Introduction to Aerodynamics

Prof. K. P. Sinhamahapatra

Department of Aerospace Engineering

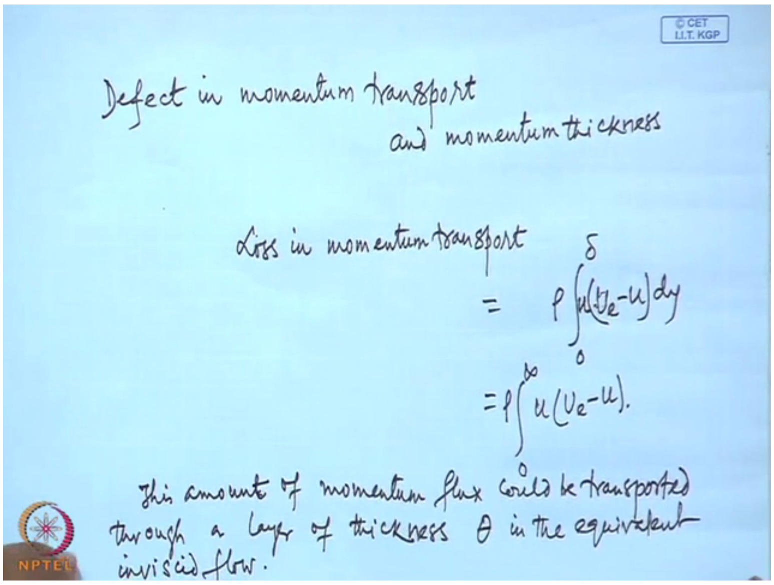
Indian Institute of Technology, Kharagpur

Lecture No. # 45

Boundary-Layer Theory (Contd.)

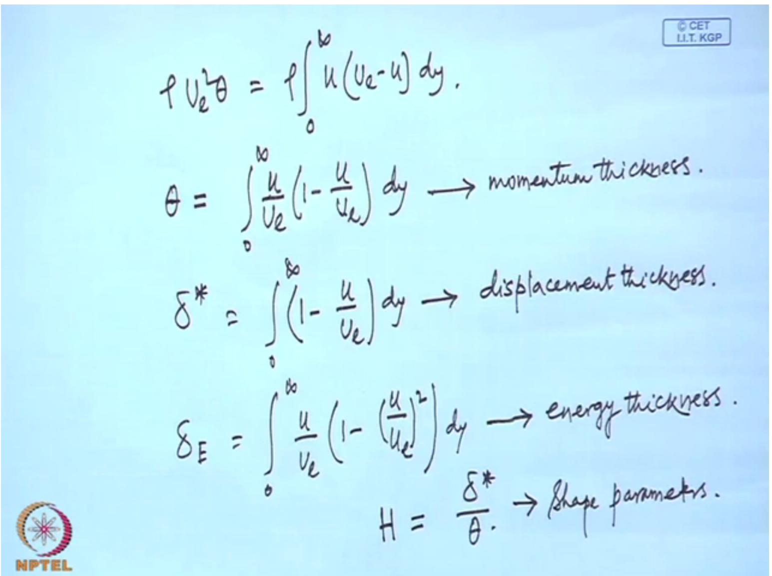
We discussed boundary layer displacement thickness yesterday. Similarly, due to the presence of the boundary layer, the momentum transport through the flow will also be reduced that is the momentum; that could have flown in the absence of the boundary layer, we will now be little less that it will be able to flow. So, there will be defect in momentum transport or deficit in momentum transport.

