Introduction to Aerodynamics
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Lecture No. # 31
Potential Flow-Combination of Basic Solutions (Contd.)

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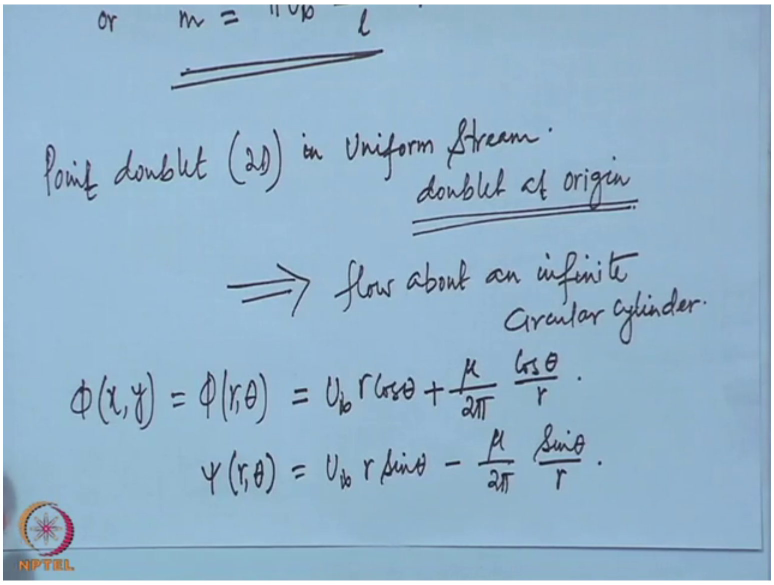

So, once again let us write the potential function and steam function at a point, so potential function at any point x, y. What it will be? It will be convenient to works now in terms of r and theta. So, we will write r, theta, the potential function in terms of r, theta is. How much? Once again you uniform stream along x direction. So, the potential function is u infinity x or u infinity r cos theta, and doublet mu by 2 pi, mu is the doublet strength mu by 2 pi cos theta by r, if you want to write psi...

What is the zero streamline? What will be the zero streamline? See eventually as we should be take note of this point also, that we can add any constant to this phi or psi does not alter the solution, because when we even if you have a constant here, those are velocity components, those will not have that constant. When you differentiate to find the velocity field, then the velocity field will not have that constant. So, velocity field is still unique, phi might have some (( )) constant that is of no importance, anyway with this what will be the zero streamline?

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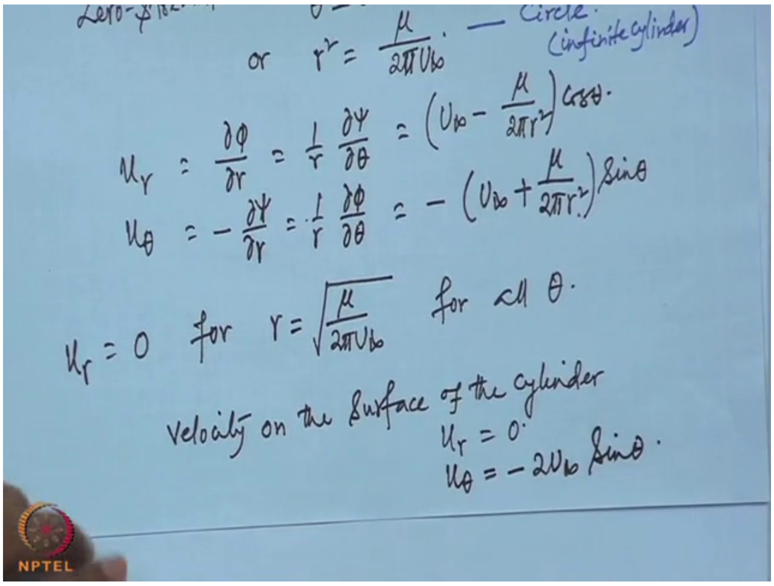

If, we take that constant to be zero. What is zero streamline?

