Introduction to Aerodynamics

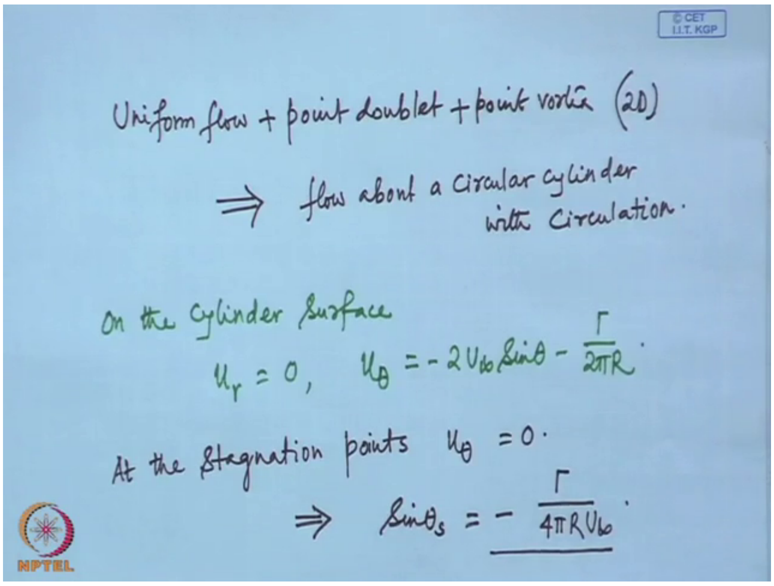
Prof. K. P. Sinhamahapatra
Department of Aerospace Engineering
Indian Institute of technology, Kharagpur

Lecture No: # 32

Potential Flow-Combination of Basic Solutions (Contd.) Lifting Cylinder

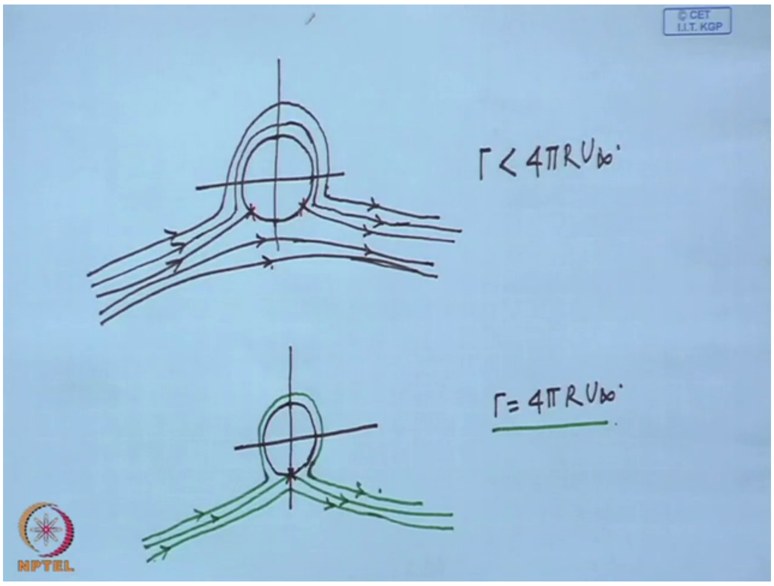
So, we are discussing the combination of uniform stream, and in finite line doublet and infinite life vortex or in two-dimensional sense a point doublet point vortex. We saw that the resulting flow is a flow about circulars cylinders with circulation. We have considered deliberately a clockwise circulation; the idea is why we have consider clock wise circulation, so that we want to have higher velocity on the upper half upper part of the cylinder, then the lower part of the cylinder; we have seen without circulation, the flow about circular cylinder is perfectly symmetric. The flow velocity on the upper half and the lower half are exactly identical, we have used a clock wise circulation as a result you can see if you look to the velocity expression, which we have written on the cylinder surface, we have written say lets write it uniform flow plus point doublet or in two dimensional sense..