(Refer Slide Time: 01:01)



We will call it defect in momentum transport and boundary layer momentum thickness. Now what is this? loss in momentum transport, how much? How much it will be? Loss in mass transport we have already seen, rho into e v minus u that rho, of course we dropped, but in actual calculation we can keep rho. See this will be straight away if you remember the loss in mass or volume transport each what we did yesterday zero to delta, this is the loss momentum transport, in mass transport. So, momentum transport will be see, these multiplied by the velocity u. And once again this, that rho we have dropped, because we have considering incompressible flow that rho we have dropped, actually there will be a rho also. So zero to infinity, or if you want to keep that rho, let us keep rho. Because these density is constant we have not taken it inside the integration, if you want consider density variation then these density will be there inside, here rho e u e minus rho u. And if we assume certain thickness in the inviscid flow, across which this momentum can be transported. So what will be that thickness, think about an equivalent inviscid flow having certain thickness through which this amount of momentum is being transported. So how much will be the momentum transport in the inviscid flow, if we consider a thickness of say eight, how much it will be, or what would be the momentum transport per unit thickness of the fluid layer, straight away rho u e square. So this amount of momentum…

(Refer Slide Time: 06:03)

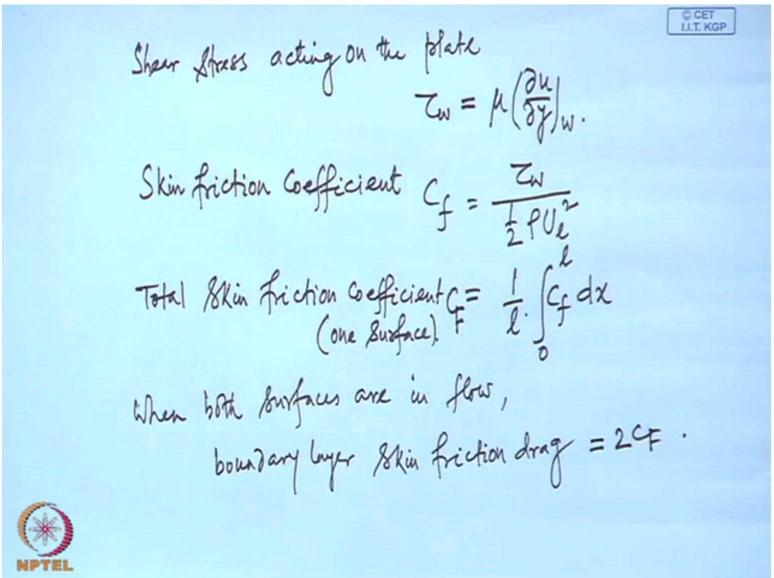


So we can write that, rho u e squared theta equal to what we said that, if we consider an equivalent inviscid flow, then through a layer of thickness theta, similar amount of or same amount of momentum could be transported, that is as if we have lost a thickness of theta, due to the presence of the boundary layer as for as the momentum transport is concerned. And mathematically that, this rho u squared theta is this or we have theta, which is called the momentum thickness this is what is the definition of momentum thickness.

So we have already defined delta star, which is one minus u by u e; integration of one minus u by u e and the momentum thickness, which is integration of u by u e into one minus u by u e. If the density is variable, can you say what type of change will be there in this definitions, if the density is a variable, let us say delta star. Delta star we have already defined, the displacement thickness. If the density is a variable one by rho, no see this will be simply rho u by rho e u e. Because what will be defect in mass flow, in the inviscid flow or the mass flow would have been rho e u e, in the viscous flow, the mass flow or the density changing the mass flow would have been rho u. So it is the difference of that, rho e u e minus rho u, that would have been the mass flow defect, integrated across the boundary layer, rho e u e minus rho u. And then, it would have come one minus rho u by rho e u e, this definition. In the momentum thickness also, this u by u e, this will be changed to rho u by rho e u e, because that is again coming from that mass flow defect multiplied by the velocity for the momentum, that will not change. Anyway similarly, you can also define an energy thickness, we are not going to use it, but an energy thickness…you can defined an energy thickness also like this.

And a very important parameter, which is called the shape parameter or shape function is defined as delta star by theta, the ratio of displacement thickness to momentum thickness is called the boundary layer shape parameter… And as it happens, that this shape parameter is always greater than one; that is delta star is always greater than theta; displacement thickness is always more than momentum thickness.