Student: (( ))

Theta equal to 0 is of course, a zero streamline.

Student: (( ))

Or.

Student: (( )) r square equal to mu by 2 pi u infinity

r square equal to mu by

Student: 2 pi u infinity

2 pi u infinity. What is this? r square by r square equal to mu by 2 pi u infinity; that is a circle. So, zero streamline either the x axis itself, the x axis itself is a zero streamline and also a circular, still let us make it, let us, confirm whether these circle can be represented by a solid body, if it is a solid body then, it will satisfy the boundary condition also if we want to represent this zero streamline or the circle by a solid body then our boundary condition also

must be satisfied. What are the boundary conditions? That the flow must be tangential, or the normal component of the flow velocity must be zero. So, let us find the velocity component. What are the velocity component u r and u theta, let us say u r and u theta u r you can find either as a d psi d r d phi d r or 1 by r d psi d theta 1 by r d psi d theta either fine d phi d r or 1 by r d psi d theta. What is u r? And similarly, u theta. So, is the normal component of velocity zero 1 that circle?

Is it not? That u r is zero and u r is eventually the normal velocity, the radial component is normal to the circle. So, the normal component of velocity is zero; when r is this. So, on that circle particularly that closed circle the normal component of velocity is zero. So, that it satisfies the boundary condition there and that circle can be replaced by a solid body and then the doublet does not exist. That circle is the effect of the doublet while the actual doublet is not there, as in case of those oval and all when you replace the oval that closed curve by a real oval, those point source and point sink they do not do not exist, but their effect is there given by the oval similarly, once we have this circle, if you have a circle that gives the effect of the doublet, if we have a circle in the uniform stream then, this circle is the same as the effect of a doublet.

So, this is now flow past infinite circular cylinder, the velocity field on the surface of the cylinder, now we will call it the cylinder. So, the velocity on the surface of the cylinder the radial component is zero and u theta is of course, now the tangential component. The tangential component is given by this you can substitute now what is r.

How much is u theta? How much is u theta?

Student: (( ))

(( ))

Plus or minus, plus or minus minus 2, this is remember, this is only on the surface of the cylinder, or circle now the complete flow field, the complete flow field of course, now we can write the complete flow field for a circle. the complete flow field for a circular cylinder

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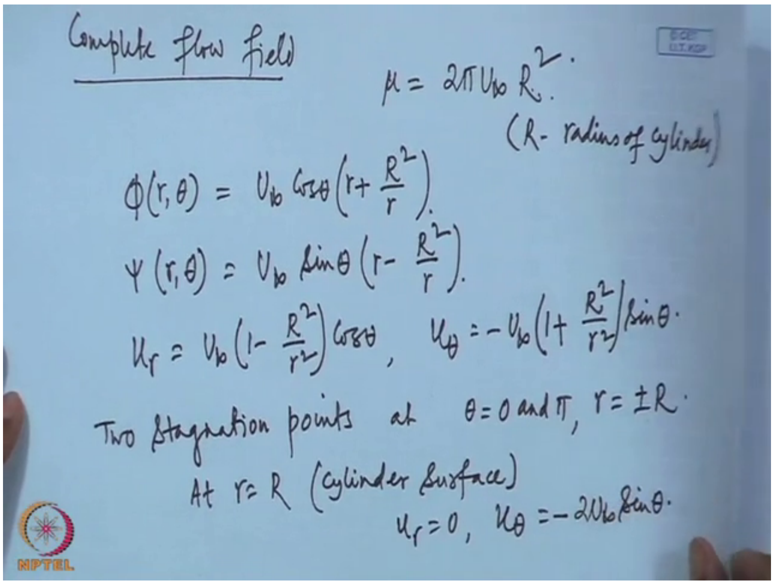

What should be phi and starting from... So, we have a cylinder we do not have any doublet. So, we should not use mu, but what is mu? mu equal to 2 pi u infinity r square, instead of this small r we are now writing that small r is a variable it is just a coordinate for the cylinder the radius is r. So, what will be phi?