(Refer Slide Time: 01:33)



And I think the velocity expression we computed the velocity expressions were on the cylinder surface. And the tangential component of the velocity is and this of course, satisfy the boundary condition that the radial component of velocity which itself is the normal velocity is 0 and this tangential velocity. Now first let us first locate the stagnation points, now u r is 0 at all points on the cylinder so at the stagnation point what we need is that u theta should be zero. So, at the stagnation point u theta equal to 0. And if the stagnation points are denoted by theta s, then what is theta s. What is the location of the stagnation points yes sin theta s equal to this gives the location of the stagnation points.

(Refer Slide Time: 05:02)

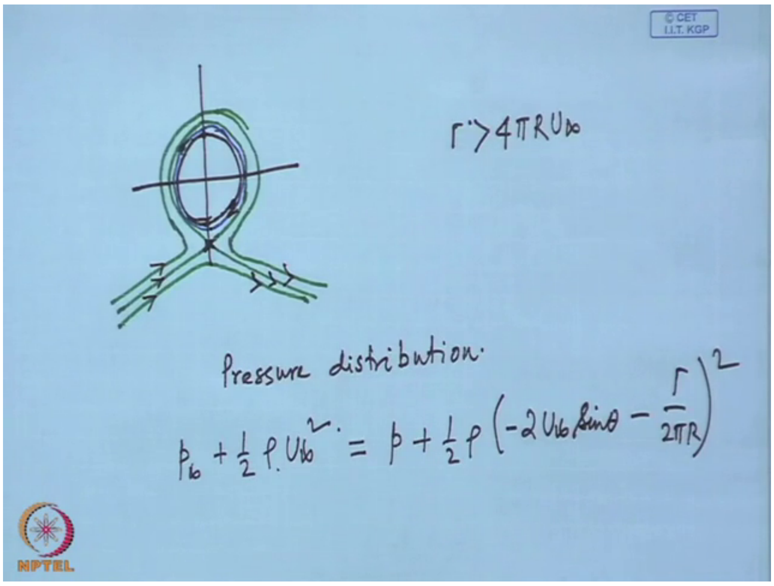


So, where will these stagnation points be if we where are these stagnation points. Third and fourth quadrant so there will be two stagnation points are the third and fourth quadrant. Say here let us take it one is here the other is let say here, and the streamlines will be now this. Now this of course, we cannot say what will be the exact value of gamma, we have nothing here to say what should be the value of this circulation gamma, because we really do not know that for a given cylinder where will this stagnation point be. So, depending upon the value of the circulation gamma depends on the location of the stagnation point or vice versa, the location of the stagnation point depends on the value of the circulation. Without knowing one we cannot determine definitely say, what is the value of the other? As an example, if gamma is equal to four phi r u infinity what will happen then?

See, this is what we have seen when gamma is less than 4 pi r u infinity this is the situation when gamma is less than 4 pi r u infinity. If now gamma is at 4 pi r gamma equal to 4 pi r u infinity, then you see that this sin theta s has value minus 1 meaning that both the stagnation point will be then here. There two stagnation points now coincides here, and if we want to have some streamlines these are the flow streamlines two stagnation point now coincides.

If the value of circulation is higher than this, there will be no stagnation point. There will be no stagnation point on the surface of the cylinder the value of theta is then imaginary however the stagnation points as it happen it will come down on this particular axis, but below the cylinder.

(Refer Slide Time: 10:37)



You can even show a situation that you see we have a here a circulars streamline some circular streamlines around the cylinder. So, you see that means, that the fluid here, in this part fluid in this region will not flow it will just rotate with the cylinder rotate around the cylinder. So, these are not imaginary situation the of course, the situation differs slightly, but this can happen this circular cylinder with a circulation we you can create that flow, just by rotating cylinder flow about a rotating cylinder or a spinning cylinder is an example, by this and where that rotational speed of the cylinder will represent this circulation, as you can increase the rotational speed by which is equivalent to this gamma you can create all this situations.

Even you will find that, if you increase the spinning velocity of the cylinder or the rotational speed of the cylinder a situation will come when some fluid close to the cylinder, will just rotate with it will not flow, because that corresponds to this situation. Where the streamlines representing the general flow they are these but, there are some fluid within this close cylinder close streamlines. Now the general nature of the flow say these are there are many practical examples, also and some of these perhaps you have heard of it or you have used it, you have seen it may be in case of a cylinder or sphere. The sphere as you have seen in that non circulation case without the circulation for sphere and cylinder we have seen what is the difference in the flow pattern, as such the flow pattern is similar only in three dimension case it is the flow is little relieved, not as much stressed as in two dimensional case, because of the additional direction available to it.

