Introduction to CFD
Prof. Arnab Roy
Department of Aerospace Engineering
Indian Institute of Technology - Kharagpur

Lecture - 58
Basics of Turbulence Modeling (continued)

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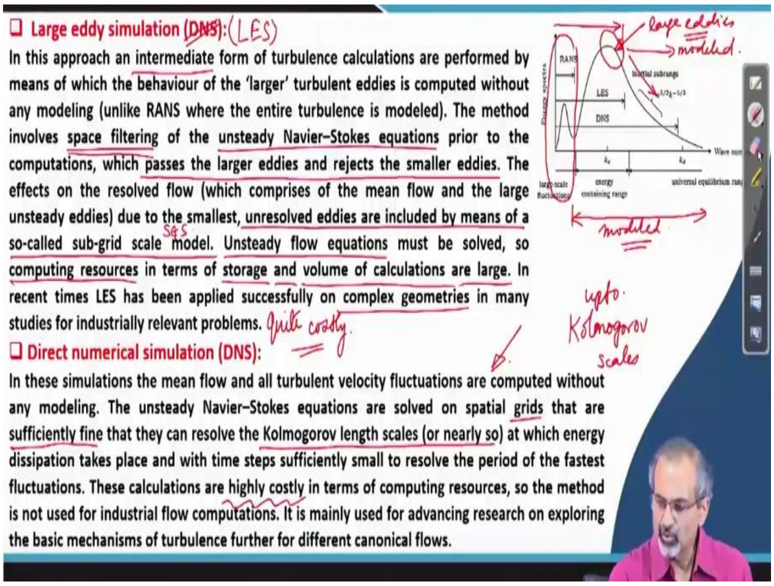

In this lecture, we continue our discussion on the different turbulence models. So, in the previous lecture, we had ended with a brief discussion on RANS based turbulence models. Today we begin the lecture with another two approaches by which turbulent flow fields are often computed. So, one approach happens to be the large eddy simulation. I am sorry, this is this should be L E S. And in this approach, we take an intermediate stand.

So, the intermediate stand is if you remember what we did in the RANS approach, we had covered up to the large scale fluctuations and all the rest of the eddies were modeled. This is essentially the approach followed in RANS based calculations. Why it happens is that the moment you do a Reynolds averaging that means you do a time averaging all the turbulence gets accumulated in the form of Reynolds stresses which have to be modeled in some manner.

And then you no longer tend to compute them directly using your numerical methods. You model them all. In that case, whatever large scale fluctuations are there in the mean flow itself below the turbulence effects are only captured while the turbulence is entirely modeled.

While there are approaches which can go further into a portion of the turbulence or the entire turbulence and try to capture it directly.

So, large eddy simulation tries to capture a portion of the turbulent activity directly and model a part of the rest. So, in this diagram for example, you have shown that up to a certain range large eddy simulation tends to capture. So, what is that range? It includes large scale fluctuations which are non turbulent while also large scale fluctuations which are turbulent.

So, this region is already into the turbulent region where you see the large eddies which are extracting energy from the mean flow and putting it into the energy cascade. So, largely eddy simulation captures up to that and what it does not capture is beyond that, that means it is modeled beyond that. So, the intermediate and the smaller scales are modeled. So, that is the basis of larger dissimulation.

And where does it cut off? It cuts off at a setting which is basically decided by a so called filter. So, large eddy simulation uses some kind of filter based on which it cuts off up to which range of wave numbers it is going to directly capture. Beyond that what happens is it models these unresolved eddies by means of a so called sub grid scale model. That means there are a whole lot of eddies lying beyond that which have to be modeled which is modeled through a sub grid scale model, often referred as S G S model in brief.

So, we talked about a space wise filtering of unsteady Navier Stokes equations. And this filter would pass the larger eddies and will filter off or reject the smaller eddies. And in large eddy simulation by default, you have to do unsteady flow calculations, because you are capturing not only the large turbulent eddies, but also say large scale structures, which are non turbulent. Therefore, inherently it has to be unsteady.

