Introduction to Aerodynamics

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Module No. #01
Lecture No. #07
Kinematics of Fluid Motion

Next, we will discuss certain kinematical consideration of fluid motion. Now, we have already assumed that continue on hypothesis, that fluid is a continuous medium, which, of course, as we mentioned earlier, these are quite natural. Because, in fluid dynamical analysis we do not, we are not concerned with, what is happening in the microscopic level or in the molecular level. Now, the first consideration in kinematics is of course, to describe the fluid motion. For solid and rigid body which, we are quite familiar with, the obvious description is the velocity of the body or the velocity of a particle, for a body which is usually used as the velocity of the center of mass. The same approach can of course, be used. Here also, we can define a material element of fluid and then, define its velocity. In that case, how do we define the velocity for a particular fluid element? We identify the fluid element by its initial position; that is, the position that it was occupying at certain time t 0. And then, how, what subsequent time is position vector is changing and the rate of change of that position vector in the velocity.

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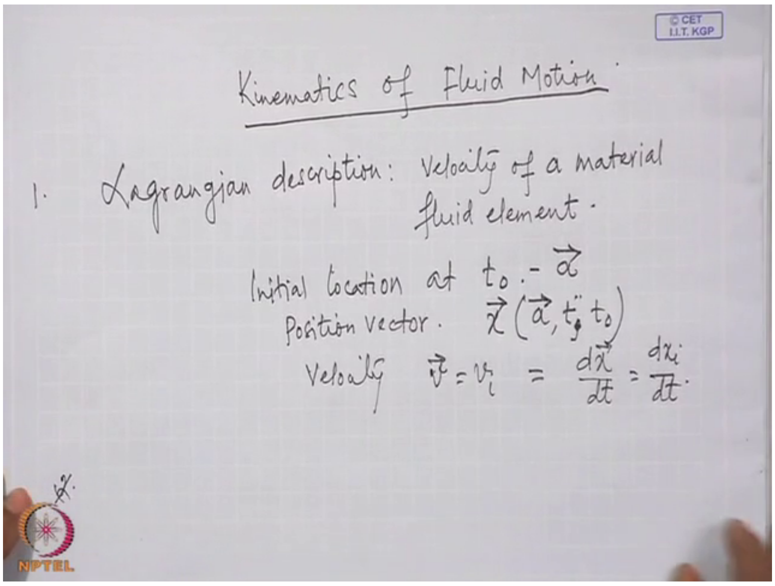

So, this type of description if used is called Lagrangian description, eulerian description. So, there is the third line which is known as streak lines, which is very important as far as physical observation of the flow. If you want to observe the flow, in your experiment, this of course, you will be doing in your experimental or laboratory classes in aerodynamics and they are called the streak lines. Can you suggest, how can you see a flow of air? Inject some color dye, inject a dye. Now, you will inject at certain point? You will inject a certain point. Just think that your injection facility such that your injecting at a point. Then, what you will see? You are injecting dye at a particular point in the flow, what we will see? This dye will spread, since, you have injected at a point, it will spread in a line. What is this line? This line is called the streak line but, what is this line? Is it path line? See, path line is the path of a particular trajectory. Is this streak line is a path line? Not in general, for special case, it can be. We will tell you, what the special case. But, in general, it is not. See, in this case, you are not looking to the single particle, that path line means you are looking to fixed particle, a single particle as an example or single material element, containing almost infinite number of molecules. But, no. you are looking here, all those material element which has passed through that injection point. Is it not? you are looking to those element which has passed through that injection point. So, this streak lines are path of the elements which has passed through a fixed point. It is all the elements that has passed through that that point. So, streak line through a point is the collection of all the fluid elements that has passed through that point.

That passed through the, all these lines coincide, if the flow is steady; that is streamlines, path lines, streak lines, they become same, when the flow is steady.

