

**Introduction to CFD**  
**Prof. Arnab Roy**  
**Department of Aerospace Engineering**  
**Indian Institute of Technology - Kharagpur**

**Lecture - 53**

**Basics of Interface Capturing Methods for Application in Multiphase Flow (continued)**

(Refer Slide Time: 00:29)

**Advecting a fluid interface (cont'd)**

Methods using marker function are called front-capturing methods.  
 Methods using marker points are called as front-tracking methods.

To identify whether a given fluid  $i$  is present at a particular location  $x$ , we use a Heaviside (step) function  $H_i(x)$ , defined by

$$H_i(x) = \begin{cases} 1, & \text{if } x \text{ is in fluid } i \\ 0, & \text{if } x \text{ is not in fluid } i \end{cases} \quad i = 1, 2, \dots, n \text{ (number of phases)}$$

In the absence of phase change, each fluid parcel retains  $H_i(x)$  value during its motion. Therefore, the material derivative of  $H_i(x) = 0$ , i.e.

$$\frac{DH}{Dt} = \frac{\partial H}{\partial t} + \mathbf{u} \cdot \nabla H = 0$$

As the interface moves, the shape of the region occupied by each fluid changes, but each fluid particle retains its identity. Thus, the material derivative (following the motion of a fluid particle) of  $H$  is zero.

The volume fraction or colour function  $C$  is defined as the spatial average of Heaviside (step) function  $H_i(x, y)$ , in each computational cell for a 2D domain,

$$C_{i,j} = \frac{1}{\Delta x \Delta y} \int_A H(x, y) dx dy$$

$H(x, y) = \iint_A \delta(x-x') \delta(y-y') dx' dy'$

- Heaviside function is generated by integrating delta functions
- Both functions fall under the category of generalized functions, by means of which discontinuous functions are represented like smooth functions

- Here,  $C = 1$  for the cells away from the interface if full and,  $C = 0$ , if empty.
- If the interface is located somewhere in a given cell,  $C$  for that cell has a fractional value.

In this lecture, we continue our discussion on advection of the interface. So, in the previous lecture, we had discussed about how we introduced the concept of advection of a fluid interface. And then further we had distinguished two different techniques in the sense that there are a set of techniques called as front capturing, which are based on marker functions and another set of techniques called front tracking which are based on marker points.

So, currently we are discussing about the marker function. So, we already talked about the concept of substantial derivative. And the substantial derivative of the Heaviside function when it is a sharp interface goes to 0 which means, a particular fluid particle contains information about whether it has a marker  $H = 0$  or  $H = 1$  and gets advected in the flow. We discussed briefly about the volume fraction or color function  $C$  which is built on the Heaviside function by integrating it area wise in a two dimensional domain.

And then we understood that  $C$  also would occupy values anywhere between 0 to 1. And if a cell is empty, then this volume fraction or the color fraction, which is acting as the marker function will be 0. While if it is full, it will occupy a value of 1 and if it is in the interface

region, it will have a value anywhere between 0 to 1. So, we will discuss more in this lecture on how to tackle such advection of color function C.

**(Refer Slide Time: 02:33)**

$I(x, y) = \iint G(x-x', y-y') H(x', y') dx' dy'$

Indicator function I is a smoother version of colour function C. G is a smoothing kernel.

Another possibility is using a smooth function F, where  $F > 0$  in one fluid and  $F < 0$  in the other. The interface is therefore identified by the level-set  $F = 0$ .

**Representing an interface on a one dimensional grid by different approaches**

- ❑ H(x) is the discontinuous function shown by a solid line that changes abruptly from H = 1 to H = 0 in cell j, exactly where the interface is located.
- ❑ The color function C<sub>j</sub>, the average value of the marker in each cell, is shown by the shaded area.
- ❑ H and C are identical everywhere except in the interface cell j
- ❑ The smooth approximation I(x) is shown by the dashed line. 'I' approaches a uniform value more slowly.
- ❑ The level-set function F(x) is defined such that  $|F|=D$ , where D is the distance from the interface. The level set F=0 is located at the interface.

So, here we are talking about an interface region. You can understand this is the interface region where the Heaviside function has dropped down sharply. And you can see that because color function is essentially an integrated effect of the Heaviside function how it is defined within a certain cell. Therefore, the color function occupies this gray region as though the cell is getting filled up from bottom up.

As the Heaviside function moves into the cell, the color function moves up from the bottom to the top to fill up that cell. So, you can very well understand that there is a area conservation between what is covered by the Heaviside function within that cell and what is covered by the C<sub>j</sub> function that is the color function within that cell j. Now, there are other ways apart from color function also to indicate the interface.

