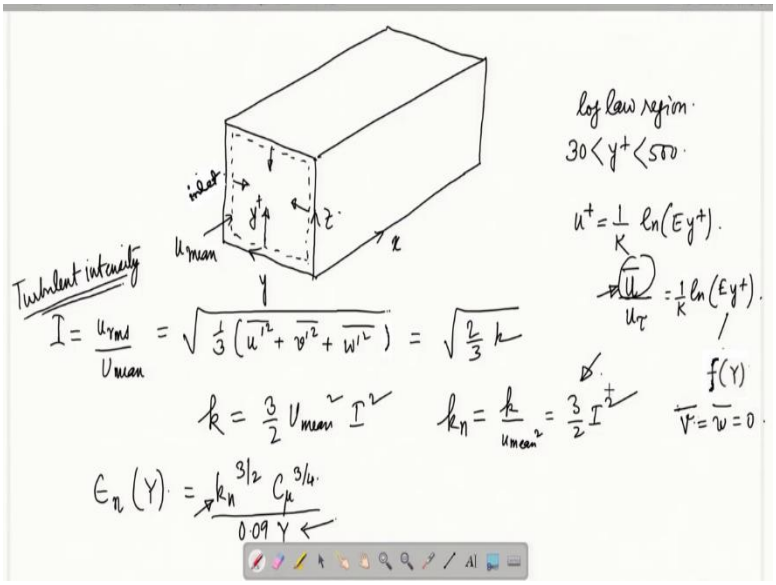

So, turbulence is inherently three dimensional. So, let us try to set up the problem in a three dimensional manner like this. So, this is essentially a channel with walls all around. So, computationally when you simulate it you try to set the boundary cells first of all in order to set the mostly boundary conditions along that dotted surface. And then you try to simulate a flow through this.

So, you have a mean velocity which is approaching the channel. And then the moment it reaches the channel, it will be constrained by the walls of the channel. And then there will be a turbulent flow developing them. So, when you try to solve for a problem of this kind, you define turbulent intensity at this boundary through which the flow enters the domain. So, it is basically the inflow boundary.

So, turbulent intensity is nothing but the rms velocity say u rms by the U mean and u rms is nothing but this. So that can be easily shown to be equal to two third of the turbulent kinetic energy and therefore, it is easy to show that k works out to be 3 by 2 U mean square I square. So, when it comes to the non dimensional k it is k by U mean square and therefore 3 by 2 I square.

So, turbulent intensity if it is on the lower side, it could be really low, it could be well below 1 percent. But in a highly turbulent scenario it may be well into 5, 10 percent turbulent intensity so that if you fix it to a certain value depending on the nature of turbulence that you want to feed in the inlet. Accordingly the k gets defined. You can define epsilon also in a normalized or non dimensional manner as a function of distance from the wall.

So, one possible way of doing it is using a relation of this kind. So, depends both on turbulent kinetic energy at the inflow, the coefficient C mu and the distance from the wall. Again, distance from the wall is normal to the wall. You have different walls and therefore, different normal directions away from the wall. Again, you have the non dimensional y which is the y plus which is a very important parameter which you have to keep in mind.

So, we have talked about k epsilon we also have to talk about what velocities you set at the inlet. So, if you imagine that the first point that you set from the wall is having a distance y in such a manner that you are already into the log law layer. That means you remember we have discussed earlier that you need to be in this range in order to be in that log law region.

So, in that case, you can conveniently use the relation in order to set your u because u plus is nothing but u by some u tau. And if this tau is again a non dimensional tau, then you will get the u in a non dimensional manner. And the y plus on the right hand side of course, is dependent on the actual y away from the wall. So, we will briefly show how to compute the y plus, but before that we just recall.

So this is how you define u and remember that at the inlet you can set the v and ah w both of them to 0. Of course, as the flow gets into the channel, the v's and w's will become nonzero because of the boundary layer formation and so on. Now, a very brief idea about how to compute the y plus. So, let us say the first grid point away from the wall that you are locating is at a distance of y p.