Again the stagnation point will be at the 2 tip of the diameter along x axis, at the tips of the diameter along x axis just like in case of oval. So, that is at theta equal to zero and theta equal to phi, 2 stagnation point at theta equal to zero and theta equal to phi, x equal to plus minus r which you have already done on the surface of the cylinder.

Let us now find the pressure on the surface of the cylinder at any point in the flow field of course, you can find using this u r and u theta, but it is more important to find them on the surface of the body. So, we will find pressure on the surface of the body.

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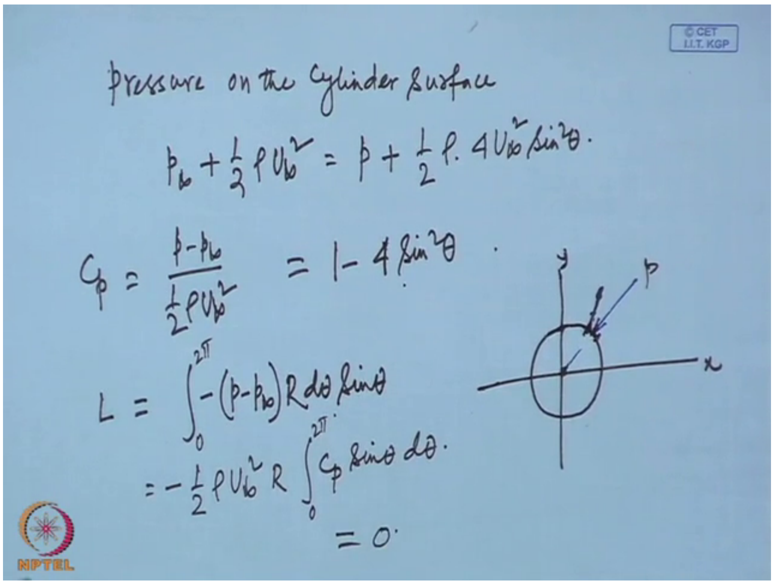

Taking 1 point at infinity, or faraway p plus half rho u r square plus u r on the cylinder surface is zero. So, only u theta square and what is u theta square? 4 u infinity square. It is usual to find pressure in terms of pressure coefficient, or either in aerodynamics when someone says pressure, he actually means pressure coefficient. So, if you are sometime asked to find pressure you can leave it at pressure coefficient only, or rather try to find the pressure coefficient not the absolute pressure. To the pressure coefficient as you know we defined as c p is p minus p infinity by half rho u infinity square. So, what this will be?

Student: (( )) 1 minus 4 sin square theta.

1 minus 4 sin square theta, 1 minus 4 sin square theta. So, at is obvious that at the stagnation point pressure coefficient is 1 because at stagnation point p minus p infinity is half rho infinity square, at stagnation point since the velocity is zero. So, p minus p infinity equal to p. So, at stagnation point the pressure coefficient is 1, which we can see from here also at theta equal to zero and theta equal to pi c p is 1.

This c p becomes minus 3 c p become minus 3 at theta equal to pi by 2 and again, also at 3 pi by 2. So, on the y axis diameter at the 2 tips pressure is c p is minus 3 and negative c p means that there is suction. The pressure there is less than the pressure that is undisturbed, less than p infinity. This c p does not c p negative does not mean an absolute negative pressure that is not possible, absolute negative pressure is not possible, but this is with respect to that undisturbed or reference pressure. So, pressure has decreased there meaning that flow there is accelerating and there is suction, but this is same at theta equal to pi by 2 as well as 3 pi by 2.

Similarly, if you take all this corresponding points, or the geometrically similar point we will see that pressure will also be similar, velocity will also be similar. Meaning we are we will expect a zero force on it. However, we can since we now have a very simple expression for our pressure coefficient, or pressure we can completely calculate the forces. What will be the lift force?