In unsteady flow or in time dependent flow, the lines are separate and each set of lines changes with time; that is, the path lines also changes with time, the streamlines also changes with time and streak lines also change with time. In our fluid dynamics analysis, most often, we will consider two-dimensional flow. Now, we say what exactly we mean by a two-dimensional flow. When we say two-dimensional flow, considering your cartesian coordinate system x y z, it does not mean that there is no z direction. No, a two-dimensional flow, we can say that, if the flow velocity is everywhere at right angle to a certain direction, the flow velocity is everywhere at right angle to a certain direction, so that, we can define our coordinate system such that the component in the third direction or in that direction to which it is perpendicular is zero. That is, as an example, let us say that the velocity is everywhere perpendicular to the z direction, everywhere perpendicular to the z direction, then we can define the velocity field, simply by the other two components, even v and the flow field is two-dimensional.

In other way, if there is a direction in which there is no change in flow, then it is two-dimensional. It is not that the third direction is not there or we are thinking about only a plane surface, no. it is that, there is no change in the third direction, then its two-dimensional. And as an example, in general in many cases, this can happen. If the third direction is infinite, not zero but, infinite and that is what, we will mean by two-dimensional; that the third direction exists but, it is infinite in length. So, that there is no change in that direction, even when we go for experiment, we would like to simulate or you want to simulate this two-dimensional two-dimensional flow and there also we can make it. So that, there is no end in the third direction, that is the third direction is mathematically infinite, there is no end to the third direction. We will see later on, that this we do by in an wind tunnel experiment by making the model fixed to the walls. So, that for the flow, there is no end to the body or the model.

Of course, there are certain other aspects showing, when you come to experiments but, that is the way two-dimensional two-dimensional flow is simulated in wind tunnels. When the model is up to the wall itself, fixed to the walls on both sides then, for the flow there is no end in the model. Model is not ending anywhere, in the third direction. So, that is what, is we call two-dimensional flow, when the flow is everywhere at right angle to the, to a certain direction and the velocity field can be expressed by simply two components and it simply means, that there is no change in the third direction. And mathematically, this means that the flow is infinite in that third direction.

Another simplification in flow is sometime achieved, when we consider the flow to be axisymmetric; that is, the flow is everywhere symmetric about or certain axis. In terms of cylindrical coordinates cylindrical coordinates x r theta where, the say x is the along the axis, this simply means an axisymmetric flow is that, it is symmetric about the axis; that is, there is no change in the azimuthal direction, there is no change in the azimuthal direction or in the theta direction. So, at any x r plane, the flow field is same. Of course, there are, in this case, there may be various situation, the azimuthal component of the velocity that may be zero. That may be nonzero. If it is nonzero, of course, it has to be fixed constant because, an axisymmetric flow cannot have variation in the azimuthal direction. There is no variation in the azimuthal direction; that is what axisymmetric flow is. So, when the azimuthal component of the velocity is nonzero, it has to be constant. So, either it is zero or some nonzero constant, azimuthal velocity.

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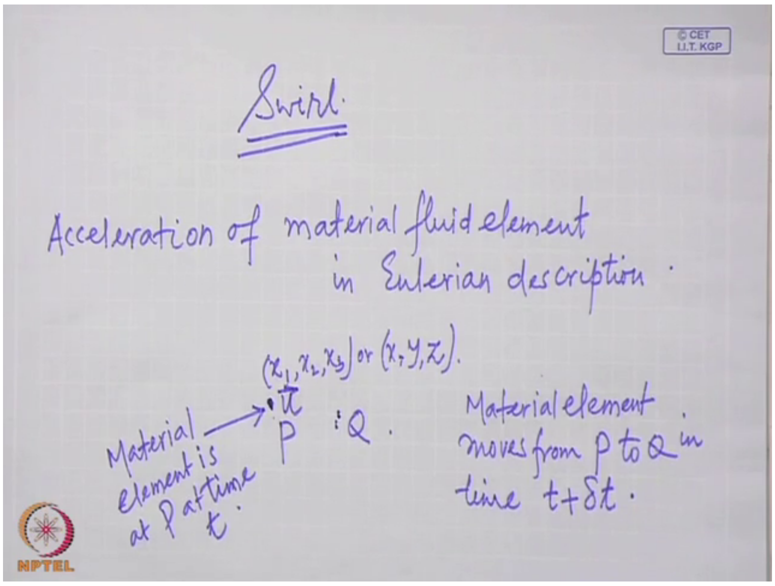

The azimuthal velocity is known as swirl, the azimuthal component of velocity is known as the swirl velocity. Let us now, come to the definition or how to define acceleration of fluid element, when you are using eulerian concept; the acceleration of a material element in eulerian description.

