Introduction to CFD
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Lecture - 56
Basics of Turbulence Modeling (continued)

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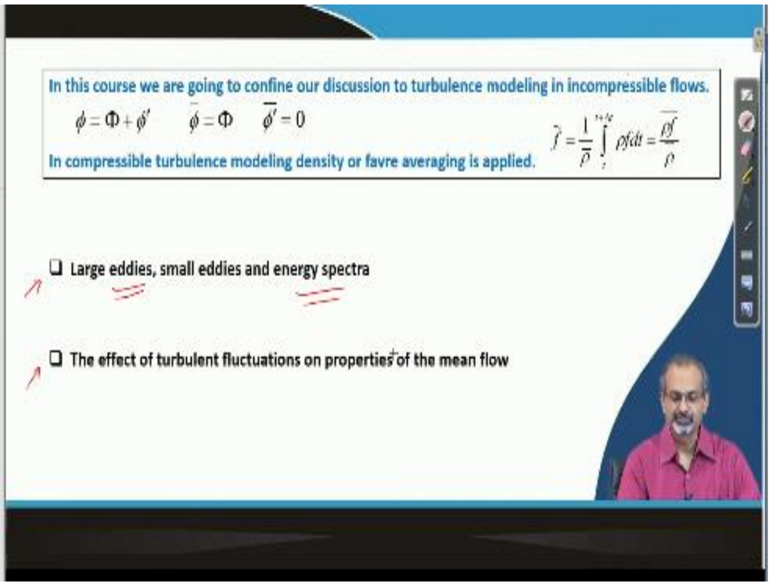

We continue our discussion on basics of turbulence modeling in this lecture. Last time, we recall having discussed about typical turbulent flow field signal which has a lot of unsteadiness and we try to indicate it with unsteady component. And we also indicated that there would be a statistical mean. And when you sum them up, you essentially get the time variation of a flow variable which we in general indicators 5.

So, the time averaging gave us the mean value and then we saw that the time average of the fluctuation went to 0. We would like to emphasize at this point that we are essentially discussing turbulence modeling from the perspective of incompressible flows. And as long as we do that this is the way we do the averaging process.

However, if you were to discuss about compressible turbulence modelling where density fluctuations are also very, very important to be accounted for, then we talk about favour averaging which is essentially taking care of the density variations. So, of every averaged quantity would be indicated like this with a tilde. So, that is essentially 1 by the Reynolds average density times the time integration of density times the variable f integrated over that time interval.

So, that is what gives you a Favre averaging of the variable f. So, we wanted to refer to this briefly. So that if you are talking about turbulence modeling in the context of compressible flows. This is how you would actually have to pursue the averaging process. However, in our lectures here, we are going to focus on incompressible flows. In the previous lecture, we also discussed quite a bit about the statistical measures of a typical turbulent flow field.

So, we are going to just recall on what all we tried to cover at the time. So, we talked about the time averaging of the variable phi and different measures of the spread of the fluctuations phi dash in the form of variance and root mean square quantities. We also talked about moments of different fluctuating variables. And then we talked about the correlation functions both in time and space in the form of autocorrelation and cross-correlation.

So, we said that there is going to be a strong correlation between fluctuations if we are talking about very close vicinity points in the form of spatial distancing or even in the form of temporal distance. Now, today, we begin our discussion with another few very important aspects of turbulence. We were talking about large scale and small scale vortex structures which are often referred as eddies and very important concept of the energy spectrum.

And then we will also have a brief look at effect of the turbulent fluctuations on properties of the mean flow.

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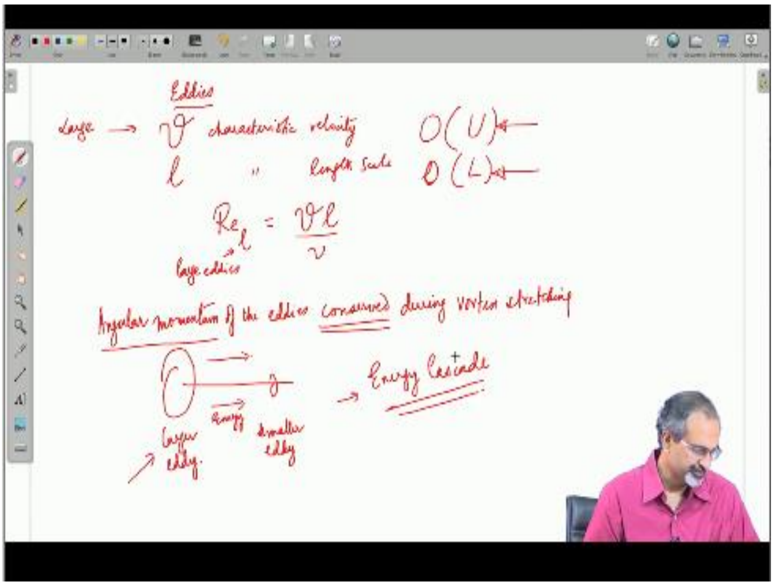