Once again remember lift force is the force component, which is normal to the direction of undisturbed flow, or normal to the direction of free stream. So, in this case, the free stream is along x. So, its normal is y. So, y component itself is the lift force, but please remember the y component or the what we call the vertical component is not necessarily the lift force. The lift is normal to the free stream direction normal to the free steam direction which happens in a real aircraft the direction of the flight.

So, the lift force is with respect to an aircraft is normal to the flight velocity, and here we will call it normal to the free stream sometime, it may coincide with your y axis; sometime may not.

So, what will be the lift force? The pressure acts always normally, it is the normal component of pressure force, but normal to the body it is not normal to the flow, pressure always is normal to the body. Normal to the body means along r. So, what will be the y component? y component of the force.

Student: (( ))

The result will be zero, but at this stage i am not asking that final result we want a general expression, which we can use for other cases where it will not be zero. This is the direction of the pressure. So, what will be the lift force consider a small length r d theta and consider a unit width because this is infinite body. So, we cannot find it completely we have to consider only for unit width. So, if we consider a unit width that is length along the z direction then, area of that (( )) is r d theta into 1. So, pressure into that area that will give you the force and then.

Student: (( ))

A component of it, that is the total force no a component of it

Student: (( ))

Sin theta. However, there will be a negative. That p minus p infinity can be written as half rho infinity square c p. Then half rho u infinity square can be taken out now this c p you can integrate and see that it is zero, integration is from zero to 2 pi.

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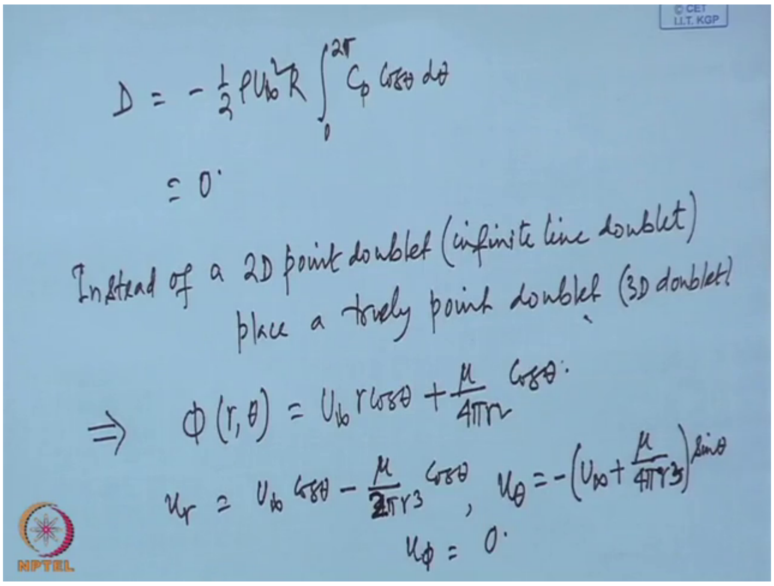

Similarly, the drag force is all the same only the cos theta component. So, here also again a situation where the flow is perfectly flow has perfect symmetry both with respect to top; and bottom; as well as front and rear. And not only that since, circle or cylinder has such a symmetry that even, if you are flow direction changes, the body still symmetry, in there are, certain cases where if you are flow direction changes then the symmetry may be lost.

So, in that consider the case of Rankine oval. In that case if the flow was in x direction only it was symmetric, but if the flow direction made some angle with respect to the x axis then it is no longer symmetry, but for a circle, if your flow direction changes and makes certain angle with respect to this the diameter that your considering earlier, but then it becomes symmetric with respect to another diameter and again with respect to that diameter of the body symmetry. So, whatever the flow direction you make it remain symmetric, and this flow about a circular cylinder we will either give lift or drag in any situation for an incompressible inviscid irrotational flow.

