Introduction to Aerodynamics

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Lecture No. # 36
Zhukovsky Transformation (Contd.)

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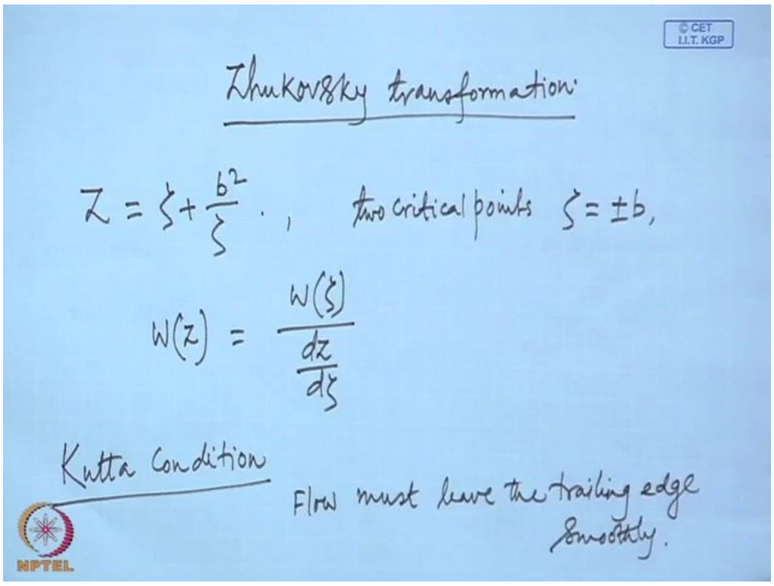

So, we consider Zhukovsky's transformation, which we considered now that the transformation will map an airfoil in the Z plane to a circle in the zeta plane. That is the geometry whatever is on the Z plane that we are calling an airfoil, and this is mapped onto a circle on the zeta plane. For the circle, we have two free parameters; the radius of the circle and its center and for this transformation, we have two critical points which are at zeta equal to plus minus b. So this transformation has two critical points.

Now of course, if we place the critical points on the surface of the circle then at those two points, the angle will not be preserved and we have seen that, the angle on the circle plane which is 2 pi will become 0 on the airfoil plane, which we have seen yesterday itself; that the angle on the circle plane at all the point is 2 pi on the surface of the circle, where sorry pi where the two curves are intersecting. This angle will transform to 2 pi and consequently, what we call trailing edge angle or the angle at that contained by the curve that becomes 0.

So, since there are two critical points, if we have both the critical points on the surface of the circle then, the transformed geometry will also have 2 such sharp corner; because at the critical points, the transformation is not preserving the shape, not preserving the angle and the smooth surface that they becomes sharp corner. So, if you if you have both the critical points on the surface of the circle then, we will get two such critical point or 2 such corner on the transformed geometry.

So at least for an airfoil which has a trailing edge, we must have one of the critical points on the surface of the circle. The other critical point of course, must not be in the circle on the surface of the circle, if we do not get, if we do not want two sharp corners. We say that the velocity the complex velocity will transform according to this rule; that complex velocity on the Z plane will become the complex velocity on the zeta plane divided by dz d zeta.

Now see, at the critical point what will happen? At the critical point dz d zeta is 0 then, unless w zeta itself is 0 then, W Z will become undefined. Now, since the flow over an airfoil is a real practical problem obviously, there will be not a point on the airfoil surface or anywhere in the flow fluid, where the velocity is infinite is not possible. In any physical problem, this infinity is not possible. So, since that point corresponding to the critical point on the circle is not a singular point on the airfoil plane, this W(zeta) must be 0 at zeta equal to plus minus V should be. That means, this will now give us or will help us to find the value of circulation. If you remember that, the circulation the value of circulation, we could not find for a cylinder, for a circle.