So, when there is circulation also the same thing happens, even the presence of circulation that three dimensional reliving effect is present rather that is that is always present three dimensional reliving effect. However this basic nature still remains the same, and you have often heard of that swing of cricket balls and all there all example, of this where some sort of rotation is introduced to the ball. So, that it behaves like a spinning sphere or rotating sphere. Even in the earlier days of navigation long back few hundred years back people use this effect to propel ships or boats by rotating a cylinder. Even now a day's rotating cylinders are used say there are some high speed automobiles which used rotating cylinders to reduce fuel consumption. The basic principle is this you will see that this will now produce a lift force; again lift force is in the direction normal to the flow. so what we call lift sometimes it can be even side rather in case of swing of a cricket ball that lift force is basically a side force, but again it is normal to the flow direction anyway lets.

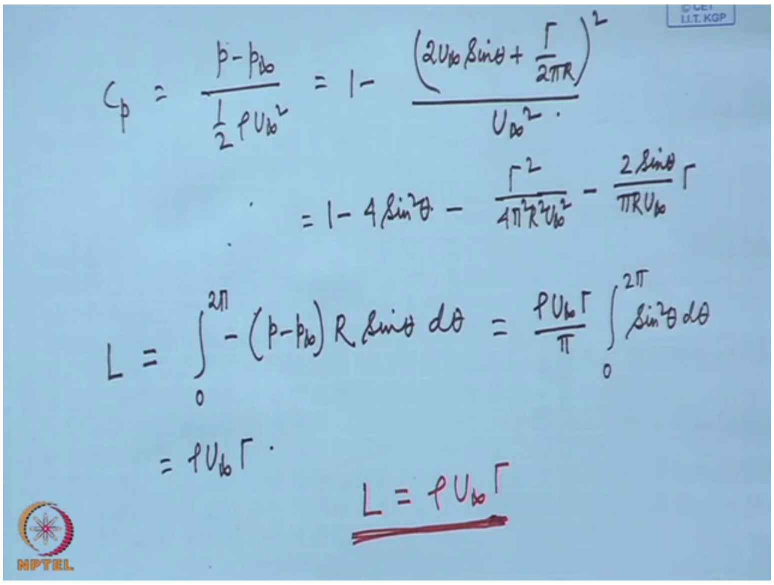
Now write the pressure coefficient. What will be the pressure coefficient? Pressure coefficient pressure distribution or pressure coefficient on the surface of the cylinder at far away we have p infinity plus half rho u infinity square we are not writing rho infinity there is not point, is on the surface of a cylinder let say at any point pressure is p and the velocity is the radial component is 0 only tangential component gives the total velocity. So, how much is that half rho how much is that velocity. Minus.

Student: Minus 2 u infinity sin theta.

Minus.

Student: Minus gamma by 2 pi r

(Refer Slide Time: 18:41)



So, what it will be p minus p infinity c p equal to p minus p infinity by half rho u infinity square 1 minus. 2 u infinity sin theta plus gamma by 2 pi r square divided by u infinity square. So, how much is that 1 minus 4 sin square theta term 1 is already is there, which was the c p for without rotation or without circulation and in addition what we have.

Student: Minus gamma.

Minus gamma square by, minus gamma square by.

Student: 4 pi r. 4 pi r u infinity square.

Yes. 4 pi r square, 4 pi r square u infinity square.

Yes minus 2 sin. Minus

Student: 2 sin theta minus 2 sin theta gamma by.

How much it is coming the radiational term?

Student: Minus 2 sin theta.

Divided by pi r u infinity.

2 sin theta by

Student: Pi r u infinity.

Into

Student: Into gamma this these say additional terms.

The pressure distribution is now no longer symmetric on the top and bottom. Now calculate again the lift force, lift force per unit span. What it was 0 to 2 pi minus p minus p infinity r.

R sin theta d theta we can substitute what is p minus p infinity and complete this integration. How much it is coming let's carry out this integration yes how much it is coming?