(Refer Slide Time: 13:04)



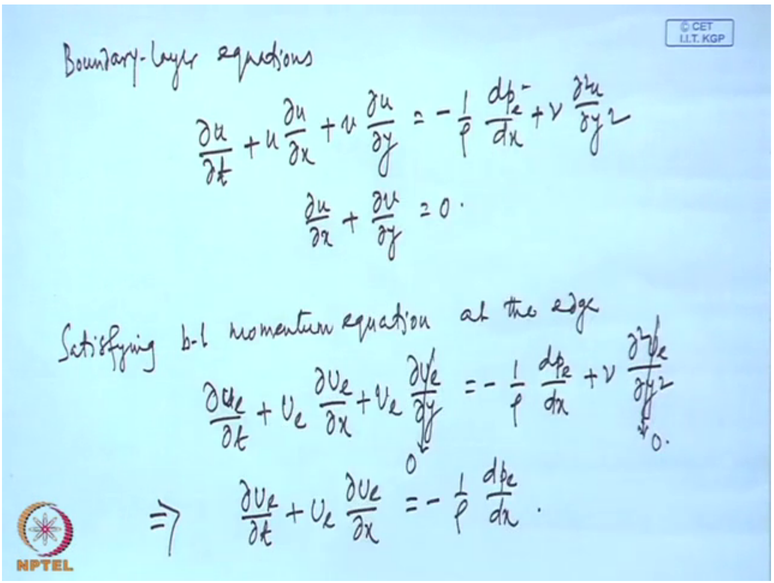
Now since there is a velocity difference or velocity is continuously varying across the boundary layer, so there is also viscous shear stress acting on the body surface. So how much is the shear stress acting…the shear stress acting on the plate, tau x y of course, it is we have only x and y, so tau x y, we have denoting is tau w, that is shear stress on the wall, that w stands for wall. How much is it, what is tau x y, if you remember that earlier definition…if you remember the definition of that stress tensor sigma i j equal to mu into du by dx j plus du j dx i minus two by three mu delta i k and divergence u, that was of course, for general case. Now you can simplify how much it is, you can check it and it will come simply as mu du dy; will come as mu du dy and since we are interested only at the shear stress on the wall, that is mu du dy at wall, that is at y equal to zero. The value of du dy on the wall and on this, shear stress is integrated over the surface, that gives the total frictional force or the frictional resistance to the flow or frictional drag, so you get a drag force. So how much is; before that lets define a skin friction coefficient and what we have seen that, if we integrate the shear stress over the surface, we get the resistance force or the viscous drag force, frictional drag force that is not the only drag, there might drag from different sources, so that is, we will not call it drag force, but only the frictional drag force.

And similarly, if we integrate this skin friction coefficient, then we will get the total skin friction coefficient or the viscous drag coefficient; skin drag coefficient. So we can write total skin friction coefficient, how much it will be; for one surface. Usually when the body is emerged it will have two side, two surface: upper surface and lower surface and both will offer these resistances. So for the total skin friction coefficient, that should be multiplied by two, but now we are considering one surface, so this is how much; integration of this over the entire length, the length we take what did you take l but, small l or capital L anyway, so that is the length of the plate, we call it c capital f. So if both surface are in contact with the flow, then this will be multiplied by two and that is called the boundary layer friction drag, boundary layer skin friction drag.

Remember, this is not the only contribution to the total drag, this boundary layer has another contribution to drag, this is the direct viscous contribution coming through friction. But in addition, due to the presence of the boundary layer, the overall pressure distribution will change slightly, not very large change, but it will change to some extent.