You can see a function I x which is much smoother than the color function. The color function is also behaving in a piecewise constant manner. But, the function I which we call as the indicator function is a smoother version of the color function. Because we are incorporating a smoothing kernel, it smooths out the step jump. Another possibility is the level set function which we discussed in the previous lecture, which occupies a value greater than 0 in one of the fluids and less than 0 in the other fluid.

And it is exactly equal to 0 at the interface. So, you can understand that this dashed line is crossing over the interface where the function goes to 0. So, you have smoother indicators defined through I and F. And of course, the level set function is defined in a manner where the modulus of the function would generate the distance information from the interface.

**(Refer Slide Time: 04:57)**

Advection of the color function

Assume that the flow is incompressible and the velocity is constant. Consider  $u = U > 0$ . The advection of the color function is therefore governed by

$$\frac{\partial C}{\partial t} + \frac{\partial F}{\partial x} = 0 \quad F = UC \quad \text{Flux function}$$

- As the interface moves to the right (at velocity U), C (colour function) flows into cell j through the left boundary and out through the right boundary.
- For cell j the left and the right boundaries are denoted by  $j-1/2$  and  $j+1/2$ , respectively.
- The fluxes that are transported through these boundaries are  $F_{j-1/2}$  and  $F_{j+1/2}$  respectively.

The above transport equation for C can be integrated over time step  $\Delta t$  as follows

$$C_j^{n+1} = C_j^n - \frac{1}{\Delta x} \int_t^{t+\Delta t} (F_{j+1/2} - F_{j-1/2}) dt = C_j^n - \frac{1}{\Delta x} \int_t^{t+\Delta t} (C_{j+1/2} - C_{j-1/2}) U dt$$

- Area under H curve in j cell = area under C curve in j cell (conservation of area)
- Since C fills up each cell from bottom up in the adjacent figure, if  $H=1$  interface has moved into cell j, C value of cell j becomes non zero and starts contributing to outflux from  $j+1/2$  face even though H has not yet reached that face.

Let us talk about an incompressible flow and where the local velocity is a constant. So, the velocity  $u$ , which in general is a variable is set to a constant value here which is called as capital  $U$  and it is kept positive. Now, we are trying to find out that under the influence of this positive constant velocity, how the color function would advect. So, the advection equation is written in this manner, where  $F$  happens to be the flux function and it is in a conservative form that has been shown over here.

And as you can understand that under the influence of the positive  $U$ , the interface would move from left towards the right as time progresses. What would happen as a consequence? That you can already understand that these cells  $j - 1$  or  $j - 2$  have been completely filled up by the color function as the interface moved and the cell  $j$  is now partially full. When will it be completely full?

When ideally, the analytical description of the interface which is defined by the  $H \times$  function would reach the  $j$  plus half face of that cell. So, when that happens, then  $H$  covers up that entire cell. And in order to conserve the area, both under the  $H$  and the  $C$  curves, you have to make sure that  $C$  also fills up the entire cell. In that case, the gray region will completely reach the top of the cell  $j$ .

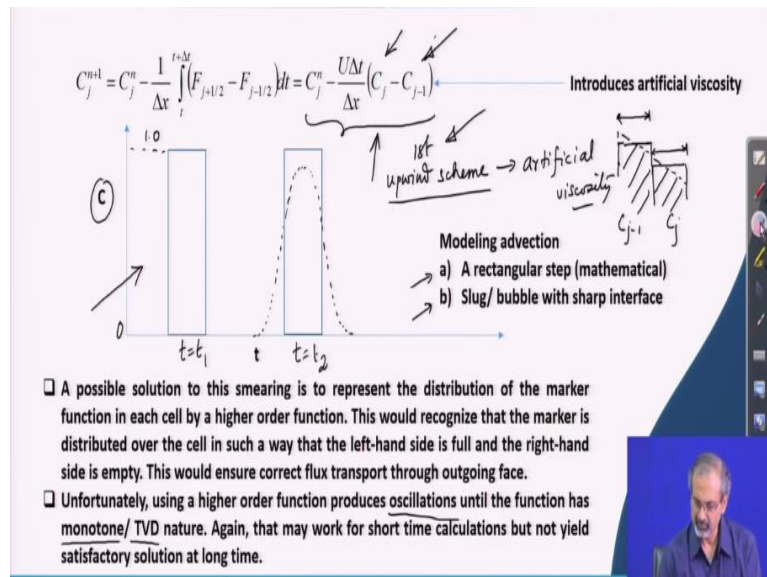
Now, we have to be careful that this picture is correctly represented when we try to work out the fluxes. If that is wrongly represented, then there will be numerical effects due to that. Now, when you discretize this transport equation, what happens is that it takes up a form of this kind. And then you can understand that when the velocity is constant, it comes out of the integral.