And then usually it will take up much more computing resources, both in terms of storage and volume of calculations which you perform. And in recent times, it has been successfully applied on a lot of complex geometries which are industrially relevant, but it still remains quite costly. So, until, unless, it is absolutely essential, such simulations are not routinely done in industry.

There is another approach by means of which you apply no modeling but capture all the length scales existing in the problem stretching right from the large scale fluctuations which are essentially laminar and beyond that all which is turbulent starting from the largest turbulent eddies to the smallest ones, which span up to the Kolmogorov level. So, obviously, it is extremely expensive to do such computations.

And also the numerical schemes that you apply there have to have sufficient accuracy to capture the entire range of scales. So, the scale separation problem has to be correctly addressed by the numerical scheme. Also you have to provide extremely fine mesh so that the numerical scheme can do a good job in capturing up to the Kolmogorov length scales on nearly Kolmogorov length scales.

And obviously, these will be highly costly and not worth applying in industrial irrelevant problems, but extensively applied in research mainly to explore further on the different basic mechanisms of turbulence. So, we have broadly seen more state of the art approaches of turbulence capture, and we cannot call them entirely modeling because, as far as direct numerical simulation is concerned, there is absolutely no modeling involved. There is a partial modeling involved in the large eddy simulation model.

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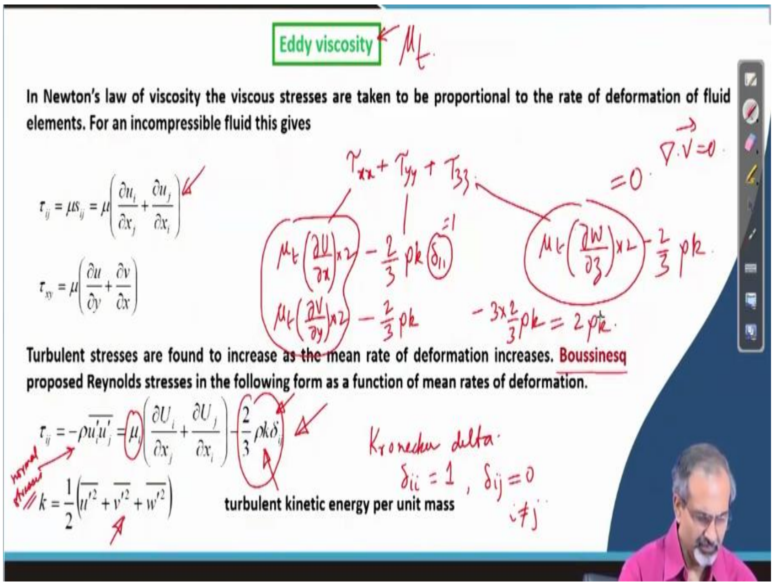

We come up with a very important concept now of eddy viscosity, which will be used further in the RANS based approach which we are going to discuss a little further in the subsequent slides. So, the eddy viscosity comes up with the Newton's law of viscosity where we see that the viscous stresses are proportional to the rate of deformation of the fluid elements, which is expressed through this equation.

And then Boussinesq proposed that turbulent stresses can also have a similar behavior. That means they also show this kind of functional dependence. And then came up with an alternative definition. But, as you can see, first of all, there is a change here. It is no longer the molecular viscosity. But with a suffix t, t standing for turbulent and that is what is called as the eddy viscosity because, it is connected with turbulent.

And therefore, the turbulent eddies. So it is an augmented viscosity, which is an effect of the turbulent fluctuations. Additionally, we have a term over here, which involves the turbulent kinetic energy which is expressed in terms of the velocity fluctuation squares summed up over the three directions and then the Kronecker delta.