Student: 2 root 2 infinity minus.

2 root 2 infinity minus. 2 now the 2 will not be there.

Student: 2

2 will not be there. So, we have got this very important result and very general results that lift is rho u infinity gamma or most importantly that lift is linearly proportional to gamma. So you see the importance of circulation.

And we have seen the purpose of these doublet source and vortex. What we have seen that doublet source vortex they are singularity in the flow, and in incompressible flow also we have saying that there cannot be any expansion where source and sink are basically expansion singularity in expansion, but you see now we do not have source or sink in the flow the source or sink we are just imagining at the center of the cylinder that is all. It is our in it is in our imagination even the singular vortex is also in our imagination we are assuming that there is a the effect of the cylinder is such that we have a point doublet and a point vortex at the center that is all.

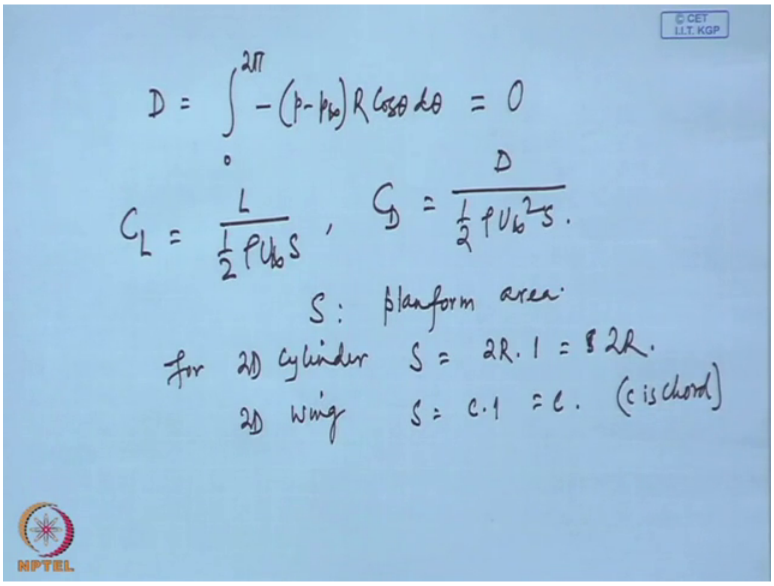
So, it does not mean, that flow over a wing later on now we will see that flow over a wing also will be getting through source sink doublet, which does not mean, that in an aircraft wing if you open the wing you will find some source and sink. Those source or sink are in our imagination we assuming that is the effect of the wing can you find the drag force.

Again you can integrate same thing only cost theta. That p minus p infinity r sin instead of r sin theta it will be r cost theta now.

Student: Sir, minus.

Yes how much is the drag force

(Refer Slide Time: 27:53)



0 that is the failure of classical theory, that no matter whatever you do or two dimensional ideal flow will give 0 direct. And one need to think that perhaps two dimensional ideal flow assumptions are not adequate. We have come here through a some assumptions know we have assumed that the flow is incompressible, flow is in viscid, and flow is irrotational. Obviously there is some force somewhere in those assumptions. We have assumed that the flow at very high Reynolds number the viscous effect will be confined within a very very thin region which is basically negligible over the when we considered the entire flow field. When we considered the entire flow field then that very very very thin reason near the body is really negligible, And we see that as far as finding velocity at least if not for circular cylinder, because the circular cylinder case is a little different even though we have solved the flow for circular cylinder, but we will tell why circular cylinder is a little different from others, but when we apply these to say airfoil like bodies we really get very good result as far as lift force is concerned even the pressure field velocity field, but when we come to drag it gives straight away 0.

Even for a circular cylinder which has a very high drag coefficient, usually this lift and drag they are expressed in terms if coefficient which we have defined earlier. Lift coefficient is lift force divided by that half rho u infinity square into an effective area. For a wing it is used plan form area for a cylinder or for a finite cylinder for finite cylinder of course, again you can take the plan form area for an infinite cylinder usually unit width is considered. So, it is only the area is simply diameter diameter into 1. So, that when we divide this lift to a drag force by that that gives the lift coefficient or drag coefficient say you can write c l as lift force by half rho u infinity square s.