So, let us briefly discussed about discuss about the eddy concept and how it is significant in the study of turbulent flows. So, when you talk about the large ones, you would like to define a characteristic velocity associated with the large ones. And again a characteristic length scale and the characteristic velocity would be of the same order of the velocity scale of the flow problem itself.

And the characteristic length scale on the larger eddies they would be of the same order as the length scale of the mean flow field which means that if you define a Reynolds number associated with these large eddies with a subscript l standing for the large eddies. Then this Reynolds number will be defined this way which will also be large. And would be of the order of the Reynolds number corresponding to the mean flow velocity and the length scale of the problem itself.

However, there could be a Reynolds number of a very, very different order if you go to the small scales. At the large scales, a very important concept is that the angular momentum of the eddies would be conserved during the vortex stretching. And what happens is as a consequence as the eddies rotate as they move and they get stretched. They may get narrowed down.

That means an idea of this scale when stretched could become smaller, much smaller in terms of its characteristic dimension. And doing this stretching exercise the angular momentum remains concept that is what we are trying to convey. And then as they get stressed, you essentially have the larger ones forming the smaller ones. This is how the length scales would undergo transformation.

Again as the length gets smaller, do you do this exercise also energy percolates from the larger ones into the smaller ones. And what essentially happens is that there is a closer connection of the mean flow with the larger radius. Because the larger eddies trapped energy from the mean flow in order to undergo the stretching exercise. And lot of the energy percolates to the small eddies.

But the smaller eddies are more closely connected with the larger ones rather than directly to the mean flow. Now, this energy transfer happens through a so-called energy cascade. So, this transmission from the larger to the smaller scales happen through an energy cascade.

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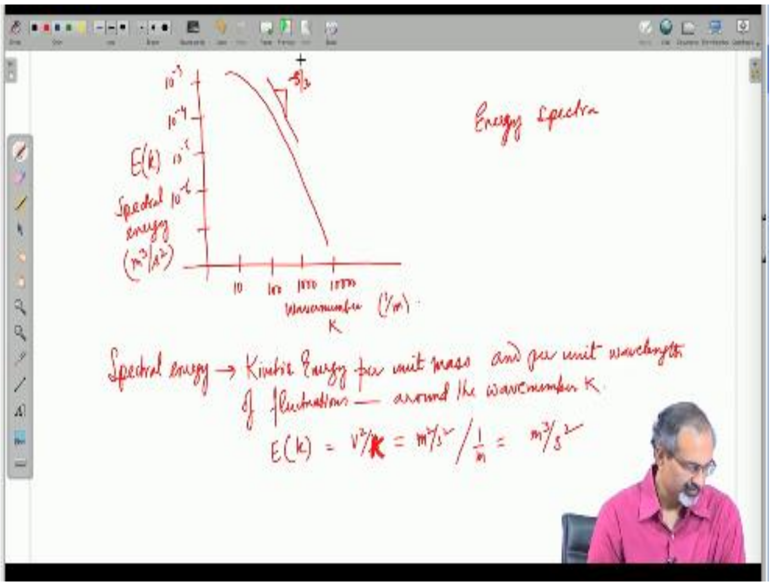

And if you were to look for a diagrammatical representation of that then what you have is the so-called energy spectrum. We often represented as E k that is nothing but spectral energy and dimensional happens to be meter cube per seconds square. And we essentially have a log, log scale here. Along the x axis, we have wave number, dimension of which is per meter.