So, as we all know that the presence of the Kronecker delta will mean that if the two indexes indices match, then in the Kronecker delta expression it will effectively become i i. And then with matched indices it will have a value of 1 and when i not equal to j, then this will be equal to 0. And therefore, as long as these are normal stresses which means the indices match you will have the contribution of this second part of this equation.

While for the shear stresses, you will have no contribution coming from the turbulent kinetic energy part. Now, we will we can do a simple exercise in order to show the validity of this that if you were to sum up all the normal stresses, then it will account for the tau xx tau yy tau zz. And what will happen is that for each one of these, let us say tau xx it will produce into 2 two times because even the second term will produce the same effect inside that bracket.

And then you will have minus two third rho k and delta ii. So, it is 1 1 here and therefore it is equal to 1. So, tau xx will produce this, tau yy will produce twice of this and tau zz will produce. So, since we are handling incompressible flow, therefore, if you sum up these three set of terms, it will give you a zero from divergence of v = 0. While the rest of the terms when you sum them up, so, it is three times two third rho k, which is two times rho k. So, this is what is going to come up.

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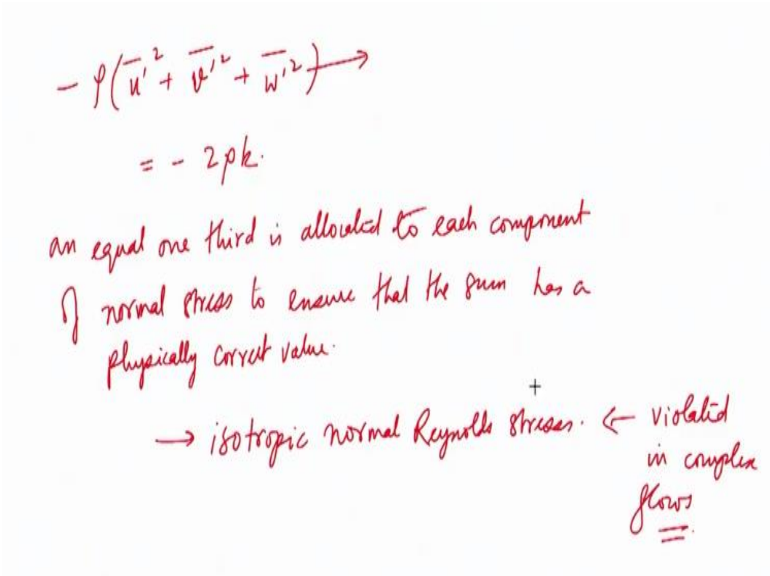

What we can further write is that the sum of the normal stresses. So, that is going to be equal to minus two rho k. And what happens essentially is an equal one third is allocated to each component of normal stress to ensure that the sum has a physically correct value. Now, this has a very important physical implication. So, this is essentially assuming isotropic normal Reynolds stresses.

That means as though each one of these stresses are isotropic. Now, that is very often not the case in complex flows. And therefore, the prediction accuracy based on these assumptions will degrade substantially. So, this is one of the major assumptions made but again a major fallacy in this concept which may add to inaccuracies. But nevertheless, this gives a very effective way of defining and augmented viscosity coefficient, which is of major use in the RANS based approaches.

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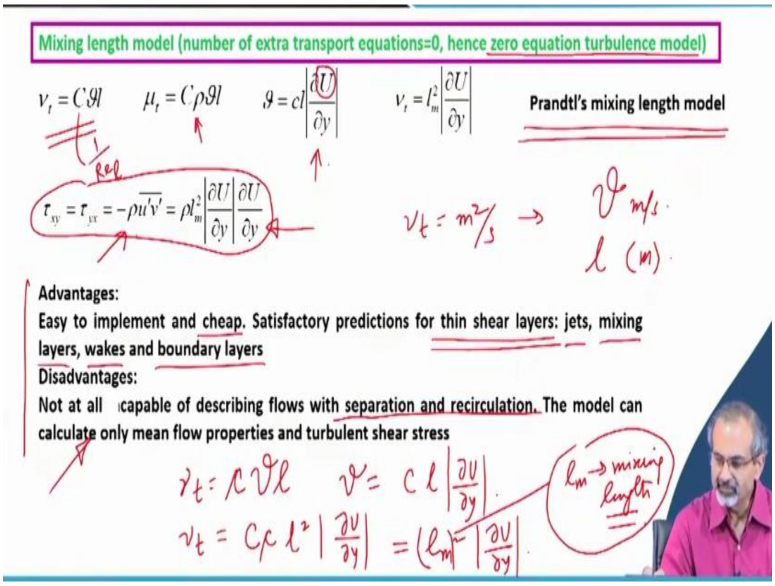