However, as you know that this is not of course, a real solution, it always gives drag it always gives drag you can never make it zero drag, lift there are many situation where it does not produce lift, but drag is always there, however we do not get it. So, first thing if we what we need to get some force that if we can make the flow asymmetric, if we can make the flow asymmetric and see this simple way to make this flow asymmetric we will be if we put at point vortex also or an infinite line vortex at the centre or at the origin where the doublet is then, you know you see how the asymmetry will be made a point doublet produces a circular streamline.

Now, you see that with respect to that doublet or if we think about say as a top, and bottom of the doublet you see the flow direction is in different flows in different direction a circular flow on the top it is along this direction on the bottom it is along this direction. So, it will increase the speed on the upper half; decrease the speed on the bottom half. And an asymmetry will be created particularly top bottom asymmetry, and we can get a lift force.

However, before we try doing that let us look to this uniform stream and doublet, but in a 3 d case. Where we have considered here at a 2 d point doublet, which is an infinite line doublet, instead of an infinite line doublet, if we consider a truly point doublet; then we can guess that our body will now be sphere, instead of infinite cylinder, we will have a sphere.

Let us check that. So, then we will come back to this again then, what will be the potential function? The potential function in the r, theta plane in the r, theta plane once again it is u infinity r cos theta plus... What is a potential for a point doublet? Potential for a point doublet mu by 4 pi r square cos theta mu by 4 pi r square cos theta and the velocity components u r d pi d r 4, or 2, it should be 2 what will be u theta?

So, here also see that the radial component of the velocity, will be zero for r cube equal to 2 pi mu by 2 pi u infinity that radial component of velocity is zero for theta equal to zero as well as for that.

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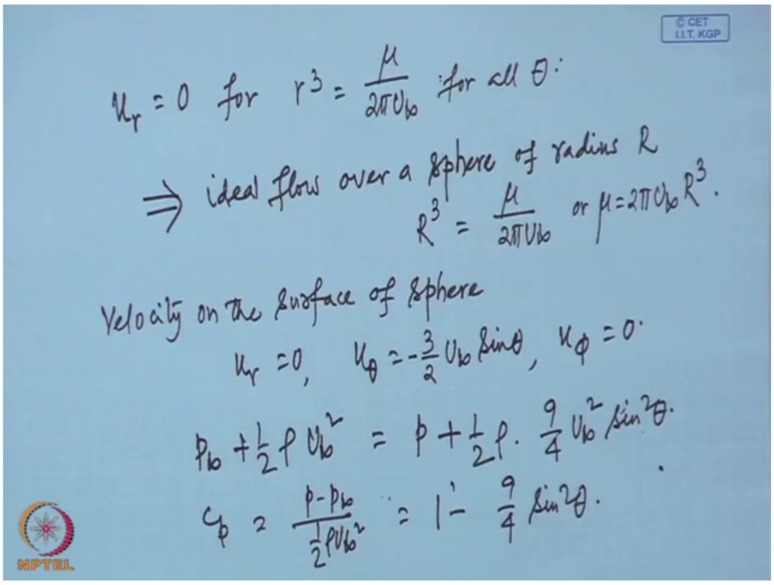

So, we get here again... Once again what will be the velocity on the surface of the sphere? So, see that the tangential velocities are less than what they were for the cylinder magnitude wise, it was 2 u infinity sin theta and now it has become; 3 by 2 u infinity sin theta. That means in case of a sphere the flow acceleration is less, the maximum is again at theta equal to pi by 2 which will be minus 3 by u infinity in case of a cylinder it was 2 u infinity. So, at theta equal to pi, the flow velocity was zero and it increases to 2 u infinity in case of cylinder, while it increases of up to 3 by 2 or 1.5 u infinity in case of a sphere.

So, flow has accelerated less and that will be reflected of course, in pressure also if we look for the pressure coefficient. What it will be once again we can write that p infinity plus half rho u infinity square equal to p plus of rho, or instead of 1 minus 4 sin square theta, we have 1 minus 9 by 4 sin square theta least suction then, what it had in case of a cylinder. Smaller pressure difference, this eventually a general result that whatever velocity increase, or pressure difference is created it in two-dimensional flow; in 3-dimensional flow it is much less.