We said if the value of circulation is this then, the stagnation point will be here. If the value of circulation is this then, the value of then the stagnation point will be here and so on we considered various possibilities. But if just a given circular cylinder we cannot say what should be the value of circulation. But now, you see that when we think the flow about an airfoil, we can enforce that what should be the value of the circulation because of this requirement that, wherever we are making the critical point provided the critical point is on the surface of the circle that should be a critical stagnation point; that should be made as stagnation point; reason that if since that point is being transformed to the trailing edge of the airfoil and at the trailing edge of the airfoil, the velocity is not infinite. Eventually, this is known as a very famous condition called kutta condition which simply kutta condition k u t t a, a kutta condition which says that for an airfoil, the flow should be such that it will always leave the trailing edge smoothly it will always leave the trailing edge smoothly. See, what is the implication of that?

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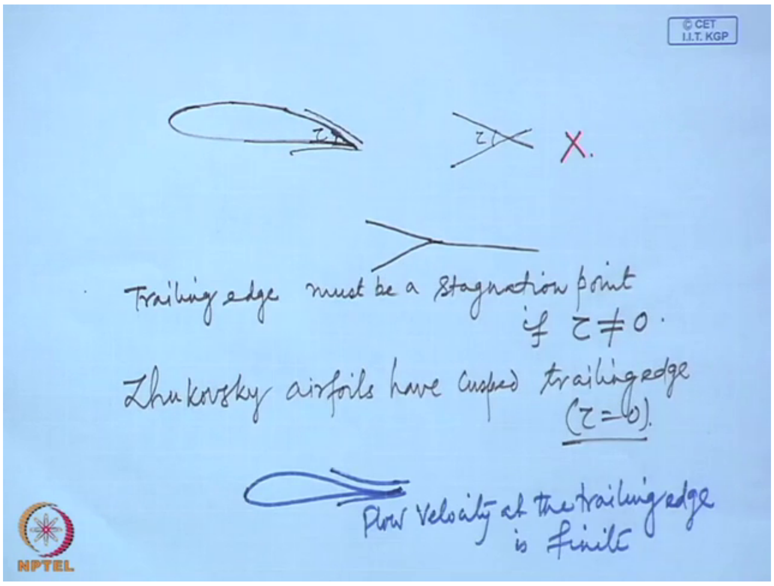

See, considering the physical problem, let us see, what do we get? Think about an airfoil or boundary condition states that, the flow must be tangential to the surface for an inviscid flow; the flow must be tangential to the surface. Of course, in a viscous flow it is no slip; that is everywhere the total flow velocity is 0; both the tangential and normal component as 0. But in inviscid flow, the flow is tangential to the surface both on the upper surface and lower surface. Meaning, that the streamline adjacent to the airfoil surface is parallel to the airfoil surface; the streamline, which is parallel to the sorry adjacent to the airfoil is parallel to the airfoil surface. So that is, if we consider a streamline on the upper surface that streamline is just say parallel to this, and on the lower surface also, this streamline is parallel to this.

So, these two streamlines what will happen following their original path; if we follow the original curve then, they will intersect at the trailing edge, but we know that 2 streamlines cannot intersect; the two streamlines cannot intersect. This gives that 1 streamline is coming like this; another streamline is coming like this. And at the airfoil, if the airfoil has a finite trailing edge angle tau, if it is a finite trailing edge angle tau then, these 2 streamlines is intersecting here. But we know 2 streamlines cannot intersect. So this is the situation that will not occur; this situation cannot occur; what will occur is something like this, but these 2 streamlines will merge and become 1 streamline smooth, and how is that possible? That is at the trailing edge, the velocity must be 0 at the trailing edge the velocity must be 0 or the trailing edge must be a stagnation point. That is when the trailing edge angle is finite; the trailing edge must be a stagnation point.

That is what is the meaning of that the flow must leave the trailing edge smoothly. However, we have seen already that Zhukovsky airfoils have a cusped trailing edge; that is the trailing edge angle is 0. What will happen in that case? The Zhukovsky airfoils are cusped and what is the meaning of this? Then let us see that consider a Zhukovsky airfoil the trailing edge angle is 0.