And a formal definition of spectral energy would be as follows. It is associated with the kinetic energy. So, it is kinetic energy per unit mass and per unit wavelength of fluctuations around the wave number which we indicate as K. So, that is how it is defined. So, E k if you are interested to know dimensionally, this is indicated by K. So, it would be like v square by K. So, dimensionally you can look at it this way.

So, that is what gives you a dimension of meter cube per seconds square. Now, having said that the nature of the curve would be somewhat like this. But if you try to find out the slope, we will find that it conforms closely to minus 5 by 3.

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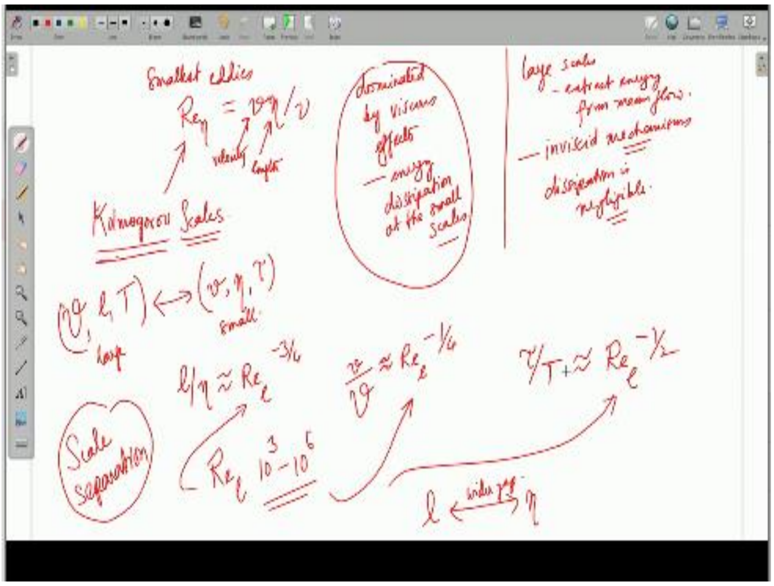

As far as the smallest eddies are concerned, we have a Reynolds number associated with them also. So, we have a small v indicating the velocities associated with the small eddies; eta indicating the length scale which gives you a small eddy Reynolds number. And the smallest eddies are dominated by viscous effects. And there is energy dissipation as a consequence at these scales.

Now, if you recall when we talked about the large scales, we said that large scales extract energy from mean flow and not only that they primarily function through inviscid mechanisms. Because of the large Reynolds numbers associated with them. So, viscous dissipation is not an issue at those scales. So, dissipation is negligible. That is how energy is adequately supplied into the energy cascade at those scales.

It is not dissipated at those scales. It moves down the line into moderately large eddies, smaller eddies and the smallest eddies. And the dissipation essentially happens at the smallest eddies. And because dissipation happens, there is a supply into the energy cascades at the larger scales and dissipation at the smallest scales. And if these two balance each other, then there is no accumulation of energy at any intermediate scales.

And for a large number of problems that is broadly true. However, very rapidly changing flows with huge amount of unsteadiness may sometimes at least locally violate this assumption. Now, if you were to compare the scales between the large and the small, and also try to qualify the smaller ones. Further, we will talk about firstly the Kolmogorov scales which are associated with the smallest eddies.

And also the scale separation between the largest eddies and the smallest eddies. So, the Kolmogorov scales are the ones associated with the smallest eddies. And we can now try to link the large and the small ones in the form of some kind of a mapping. Let us say, the large ones are having velocity scale, length scale, and say, a time scale given by this. And you are trying to compare with the velocity length and the timescale on the small one.

So, this is the nomenclature we might use. And then Kolmogorov showed that these relationships broadly apply in terms of the length scales, in terms of the velocities and expressed as a function of the Reynolds number associated with the large scales. So, you have wonderful scaling laws available here with which you can at least try to get an estimate of the separation of scales between the large and the small eddies.

Now, when you consider the large eddies Reynolds numbers may be significantly large, say of the order of 10 raise to the power of 3 to 10 raise to the power of 6, a large number of industrial irrelevant problems. And therefore, if you fit that into these equations, then you will understand how the length velocities and times separate. So, this is what is called as the scale separation.

As the large eddies Reynolds numbers become larger, there is a widening scale between say the large eddy length scale and the small eddy length scale. There is a wider gap in dimensions.

