Introduction to CFD
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Lecture - 55
Basics of Turbulence Modeling

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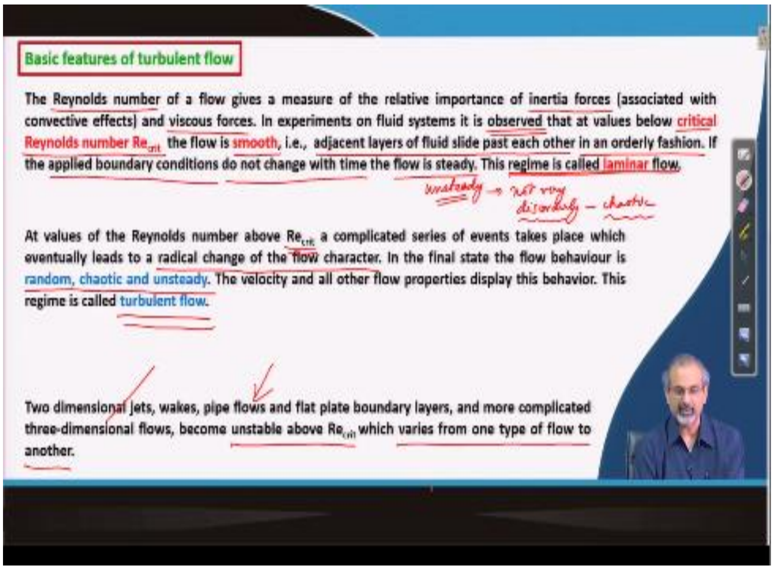

With this lecture, we begin our discussion on basics of turbulence modeling. In this lecture, we will mainly deal with some basic features of turbulent flows. You may have come across these concepts in fluid mechanics courses, may be slightly advanced courses in fluid mechanics. Even, introduction to turbulence is often taught briefly in basic fluid mechanics courses these days.

So, we recall a very, very important non dimensional number, Reynolds number once again which is a measure of the relative importance of inertia forces associated with convective effects and the viscous forces. So, it is essentially a ratio and it is observed through a lot of experimental evidence from a long time. And these days with enough numerical evidence as well that below a critical Reynolds number, the flow usually is smooth adjacent layers of fluid slide past each other in a very orderly fashion.

And if you tend to disturb the flow a little further as long as you are well below this critical Reynolds number, the flow seems to stabilize back again. It seems to dampen out such perturbations and remain orderly. And the applied boundary conditions usually do not change with time and the flow is steady or sometimes unsteady as well but it is never very disorderly. Often in literature, this is said as chaotic.

So, laminar flow is what we have here. So, laminar flow is not disorderly or very less disorderly, it is not very random or chaotic. And it generally gives a very smooth behavior to the flow at values of Reynolds number above this critical Reynolds number a complicated set of events take place which eventually lead to a radical change in the flow character. In that case, the flow becomes random, chaotic, unsteady and that is what is called as turbulent flow.

We come to more concrete definitions of how you quantify turbulent flows but this is how we generally talk about it in terms of few sentences of few words spoken to distinguish a turbulent flow from a laminar flow. Usually, when you look at two dimensional profiles of canonical flows like jets, wakes, pipe flows, flat plate boundary layer, you may have looked at these kind of flows in your fluid mechanics courses or even more complicated three dimensional flows.

Let us say flow past a sphere; flow past a three dimensional body like a car or a chimney, an aircraft and things like that. All such flows can become unstable above a certain critical Reynolds number but then you do not have a universal value for critical Reynolds number for each flow. That means for each flow it would be a different number. So, it is flow dependent as well.

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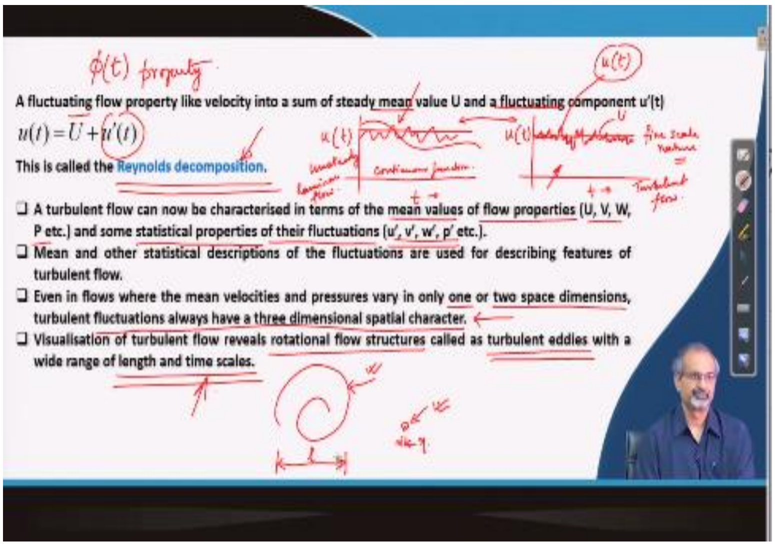