Now, the streamline on the upper surface and lower surface; we see they are now not intersecting. The streamlines on the upper surface and the lower surface they are parallel to the airfoil surface. Upper surface streamline is parallel to the upper surface; lower surface streamline is parallel to the lower surface and the upper surface and lower surface of the airfoil they are parallel to each other. Meaning that, these two streamlines are also parallel to each other. Then, it is not essential to have that velocity 0, but both the streamlines must have must represent same velocity because, this trailing edge point which is1 point at one point of course you cannot have two velocities. So, there will be only single velocity, but it need not be 0, it can be anything any finite value.

So in this case, this flow velocity of course, flow velocity at the trailing edge is finite; it is finite. Sometime it may be 0, but it need not be 0 in general. It can be anything a finite value. So, this is the implication of that kutta condition that the flow must leave the trailing edge smoothly. Essentially that, translates to that if the airfoil has a sharp trailing edge then, the trailing edge must be a stagnation point, but if the trailing edge is cusped trailing edge trailing edge angle is non-zero. In that case, the trailing edge need not be stagnation point, but the velocity at the trailing must be a finite value.

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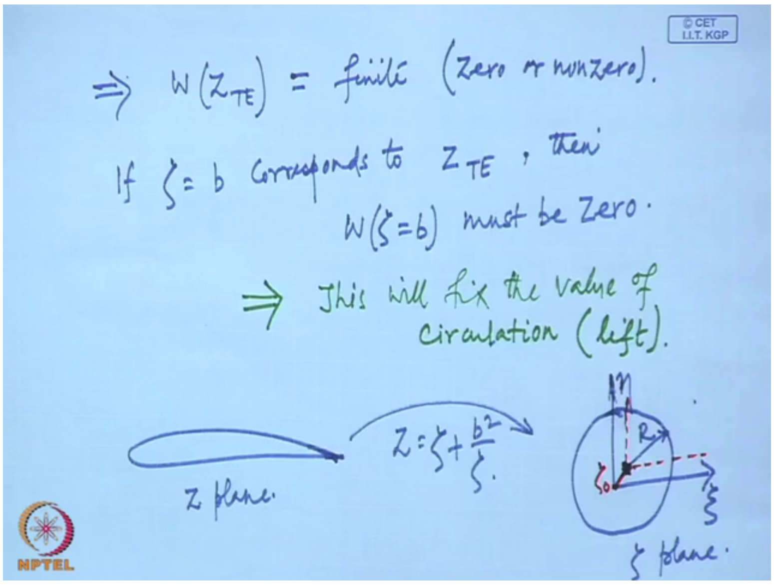

So now, going back that W corresponding to the Z trailing edge is finite; either 0 or non-zero finite 0 or non-zero. So, if zeta equal to b corresponds to sorry no. Let us come to this little later first of all point. So this is then and let us consider this is that b; this is what is b then, this point will become the critical point, and this is what will become this trailing edge point, and this angle let us say, take beta. The center of the circle is at zeta naught then, any point on the circle, how they are obtained? What is equation of the circle?

If the center if the center coincides with origin then, the equation of circle is Re to the power i theta; if the centers of the center coincides with the origin of the coordinate system then, it would have been zeta equal to Re to the power i theta, but now the center is at Z naught zeta naught then, what is what is the equation of the circle? Yes.

Zeta minus zeta naught equal to Re to the power I theta. Zeta minus zeta naught equal to Re to the power i theta. So that is, what is the point on the circle? What is the trailing edge point? Now, if we look back to that expression for the complex velocity on the circle plane complex velocity on the circle plane; circle plane or zeta plane. Remember that, expression W Z zeta equal to Q infinity e to the power minus i alpha the general case plus; plus what we had? i gamma by 2 pi 1 by zeta minus zeta naught plus or plus or minus; what are the expression? Minus Q infinity Q infinity R square e to the power i alpha i alpha know divided by zeta minus zeta naught square.