So whatever the pressure distribution acting on the body, due to the presence of boundary layer, that pressure distribution will be slightly modified and that modified pressure distribution will also give a drag force. So due to the boundary layer, there are two contribution to drag force: one coming through pressure, one coming through viscous stress. So that coming through the viscous stress, that of course we can get directly from the boundary layer solution. But what is coming through the pressure distribution, that we need to solve the two flows together in a composite manner, which of course cannot be done analytically. Except perhaps for very simple flat plate case, then also it is not possible even analytically for that. So we cannot give a formula or anything for that boundary layer pressure drag. Of course, there are other contribution to drag as well, which we mentioned earlier that; when body is finite, that is a three dimensional body, then there is another contribution to drag force, which will be discuss in later in your other courses in aerodynamics called as induced drag or lift induced drag, which actually is basically a component of the lift acts as a drag and then at a very high speed; particularly in supersonic flow, there is another type of drag, which is known as wave drag. So these are different contribution to drag and together give the complete drag force. So what we have got here now is the boundary layer skin friction drag, in addition there is a boundary layer pressure drag and these two together sometime called as the form drag, this boundary layer skin friction drag and the boundary layer pressure drag together, often called as the form drag.

(Refer Slide Time: 22:38)

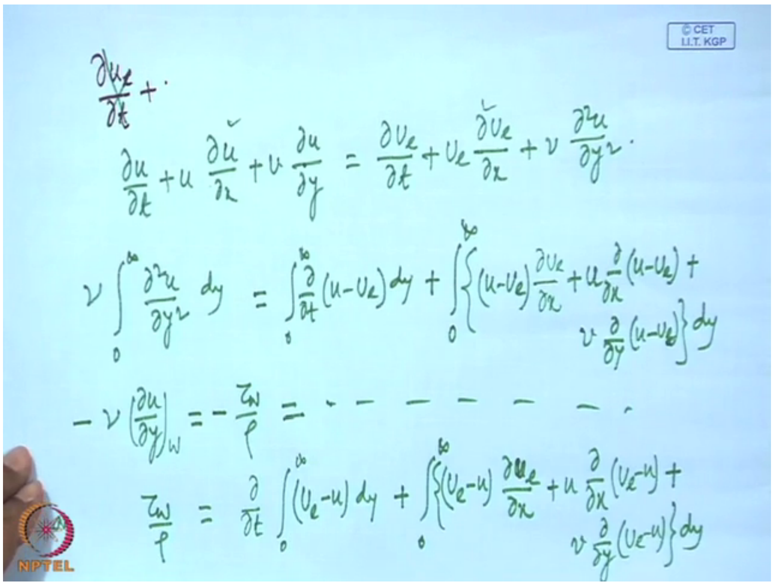


Anyway, now let us come back to the boundary layer equations once again, the boundary layer equations…we have seen that the boundary layer flow can be solved by these approximate form of the navier stokes equations.

So these are the two equations, that are required to be solved; for solving the boundary layer problem subjected to the boundary condition, which is the no slip condition on the solid wall, the merging of the inviscid and viscous flow at the age of the boundary layer and in addition and initial condition at certain x location, we have to specify how u is varying with y. At certain x location, how u is varying with y, that is to be specified. So find all these are specified, theoretically these can be solved and there are two unknown here: u and v with two equations, they can be solved. The pressure, here in this equation is taken as known, from the solution of the outer inviscid flow.

Now let us see, how that is; so at the age of the boundary layer, this equation is satisfied. Now at the age of the boundary layer what it is; boundary layer momentum equation, at the age we have du e dt plus u e; du e dx, we know that u e is not a function of y; u e is not a function of y. We can write it here; v e and this is zero. This is also zero. So we get, this now we can substitute in this equation, in the x momentum equation.

(Refer Slide Time: 27:01)



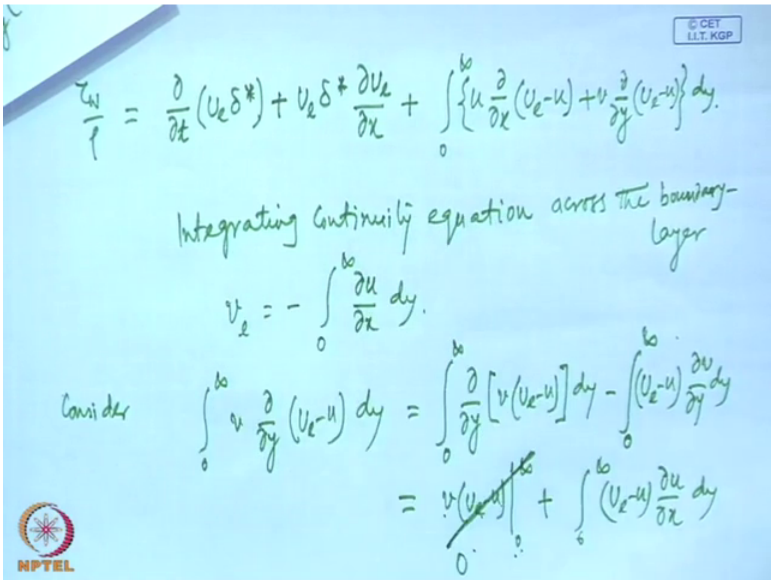
See, if we substitute this in the x momentum equation what do you get.