In Lagrangian description, as we have mentioned, it is quite straightforward. You have the velocity of the fluid element, material element, you simply differentiated with respect to time or you differentiate twice the position vector of the element. In Eulerian description, it is little different. First of all, let us consider a position here. It is called this position p. consider the fluid element that is passing through this point P x 1 x 2 x 3 or if you want you can write x y z, whatever it is. This is the position at, think about a material fluid element which is passing through this point at a time t. A little time later, at t plus delta t, this element will move to certain other location. Let us say to another location Q here. The material element material element moves from P to Q in a small time interval delta t.

Now, at point Q, let us say the velocity at point P is u, the velocity at point P is u. Of course, it is a function of those x 1 x 2 x 3 and t but, every time, we will not write it. Here, of course, we have to write anyway and that Q, it is a different point, the velocity is different.

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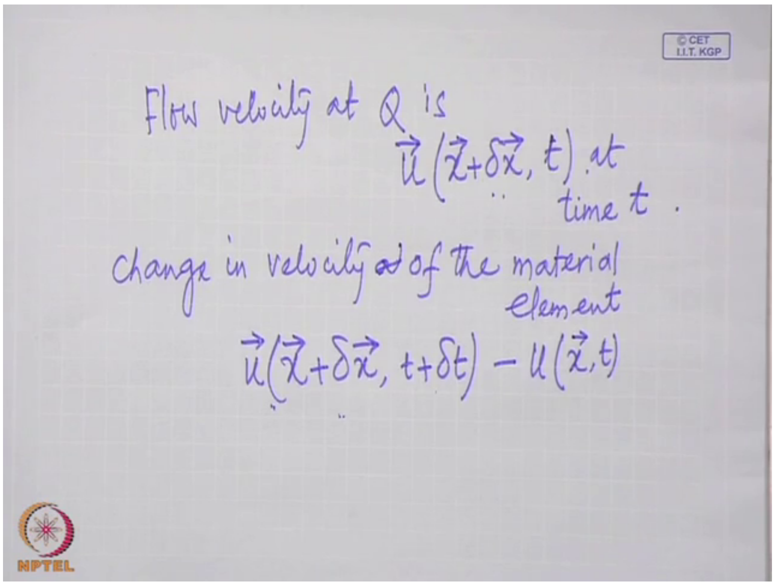

Now, what will be the velocity at Q? You can write flow velocity at Q.

At same time at t, at time t. this is the flow velocity at Q, at time t; however, the material element is reaching to this point is not at time t but, it reaching a t plus delta t. So, the change in velocity of the material element, change in velocity of the material element will be how much? This is the velocity at point Q at time t plus d t. The velocity of, at point Q, at time t plus delta t minus the velocity of, at point x at time t. Now, this difference can of course, be very easily found by Taylor series expansion. You are familiar with Taylor series expansion?

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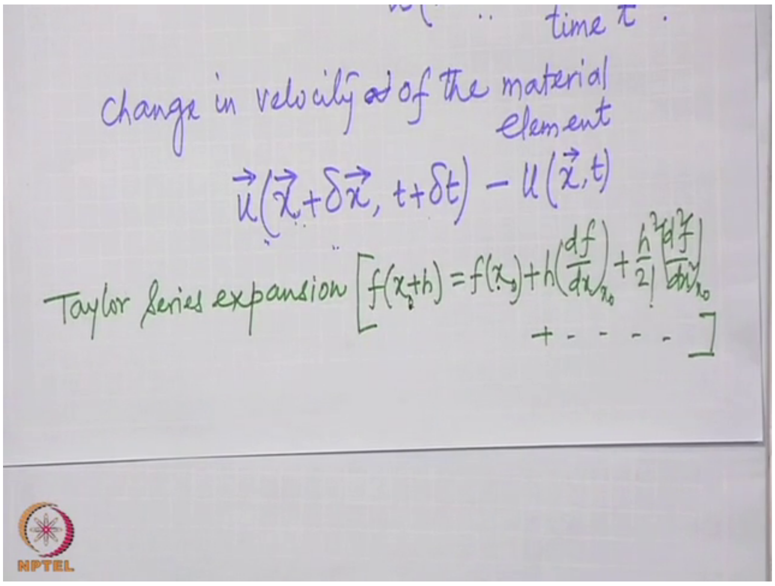