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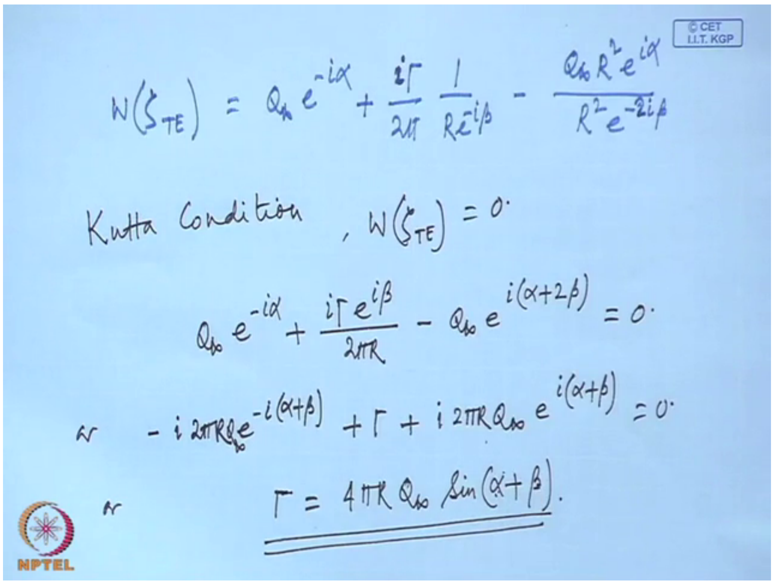

So, let us find out the velocity at the trailing edge. The first term of course, second term i gamma by 2 pi; so this is not 2; i gamma by 2 pi; the second 1 write everything Q infinity R square e to the power i alpha; R square e to the power minus 2 i beta. Now, according to kutta condition this is 0 Multiply it multiply throughout by minus i e to the power minus i beta 2pi R; multiply throughout by minus i 2 pi Re to the power minus i beta. What happened, if you multiplied by that minus i e to the power minus i beta 2 pi R and this Q infinity I have missed. So what is gamma now? Gamma equal to. 2 pi R Q infinity so much?

So much; look to that first and last term both of them are nearly same.1 is multiplied by e to the power minus i alpha plus beta; the other one is e to the power plus i alpha plus beta show out.1 either the cos or the sin will get cancelled, which one? 1 of them is multiplied by minus i, the other one is multiplied by plus i. First of all, you must remember that, this gamma without doing anything that this must have a real value. It will not have a complex value. It will have a real value and since these are multiplied by i that the sin will be associated with the real; sin will give the real part. So this is 4 pi R Q infinity.

4 pi R Q infinity sin alpha plus beta. So, this will be the circulation produced for the Zhukovsky airfoil or for the airfoil; 4 pi R Q infinity sin alpha plus beta. You see that, the circulation of course depends on alpha which is the angle of attack or the flow inclination with respect to the chord and also a beta. A relative location between the centre and the critical point or which has later become the trailing edge point of the airfoil. We will see later

on, what is the meaning of these; what it says? You see, at this stage, it tells the circulation only depends on the free stream velocity all right, angle of attack and this relative location between the centers and (( )) critical point or the trailing edge or the rear stagnation point; the centre and the rear stagnation point.

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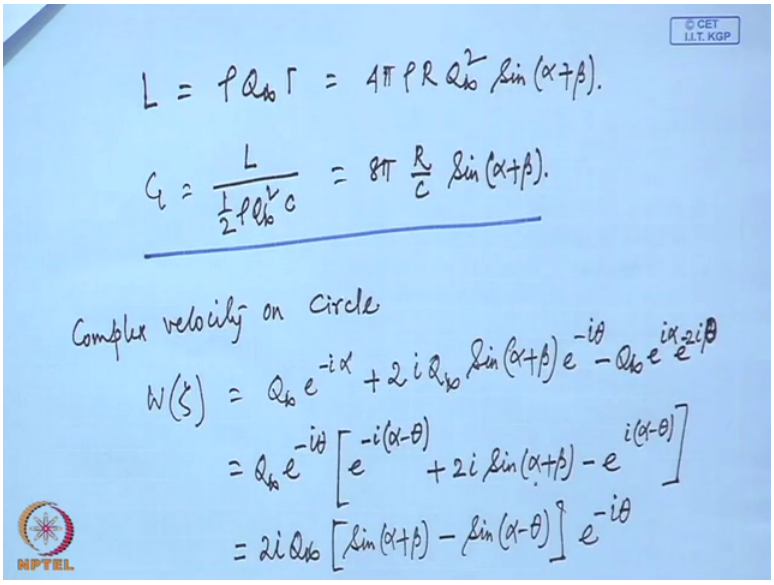