The pressure difference that is created in three-dimensional flow is less than what it happens in two-dimensional flow, that means in three dimension the flow is much relived than in 2 dimension on this is eventually known as a three-dimensional releasing effect. The flow is not that stressed as in case of 2 dimensions it is much more relieved, much more relaxed why does it happen?

Because in 3 dimensions the flow has 1 additional direction to adjust itself, while it is not there in 2 dimensions. In 3 dimensions the flow has 1 more dimension or 1 more direction in which it can adjust itself. In 2 dimensions it does not have that liberty it cannot adjust in the third direction because in third direction it is infinite. So, it cannot change anything, but in 3 dimensions it can change in the third direction also and consequently it is much more relaxed or relieved.

So, it is a general result not restricted to only inviscid, incompressible, irrotational, or ideal flow, it is true or all flows. However, the magnitude of relaxation or magnitude the relieving effect, that might change little bit, but the releasing effect is always present even in real viscous flows rotational flows or even in compressible flows. In all cases this releasing effect is present, the amount of relief may not be same is or rather not same it is different, but the relieving effect is always present. So, as far that qualitative result is concerned that when we go to 3 dimensions the flow is relieved then in 2 dimensions that general result is valid under all circumstances.

Now, let us come back to that cylinder once again as we said that we will now place a point vortex, or an infinite line vortex at the location of the doublet itself that is doublet and vortex are at the same location.

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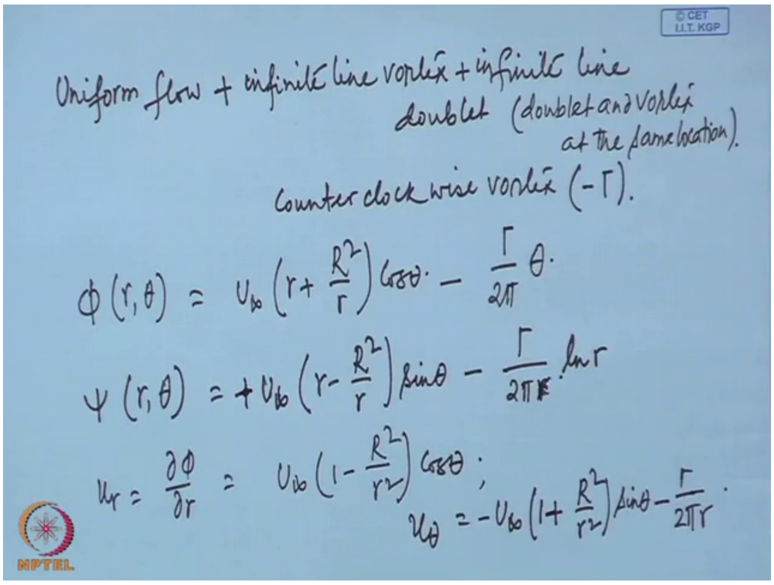

So, uniform flow of course, in the x reaction let us, take at the time being, plus infinite line vortex sorry plus infinite line doublet, and we will consider a counter clockwise doublet that is minus gamma, we will consider a counter clockwise vortex. So, the strength is minus gamma, as before this uniform flow plus infinite line doublet, we will create that flow vortex circular cylinder, and if the doublet strength is mu, or if the cylinder radius is r then the doublet strength should be 2 pi r square, 2 pi u infinity r square, that part remain valid because this is just a superposition.

The whatever, we have found earlier that is that remains; it is not this is going to change everything. It is a just linear superposition. So, whatever we had with uniform flow plus infinite line doublet that remains as it is, that uniform flow plus infinite line doublet gives flow past circular cylinder, and the radius of the circular cylinder is related to the doublet strength, that part still remains, that is not changed.