We said that it is very random. That means how does a turbulent flow signal look like and in comparison how would a laminar flow signal look like. Somebody used a probe to pick up signals of see some velocity at a location. Then two different signals may look like this. By looking at them, you can certainly distinguish one thing that though this signal has some time variation.

It is not so rapid and again it seems that you can describe it with a continuous function. However, representing this kind of a variation may be very, very difficult in terms of a functional description. Also, there seem to be very, very rapid changes; sometimes they are small; sometimes they are large but it has a very, very fine scale nature. Typically, this is a unsteady laminar flow.

More simpler laminar flows may have more orderly representations of this kind or may be absolutely steady this kind and so on. But they will never tend towards the right plot. If they tend towards the right plot, we no longer have a laminar flow in place. We have a turbulent flow. This is a very naive way of looking at signals and trying to interpret out of the signal that whether it is broadly laminar or turbulent.

Again in turbulent flow, you can understand that there is a continuous change of the property in time, sometimes very rapid changes. So, the question is that can we extract any useful statistical information from this apparently random chaotic signal? The answer is, and fortunately the answer is yes. That though this signal looks so disturbed, there is an underlying order somewhere.

So, there is a mean value of this velocity which we may call as capital U. Sometimes people put an over bar here and then there is a fluctuating part. So, there is a mean component and a fluctuating component. If you decompose the time varying velocity or for that matter any property phi of the turbulent flow field in this manner, the first part being the mean; the second part being the fluctuating component.

Then we have the famous Reynolds decomposition. You will see later that if you take a time mean of the fluctuating part then that equates to zero. A turbulent field can be characterized in terms of the mean values of different flow properties like velocities or pressure and the fluctuations. But when we deal with fluctuations, we do not talk about their instantaneous values.

We always talk about their statistical properties because instantaneous values make or hardly make any sense. Another very important aspect of turbulent flow is that though the mean part of the flow can be just one or two dimensional but the fluctuations inherently are always three dimensional. How do we visualize turbulent flows? Turbulent flows are usually rich laden with vortex structures.

Vorticity is associated with rotationality of the flow, called of V. These rotating chunks of fluid are often called as turbulent eddies and we will see later that they have a wide range of length and time scales. Length scale means a large eddy and a small eddy that is length scale. Typical dimension of this is very large. This is very, very small. Again, how long does this structure remain in the flow? How long does this structure remain in the flow? How rapidly does this vary? How rapidly does this vary?

These would define the time scales associated with the different scales of eddies which are available in a turbulent flow field. Again, remember that very fine scale signal probably has its roots to these widely varying structures which are moving around in the turbulent flow field and because laminar is more orderly. You do not have so widely varying features and that is why a smoother signal because the probes that you are putting into the flow are hit by these structures.

And that is why you get the changes in the signal. How often are they being hit? And they are being hit by larger structures or smaller structures or larger structures alone would probably decide a lot about how the signal looks.

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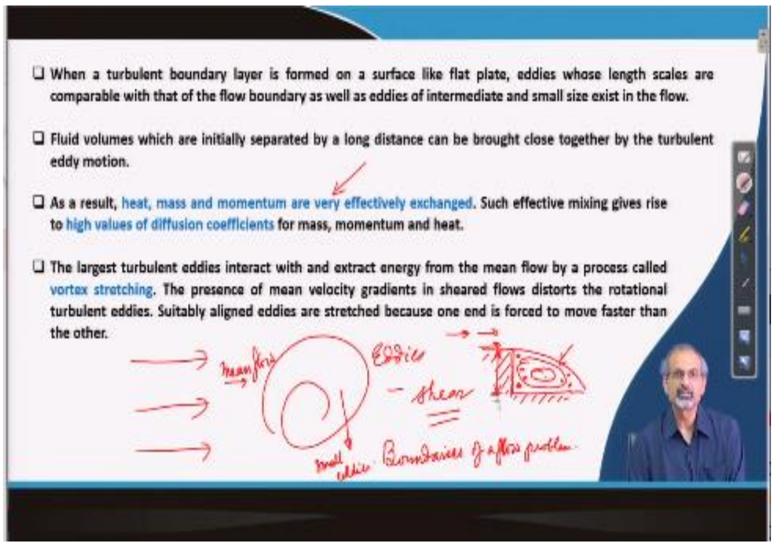

When a turbulent boundary layer is formed on a certain surface say like a flat plate on an airfoil. Eddies of different length scales can coexist large small intermediate that we already talked about. And fluid volumes moving around on that surface are initially separated by long distances. But at some point of time can be brought together because there is a lot of fluctuations in the velocity components which can augment the transport, transport of heat mass momentum everything gets augmented.

