Introduction to Aerodynamics
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Module No. # 01
Lecture No. # 43
Boundary-Layer Theory (Contd.)

So, what you see that the development of boundary layer on a solid wall in one sense supports the assumption of inviscid irrotational flow, that over the large part or over almost the entire flow domain the flow is basically inviscid irrotational. However, it say that there is a very thin region where the effect of viscosity cannot be neglected, whatever high the reynolds number may be whatever high the reynolds number may be, there will be an extremely thin region where the effect of all these effect; the effect of no slip boundary condition, the effect of vorticity, the effect of viscosity; they cannot be neglected, over a very thin region. And it suggests that this thin region must be solved separately this is an extremely thin region, but this must be solved, this cannot be completely ignored.

What we did earlier this cannot be completely ignored this has to be matched or added with that earlier solution that we are discussing about the inviscid irrotational flow. However the very nature of boundary layer helps us in simplifying the flow equation to some extent it will not be as simple as the inviscid irrotational flow equation, but it will still help to simplify to some extent. This comes from the fact that in the normal direction the normal to the wall in that direction the changes occur over a very small distance over the thickness of the boundary layer. We of course, do not exactly know what is the thickness of this boundary layer, or whether we will be even be able to define exactly a thickness of the boundary layer.

We simply know that it is of the order of inverse square of Reynolds number nearly, but see these are in a real situation, in a practical flow situation there is no two different region. It is just only one flow this is our mathematical nature that description that near the wall there is a region where velocity gradients are occurring very quickly velocity is changing very quickly and after that is not changing. But in the real case there is no discontinuity, it is a continuous flow from the solid wall to infinity it is a continuous flow. Within that continuous infinite region we have separated one thin region and one the remaining part. Say one is say about point .01 percent and the remaining is 99.99 percent, but those two are not to distinct identity that you must remember.

Basically that is one flow with some distinctive properties and we are making them two different flow, but in reality they are not two different flow only one flow. Anyway now along the normal direction the changes are very rapid the velocity changing from zero to characteristic velocity or what would have been the inviscid flow velocity over a small distance while in the stream wise direction or in the flow direction. The changes are occurring in the stream wise direction also the change in velocity is of the order of u infinity that you have seen the change in velocity in the stream wise direction is also of the order of u infinity.

We have seen the maximum velocity can reaches two times or three times minimum velocity may become zero stagnation point so, the change is of the order of u, but this characteristic change is taking place over a larger distance l. While in the normal direction or in the direction of boundary layer thickness the change is taking place over a small distance so the gradients are much larger in the normal direction or y direction.

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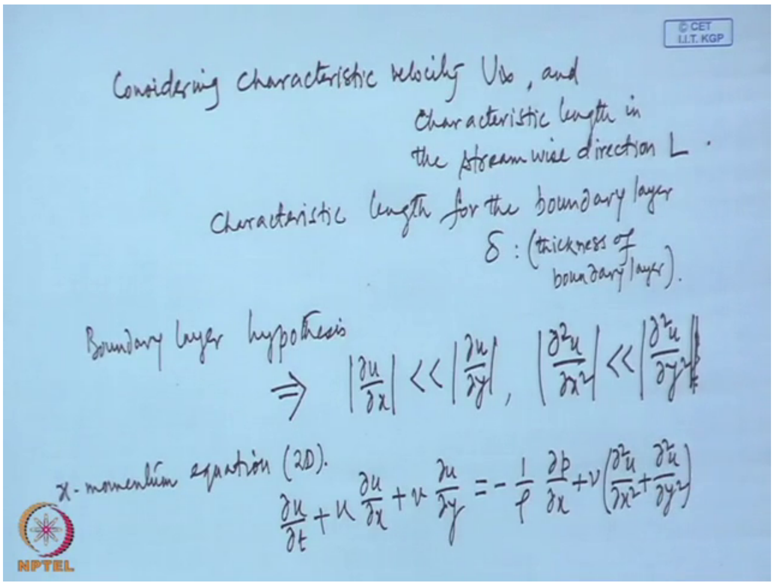

Let us consider each of the eq1autions separately first consider the x component of the momentum equation can anyone tell what is the x component of momentum equation? x momentum equation anyone remember x momentum equation? We will consider two dimension not three dimension d u, d t plus u, d u, d x plus v, d u, d y equal to minus 1 rho d p, d x plus nu into d 2, u, d x 2.

boundary (( ))

If you if you if you measure the velocity we

Will be it distinct (( )) jump from (( ))