Similarly, C D is s is that plan form area usually or some reference area. So in this case of course, we find C D is 0. C D is 0, for wing or so the usual c d is of the order of 0.01. So, instead of 0.01 if we get 0 we can say it is a perhaps very close to it, but in case of a cylinder, the c d is of the order 1 or even higher of the order of 1 or higher. That also we are getting 0 eventually anything we try C D will come to be 0 the drag force will come to 0. The reason at this stage I can tell you that what we have neglected is 1 boundary condition not that we have dropped one term in the equation that is not that serious. The serious thing, because we have dropped one boundary condition we are not satisfying no slip boundary condition. On a body surface we have said that normal velocity is 0, but in actual case in real viscous flow it is both normal velocity as well as tangential velocity both should be 0 on the body surface, which we cannot satisfy in our in viscid flow calculation.

Because, we have dropped that second order term as a result we are not been able in a position to satisfy two boundary conditions. So, even though viscous effect is really small the effect is really small, as far as lift is concerned the effect is really small. It hardly changes the lift coefficient may be a few percentage five to six percent decrease in lift coefficient, because of that viscous effect. So, as far as lift is concerned this theory gives a very very good estimate of lift only a small error, but as far as drag is concerned it gives 0. Of course, later on when you go up to three dimensional computations we will not do it here, we will see that we get a different type of drag, but that is all together a different proposition.

Now what else similarly, superposition of these singular solutions or basics solutions, we can produce some other flow features or some other important flow, but we will not go to this we will conclude this discussion with this warning that. We can get flow over certain bodies by just superposition, but to give when you have given a certain body that means, if you are asked to find the flow over a certain body, you are given the body you cannot say what superposition you are going to use here it is. We have a combined uniform stream and a doublet we have got a cylinder, but given any general shape you cannot say what should be the combination straight away and cannot write this.

So, if you want or interested in finding flow about an airfoil or wing. In general you would not be able to say that what should be the combination or what should be the correct combination or whether there is only one simple combination, that will give us the flow about a airfoil or wing. In this case as it happens that it is more or less like an accident, that we have assumed we have combined something and which we have seen satisfying some boundary condition. Like a uniform stream and doublet it satisfies the boundary condition for flow about a cylinder. We did know it before starting. Similarly, for a rankine oval we have seen that, we have a uniform stream source and sink that satisfy the boundary condition for an oval separate body.

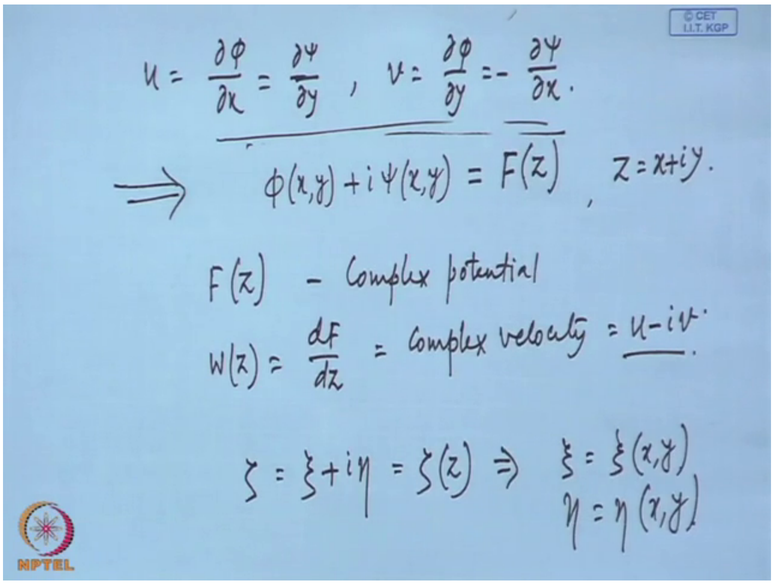
So, it took that to be the solution of an oval separate body. Now given an airfoil again the boundary condition is at normal velocity is 0 normal component of velocity on the airfoil surface is zero. Of course, now this boundary condition become complex, because of the direction of the normal, which is changing from point to point of course, we can write what would be the mathematical expression for that, but any way this boundary condition is now little much more complex than the earlier boundary condition where we would say that radial component of velocity is 0 for a circular cylinder.