Let us say Taylor series expansion. You have done it for say several variables or for single variables? Several variables, first of all, let us write it for single variable. The several variable is just an extension Taylor series expansion; that is a function of x plus h is how much? function of x or say x 0, if you call it, x 0 plus h, d f d x at x 0 plus h square by 2 factorial d 2 f d x 2 at x 0 plus, so on. If we take the first term on the right hand side to the left, that is what, this velocity difference expression is. So, the difference can be obtained by writing all these terms.

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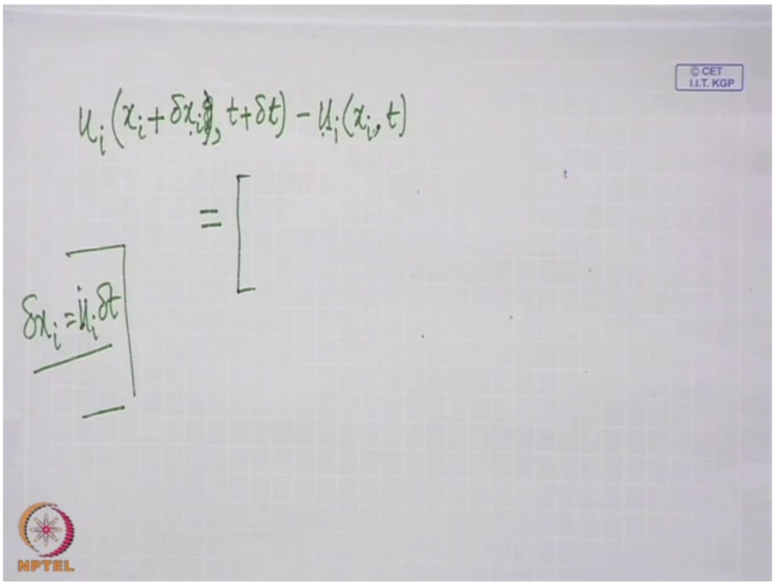

Now, see, this delta x I, what are they? The particle has moved, this much of distance, that fluid element has moved that much of distance, in time delta t. At point P, the fluid element has velocity u 1 u 2 u 3 or u v w, whatever you call. And in time delta t, it has moved the distance from P to Q. So, P to Q distance are simply u 1 delta t u 2 delta t u 3 delta t. So, this delta x i can be written as u i delta t where, u i is the velocity at x and t. This u i is this u i. Then, what will be the Taylor series of expansion of this? Now, instead of writing delta x i or delta x 1delta x 2 delta x 3, we will be writing u 1 delta t u 2 delta t u 2 u 3 delta t. See, if delta t is a small time element, small time interval, so, from P to Q, we can say the distance is also very small and over this distance, the velocity is approximated as uniform, what it was at P. So, this delta x 1 delta x 2 delta x 3 are written as u 1 delta t u 2 delta t u 3 delta t. So, what is this? In Taylor series, the first term, the derivative term, there are four variables now. There are four variables now, delta t delta x 1 delta x 2 delta x 3.