In addition to that we know have an infinite line vortex. So, the potential function as before we can write phi say r, theta is what uniform flow plus infinite line doublet how much was that? u infinity cos theta, r plus r square by r or u infinity r plus, R square by r cos theta, this is uniform flow plus infinite line doublet where that representing as cylinder of radius r. Now, we have counter clockwise vortex. So, what will be the potential for the counter clockwise vortex? Potential for a vortex potential for a vortex or same function for a... This we did earlier. Stream function for an infinite line vortex, stream function is gamma by 2 pi r, and vortex potential function is gamma by 2 pi theta. Where gamma is the strength, we have taken a counter clockwise so it is minus gamma by 2 pi theta.

Similarly, psi we can write sorry u infinity. What are the velocity components? See a point doublet it gives only tangential velocity, it gives circular streamline, or just a point vortex alone gives only circular streamline, and only velocity tangential component of velocity. So, the radial velocity component remain unchanged whatever it was, and obviously it remains that at r equal to r, this is zero, that means it still remains flow about a circular cylinder of same radius. That addition of point vortex at the centre does not change it; it still remains flow about a circular cylinder with of radius r.

However, that tangential velocity component changes what will be the tangential velocity component? What will be the tangential velocity component this part remains minus u infinity 1 plus R square by r square sin theta sorry this is. So, c is the tangential velocity component changes, so on the surface of the cylinder, on the surface of the cylinder, which we are more interested.

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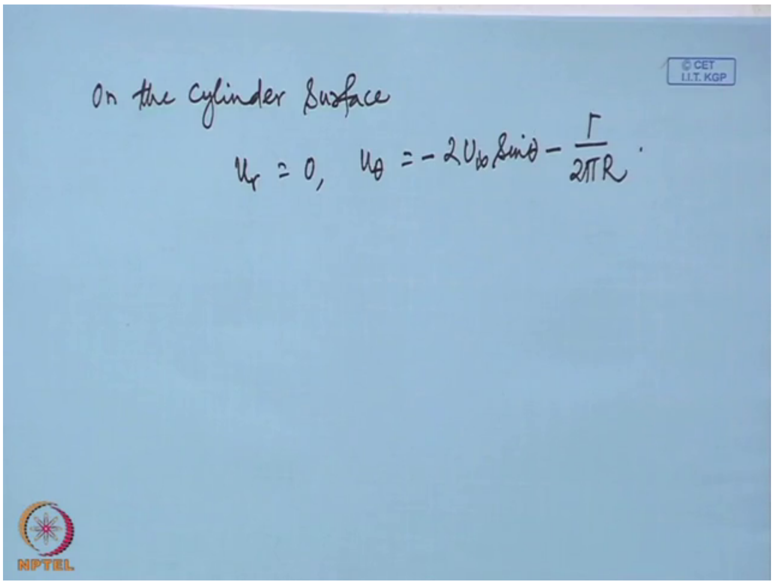

We see now the stagnation points are no longer at theta equal to zero and theta equal to pi, the stagnation points are no longer at theta equal to zero and theta equal to pi and for this counter clockwise sorry clockwise sorry I by mistake I have written... It should have been clockwise vortex, please make that correction by mistake I wrote that we are adding a counter clockwise vortex, it should have been clockwise vortex.

We are adding, we will later on see why we are adding clockwise vortex as far as the mathematics is concerned, if we add a counter clockwise vortex also nothing will change only that sign will be plus other than that nothing will be changing, but why we are using clockwise that we will see later on. The first thing we can see here that the stagnation points has changed they are no longer at theta equal to zero, and theta equal to pi and for clockwise circulation vortex, or clockwise circulation we will see that these stagnation point are coming below the axis, coming to the bottom of and the location is of course, dependent on this value of gamma, at different value of gamma the stagnation point will be located at different position, and not only that it may, so happen that the stagnation point will no longer be on the surface of the cylinder, anyway we will continue that in the next class.