So, what we see that for a given body it is usually not possible straight forward manner to specify what should be the combination of the basic solution. If we want to do it, we have to assume some sort of combination and then, forcefully satisfy the boundary condition to find what would be their strength. That approach of course, we will be doing in the next course not in this course, where for any given body we will just assume some sort of distribution or whatever you feel like, and then force the boundary condition on it. So, that we get the appropriate strength and your flow problem is solved, but that needs our numerical technique it is not an analytical technique it is a numerical method. However alternately we can think that the solution that we know, if by some mathematical technique from this itself we can find the flow about another body.

Like say we know the flow about a circular cylinder lifting non lifting when there is no lift the flow is called non lifting when there is lift the flow is called lifting. So, we have both lifting flow and non lifting flow lifting flow is uniform stream doublet vortex non lifting flow is uniform stream plus doublet. Now if we can somehow transform this cylinder to another body let us say we are interested in airfoil, if we can transform this cylinder to an airfoil and consequently the flow field is also transformed by that transformation, then our solution is complete we have the solution. And this was one approach perceived long back perceived long back and we will consider one such approach, one such transformation technique by which we will be able to transform the flow field about the circular cylinder to flow field about a airfoil.

However to apply the transformation we will again go back to our complex analysis. If you remember earlier we talked about that by definition that stream function and potential function satisfy the kosirina condition for analyticity.

(Refer Slide Time: 40:56)

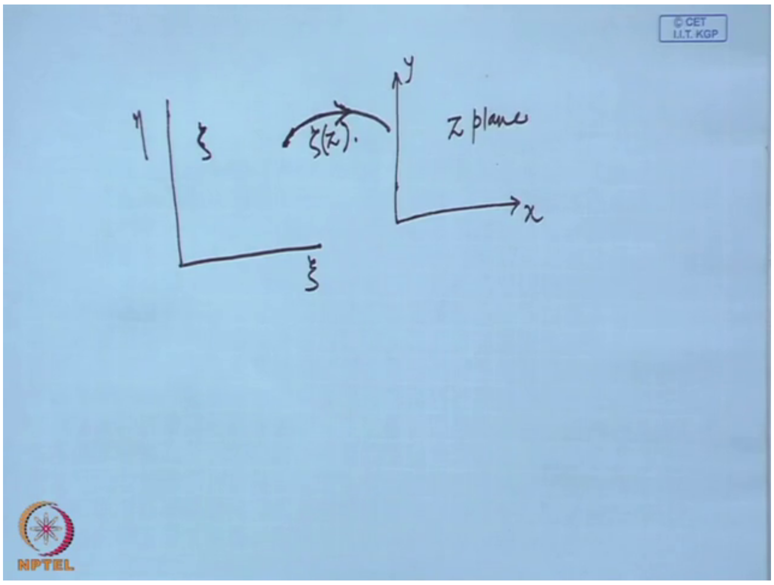


If you remember that u equal to d phi d x equal to d psi d y and v equal to d phi d y minus d psi d x which implies that phi which is a function of x and y and psi which is also a function of x and y. That phi plus i psi is that this f is an analytic function of z. The phi plus i psi is an analytic function of x plus i y, that is what the meaning of this, in terms of complex analysis the meaning of these is phi plus i psi is an analytic function of z where z is x plus i y. And then you defined this f z to be the complex potential and d f d z to be the complex velocity, which happens to be u minus i v. So, f z is complex potential and we defined w z I think where we write, remember this is even though we are calling complex velocity it is not u plus i v it is rather u minus i v. This complex analysis is of course, restricted to only two dimensions we cannot use it for three dimension.

Now, any function can be thought of as a mapping, any function can be thought of as a mapping even your simple function y equal to x square, with this you map x onto y x become y, because of this transformation. So, it is a transformation. So, any function as such can be thought of as a transformation. So, if we have a complex function of z say another complex function of z, let's say zeta zeta z where zeta is xi, xi plus i theta let us say zeta. So, this is a transformation this is a transformation from z and we can call it like say z x y defines a coordinate plane which we call the z plane in the complexed analysis that is what is called the z plane. Similarly, this zeta zeta plane in which xi and zeta can be thought of as the coordinate directions, x and y are the coordinate axis or coordinate direction in the xi z plane. Similarly, xi and zeta can be thought of as the coordinate axis in the zeta plane.