And within the integral you are left with the color function the difference of the color function values between the two faces of the cell which are located at  $j + \frac{1}{2}$  and  $j - \frac{1}{2}$ . Now, there are different means by which you can approximate this equation by choosing appropriate values of these color functions at these respective cell faces. Now, the point is that if we do not choose it appropriately, then there may be artificial leakage of flux through a face where the interface has not reached yet.

So, let us look at the point here. So, since  $C$  fills up each cell from bottom up in the adjacent figure, if  $H$  is equal to 1, interface has moved into cell  $j$ . That means, this is the region where  $H$  is equal to 1 and that has moved into the cell  $j$ . And  $C$  value of cell  $j$  becomes nonzero and starts contributing to outflux from the  $j + \frac{1}{2}$  face even though  $H$  has not yet reached that face, which means, the interface strictly has not reached the face  $j + \frac{1}{2}$ .

But because you are filling up the cell already with the color function, you would get a wrong interpretation that the flux has already reached the  $j + \frac{1}{2}$  face. And this is where the whole problem lies. That means we have to find a way around this problem so that we do the flux calculations correctly.

**(Refer Slide Time: 08:49)**



Now, let us say we followed the idea that we placed before that way  $F_{j+1/2} - F_{j-1/2}$  may be represented in this manner. And if you do that then what happens is that when you are trying to model the movement of a rectangular step like what we have shown over here. Let us say this is at  $t = t_1$ . You have defined the step function and the y axis essentially indicates the color function. So, strictly speaking this is 1. And of course, this is 0.

And you are trying to track its nature at some other time  $t = t_2$ . Then if you are not accurately monitoring and modeling the fluxes, what it will do to the step function is it will artificially diffuse the step function. And the above discretization that we have written will exactly do that. Because it has an effect similar to what first order upwind schemes do in discretizations we have discussed earlier in the context of say linear advection equation.

And it is essentially an influence of artificial viscosity which comes into the picture. Now, here mathematically we are modeling a step function, but if you talk about a mechanical or application orientation to this problem. This distribution of color function may actually mean a bubble a presence of a bubble or a slug inside the pipe. And that is being indicated by the sudden change in the color function.

Because there is a face change as you cross from the liquid to the bubble and then bubble to the liquid back. Now, if you are modeling it as a sharp interface, and then you find that the numerical scheme is diffusing it artificially what it means is that artificially the bubble will get elongated whereas, physically it should not be according to the modeling law. So, that is

where the numerical scheme plays a major role in predicting the face boundaries correctly as they advect through the fluid.

So, often these advections have to be tracked over longer times and we have to make sure that the errors do not  $(\infty)$  **(11:19)**. Now, possible way out of this is to replace this first order upwind scheme by a higher order reconstruction there. So, here it essentially means that you are taking contributions from each cell in the form of a constant value that means, step wise distribution.

So, instead of doing this step type of distribution say  $C_{j-1}$  is looking like this.  $C_j$  is looking like this. Higher order reconstructions usually fit linear distributions which are piecewise that means, in each cell there would be a piecewise distribution defined. And there are some geometrical basis on which these piecewise reconstructions are done. So, we will discuss about it shortly.

However, even if you do that the higher order reconstructions often bring along with them oscillations in the solution, which we have seen even in other transport equations that we have dealt with. So, if you are able to incorporate monotone or TVD nature in those reconstructions, then you will not have any oscillations in the distribution of  $C$ .

**(Refer Slide Time: 12:57)**

**(A) The volume-of-fluid (VOF) method**

In one dimension,  $C_j$  is either 0 or 1, except in an interface cell, so the value of  $C_j$  in the interface cell yields the exact location of the interface. If  $U \geq 0$ , then the flux through the left boundary is always  $U$  and we can compute exactly the flux through the right boundary, since we know the exact location of the interface.

$$\int_t^{t+\Delta t} F_{j+1/2} dt = \begin{cases} 0, & \Delta t \leq (1-C_j)\Delta x/U; \\ (C_j-1)\Delta x + U\Delta t, & \Delta t > (1-C_j)\Delta x/U \end{cases}$$

Noh, W.F. and Woodward, P. (1976). SLIC (simple line interface calculation), in *Proceedings, Fifth International Conference on Fluid Dynamics* (eds. A.L. van de Vooren and P.J. Zandbergen), Lecture Notes in Physics, volume 59, Berlin, Springer, Berlin, pp. 330-340.