Not distinct jump it is no longer a jump it is a continuous variation smooth variation the velocity variation or what is known as the velocity profile see the velocity will vary later on. We will see it if we call this is that distance y and this is the velocity u within the boundary layer the velocity will vary something like this, it may be something like this or it may be something like this way. It will go on a smooth variation no jump was at t equal to zero only, now you see this what essentially we can do from this equation based on the boundary layer hypothesis that d 2, u d x 2 which is much smaller than d two u, d y two can be neglected because this is much smaller compared to this that is what the boundary layer hypothesis says so, we can remove this from the term.

u by

U by l so, in this and d v, d y as we can say is v by delta. Now if they have to be the same let us say u by l is of the order of one. Let us say that u by l is what we call one then v by delta should also be of the order of one or v will be of the size of delta of course, remember we are saying of the size same two different quantity v and delta. So, v cannot be delta do not write equal cannot be equal to delta v is a velocity and delta is a length so they two these two quantity can never be same a velocity a velocity may be two meter per second and length may be two two meter, but they are not same still so never write v equal to delta remember even if their values are same.

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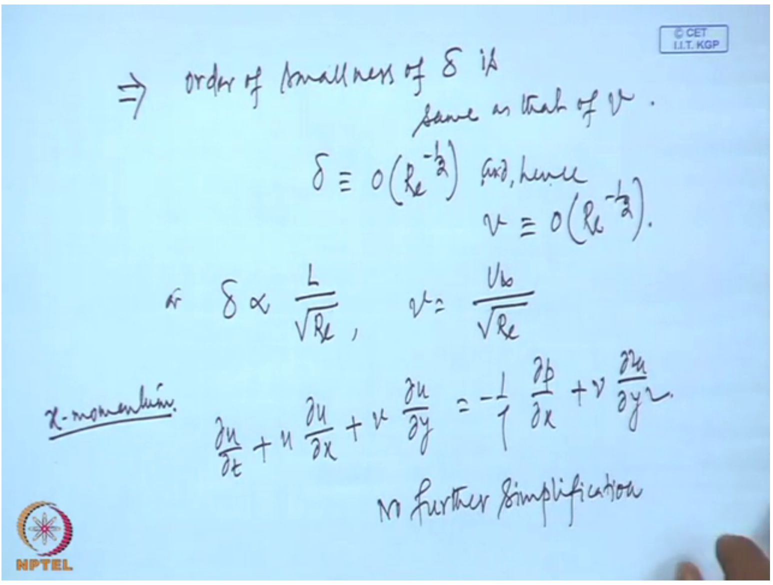

We already have order of u is infinity, of order of x l, order of v is infinity, order of u infinity, R e to the power minus half order of y also u infinity, R e to the minus half looking that what will be the order of each of these term time term also you forget.

Student: (( ))

(( ))

What is the order of this equation in terms of reynolds number

minus one

So, it is u infinity

Student: (( ))

By l

By l into

Into R e to the power minus one here.

Student: (( ))

First one

Student: (( ))

Second one

Student: No sir (( ))

This one

Student: (( ))

Left hand side

Student: (( )) R e to the power minus (( ))

It also should be R e to the power minus half not minus one, y is also there no v d v, d y 2, v 1, y 1, y both are R e to the power minus half v is also R e to the power minus half y is also R e to the power minus half, so one R e to the power minus half get cancelled right hand side here in the first term. Ever tried then this term needs to be retained beyond second-order no other boundary layer theory exits as of now either first-order or second-order in most cases first-order in some special cases second-order, but second-order is only special not classroom or study regular study material as yet. So, what it gives this boundary layer theory another implication that the pressure the pressure within the boundary layer remain constant it does not change meaning whatever is the pressure at the edge of the boundary layer or what we get from the inviscid flow solution that itself is the pressure on the wall that itself is the pressure on the wall.