It also adds to the stresses, the viscous stresses in the flow field and therefore higher losses. So, turbulent flows are also associated with higher frictional losses. So, in order to run a turbulent flow, you have to spend more power. To make your way through a turbulent flow, you have to spend more power. The largest eddies which interact with the mean flow, we already drew the large eddies if you remember.

And if there is a flow coming in into a certain region which creates eddies. How are the eddies created? Most often created by shear effects in the flow. Then the large eddies which are created are generally created by the boundaries of a flow problem. Let me say that suppose, there is a flow approaching a backward facing step. So, the flow comes in from left. It faces a certain step.

A viscous flow will separate from here and it will create a recirculation region. So, this is an eddy. This is a large structure. So, that is formed by the definition of these dimensions, the taller the step usually the larger the bubble but what about the smallest bubbles which are moving around along with these large bubbles. How are they formed?

And how is the energy which the larger eddy which has extracted the energy from the mean flow that energy is percolating into the smaller eddies. These are very fundamental questions to ask in turbulence modeling because the governing equations which you are going to solve actually are modeling these physics.

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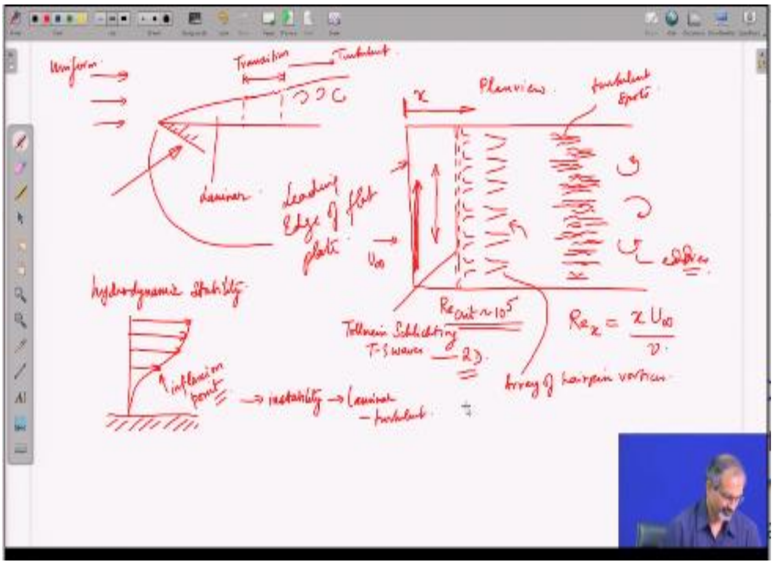

Let us try to make a small sketch of a flat plate over which a uniform flow is impinging. So, this is the leading edge of the flat plate. You know that there is a formation of a boundary layer. Initially, it remains laminar then there is a region where it starts developing disturbances and then there is a region where it has become completely turbulent and therefore eddying structures fill the flow.

This intermediate region is often called as transition and finally what you have is a turbulent flow. How would this flow look like? If you were looking at the plate from top as though you have an option of taking a plan view of the happenings. What you will find is that this is essentially the edge of the flat plate, the same edge. We often call it as a leading edge. So, as the flow hits the leading edge and moves downstream.

Some distance downstream it reaches the so called R e critical for flat plate. It is of the order of 10 to the power of 5 where some instabilities which are essentially two dimensional start developing. These are called as Tollmien Schlichting or T-S waves. In due course, they tend to form some three dimensional distortions along the direction of the flow. That means they are gradually losing their two dimensional character.

So, these are two dimensional in nature that means they do not have any variation along the span wise direction of the plate. And what is the span wise direction of the plate in this view? It is along the direction normal to the plane of this board. So, if we went depth wise in that view then we would be able to see this direction which we are seeing in the plan view. So, if you see no structural changes along this direction that means it is a two dimensional structure.