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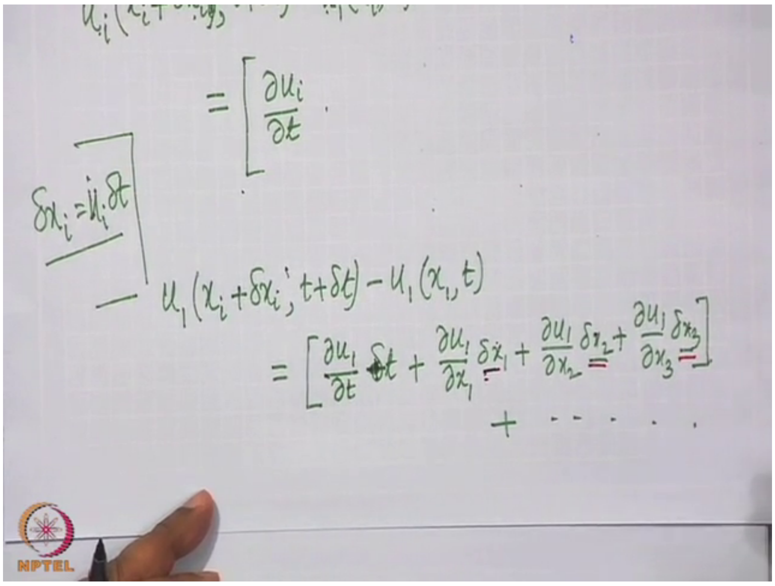

The first series, delta t has of course, no problem. It is simply d u i d t into delta t. for the time variable, this is straightforward, d u i d t delta t plus, it will be again d u i d x 1 d x 2 d x 3, not simply d u i d x i d u i because, for u equal to u 1. u 1, it will be differentiated with respect to both x 1 x 2 x 3. Similarly, u 2 will also be differentiated with respect to x 1 x 2 x 3. So, the index of i and index of x is not same. That is different, for every index of u, index of x will vary, if we write in as scalar form. Let us say, that this is u 1. Just think that this is u 1 then, here you will have d u 1 d x 1 plus d u 1 d x 2 plus d u 1 d x 3. And each case, they will be multiplied by d u 1 d x 1 into delta x plus d u 1 d x 2. Should we write it for one? We will fill in the letter, first write it for say 1 u 1 x i plus delta x i t plus delta t minus u 1 x 1 into t will be d u 1 d t. See, this delta t, I am not writing. Why, we will come to it later.

The second term will or let us write it into delta t plus d u 1 d x 1 into delta x 1 plus d u 1 d x 2 into delta x 2 plus d u 1 d x 3 into delta x 3. This is the first derivative term for of the Taylor series; that first term h d f d x. This is what, is this. Since, there are four variables; we have four first derivative terms. Plus, of course, the second derivative term. So, we are not writing. The second derivative terms, we are not writing. Now, in this this delta x 1 delta x 2 delta x 3, these replace them by u 1 delta t u 2 delta t and here u 3 delta 3. Sorry, u 3 delta t. We can do that, this delta x 1 is replaced by u 1 delta t, this delta x 2 is u 2 delta t, this delta x 3 is u 3 delta t. So, all these terms have delta t. So, there delta t comes out and these becomes

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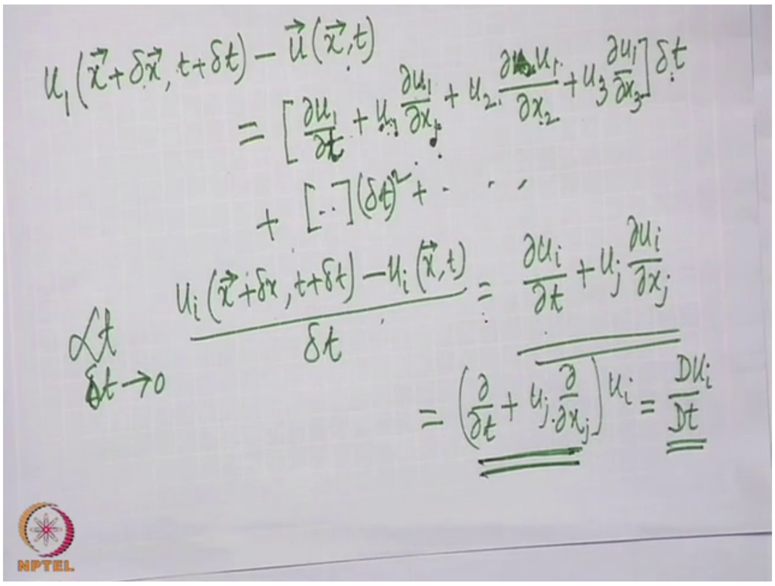