Now, before we go into discussing more advanced RANS based approaches, we actually begin our discussion with a very simple turbulence model which is often called as the mixing length model, which is essentially inspired by the Prandtl's mixing length model or the mixing length hypothesis.

So, in this approach, we make certain assumptions based on dimensional grounds that the kinematic viscosity, which has the dimensions of meter square per second can be expressed as a product of a turbulent velocity let us say capital V which has dimensions of meter per second and a turbulent length scale let us say l it has a dimension of meter. So, purely based on a dimensional argument, you can actually come up with a expression like this.

Now, that essentially means that this coefficient C is a reciprocal of R e l, if you look back at the expression of R e l. And then obviously mu t comes from there by multiplying it by the rho the density. And if we accept this hypothesis that there is a strong connection between the mean flow and the behavior of the largest eddies, then we actually can attempt to connect the characteristic velocity scales of the eddies with the mean flow properties through an expression of this kind.

So, this is essentially the mean flow influence. And then comes, the subsequent exercise that means trying to get a definition for nu t. So, if you want to do that you first write nu t equal to C times v times l and then further v is equal to C times l times del u del y and then nu t is nothing but C into the large C and the small C and then l square del u del y. And that can be represented as l m square times del u del y.

So, this l m is nothing but the mixing length. So, now, when it comes to the turbulent stresses, you already have a definition for nu t based on which you can actually have an expression for turbulent stresses. Now, this gives a very simplified approach by which you can actually compute the turbulent stresses in terms of the mixing behavior of the eddies. And it needs no additional transport equations in the form of partial differential equations.

So, therefore, it is often called as a zero equation turbulence model. So, there are no extra transport equations to solve here and therefore, zero equation. Now, there are obvious advantages in terms of simplicity and cheapness of the calculations. And there are satisfactory predictions in many thin shear layers involving jets, mixing layers, wakes and boundary layers. So, all these flows we have briefly discussed in our previous lectures.

So, you can be happy that a very simple model of this kind, predicting Reynolds stresses with simple equations involving velocity gradient and mixing length using a form like this can predict the turbulent nature of the flow. However, there are obvious disadvantages that because it is a very simplified model. Ah, if you have more complicated transport in complex flow problems involving separation and recirculation.

It is very difficult to define this mixing length. Because we have not got an ad hoc definition of the mixing length. So, you may actually have to do some kind of experiment based tuning of these mixing lengths for different types of problems. And then it will work as long as you operate within the purview of that problem, but the moment you try to fit it to other problem scenarios, it fails.