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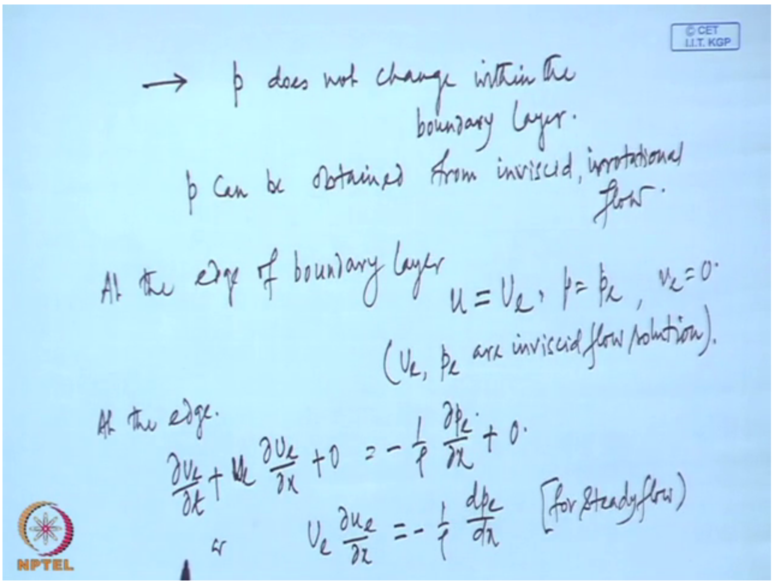

So, you see this pressure is now basically unknown parameter once we have the inviscid flow solution that inviscid incompressible irrotational flow solution then the pressure is known so, in the x momentum equation the pressure term that is present is basically known. We will also see the experimental verification of these statements.

If you consider steady flow this is what is give this is what is your conventional Bernoulli's equation. This is what is the Bernoulli's equation no p plus half rho square equal to 0 solving a boundary layer equation now means what we need to solve boundary layer flow. What we need to solve the equations is the continuity equation. These are the two equations that you have to solve subjected to the boundary condition what are the boundary condition.

Student: (( ))

We have no slip boundary condition on the wall which is u equal to 0, v equal to 0 at y equal to 0 for all x for all t anything what else.

Student: At y equal to (( ))

At y equal to their delta u equal to u e, but do not know what is or say let us write it this way u at any x that at any y at x t, but we do not know delta this for very large y or very large y anything else.

Student: (( ))

Any other boundary condition there is a very important thing that this boundary layer flow is not distinct from the other flow there is no genuine edge of the boundary layer. The boundary layer at the outer inviscid flow they are same and only one identity they do not have separate identity. So, they must merge very smoothly the outer inviscid flow and the boundary layer flow smooth and for smooth merge smoothing, what do we need that all derivative at the interfaces must be equal all the derivatives first, second, third, fourth as many as you want. Of course, we would not be able to satisfy all up to infinite boundary derivative.

So, we will create some interface where might be some derivative discontinues, if we make first derivative continues the second derivative can still be discontinues which means not smooth merging. So, how many we can make that that will determine our smoothness the solution smoothness, but anyway what is required is that the derivative as you obtained it from two sides from the boundary layer side and the from the outer inviscid side is same or this may this may little confuse. You let say let us write it this way and then finally, we will write it let d u, d y at y equal to delta equal to d v d y, but this at y equal to delta this is equal to zero and similarly.

The second and third and all are can be written as zero actually if the required condition is that these two derivatives should be equal and we know that these are zero a smooth merging for smooth merging of two layers boundary layer and outer inviscid flow these are the required condition for smooth merging of the two in general that is for any general when there are two different thing and they merge smoothly all the derivative should be equal in this special case, we know that this derivatives are zero.

So, these are the equations that to be solved to solve the boundary layer flow and the outer inviscid flow, which we have discussed earlier how to solve outer inviscid flow of course, there will be many more methods that will be discuss later on, but at least we have discussed some of the basic things and once these two solutions are complete. We can say the solution flow solution is complete to some extent not complete, but almost complete because these two now as you can see at this will affect the others one will affect the other as you see this boundary layer solution. Obviously depends on the inviscid flow solution so, this term the inviscid solution comes in here.

Similarly, what this viscous solution going to affect the inviscid flow going to affect the inviscid flow. So, there is an interaction this interaction sometime may become weak sometime may become strong and if necessary these are this process has to be continued the solve inviscid flow find the inviscid flow pressure distribution use that inviscid pressure distribution to find the viscous flow solution. Once you find the viscous flow solutions your boundary condition for the inviscid flow solution will change so, recalculate your inviscid flow obviously you are going to find a new pressure distribution means a new boundary layer development until they converge and once you get that that is the complete solution within the framework of the boundary layer hypothesis.

However, all flows are not boundary layer flows all viscous flows are not boundary layer flows as you have mentioned earlier that only for high reynolds number that thing happen and even there are certain cases when even at high reynolds number. They are may not be a boundary layer flow the boundary layer to develop, we need opposition between convection and diffusion sometime they do not oppose there are some flow situation where they do not oppose in that case there will be no boundary layer.