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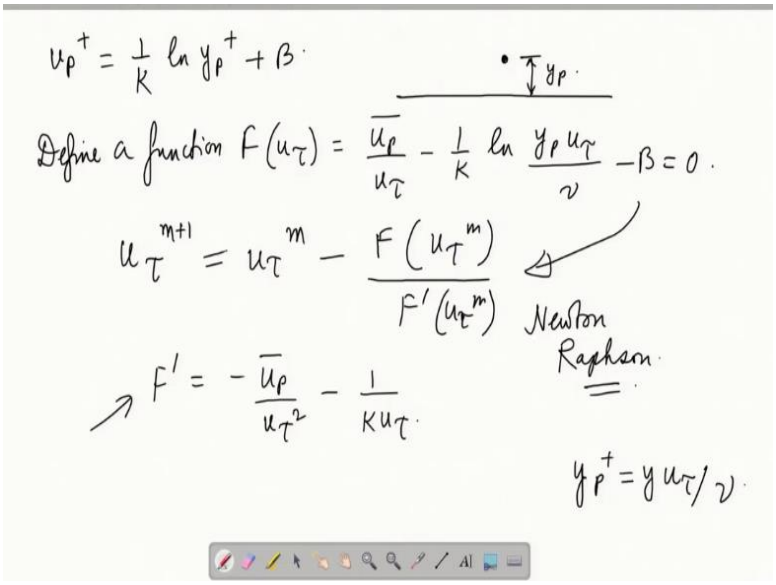

Then you can use this log law equation and then define a function say capital F which is a function of u tau. And then you use the Newton Raphson method in order to refine the value of u tau for which you also have to have the derivative of the function and m is essentially the iteration level of finding the root or defining the root. Note that F dash is. So, if you feed these expressions into this equation you can refine the value of u tau till it converges.

Once it converges, you can feed it back to the y plus equation and check whether you are actually within the log law layer or not, if not, you have to tune the distance y p to make it greater or lesser accordingly so that you finally fit into that range. So, this is essentially the way we locate the first grid point so that it is within the log layer.

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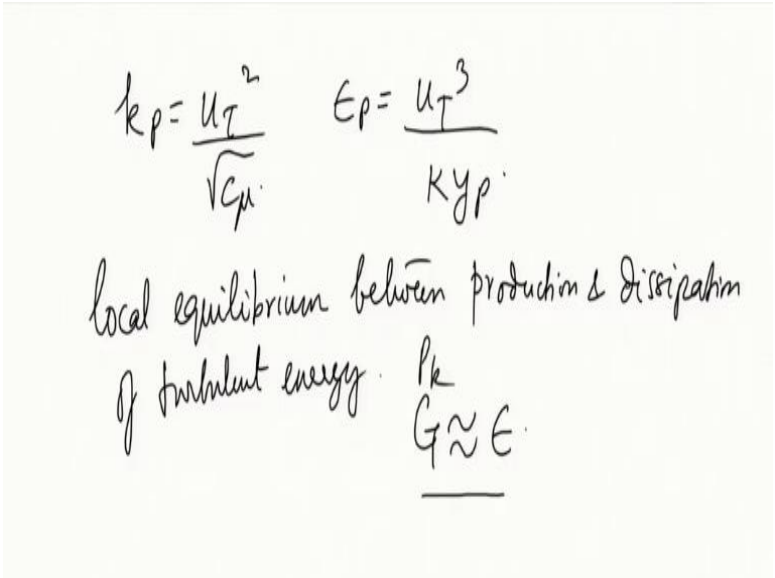

Once we have done that in terms of that u tau also you can compute the k and epsilon which is based on the concept of local equilibrium between production and dissipation of turbulent energy which means, G is approximately equal to epsilon. G or P k whatever you call it is approximately equal to epsilon. So, that is when they are in equilibrium approximately. Again this is good time to aah relook at the structure of the RANS equations that we are using.