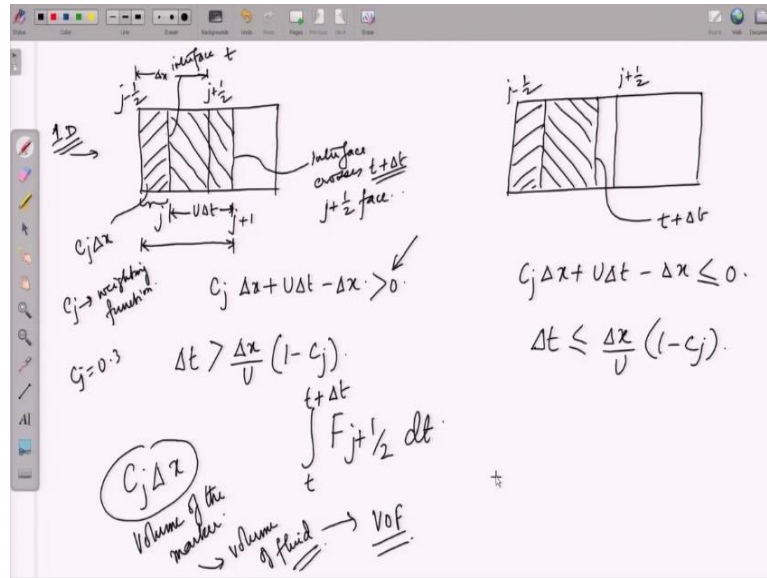
- Extending this concept to multidimensions is a non-trivial problem. The first approach to do so was the **Simple Line Interface Calculation**, or **SLIC** method of Noh and Woodward (1976). In this approach the marker function is advected by **time splitting**.
- In 2D flow, **advection is performed first in one coordinate direction and then in the other.**
- For **advection along x direction**, an interface cell is divided by a vertical line into a full part and an empty part.
- The decision of which side is empty and which side is full depends on the volume fraction in the cells to the left and the right.
- Once the location of the interface has been determined, the time integration of the fluxes is done using the above equation.
- Subsequently, the **interface is advected along y** by dividing the cell by a horizontal line into a full and an empty part.
- In 3D the interface is additionally advected along z.

So, with this introduction of the color function or the marker function, let us go into the description of the volume of fluid method, which is one of the most powerful methods in this category. So, as we discussed earlier that in one dimension the  $C_j$  value can lie anywhere



between 0 to 1. And in the volume of fluid method, we define the flux at the j plus half cell in this manner, which maintains the advection of a sharp interface. So, it eliminates the problem that we just saw in the previous slide. So, let us try to understand what it actually does. We will talk about two situations.

**(Refer Slide Time: 13:47)**



Let us make a small diagram here or rather a pair of diagrams, so, that we can see the two scenarios. So, these are the two cells. Let us say, this is the  $j$  cell; this is the  $j + 1$  cell. So, this is  $j$  plus half interface; this is  $j$  minus half interface. And let us assume that at this point of time the  $C_j$  has been defined this way that  $C_j$  times  $\Delta x$  shows the dashed region. Now, remember that  $C_j$  is essentially acting like a weighting function.

When you multiply it with  $\Delta x$ , what it does is it fills up a certain fraction of a certain of a cell a certain fraction of the width of a certain cell. Again, when it is a one dimensional advection, watch carefully that this time we have oriented the  $C_j$  distribution in the vertical manner, unlike what we did previously, where it was oriented in a horizontal manner that means it was filling bottom up.

So, here we are forcing it to fill it from left to right rather than bottom up and that makes a difference, you will realize soon. Now, as we said that if  $C_j$  is equal to 0.3, this basically means that  $C_j \Delta x$  gives you 0.3  $\Delta x$ , which means you are exactly filling up 0.3 times the width of the cell and that is where the interface exactly lies at a certain time  $t$ . Now, you are interested to know that with elapsing of some  $\Delta t$  time where the interface will stand.

Now, let us say that with passage of time, the interface moves ahead and the area that is now covered is  $U \Delta t$  because it is advecting at constant velocity  $U$ . Now we will try to work out this expression  $C_j \Delta x + U \Delta t - \Delta x$ . So, this is  $C_j \Delta x + U \Delta t$  that means this total length that is covered by the (0) (16:14) region minus  $\Delta x$ , which is this cell width.