Now we will integrate this equation across the boundary layer, we keep this term on one side, the rest in other side; integrating across the boundary layer. See this and this can be combined, this and this; combination of these two have written in this manner and in this v term; we have inserted one, included one u e, but that is all right, since d dy of that is zero; that u e is not the function of y, so we can take it for convenience.

What is the integration of this one, first one; integration of this is simply du dy. Now du dy, for any value of y greater than delta is zero, u is not changing once y is more than delta. So, at the upper limit the value is zero, in the lower limit; it is du dy at y equal to zero or du dy wall. what is that; mu into du dy that is, that skin friction by rho, tau w by rho. So the first term is becoming, the one negative sign will send or let us write first one line, this is what it is coming, for y equal to zero, that zero we are replacing by wall. And this is what is; and the right hand side remain as it is or what do we have, tau w by rho.

The first term is simply u e into delta star. Now we have one v here in this equation, we can replace this v using the continuity equation.

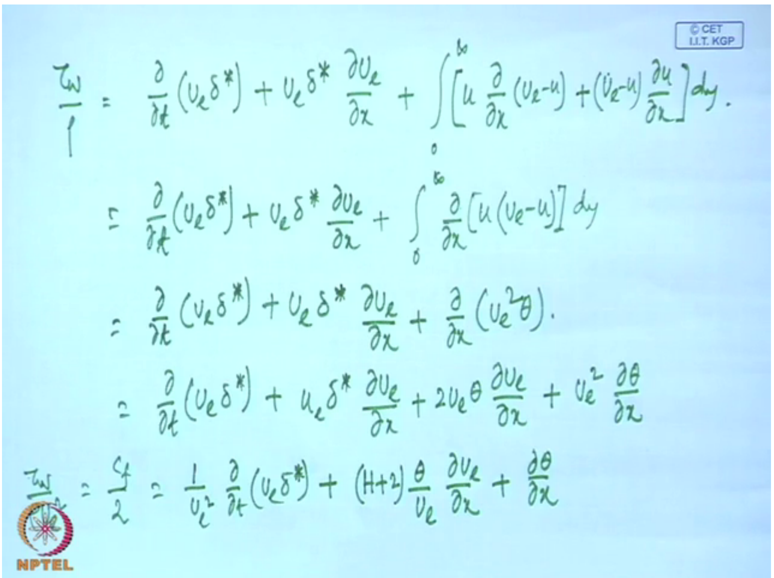
(Refer Slide Time: 34:24)



Say we will do that first, let us write it once more, tau w by rho equal to d dt of u e delta star. We will also take the second term, that is the first term in this three integration, du e dx is not a function of y, so it comes out of the integration. So what it remains is, u e minus u with respect to it and what is u e minus u, again u e into delta. So these term becomes u e into delta star into du e dx. So there is no point in writing of any partial derivative here, because u e is not a function of; but anyhow we are considering unsteady flow, so u e can be a function of time, it is not a function y. Their next two terms of course, let us keep it now. If we integrate the continuity equation across the boundary layer. Now let us consider this integration, v into this.