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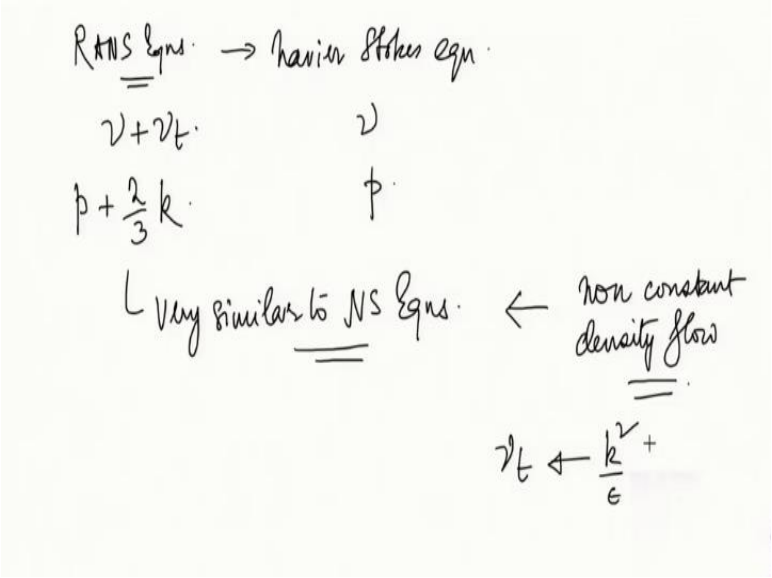

So, the RANS equations when you compare them with Navier Stokes equations you will find primarily the difference lies between Navier Stokes using nu and RANS using nu plus nu t. That is the primary difference. The other more subtle difference lies in pressure alone over here and pressure plus two third turbulent kinetic energy over here. Those are the only two things which make them different.

So, broadly speaking RANS equations are very similar to the Navier Stokes equations structurally. And which is of course, quite obvious, because you have averaged the Navier Stokes equations to come to that. Now, that basically means that the strategies that we use for solving incompressible Navier Stokes equations also strategies apply in solving the RANS equations. So, that is very important.

The main thing is that we solve for as do a non constant density flow, but again not in a compressible sense. The density changes essentially due to the effect of turbulence because of augmented stresses due to Reynolds stress effects. So, after solving the RANS and the continuity equations, we solve for the k and epsilon equations so that we can assess the nu t, the turbulent the eddy viscosity.

So, that updated viscosity is again fed back to the RANS equations and then this computation goes over and over. As you do that you can realize that because RANS is being solved in the beginning. So, what will happen is, you have a nu t term in the RANS equation also which is dependent on k and epsilon. And you have not solved the k and epsilon equation at that time step.

So, you are not aware of the updated value, but you can always use the previously available value. And therefore, the previously available value helps you to assess the nu t and that way the computations can be made sequential. So, when it comes to time stepping the k epsilon equations. Again their structure is very similar to general transport equation.

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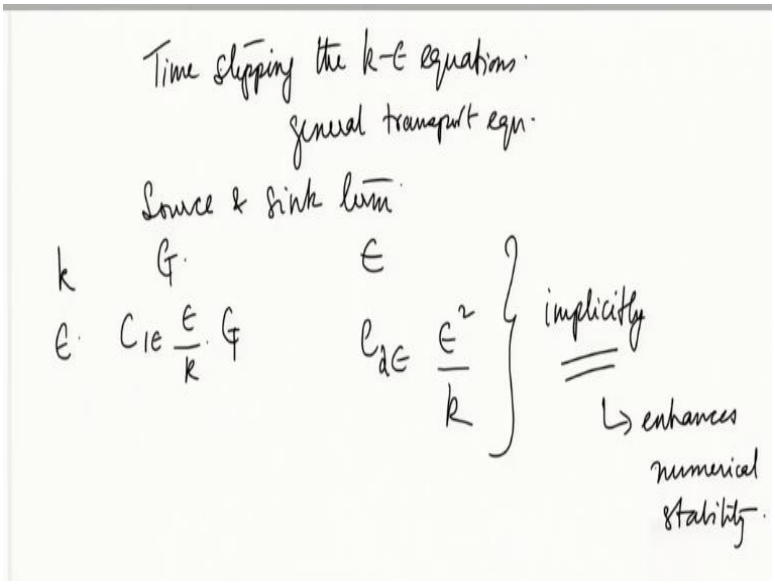

So, the different discretization techniques that we have learned earlier should generally fit in. For example, you have terms related to advection, diffusion, which we have dealt with very elaborately in our module on advection diffusion equations. So, those terms can very well fit in into the left hand side and the right hand side of the k epsilon equations. But the newer ones are the source and sink terms.