Now, if this is greater than 0, then what happens is that the interface crosses the  $j + \frac{1}{2}$  face. So, it has filled up the entire cell  $j$  and then moved on. Now that is possible if  $\Delta t$  is greater than  $\Delta x / U(1 - C_j)$ . So, as long as  $\Delta t$  satisfies this condition the interface will cross that and in that case, we know that how to calculate the flux through that interface  $j + \frac{1}{2}$  because it has reached and it has crossed that interface over that time.

Now, what if the other happens? So, this is the interface location at  $t + \Delta t$  under this condition. And let us say we have another situation where the  $U$  is such that the interface does not cross that cell at all. So, this is the  $j + \frac{1}{2}$  this is the  $j - \frac{1}{2}$  and interface stands here at  $t + \Delta t$ . So, that can happen if in the limit the interface can just go and coincide with the cell face but not cross it.

And as long as it does there is no outflux from that  $j + \frac{1}{2}$  face. So, that is possible if  $\Delta t$  is less than equal to  $\Delta x / U(1 - C_j)$ . So, if this is the basis on which you are calculating the flux transfer then you will never have any artificial diffusion. And that is exactly what we indicated in the previous equation for  $F_{j + \frac{1}{2}}(t) \rightarrow F_{j + \frac{1}{2}}(t + \Delta t)$ . And another thing that we need to remember here is that the product  $C_j \Delta x$  it is essentially indicating a volume of the marker.

And therefore, it is volume of fluid as well which is coming from the gray portion that fluid that face, how much volume has that face filled up of a certain cell and therefore, the technique volume of fluid the name of the technique which comes up. Now, this is somewhat easily done in one dimensional sense. It gets more complicated when you do it in a two dimensional or three dimensional sense.

**(Refer Slide Time: 19:21)**



**(A) The volume-of-fluid (VOF) method**

In one dimension,  $C_j$  is either 0 or 1, except in an interface cell, so the value of  $C_j$  in the interface cell yields the exact location of the interface. If  $U \geq 0$ , then the flux through the left boundary is always  $U$  and we can compute exactly the flux through the right boundary, since we know the exact location of the interface.

$$\int_t^{t+\Delta t} F_{j+1/2} dt = \begin{cases} 0, & \Delta t \leq (1-C_j)\Delta x/U; \\ (C_j-1)\Delta x + U\Delta t, & \Delta t > (1-C_j)\Delta x/U \end{cases}$$

Noh, W.F. and Woodward, P. (1976). SLIC (simple line interface calculation). In *Proceedings, Fifth International Conference on Fluid Dynamics* (eds. A.I. van de Vooren and P.J. Zandbergen), Lecture Notes in Physics, volume 59. Berlin, Springer, Berlin, pp. 330-340.

- ❑ Extending this concept to multidimensions is a non-trivial problem. The first approach to do so was the Simple Line Interface Calculation, or SLIC method of Noh and Woodward (1976). In this approach the marker function is advected by time splitting.
- ❑ In 2D flow, advection is performed first in one coordinate direction and then in the other.
- ❑ For advection along  $x$  direction, an interface cell is divided by a vertical line into a full part and an empty part.
- ❑ The decision of which side is empty and which side is full depends on the volume fraction in the cells to the left and the right.
- ❑ Once the location of the interface has been determined, the time integration of the fluxes is done using the above equation.
- ❑ Subsequently, the interface is advected along  $y$  by dividing the cell by a horizontal line into a full and an empty part.
- ❑ In 3D the interface is additionally advected along  $z$ .

Now, when the interface remains as a straight line like we discussed a few minutes back, then the method that you have essentially is called as the Simple Line Interface Calculation or in brief SLIC. And when it is applied in multidimensional sense, it is done usually by time splitting. That means you take care of advection along  $x$  separately along  $y$  separately and along  $z$  separately in a three dimensional situation (()) (19:56). Again, there were other methods which evolved over time. Let us have a brief look at that.