Now of course, we can find what will be the lift force what lift force is and the lift coefficient; the lift coefficient for the airfoil of course is the lift by half rho Q infinity square into c. The plan form area of the airfoil is c into 1 unit span; this lift is also per unit span and this gives what? Rho Q infinity square gets cancelled; it becomes 8 pi R by c.

So this is, what is the lift force or lift coefficient. Now, let us find the velocity at any other point what should be the velocity; now that we have circulation known what will be the velocity at any point on the first, the surface of the circle then, what will be the velocity on the surface of the airfoil. So the complex velocity on the surface of the circle; what we have? W zeta equal to Q infinity e to the power minus i alpha i gamma by 2 pi i gamma by 2 pi; gamma is 4 pi R Q infinity sin alpha plus beta; by 2 pi this becomes simply 2; 2 R Q infinity sin alpha plus beta, and what is that zeta minus zeta naught? R e to the power i theta

Now it is it i theta not beta; that R also get cancelled. So, what remains? What remains? 2 i Q infinity sin alpha plus beta into e to the power minus i theta, and what happens to the last term? Last term is minus Q infinity e to the power i alpha e to the power 2 i beta. Simplify; not 2 i beta, this will be 2 i theta know; beta is only for the trailing edge; you can simplify it.

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minus.

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Watch the last term. Yes, what happens if you simplify it? Let us take out this Q infinity e to the power minus i theta. Then, the first term becomes what e to the power e to the power minus i alpha minus theta. The second term remains 2 i sin alpha plus beta and the last term remains; the first and last term first and third term, what do they become? Minus sin alpha minus theta. Of course, a 2 i; so that 2 i also comes out; these 2 also gives 2 i sin alpha minus theta; so that 2 i comes out. So it becomes 2 i Q infinity sin alpha plus beta minus sin alpha minus theta into e to the power minus i theta.

So this is now, the velocity on this surface of the circular cylinder which corresponds to the airfoil. Remember, you cannot use it if you are dealing only with a circular cylinder. This is these you can use only when you are dealing with an airfoil that airfoil has been mapped to circular cylinder. Just a simple circular cylinder and you are talking about flow about a circular cylinder then, you cannot use this because, and in that case the value of circulation is not fixed. The circulation is fixed because of that airfoil; the circulation the value of gamma is fixed because of that airfoil not because of the circular cylinder; with the circular cylinder you cannot fix gamma.

So, these none of these formulas can be used for a just a circular cylinder. Now, let us find the velocity at the trailing edge quickly;

