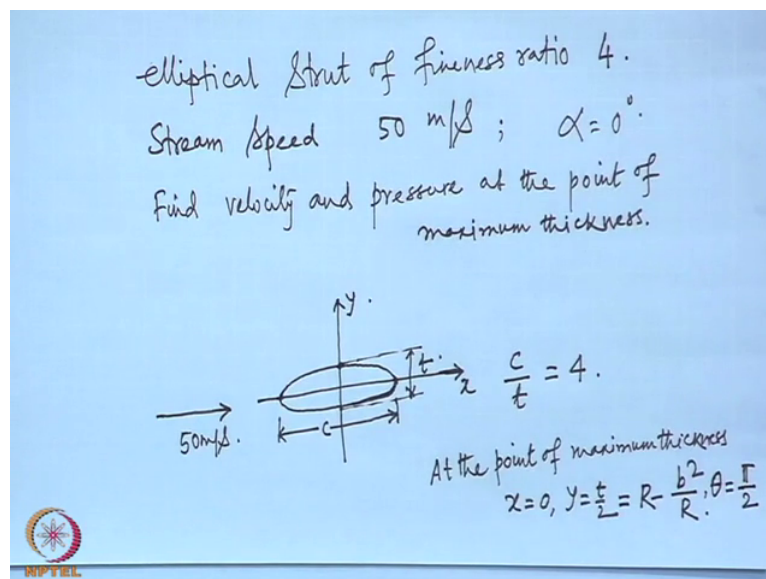


**Introduction to Aerodynamics**  
**Prof. K. P. Sinhamahapatra**  
**Department of Aerospace Engineering**  
**Indian Institute of Technology, Kharagpur**

**Module No. # 01**  
**Lecture No. # 41**  
**Transformation (Contd.)**

We will consider another problem it is consider a flow at speed 50 meter per second flowing over an elliptical strut at an angle of attack zero. We have to find the velocity just outside the boundary layer which in inviscid flow is happens to be on the surface of the ellipse, at the point of maximum thickness the ellipse has a fineness ratio of four. Elliptical strut that is a column with cross section is ellipse that is what is an elliptical strut is of fineness ratio 4 the free stream speed is 50 meter per second angle of attack is zero degree find the velocity at the point of maximum thickness.

(Refer Slide Time: 01:51)



So, you have elliptical strut .. find velocity and pressure at this point. This also comes directly from the velocity formula for the circle that we did earlier. So, if you remember you can directly use it instead of writing this expression you can directly write as the velocity at that

point is now what is the velocity on the ellipse this divided by  $d z d \zeta$  at theta equal to pi by 2.

(Refer Slide Time: 12:51)

Velocity on the ellipse  $w(z) = \frac{w(\zeta)}{\frac{dz}{d\zeta}}$

$$\frac{dz}{d\zeta} = 1 - \frac{b^2}{\zeta^2}$$

At  $\theta = \frac{\pi}{2}$   $\frac{dz}{d\zeta} = 1 + \frac{b^2}{R^2} = \frac{8}{5} = 1.6$

Required velocity  $v = \frac{100}{1.6} = 62.5 \text{ m/s}$

pressure  $C_p = 1 - \frac{v^2}{v_\infty^2} = 1 - \left(\frac{62.5}{50}\right)^2 = -0.56$

The slide also contains two diagrams of the complex plane. The left diagram shows a circle in the  $\zeta$ -plane with a point on the circle. An arrow points to the right diagram, which shows an ellipse in the  $z$ -plane, representing the image of the circle under a conformal mapping.

pi by 3 and (()).

Student: (()).

Pi by 3

Student: Pi by 3

Pi by 3

Student: Pi by 3

(()) pi minus 1 by.

Student: (()).

We have already used theta equal to pi by two now you are saying it is pi by th[ree]- three.

Student: (()) Pi (()).

Um.

Student: Root pi by three (()).

Which point is that.

Student: (( )).