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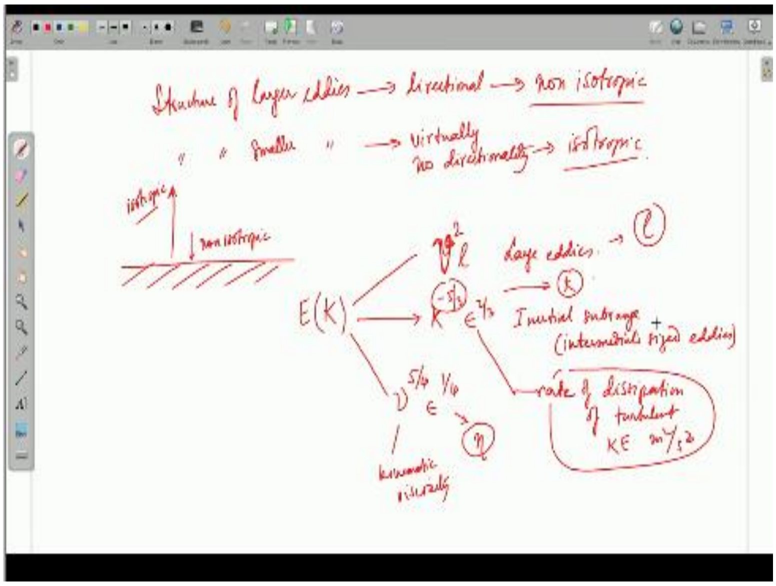

Structure of the large eddies is highly directional. This is what we call as non-isotropic. That means if you look at different directions standing at a certain point in turbulent flow and we are looking at the largest eddies, then you will find that their structures have very rapid variations as you move along different directions. That is what is meant by non-isotropic. While the smaller eddies have virtually no directionality, this is what is called as isotropic.

We, let us see that in many of the wall bounded flows in turbulent regime. Let us say flow past a surface like this. There can be a lot of directionality in the turbulent structures because of the presence of the wall. So, that means the flow structures as you approach the wall can become increasingly non-isotropic. While as you move away from the wall, they may be becoming increasingly isotropic.

So, the behaviour of these eddy structures; not only in terms of their scales but also whether they will be having an isotropic or non-isotropic nature depends on the boundaries of the flow and the flow configuration. Before, we end our discussion on the larger, smaller eddies and energy spectra, we just try to have a perspective in terms of how the energy spectra scales in the different regimes.

So, for the large eddies, the energy broadly varies with V square times l in the intermediate sizes which are often called as inertial sub-range which essentially means the intermediate sized eddies; not too large, not to small. It roughly varies with K the wave number to the power of minus 5 by 3 times epsilon to the power of two third; epsilon stands for rate of dissipation of turbulent kinetic energy and its unit happens to be this meter square per second cube.

And you may recall that the energy spectrum curve had a slope of the order of minus 5 by 3 which is showing up here. And when you come to the smallest eddies, the smallest eddies have energy which scales with Neu which is kinematic viscosity and again epsilon which is the dissipation. So, apart from the largest eddies, you see that the dissipation comes into the picture. That means energy is actually decaying out as it percolates to the different scales.

And what are the length scales of interest in these 3 regions, here it is l; here it is eta; we already know that. And in the intermediate range, it scales with the wave number itself. So, this is how the length scales are associated with the wave number range. So, we had a broad overview about the nature of eddies and their connection with the energy spectrum.

Another very, very important aspect is the effect of turbulent fluctuations on properties of the mean flow. So, if you try to understand what this means.

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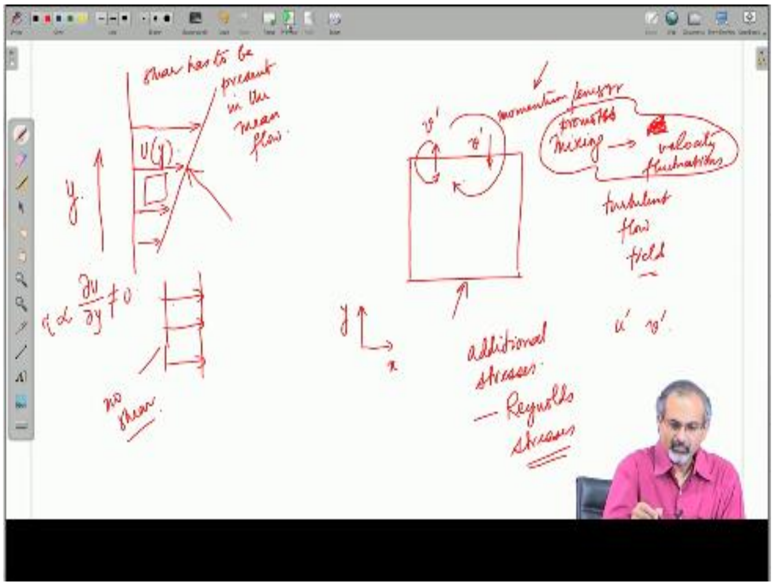