But gradually, it loses its two dimensional structure and then it gives rise to structures of this kind gradually as you move further downstream. So, remember that you are moving further downstream in terms of the length. So, any location x will give you the Reynolds number at that location, the local Reynolds number which is like x times say U if U is your velocity or U infinity, the free stream velocity by Mu. That is your local Reynolds number.

And when we say R e critical, it is the local Reynolds number at that location which has reached the order of 10 raise to power 5. Now; what you see over here is essentially array of hairpin like vortices and then the disturbances keep growing gradually these hairpin vortices give rise to what are called as turbulent spots and they also tend to lose their orderliness. So, they are more regular here but they are more disorderly over here.

So, such streak like structures which are called as turbulent spots develop and further downstream the flow becomes completely turbulent and filled with eddies. So, this is what happens in plan view more or less. Very often the change over from laminar to transitional turbulence is treated within the purview of hydrodynamic stability which is a subject in itself.

And a very basic information that you should always keep in mind when looking at changeover of a flow from a laminar to a turbulent 1 is that locally due to some disturbance whether velocity on a surface like this encounters an inflection point or not. So, inflection point is a point where slope would change sign. The slope of the velocity would change sign.

So, if there is presence of an inflection point this would probably lead to instability and this instability may finally lead to a laminar flow turning turbulent.

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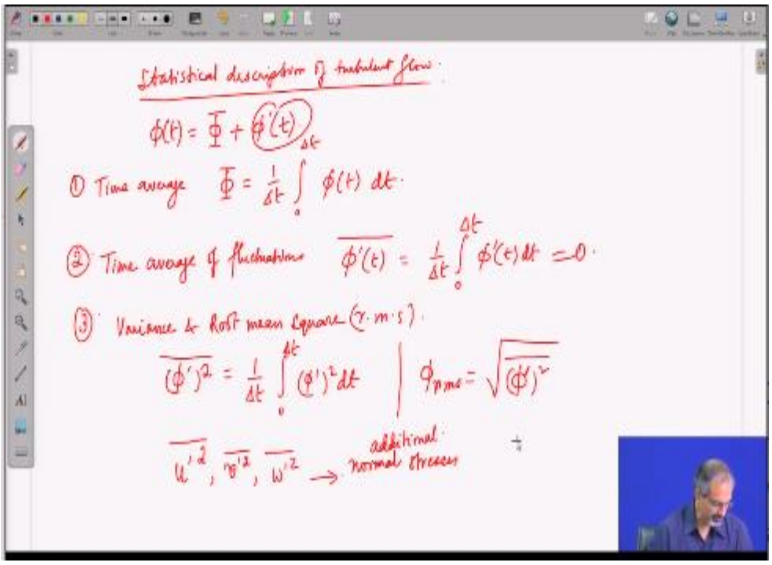

Let us look at a bit of the statistical description. We already talked about the Reynolds decomposition. So, for a property phi, let us say the mean is capital phi and the fluctuating part is phi dash. So, the time average which is the mean is calculated through a time integral like this. Where delta t is a sizable short time over which you have enough information to get a statistical average.

What about time average of fluctuations? So, this is the fluctuating part which we are trying to do a time averaging. When we do a time averaging, we put a over bar. So, when you average the fluctuations, they give you a zero. That is the sum total effect of those fluctuations always goes to zero by definition. How are these fluctuations spread around the mean value?

For that you need to know the variance and the root mean square often called r m s. So, the variance would be square of the fluctuation mean and the r m s will be its root mean square. This is very, very important for us in the sense that when it goes to the velocity fields, you will be able to get the variance information coming in, in terms of quantities like this which are square of the fluctuations with a time mean.

And when we do a time averaging of the Navier Stokes equations, we later show that these quantities are going to give us additional momentum flux due to turbulent transport and they contribute to additional normal stresses.

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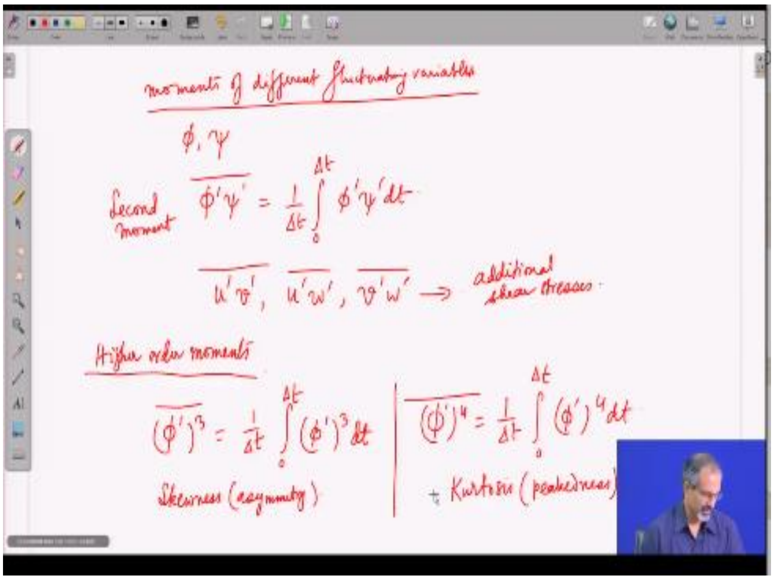