So, this is the transformation let us have it once again. So, this is the point where we are interested to find the velocity to which point this has transformed theta equal to pi by 2.

Student: (( )).

This point.

Student: (( )).

What is this point?

Student: Pi by (( )).

Pi by (( )).

Student: i R.

Pi by 3.

What pi by 3?

Student: No pi by 3 (( )).

So, it is i R this becomes at theta equal to pi by two.. these I think will come as something see 62.5. Another pressure how to find the pressure. Let us try one more before we conclude this topic at this we will good example.

A 20 percent thick Zhukovsky airfoil set at zero incidence to a free stream of 10 meter per second a twenty percent thick Zhukovsky airfoil set at zero incidence. We have free stream of 10 meter per second has a lift coefficient of 0.3 find the lift coefficient at 5 degree incidence find the lift coefficient at 5 degree incidence and the velocity at the nose of the airfoil and the velocity at the nose of the airfoil

Next you have to find the velocity at the nose of the airfoil. So, once again same procedure you have to find what is the velocity on the corresponding circle at that point and divided by  $d z / d \zeta$  at that point. So, what is the point at nose at airfoil nose how much is theta at airfoil nose how much is theta?

Student: Pi.

(Refer Slide Time: 28:46)

points on transformed circle

$$\zeta = \zeta_0 + R e^{i\theta}$$

$$= -b\epsilon + i R \sin\beta + R e^{i\theta}$$

The point corresponding to airfoil nose is

$$\zeta_N = -R - b\epsilon + i R \sin\beta, \quad R = b(1+\epsilon)$$

$$= -1.308b + 0.0477bi$$

$$\left. \frac{dz}{d\zeta} \right|_N = 1 - \frac{b^2}{\zeta_N^2} = 1 - \frac{b^2}{b^2(1.7086 - 0.1248i)}$$

$$= \frac{1.2264 - 0.1248i}{2.935}$$

Pi points on circle points on transformed circles are given as zeta equal to zeta naught plus r e to the power i theta what is zeta naught?

Student: (( )).

Zeta naught is minus b epsilon plus R sin beta i R sin beta zeta naught is minus b epsilon plus i R sin beta plus R e to the power i theta. So, the point that corresponds to airfoil nose epsilon is known beta is known theta is pi. So, substitute those values theta is pi e to the power i pi. How much is that?

Student: Minus (( )

Minus.

Student: Minus 1

Minus 1

So, it becomes minus R. R also you can express in terms of b see there are two R and b. We have a relationship between R and b. R is how much?

Student: B into one plus epsilon

B into one plus epsilon

So, we will have only b that of course, we cannot find it will remain as b and you please check the calculation what I have got is something like this minus 1.3086 b plus 0.0477 b i. This zeta just to denote that this is this corresponding to the nose point we will just give a subscript n that this is the point corresponding to the nose of the airfoil this is the point on the circle that corresponds to the nose of the airfoil. Now we can find what is  $dz/dzeta$ ,  $dz/dzeta$  corresponding to that nose is 1 minus.. This has come to be I have left it like this 1.2264. Please check these calculations..

(Refer Slide Time: 34:51)

Velocity on the airfoil  $w(z) = \frac{w(\zeta)}{\left(\frac{dz}{d\zeta}\right)}$

$w(\zeta) = 2iQ_{\infty} [\sin(\alpha+\beta) - \sin(\alpha-\beta)] e^{-i\theta}$

$= -4.3086 i \text{ m/s} \cdot \left[ \alpha = 5^\circ, \beta = 2.37^\circ, Q_{\infty} = 10 \text{ m/s} \right]$

$w(z) \Big|_{\text{nose}} = \frac{-12.445i}{1.2264 - 0.1248i} = 1.04 - 10.2i \text{ m/s}$

$q = 10.26 \text{ m/s} \cdot \left[ \begin{matrix} u = 1.04 \text{ m/s} \\ v = 10.2 \text{ m/s} \end{matrix} \right]$

Now the velocity on the airfoil.. free stream is 10 q infinity is ten alpha 5 beta 2.37 theta pi. So, if you substitute that this has come to be minus 4.3086 i.. Excuse me sir the q infinity value that we used here as 10 meters so, that kind then dividend or the z plane. So, should it be transformed (( )). The free stream that q infinity will not change free stream speed will not change the free stream speed will not change.