**(Refer Slide Time: 20:09)**

- ❑ **Hirt and Nichols (1981)** proposed a method where the interface was still approximated by straight lines, parallel to the coordinate axis, but the same orientation of the interface was used for the advection in the different coordinate directions. Hirt, C.W. and Nichols, B.D. (1981). Volume of Fluid (VOF) method for the dynamics of free boundaries. *J. Comput. Phys.*, 39: 201-226.
- ❑ To determine whether the interface should be horizontal or vertical, normal to the interface is found using values of  $C$  in the neighboring cells. Orientation of the interface was selected depending on whether the normal was more closely aligned with the horizontal or the vertical axis.
- ❑ One of the limitations of the above SLIC, Hirt & Nicols methods is that they distort the interface and portions of the interface break away in an unphysical way.
- ❑ Subsequent methods demonstrated that improving the behavior of the advection scheme is possible through reconstruction of the interface in each cell. This is achieved by using the value of the marker function in each cell, along with the value in the neighboring cells.
- ❑ In the Piecewise Linear Interface Calculation (PLIC) method introduced by DeBar (1974) and Youngs (1982), the interface is approximated by a straight-line segment in each cell, but the line can be oriented arbitrarily with respect to the coordinate axis.
- ❑ The orientation of the line is determined by the normal to the interface, which is found by considering the value of  $C$  in both the cell under consideration and in the adjacent cells.
- ❑ Once the interface in each cell has been constructed, the fluxes from one cell to another are computed by geometric considerations.

• DeBar, R. (1974). Fundamentals of the KRAKEN code. Technical Report UCIR-760, LLNL.  
• Youngs, D.L. (1982). Time dependent multmaterial flow with large fluid distortion. In *Numerical Methods for Fluid Dynamics* (eds. K.M. Morton and M.J. Baines) Academic Press, New York, pp. 27-39.

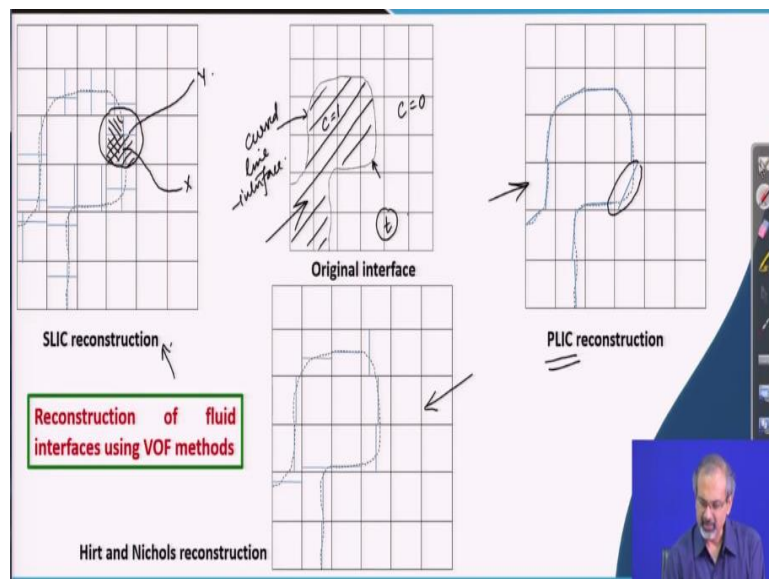
So, there was another technique which was related to the SLIC technique. But somewhat more simpler than that which was proposed by Hirt and Nichols. And there what they did was that they looked at the interface as any one particular orientation, either horizontal or vertical. So, you remember that when we first showed you the gray color diagram, there, the interface was horizontal.

Then in the calculations, we showed that the interface if you keep it vertical, then it is easier to track fluxes when it is a one dimensional movement. So, you can imagine in a multidimensional problem, if it is oriented along y, then it is easier to calculate fluxes along x. If it is oriented along x, then it is easier to calculate fluxes along y and so on. So, SLIC scheme did different orientations, when they did the time splitting and calculation of fluxes along x or y directions.

But in the Hirt and Nichols scheme, they used one particular orientation only to do the flux calculations along different directions. And how was that direction chosen? They selected based on whether the normal to the color function front at that point was lying closer to vertical or closer to horizontal. Based on that they chose how to orient the interface.

Then further came a more sophisticated method which was more accurate than these two previous methods of SLIC and Hirt and Nichols, which is called as Piecewise Linear Interface Calculation and that define the interface as a slanted straight line. That means the slope of the straight line could be other than just purely horizontal or purely vertical, it could have intermediate slopes also. And then that give more accurate calculations of the interface advection. So, we briefly look at a few diagrammatic representations so that these concepts are clearer to us.

**(Refer Slide Time: 22:30)**



So, this is where we have a brief look at how the different schemes look in perspective. So, this is essentially the original interface as you can understand that this curved line represents

the interface you can say that inside that curved line, you have a certain fluid or a certain face outside that line you have something else. So, you can say that strictly say  $C$  is equal to 1 here and  $C = 0$  here. And the sharp jump essentially comes here.