So, in a turbulent flow, we already briefly discussed that there needs to be some shear in the flow in order to sustain turbulence. So, shear has to be present in the mean flow. What does it mean that? Here, for example, as you move in this direct ion if you call it y, then this U component of velocity is a function of y. If you call calculate, d u d y is not equal to zero. That means the flow has shear unlike a flow like this which has a constant value.

This has no shear. Why? Because as we know that shear stress is proportional to this velocity gradient. Now, if you pick up a small element of fluid in that region and try to understand what is going on keeping in mind that this is a turbulent flow field. That means there are fluctuations in the flow in all 3 directions. So, even if we confine it to a 2D situation that means we just take a plane in the three dimensional flow.

Still, we will see the u component of fluctuation and v component a fluctuation. Now, let us say that we are monitoring these fluctuations across the contour of this control volume. Then there may be an eddy which is interacting with this face of the control volume which tends to have v fluctuation pointing downwards. What does it do? It brings along with it momentum or energy with it into the control volume from the upper layers.

Now, if the velocity profile looks like this, from the upper layers it brings in more kinetic energy because the upper layer is at a higher velocity. Therefore, momentum injection takes place at a lower value of y; this way. Again, if there is an eddy which is operating this way. That means then the fluctuation in v is pointing upwards and therefore, momentum is moving out from this control volume into the neighbouring one which means, it may actually slow down the higher velocity which exists on top.

So, this way it promotes mixing. So, what promotes mixing? The velocity fluctuations in a turbulent flow field promotes mixing. And mixing essentially means exchanges in momentum energy also mass. But mass mixing still means that some mass if it moves out of this control volume is compensated by again mass influx from other pahses of the control volume. So, that is how it needs to be balanced.

So, like we showed it in this phase of the control volume, it happens in other phases as well. And thereby, there is a continuous exchange. And this is what brings in additional stresses which we will see later on as defined to be Reynolds stresses which have to be overcome in a turbulent flow field. And therefore, turbulent flow fields usually consume more energy to keep running.

And as long as you keep feeding the shear into the flow, turbulence will be alive. And these fluctuations will be alive and the mixing will be alive. So, this is how the turbulent fluctuations affect the properties of the mean flow. And therefore, in mean flow in whatever manner it begins may actually get modified as turbulent fluctuations keep interacting and then take it to a different equilibrium.

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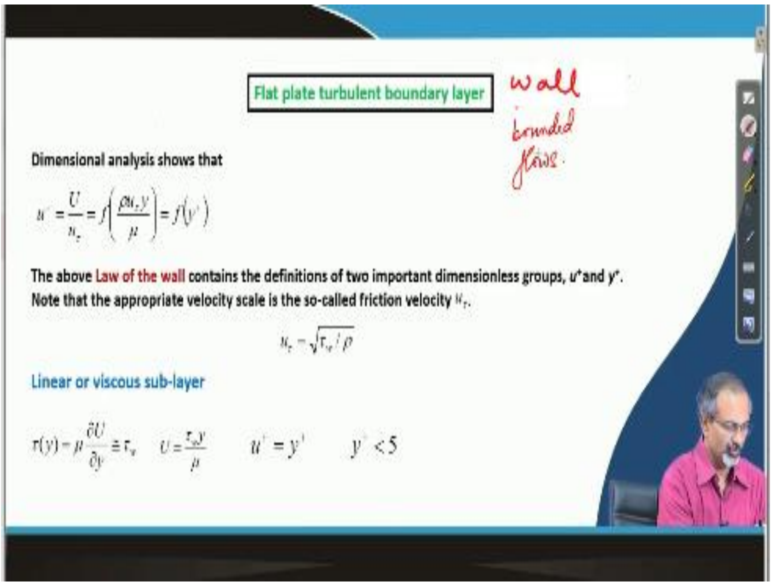

Now, there are different kinds of turbulent flows. As we said before that there could be wall bounded flows. And we can therefore, discuss about flat plate turbulent boundary layer which we will do soon. And in our previous lecture, we also have discussed about how turbulence at all initiates on a flat surface through a sequence of processes. But then apart from wall bounded flows, you can also have flows which are essentially free turbulent flows.