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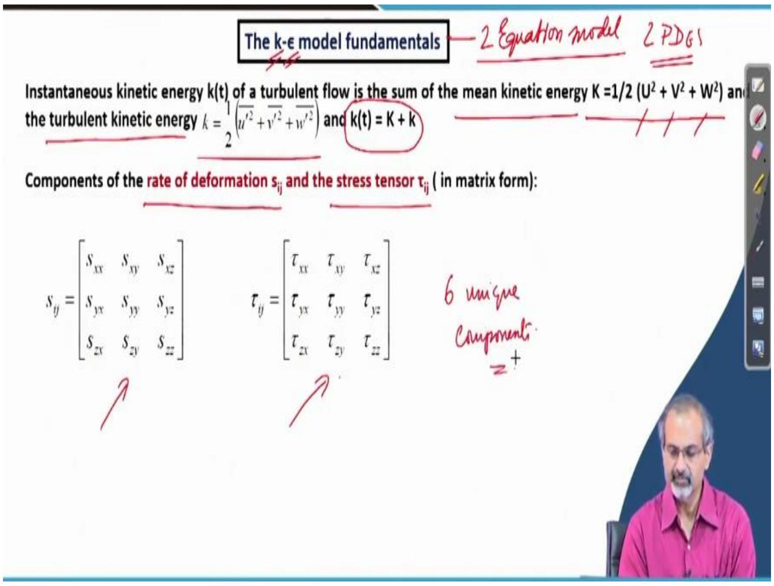

Coming to the k epsilon model, we already said that this is a model which involves two separate partial differential equations one for the turbulent kinetic energy and one for the dissipation, turbulent energy dissipation. Therefore, it is a two equation model which means, apart from the Reynolds averaged Navier Stokes equations continuity equation you are also solving 2 transport equations 2 partial differential equations.

Now, this kinetic energy the instantaneous part of the kinetic energy is comprised of the contribution coming from the mean kinetic energy that is the mean flow field indicated through the upper case values of U V W and the turbulent kinetic energy which resides in the fluctuations. And k t is essentially a sum of the two components. Now, to take the concepts further we need to relook at the rate of deformation and the stress tensor.

So, both are tensors which are represented in this manner. And as discussed even in the context of Reynolds stresses, we said that there are actually 6 unique components. So, if you look at the rate of deformation tensor, the different components look like this. So, these are the normal components; these are the shear components. And again as you can see the mean part and the fluctuating part have been segregated.

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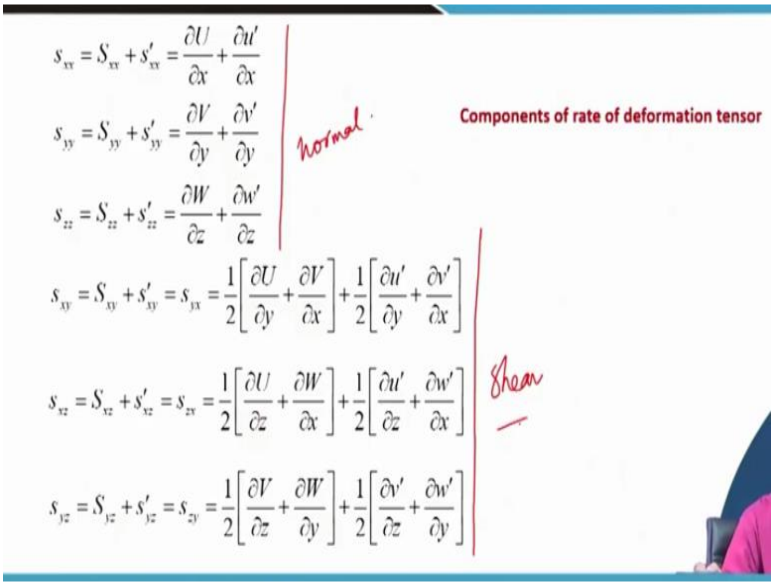