So, this function zeta equal to zeta is a function of z transform z to zeta. Like the function y equal to x square transform x to y similarly, z zeta is a function of z transform z to zeta. So, if we have a curve in the z plane, it will be transformed to another curve in the zeta plane. Now let us say this transformation function is at analytic function, the function which is used for transformation is an analytic function. Then what can you say about the property of the transformation. Yes.

(Refer Slide Time: 46:36)



The complex number z x plus i y represents a point in the z plane. Similarly, the complex number xi plus i theta represents a point in the zeta plane. A point in a transformation of course, there the transformation need not be 1 to 1 there is an 1 to 1 transformation and 1 to many and many to 1 there are different type of transformation. 1 to 1 means, for a single point if you get only point it is one to one. If your transformation is such that for each z point you get one zeta appoint then it is one to one transformation or one to one correspondence, but you may have one to many for a given one point one given point you may have two points. Like as simple as that y equal to root x. If x is say 4 for y you get plus minus two both, y equal to x square is many to one. You give x equal to plus two or minus two you get y equal to 4 many to one.

So, all transformation can be different type but, for say time being let us consider that one to one for simplicity. So, if you have one point in z plane by this transformation we will get one point in zeta plane. So, if you have a curve curve is of course, just a combination of a set of points. So, we will have another set of points on the zeta plane which will also give a curve, the shape of the curve may be different. Now what we are saying that these functions the transformation function is analytic function, which satisfy a very special property. What can be that property you have already you have through a course on complex analysis? Laplace equation. Cauchy-Riemann condition.

Cauchy-riemann condition is the condition for analyticity, whether a function is analytic or not you check it by cauchy-riemann condition. What is that we have already written for phi psi. The most important properties of transformation through analytic function is, that this transformation is angle preserving. This transformation is angle preserving it preserve the angle. As an example, think about you have two intersecting curves. Let us consider that you have two intersecting curves, if you transform it again you will get two curves, but in general transformation that those two curves may not also intersect is it, but if it one to one they will intersect they will intersect after the transformation also, because the point of intersection will be transformed to 1 point only they have to lie on both the curve. And after this transformation the angle between the curves will remain the same.

So, now think about these two curves, these two curves need not be an isolated curve even one curve can be thought of as intersection of two curves. What you are seeing as one curve, say arc of a circle, to it appears as one curve, but I do not think that it is also a intersection of two curves interaction of two curves, half of the arc I take as one curve another half I take as another curve. So, at any point we can think that two curves have intersected and the angle there will be preserved. Try to think what will be the implication of it transformation through analytic functions have a very popular name it is called conformal mapping or conformal transformation. Transformation by analytic function is called conformal mapping or conformal transformation. And the literal meaning of conformal is angle preserving that is why this name. Think it this way now think about that you are a transforming 1 curve in the z plane, let us say in this z plane this curve is a circle. And you are transforming it by a conformal transformation or by an analytic function. At each point on the circle you can think that two curves have intersected, and after transformation these two curves will again intersect and the angle will remain same. What type of curve you can get. Same shape same shape.

Same shape not necessarily the shape is preserved in infinitesimal sense not in large sense and infinitesimal portion of the curve will retain it is shape. So, an infinitesimal straight line will remain as infinitesimal straight line, but when it become finite it may change. It will remain closed, remain closed. It will remain closed it is definitely.

It will remain closed infinitesimal shape will remain same, but size need not be. The size need not remain same then, now can you tell what what we can get. yes At any point on a circle two curves are intersecting at an angle pi the angle is pi. After transformation also it will intersect at pi, if it is a transformation through an analytic function. The shape in the infinitesimal sense will re[main]- remain same that is infinitesimal straight line and infinitesimal part of the curve is a straight line it will remain straight line. But its size may be in changed. It will become an ellipse circle. It become it will will become ellipse circle is of course, a special case of ellipse only anyway we will continue this in the next class. Conformal mapping.