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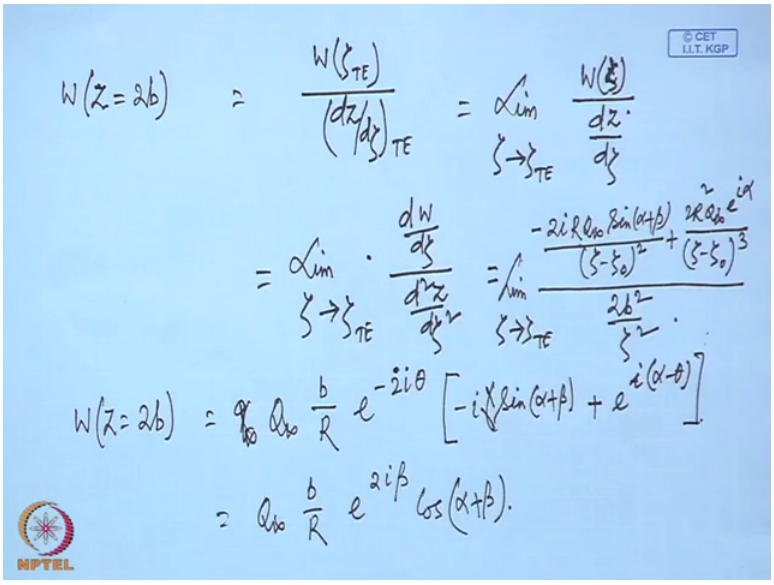

How much will be the trailing edge in the Z plane location of the trailing edge? You see, the trailing edge zeta equal to b; so the transformation gives Z equal to 2b. This of course, is indeterminate this of course, is an indeterminate. So, we take it as a limiting case, and this will be what? W sorry W; how to find the limit then, it is known to you; you can use that (( )) formula and that gives. Now, we can find what it is. Then of course, you can substitute zeta minus zeta naught as R e to the power i theta, and gamma is simply sorry Q infinity b by R e to the power minus 2 i theta. This is 2 that dot as nothing it is, but 2 i theta into.

So, as we discussed that at the beginning for a Zhukovsky airfoil which is the cusped trailing edge, the velocity at the trailing edge is not 0, but is a finite; you can see this is the finite value. Now, we will consider some specific cases. Right now, we have considered an arbitrary location for the centre and the trailing edge; now we will consider certain specific cases. As an example, if we if the centre of the circle coincides with the origin or if it is shifted somewhere along the real axis or along the imaginary axis or at certain angle to the both, what type of airfoil or what type of shape we are likely to get, we will consider those specific examples.

The first case we will consider is, where the origin coincides with the centre; that is that zeta naught is 0, and the parameter b equals R; what will happen then, if the parameter b equals R the radius of the circle then, both the critical point are on the surface of the circle. When b equal to R and the origin is placed at the centre then, both plus b and minus b they are on the surface of the circle; the front point and the rear point and, obviously the geometry in the real plane or the in the Z plane will have 2 sharp corner corresponding to Z equal to b, and corresponding to Z equal to minus b both.

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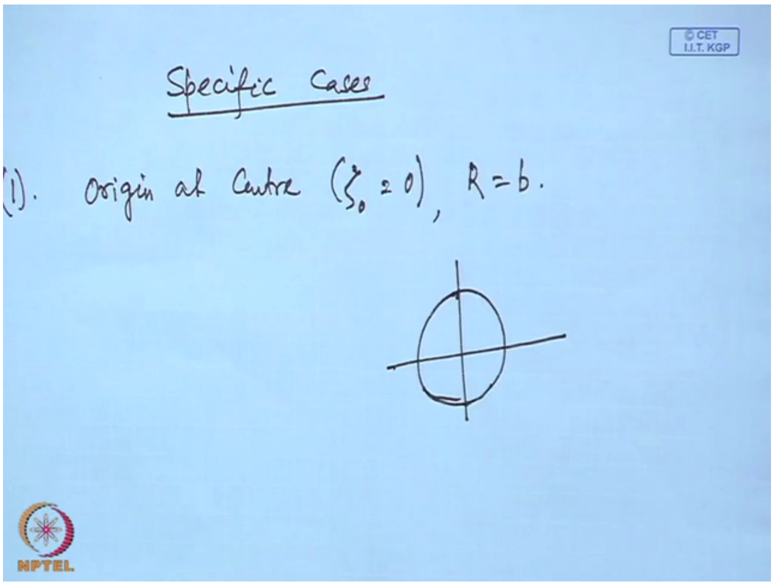

Now of course, we will see what that geometry looks like otherwise. Anyway, we would not be able to finish it today. So, we will stop here and we continue with this in the next class.