The first term is of the order of delta t and the second term, without writing all details, we will simply write as something multiplied delta t square and so on. Delta t, yes. Now, look to these indices. You see this, u 1 d 1 d x 1. So, this multiplicating component, this multiplying component has the same index with x. While this quantity u, it has a fixed index 1. But, this is u 1 x 1 u 2 x 2 u 3 x 3. So, we have got what the general index will be.

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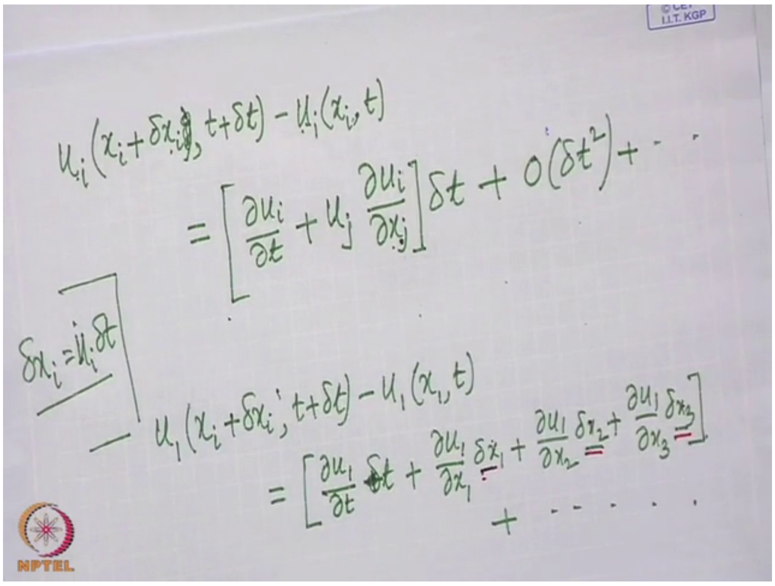

So now, we can come back here and complete this equation. This, instead of writing all those terms, we can write, if you look to that u j d u i d x j, agreed. This multiplying u has the same index as the x; meaning, it is a sum over then. And we are getting all these three terms by writing this into delta t plus something, which is of the order of delta t square and so on. So, this is the change in the velocity of the fluid element over a small time interval delta t. Divide this by delta t and in the limiting case of delta t approaching zero, we get the acceleration. So, acceleration of the fluid particle then, becomes what? This divided by delta t; you see and then, let delta t approach zero. So, you see that these terms and these terms, all these terms will approach to zero because, when you divide by delta t, this will still remain delta t; however, there will be no delta t here. Now, we get that the limiting case of delta t approaching zero, u i x plus delta x t plus delta t minus u i x t by delta t is simply that term, d u i d t plus u j d u i d x j. So, this is what, is the acceleration of fluid element in Eulerian description.

Student: (( )).

No, see, this delta t is a very small time interval, delta t is a very small time interval.

But still our assumption there is (( )).

I understand, what you are saying that if you are, if there is an acceleration, how can this delta t for a small time but, if the time, what we are ultimately interested is that delta t is approaching zero. So, it is a time interval of that size. So, for that small time interval, we can still assume it, that if it is for a finite time interval, of course, we can do it. But, for an infinitesimal time interval, we can still do that. So, this is what is the acceleration, this can also be written as you see that d is a mathematical, instead of a simple derivative as an operator. This entire thing can be taken as an operator. And since, this is the derivative that we have obtained while following a particular material, this derivative is often called as material derivative or substantial derivative. And it is customary to write it with this notation, a capital D. So, whenever you see this capital D within the Eulerian framework, it implies that the derivative is computed while following a particular material element.