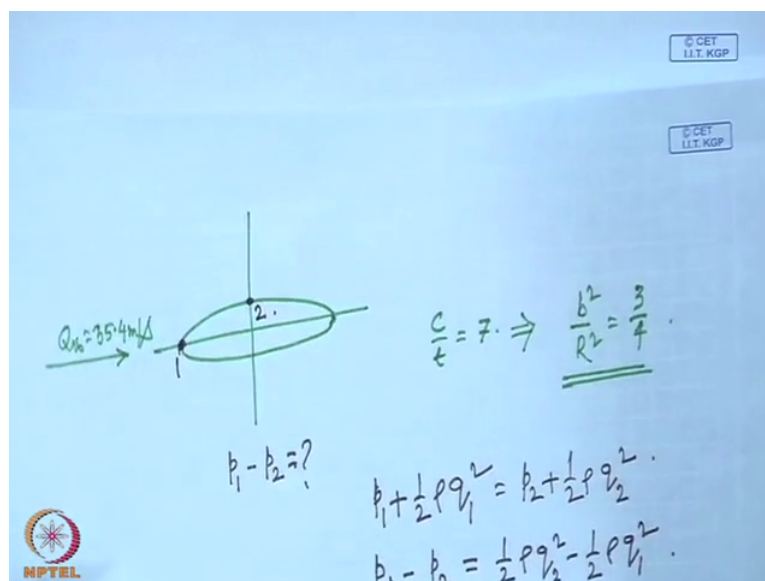
In both z and zeta that will not change we have never changed that the streamlines will change, but the magnitude of the velocity stream that will not change this has come to be 1. I

think I earlier told you how the pressure is measured pressure acting on a body how it is measured you have not done any experiment as yet see on the surface of a body or very small hole is drilled and then some probe usually it is just a tube is inserted through the body.

So, that it is flushed to that surface it is not flow through that from the surface it is just flush to the surface and through the hole that it is connected and there is of course, that tube is connected to another measuring device whatever it can be it can be a simple manometer or it can be a pressure transducer or anything. So, that whole serves the purpose of a sensor it sense the pressure and if the flow is tangential to the hole then that hole will simply sense the pressure the static pressure of course, however if the hole is normal to the flow then it will sense the total pressure the stagnation pressure which is  $p$  plus half  $\rho u$  square for incompressible flow.

So, the next problem deals something like this a long elliptic cylinder of thickness to chord ratio of 1 by 7 a long elliptic cylinder of thickness to chord ratio of 1 by 7 is set at it is set at an incidence of 0 degree which set at an incidence of 0 degree in an airstream of 35.4 meter per second calculate the pressure difference or calculate the difference between the calculate the pressure difference between calculate the pressure difference between pressure holes set in the nose and at the point of maximum thickness.

(Refer Slide Time: 43:07)



We have pressure hole here at the nose and the point of maximum thickness you have to find the difference between pressure holes pressure at these two points.. As before the way we

have done earlier these fineness ratio of 7 implies that  $b^2$  by  $R^2$  equal to 3 by 4. We have done it already exactly same this will give  $b^2$  by  $R^2$  equal to 3 by 4.

Now let us see what we have to find first before trying to do it, let us designate these two points as 1 and 2, 1 and 2 we want we want difference between pressure at 1 and 2.  $p_1$  minus  $p_2$  now let us this is an inviscid irrotational flow ideal flow so, the Bernoulli's equation applies everywhere Bernoulli's equation applies everywhere in the simplest form or what we have is  $p_1$  plus half rho if the velocity here is  $q_1$  and here it is  $p_2$  plus half rho. So, what we want in sense is  $p_1$  minus  $p_2$  equal to now  $q_1$  is zero  $q_1$  is zero that is the stagnation point of course, you can put it in the formula and check..