That is how it stands at a certain point of time. As time changes, as you can understand that this interface is going to advect, it is going to deform, it may even rupture somewhere and so on. Now, at this point of time, how are the different reconstructions of the interface working. So, as we defined SLIC reconstruction, you can pick up any one of the cells and you will see a vertical and a horizontal front which is indicated.

So, as you can understand that this vertical front will work better, as we advect along the  $x$  direction. The horizontal interface will advect better, will work out better as you advect along  $y$ . But in both cases, there are area conservations which are working that you have to ensure.

So, the locations are not arbitrary, if you have a horizontal line that then the amount of area that it fills up should be exactly equal to the area filled up by the vertical line from the other interface. And again should match with the exact area under the interface curve the exact interface curve. So, all of them equate with each other within numerical error bounds. So, this is how the definitions work.

Hirt and Nichols somewhat simplified it by using only one front. And a still better way of reconstruction was shown by the PLIC scheme which is piecewise linear that means, these can be could be slanted fronts or interfaces. Now, there is little more to this PLIC reconstruction. So, let us proceed with a little bit of discussion on that front. So, in the PLIC scheme, let us see how the calculations work out briefly.

**(Refer Slide Time: 25:03)**

2D.  $\frac{\partial H}{\partial t} + \nabla \cdot (\vec{u} H) = 0.$

$h^2 \frac{\partial C_{ij}(t)}{\partial t} + \int_{\Gamma} \vec{u} \cdot \vec{n} H(\vec{x}, t) d\ell = 0.$

$\Delta t = t^{n+1} - t^n$

$\hat{m} = -\nabla C$

$h^2 (C_{ij}^{n+1} - C_{ij}^n) = -(\Phi_{i+\frac{1}{2}j}^n - \Phi_{i-\frac{1}{2}j}^n) - (\Phi_{ij+\frac{1}{2}}^n - \Phi_{ij-\frac{1}{2}}^n)$

all cells of the domain

internal fluxes would cancel out in pairs.

$\sum_{ij} C_{ij}^{n+1} = \sum_{ij} C_{ij}^n$  conservation of total area

unphysical overshoots  $C > 1$   
 unphysical undershoots  $C < 0$  } numerical error.

So, in a two dimensional scenario, let us say you are trying to advect the color function and then the original governing equation in terms of the Heaviside function is this. But when you apply it in an area weighted sense for the color function and then you discretize it you will see that the equation works out like this essentially. So, of course, gamma is the cell boundary and the n is the outgoing unit normal.

And then when you integrate this equation over a certain time t which is say  $t^n - 1 - t^n$ . Then you will be able to show that this equation becomes like this. Remember thus that h is the grid spacing which is equal along both sides both x and y and the capital phi's essentially they are denoting the reference face areas which are crossing the different cell faces. So, they are essentially the fluxes.

Now, if you introduce this equation to all the cells of the domain and sum them up over all the cells with appropriate boundary conditions, what happens is the internal fluxes would cancel out in pairs. So, flux going out of the face i plus half j from the cell j is flux going into the cell j + 1 let us say through that face. So, that is how they cancel each other. So, then finally, you will be able to show that when you sum it over all the cells then you are able to maintain conservation of total area.

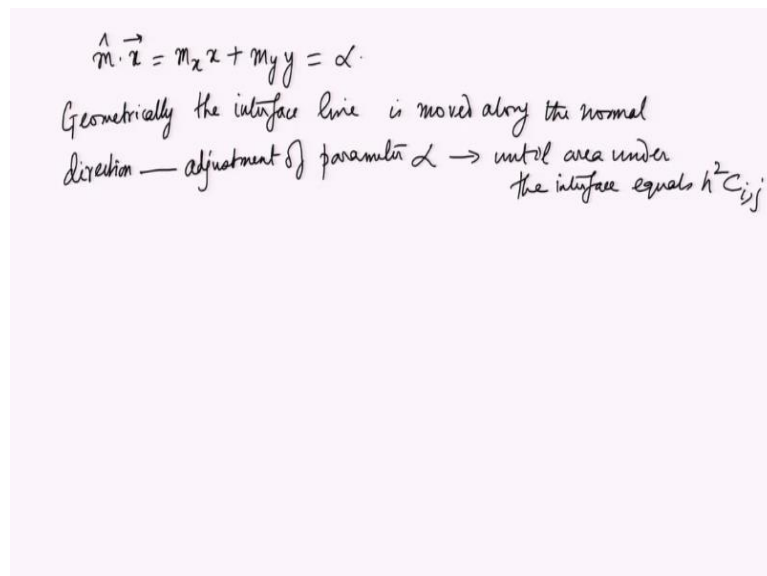
And this is a very necessary constraint which needs to be filled up fulfilled sorry. So, this ensures conservation of volumes as well. Remember that whatever numerical algorithms we are using in order to approximate these fluxes should not ever ensure any unphysical