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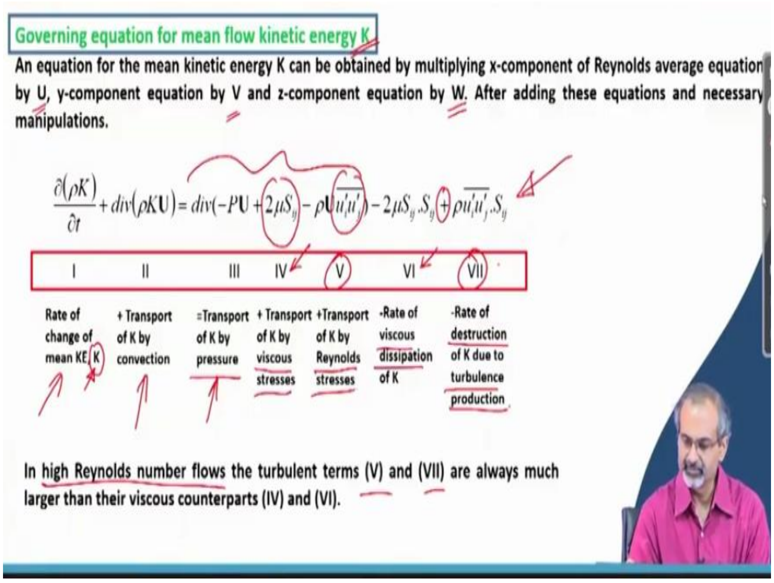

Let us look at the governing equation for mean flow kinetic energy k. So, this equation can be derived by multiplying the x component of Reynolds averaged equation by the mean velocity U, the y component Reynolds averaged equation by V and the z component Reynolds averaged equation by W. All are capital UV's and W's, because you are dealing with the kinetic energy of the mean flow. After adding these equations and doing some necessary manipulations, the equation finally comes up like this.

So, this is essentially the transport equation for mean flow kinetic energy. What does it comprise of? It comprises of seven terms in all. So, the left hand terms they comprise of rate of change of mean kinetic energy. That is time rate of change of mean kinetic energy and we are referring to mean kinetic energy by a capital K as already mentioned. The second term involves transport through convection.

The terms on the right hand side a collection of them are under the divergence. So, transport by means of pressure, by viscous stresses and by Reynolds stresses. Keep in mind that this involves mu times S ij and this involves fluctuations. Then comes the dissipation of destruction, one is viscous dissipation the other is the turbulence production and consequent destruction.

So, we should not be carried away by the plus sign here it essentially acts as a negative term eroding kinetic energy from the mean flow and supplying it to the turbulent fluctuations. Usually, in high Reynolds number flows, the turbulent terms which are five and seven. These would be dominating much above the viscous terms which are four and six.

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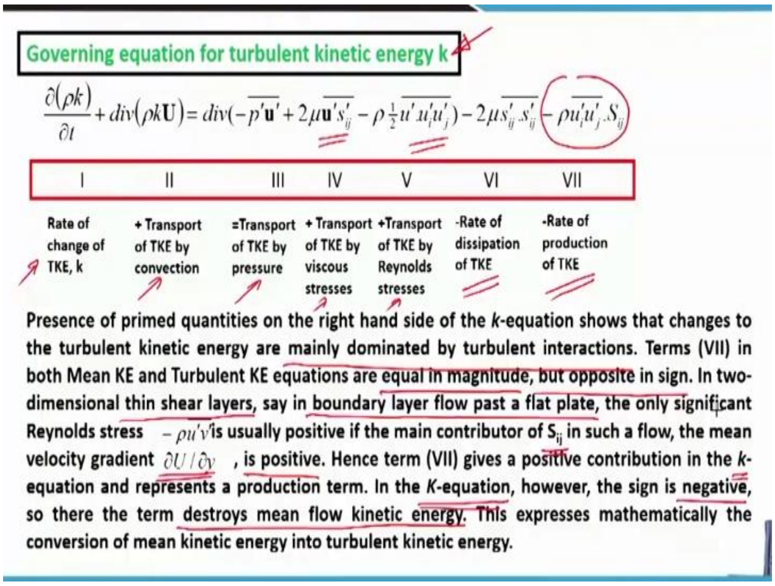

Like we have a governing equation for the mean kinetic energy field, we also have a governing equation for the turbulent kinetic energy small k. Here similarly, you have rate of change of turbulent kinetic energy and its transport through convection, pressure, viscous stresses and Reynolds stresses similar to what you had earlier.