(Refer Slide Time: 48:25)

Check  
 Velocity at the nose,  
 note  $\Rightarrow \theta = \pi$   
 Velocity at the corresponding point on the transformed circle  

$$W(\zeta) = 2i U_\infty [\sin(\alpha+\beta) - \sin(\alpha-\theta)] e^{-i\theta} \Big|_{\theta=\pi}$$

$$= 0$$

$$\Rightarrow W(\zeta) \Big|_{\theta=\pi} = 0$$
 Velocity at the point of maximum thickness.  

$$q_{circle} \Big|_{\theta=\pi/2} = \frac{q_{circle} \Big|_{\theta=\pi/2}}{\left(\frac{dz}{d\zeta}\right) \Big|_{\theta=\pi/2}} = \frac{2 \times 35.4}{1+0.75} = 40.46 \text{ m/s}$$

We can also check it let us see that this is a stagnation point, we can also check velocity at the nose now nose corresponds to theta equal to pi. So, the velocity on the corresponding point on the circle.. alpha is 0 beta is 0 alpha is 0 beta is 0 theta is pi. How much is this?

Student: 0

0

So, this divided by  $dz/d\zeta$  is again 0 so, this is confirmed now what is the velocity at the point of maximum thickness see this time. We will not repeat it again already we have done. So, we will just use the final result..

If you remember how much it was?

Student: 2, 3 upon (( )).

2 into 35.4 divided by 1 plus three by four One plus three by four or that is 1.75.

(Refer Slide Time: 52:19)

$$p_1 - p_2 = \frac{1}{2} \rho V_2^2 = 1002.67 \text{ N/m}^2 \quad \text{for } \rho = 1.225 \text{ kg/m}^3$$
$$\approx 1 \text{ kPa}$$

for Zhukovsky airfoil,

$$C_m = -4\pi \left(\frac{b}{c}\right)^2 \sin^2 \beta \quad [\text{about Aerodynamic Centre}]$$
$$\frac{x_{Ac}}{c} = \frac{z_0}{c} - \frac{b^2}{Rc} e^{i\beta} \quad [\text{location of aerodynamic Centre}]$$

We will take it from there and that comes to be 40.46 and now we can complete that  $p_1$  minus  $p_2$  of course, we do not know what is the  $\rho$  let us assume that this fluid is air and take the standard value of air density which happens to be 1.225 kg per meter cube if no density is specified, but it is air then you can always use the density to be 1.225 kg per meter cube otherwise of course, you can leave it in the term of  $\rho$  it does not matter, but using that 1.225 this happens to be something 1 kilo pascal practically approximately 1 kilo pascal.

So, as the fluid has travelled or the air has travelled from the nose to the point of maximum thickness it has experienced a suction of 1 kilo pascal consequently it has accelerated from 0 to about in case of a circle of course, it is twice, but here it is how much from 0 to 40 at the nose the fluid velocity has become 0 from there it has accelerated to a forty with a simultaneous pressure drop of nearly 1 kilo pascal.

So, with this we will conclude our discussion on this conformal transformation at the transformation technique, but oh two things I will just ask you to do if you cannot you can discuss it later that is I will ask you to find the moment coefficient the lift coefficient. We have found for Zhukovsky airfoil I will ask you to find the pitching moment coefficient and



the aerodynamic centre if you remember aerodynamic centre I think we have defined earlier aerodynamic centre is the point about which the pitching moment is independent of lift coefficient or angle of attack.

There is a point about which the moment will not change as the angle of attack changes for any other point the pitching moment will be different at different angle of attack, but when you take moment about the aerodynamic centre the moment will not change with angle of attack. So, this I will ask you to do... about aerodynamic centre and the location of aerodynamic centre  $y$  this is  $z$ .