overshoots like  $C$  going beyond 1 or undershoots where  $C$  goes below 0. This might lead to numerical error and therefore non satisfaction of the conservation.

So, these can trigger numerical instabilities. Now keeping this in mind, remember that in the PLIC approach what we were talking about was that instead of having straight lines indicating interfaces, you have curved lines. Let us give an example that in a local two dimensional region you may have a distribution of color functions. Let us distribute them in a manner like this.

Then you will see that the PLIC method may construct the interface at a certain instant in this manner. So, these are the filled up reasons. So, this is essentially the piecewise linear interface. And then we are interested to know the outward pointing normal. Let us call it say as  $m$ . So, it is known that this outward pointing normal will be equal to minus gradient of  $C$ . So, if that happens.

**(Refer Slide Time: 30:45)**



We can define the interface in this manner. So, what happens is that geometrically the interface line is moved along the normal direction through adjustment of parameter alpha. Until when, until area under the interface for that particular cell equals  $h^2 C_{i,j}$ . And that is for the new time. So, what we are trying to mean is that we have shown the discretized equation which indicates the new time  $C_{i,j}$ .

**(Refer Slide Time: 31:58)**

2D.  $\frac{\partial H}{\partial t} + \nabla \cdot (\vec{u} H) = 0.$

$h^2 \frac{\partial C_{ij}(t)}{\partial t} + \int \vec{u} \cdot \vec{n} H(\vec{x}, t) d\ell = 0.$

$\Delta t = t^{n+1} - t^n$

$\hat{m} = -\nabla C$

fixed interface

0.0	0.02	0.1
0.2	0.8	1.0
0.7	1.0	1.0

outgoing unit normal.

$h^2 (C_{ij}^{n+1} - C_{ij}^n) = -(\Phi_{i+\frac{1}{2},j}^n - \Phi_{i-\frac{1}{2},j}^n) - (\Phi_{i,j+\frac{1}{2}}^n - \Phi_{i,j-\frac{1}{2}}^n)$

all cells of the domain

internal fluxes would cancel out in pairs.

$\sum_{ij} C_{ij}^{n+1} = \sum_{ij} C_{ij}^n$  conservation of total area

unphysical overshoots  $C > 1$   
undershoots  $C < 0$  } numerical error.

(Refer Slide Time: 32:06)

$\hat{m} \cdot \vec{x} = m_x x + m_y y = \alpha.$

Geometrically the interface line is moved along the normal direction — adjustment of parameter  $\alpha \rightarrow$  unit of area under the interface equal  $h^2 C_{ij}$

$\frac{x}{\left(\frac{\alpha}{m_x}\right)} + \frac{y}{\left(\frac{\alpha}{m_y}\right)} = 1$

$\nabla C \rightarrow$  accurate calculation

And we are trying to figure out how to reach there. So, we are saying that it can be reached by adjusting this parameter alpha. And geometrically how it is done can be shown through a small diagram. Let us say, we have a certain cell here. And let us say the interface lies somewhere here. And this is the marker filled region. So, the straight line is the one which we have indicated here.

This straight line will move in or out according to how you set alpha. Because this equation can be written like. So, this equation has on top can be rewritten like this in the form of intercepts along x and y directions. So, this is the x intercept; the other is the y intercept. And so as you change alpha these intercepts are going to change and basically the interface here within the cell will change.



So, you are trying to find out an appropriate value so that it matches with this value. As soon as you reach that you are going to get the new location of the interface. Again remember that when we calculate the gradient of  $C$  it is better than by doing some averaging calculations. For example, we try to find out the value of  $C$  at different corners by incorporating the effect of neighboring cells.

For example,  $C$  over here will be influenced by a cell here, by a cell here, this cell and this cell and likewise. And therefore gradient of  $C$  can be more accurately calculated. So, accurate calculation of the gradient is very important because that decided decides the accuracy of the direction of the unit normal vector. We will discuss more on advection of fluid interfaces in the subsequent lecture. Thank you.