But increasing presence of fluctuations which are taking care of the transport, which is obvious because we are dealing with a transport equation in turbulent kinetic energy and then the dissipation and production of turbulent kinetic energy. Now, if you remember carefully the sign has changed over here. Now, if you look carefully at this equation, the presence of prime quantities on the right hand side, in this equation shows that changes to the turbulent kinetic energy are mainly dominated by turbulent interaction that means the fluctuations.

Now, the last term in both mean kinetic and turbulent kinetic energy equations are equal in magnitude but opposite in sign. So, if we refer to a simpler problem that is two dimensional thin shear layers, like say, a boundary layer on a flat plate. So, the only significant Reynolds stress usually is which contributes to S ij is the mean velocity gradient del U del y which is positive.

And therefore, this the term seven gives a positive contribution in the k equation and represents a production term. Remember, it is the small k equation. While in the capital K equation, the sign is negative. And so there the term destroys mean flow kinetic energy. Therefore, it is eroding energy from the mean flow and it is adding energy to the turbulent flow the turbulent fluctuations.

So, that is what these two terms seven are doing in this equation. It may be a good time here to spend a little bit on the dissipation of turbulent kinetic energy which comes through the epsilon term. Now, the viscous dissipation is essentially the term six. Now, this is a tensor product the scalar product of two tensors S dash ij. Now, if you had two tensors let us say a ij and b ij and you are doing a scalar product.

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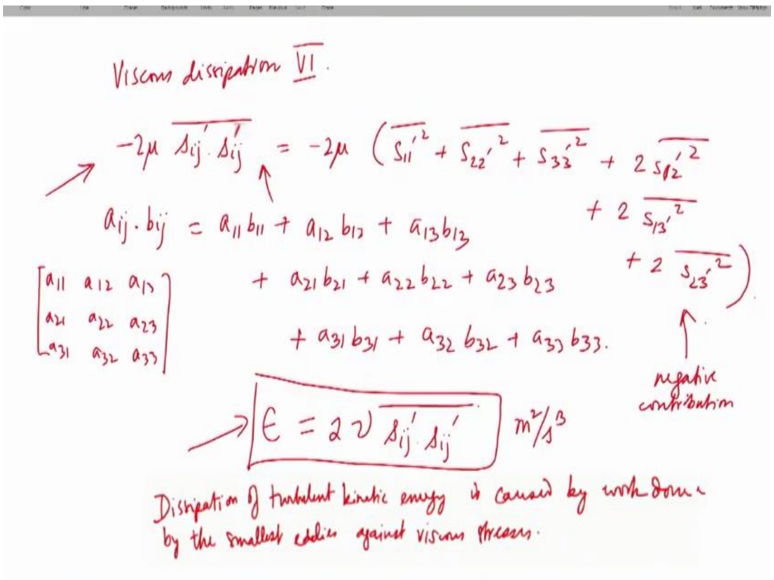

And let us write down a ij as say a one one, a one two, a one three and similar structure can be written for b ij. Then the scalar product of the two tensors would be written in this manner.

This is how we write it. Now, how is it important for us? Because it fits in here so that we get an expression for this term. This is how it fits in. Now, we can show that epsilon is essentially 2 nu times this scalar product.

Remember that because you are having square terms in this equation, they are always going to sum up to a negative value. So, this is a negative contribution. So, dissipation of turbulent kinetic energy is caused by work done by the smallest eddies against viscous stresses. And the rate of dissipation per unit volume, which is this, is normally written as the product of the density rho and the rate of dissipation of turbulent kinetic energy per unit mass epsilon.

And therefore, we have an expression for epsilon that way. Dimensions of epsilon are meters cube by second or rather meter square by second cube. And it is a destruction term epsilon is essentially a destruction term in a turbulent kinetic energy equation. And it has to be of a similar order of magnitude as the production term so that there is a balance between production of turbulent kinetic energy and its destruction.

And this is broadly how it happens in high Reynolds number flows. So, we will discuss the k epsilon turbulence model further in the next lecture. Thank you.