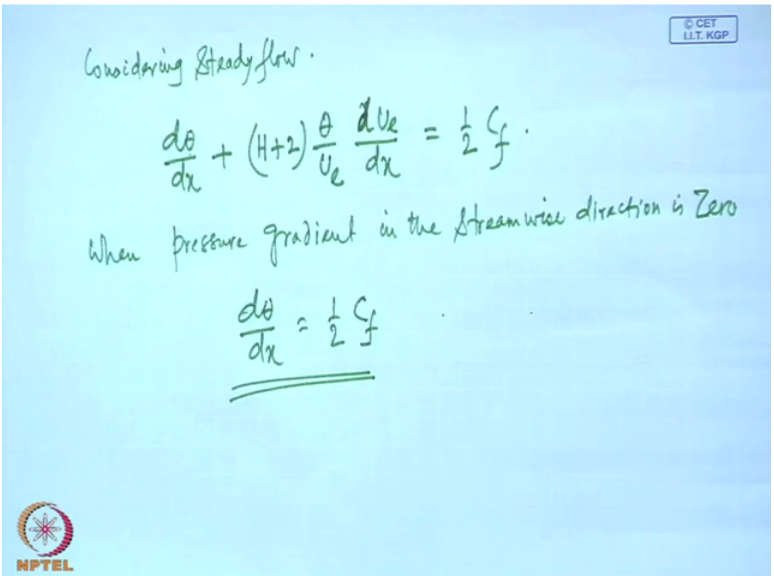
This integration can be carried out fully, the result is zero to infinity, this integration is v into u e minus u. Now dv dy minus du dx from continuity equation; dv dy is minus du dx from the continuity equation. So we can put it du dx and the minus we can make it plus, what is this; at infinity, u e minus u is zero and at zero v is zero. So this is zero, this part is; at infinity u e minus u is zero; at zero v is zero. So this last integral become this. Now you can combine these two.

(Refer Slide Time: 40:41)



So we can now write… You see this is; this can be combined very easily. Divided throughout by u e squared… This equation, which is an integral form of the boundary layer momentum equation, an integral form of the boundary layer equation is known as Von Karman momentum integral equation or boundary layer momentum integral equation.

(Refer Slide Time: 45:51)



If we consider steady flow, then this equation becomes. If we consider steady flow then all the derivatives will be ordinary derivatives, they are only function of x; not function of y, write d theta dx plus H plus 2, look to the second term, which contains that du e dx. You can see on this side. Now if you look to that momentum integral equation; momentum equation on the edge of boundary layer, this one, for a steady flow u e du e dx is what is the pressure gradient; so the second term here represent pressure gradient. So if there is a flow or there is no pressure gradient, so like a flat plate flow in which there is no pressure gradient in the stream wise direction. The flow is everywhere uniform. So in a zero pressure gradient flow, we have a very simple relation this equal to this. So this is the equation of the flat plate boundary layer; this is equation for flat plate boundary layer in the integral form. Of course, in the differential form, we will have this equation; in the differential form, this is this equation with this term dropped.

So boundary layer can be solve either this differential equation approach or we will try to solve the differential equations or we try to solve these integral equation. Now there are solutions available for either format. In the differential form, we can get solutions for some simple flow problems, for there is no pressure gradient or the pressure gradient can be expressed in a very simple form. Otherwise of course, you cannot have analytic solution. So solution of this boundary layer equation in the differential form, for simple cases, with zero pressure gradient or some specified pressure gradient; pressure distribution or specified free stream. We will postpone it to later courses. Even the solution in that integral form also will differ, we will assume that somehow, if the solution is known, then how to find the other parameters. The solution as you can see from the differential form, the solution will be how u or v varying in the boundary layer; within the boundary layer. The variation of u at a fixed x with y; variation of u with y at a fixed x location is called the velocity profile. That is how the velocity changes at a particular location, across the boundary layer how the velocity changing that velocity profile.

So if we solve this the velocity profile is the solution that is what we will likely to get. So we will not discuss how to get the velocity profile but rather we will assume that the velocity profile is known and then how to handle that result in this case. So that we will do in the next class, assuming some simple velocity distribution; velocity profile, what will be the various other important quantities that we are interested in.