When you have moments of different fluctuating variables, so, till now we were just considering one variable phi but you may have phi and psi for example and both has their respective means and fluctuating parts. So, you can do a moment which is called a second moment of phi and psi in terms of fluctuations and then it comes up like this. What does it give in terms of fluid dynamics information?

It gives you quantities of this form. How are they relevant? Again, when you do a time averaging of the Navier Stokes equations, you will find that they lead to additional shear stresses due to turbulent transport. So, fluctuations are extremely important in terms of transport phenomenon in turbulence.

If we go to other possible higher order moments, they give useful statistical information for example how is the distribution of fluctuations working out in terms of their asymmetry or peaked nature. So, from that point of view, the third moment and the fourth moments are very, very important.

So, the third moment that is called as skewness which shows the asymmetry information and the fourth moment gives you the kurtosis which is a measure of its peaked behaviour. So, higher moments give you more interpretation about the nature of turbulence.

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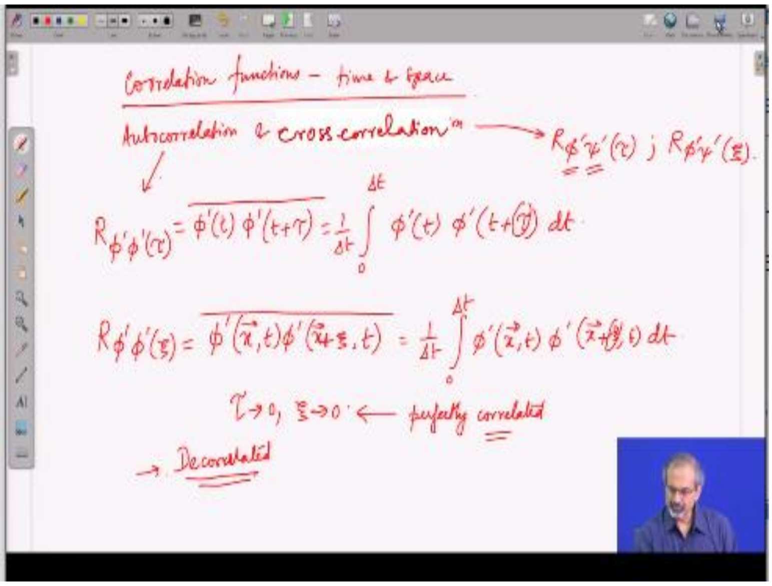

There is another very important part in terms of statistics which is called as correlation functions both in time and space. So, you can have autocorrelation and cross-correlation functions. When it comes to autocorrelation, we define it this way. One possibility is that we define autocorrelation in times in terms of a time shift. So, as you can understand t and t plus tau are two different times but tau is not a very, very large quantity.

That means you are looking at the fluctuations at two different closely space times and you are trying to find the correlation between them. So, this is an autocorrelation in time. There could also be a definition in terms of space shift. So, the space shift is in terms of the parameter Xi. So, obviously when the time shift tau or the space shift Xi, they limit to 0.

Then the value of the autocorrelation functions just correspond to the variances and then you will have the largest values of the autocorrelation functions. But that means that is the situation where they are perfectly correlated. Right. When tau tends to 0 and Xi tends to 0. But as you make these non-zero and you take information from two different points which are either stretched apart in time or stretched a part in space.

Then they get increasingly decorrelated. So, as they become small, they become perfectly correlated. As the shifts become large, they become decorrelated. That means you cannot correlate their nature as they are taken from far off points. That is the basic idea. In cross-correlation, you talk about values like this as you can understand. Again you can have it in time or space but then in one, it is phi; in another, it is Xi. Earlier it was both phi.

So, that gives you the cross-effect. So, we will discuss more on the nature of the eddy structures in the next lecture and we will discuss about the numerical and CFD aspects of turbulence modeling subsequently. Thank you.