Source in the sense say the G or C 1 epsilon epsilon by k G. This is for the k and the epsilon equation. And what are the sink terms? That means the depletion terms. These are the depletion terms, right. So, when it comes to the sink terms, it is reported in many words on k and epsilon that it is better to treat these sink terms implicitly rather than explicitly because that enhances numerical stability.

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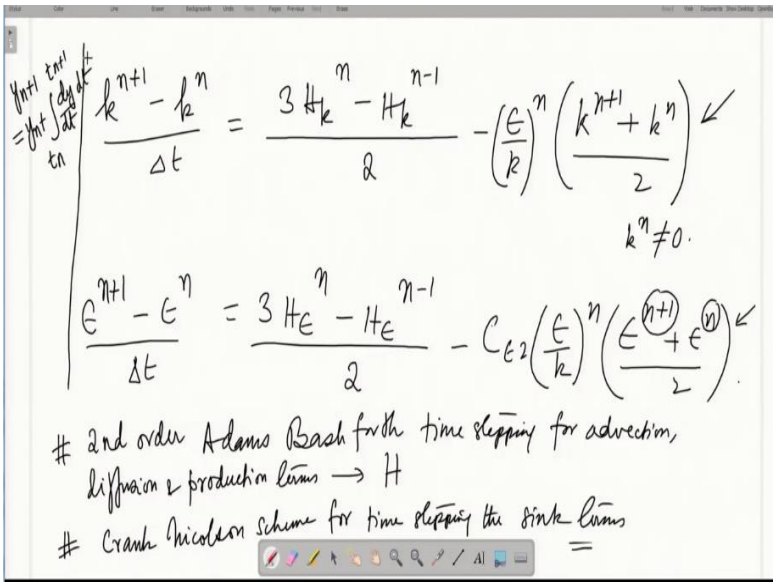

So, keeping that in mind, we will show a typical time stepping strategy. So, here the type of discretization we are using is it is a second order Adams Bashforth time stepping for advection, diffusion and production terms which are all clubbed in the terms named as H. So, advection, diffusion, production, all are clubbed into the H terms. Alright, what is left alone are the sink terms.

And we are using Crank Nicolson scheme for time stepping the sink terms. As we recall from our previous study that Crank Nicolson is a semi implicit scheme that enhances the stability. So, you are invoking from both n and n + 1th time steps that will enhance the stability. Remember that if you go back to your maths courses, where you have learned about initial value problem where you are doing a calculation of this kind.

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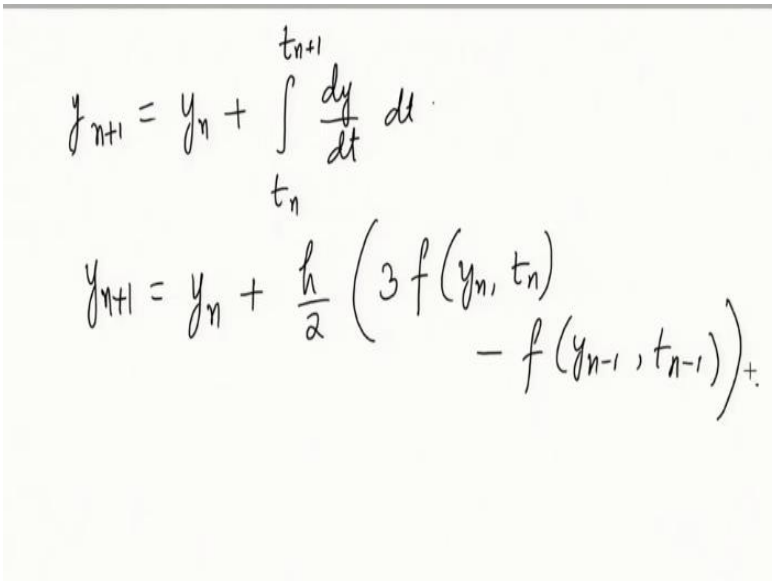

So, you are solving for y n + 1. Let us write it little more clearly here. So, Adams Bashforth does it using a formulation of this kind. So, with this we end our discussion on k epsilon model equations. We will discuss about LES and DNS in our next lecture. Thank you.