So, let us spend a little bit of time on that category where we can call them as free turbulent flows. We will spend a few minutes on them just to do a brief survey before we come back to the flat plate problem though.

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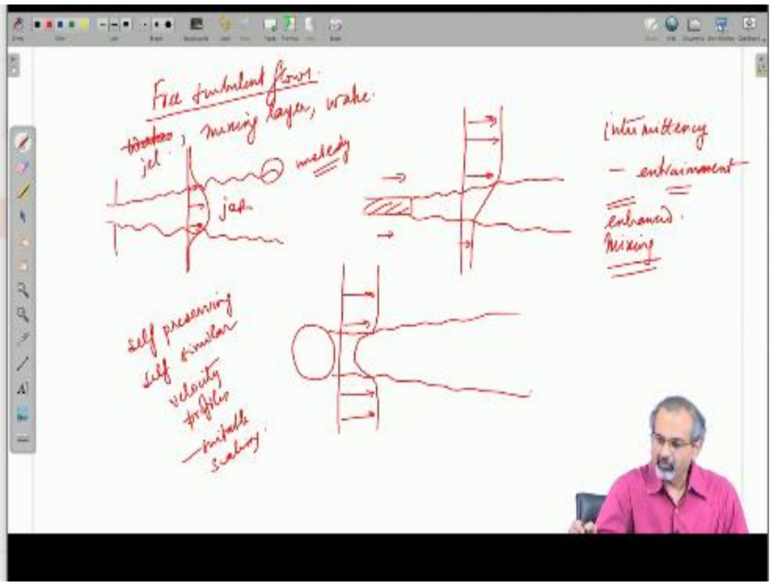

So, just briefly visiting three kinds of free turbulent flows. That means ones which are not influenced by walls or boundaries. So, there could be a jet emanating from an orifice. Sorry. So, this should have been checked the first one. So, that is one category of free turbulent flow. Another one is a mixing layer.

Let us say you have a surface like this and velocities approaching from two sides of the surface which have a jump would lead to formation of an mixing layer across which the velocity changes like this. And there could be a wake problem that I say behind a cylinder. As we know that there is a momentum deficit. And as we move downstream and we try to plot different profiles in different regions will find self similar profiles.

If we apply some kind of self preserving or self similar kind of profiles, velocity profiles if we apply certain suitable scalings. Again, all these flows essentially are turbulent and there is a lot of rich physics which occurs in these flows. Let us say, across the jet boundary, if you look closely into the structures, you will find that these structures often rupture. They are not very stable structures. They are highly unsteady structures.

And of course, they are leading laden with eddies or vortex structures and as the rupture, this happens intermittently. So, there is an intermittency associated with this turbulent activity in the outer region of the jet and with this intermittent bursts. There is also a phenomenon which is called as entrainment which occurred simultaneously. So, these are two interrelated phenomena.

So, entrainment essentially means that exchange of momentum between the jet periphery and the neighbouring stagnant food. And therefore, enhanced mixing between the two. So, we are already talking about enhanced mixing in turbulent flows. So, these are also instances where enhanced mixing occurs. And when you look at wall bounded flows, flat plate boundary layer problem is a very standard one to discuss about.

And when you discuss that, you will find that the boundary layer can essentially be divided into a few sub-regions with different kinds of properties. So, here when we deal with the near wall region, there is a so-called law of the wall which is shown through dimensional analysis that there is a non-dimensional velocity and a non-dimensional distance given by u plus and y plus.

So, the non-dimensional velocity is a function of the non-dimensional distance. And we are connected through a so-called friction velocity which is called as u tau which is defined according to an expression given here which is under root of the shear stress at the wall divided by the density. So, in the next lecture, we will discuss more about these different sub-regions of the turbulent boundary layer on a flat plate. Thank you.