Introduction to Aerodynamics
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Lecture No. # 30
Potential Flow Combination of Basic Solutions

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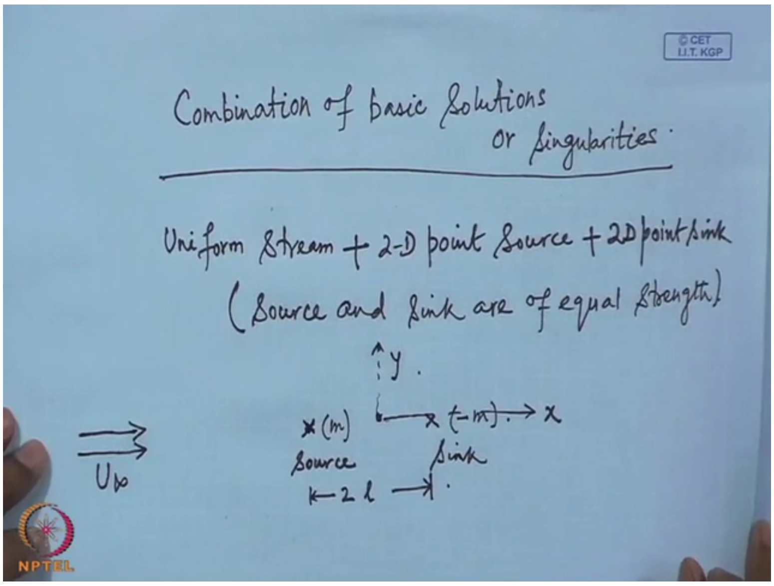

So, we are earlier discussing combination of basic solutions of laplace equation and the first combination that we considered if you remember is a point source in uniform stream. And we saw that the resulting flow represents flow over a semi infinite fairing a body which is closed at one end but, open at the other end it is closed near to the source but, it is open at the other end. So, next combination that we will look for is almost similar. Only we will add one sink to it, that is it is now uniform flow plus a point source plus a point sink. Source and sink are of equal strength.

Basic solutions or as you can see that these basic solutions that is source, sink they are all singular. At r equal to 0 or at the position of the source or sink neither the potential not the velocity are defined and hence they are called singular solutions. So, this term we will consider now a uniform stream plus a point source or infinite line source. A two dimensional point source plus a two dimensional point sink and for convenience we will take source and sink are of equal strength.

Once again let us consider a uniform stream from left to right with free stream velocity u infinity and a source and sink. The source strength is m and the sink strength is minus m and let us consider the distance between the source and sink b 2 l. We will chose our origin at the mid-point on the joining source and sink, the origin is here at the mid-point. So, this is our x axis, this is our y axis. So, what will be the combined potential for this flow? Potential for this flow? Potential function and stream function for this flow? Potential function and stream function for this flow? Yes? What will be the potential function and stream function for this flow potential function at any point x y.

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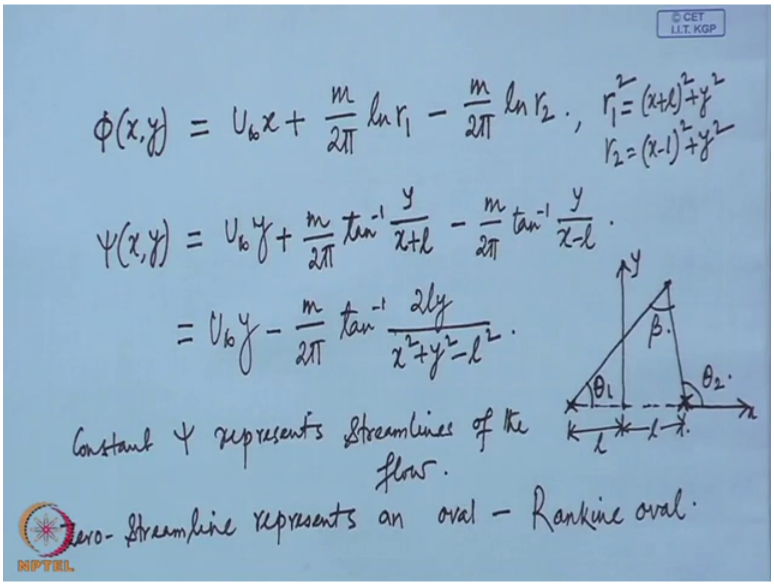

X plus l whole square plus y square. Yes?

Student: Yes sir.

And similarity r 2 is and what about this psi? The stream function, stream function due to the uniform stream is u infinity y and the other 2? m by 2 phi theta or theta 1. Yes, stream function for point source but, what will be that theta in this case m by 2 phi say in terms of x and y theta u express theta in terms of x and y? tan inverse y by x plus l y by x plus l and the other one is?

Let us work with the velocity of course, you can now evaluate using any of the 2. Let us concentrate on psi. Let us concentrate on psi, you can combine these 2 term, those 2 tangent inverse term you can combine. What will be the result? See, what is the representation? Let us take these sink and source separately this is source and sink say this is the point p x y, this is the axis, this is your x axis, this is the y axis.

So, this is what is that angle theta 1 which is tan inverse this. And this is the angle theta 2. See, if you combine theta one m by 2 phi theta 1 minus theta 2, if you write in terms of theta 1 and theta 2 this term is m by 2 phi theta 1 minus theta 2 and theta 1 minus theta 2 is this angle. See, in that form, we can straight away write m by 2 phi. Let us say this angle is beta then this term represents m by 2 pi beta.

In terms of tan inverse you can combine to tan inverse term or otherwise write what is beta using x y and l this will come as u infinity y minus m by 2 pi tan inverse 2 l y by x square plus y square and we can say that this combination a uniform stream, a source and sink of equal strength separated by distance l or distance 2 l represents flow past a Rankine's oval as you had that uniform stream and a point source represents flow over a semi infinite fairing. In this case it represents flow over a Rankine oval and you see this the point source and point sink now within that oval. So, that is not in the flow field. The oval we can now think of a solid oval so there is no flow or nothing within that oval.

And outside flow there is no singularity or nothing, it is a purely inviscid irrotational flow, incompressible irrotational flow or ideal flow. So, there is no extension there is no rotation in the flow field, whatever extension rotation that we incorporated that remain within the body. So practically they does not exist in the flow as far as the flow is concerned. If you plot this curve you will see that this is a closed curve not like the semi infinite fairing for it is closed at one end but, open at the other end.

It continues up to infinity never closing. In this case it closes, it is a closed body.

Student: Sir how do we decide the flow inside the oval.

That is of no practical interest. See, there will be no flow inside the oval. We are interested only flow outside the oval. Think about an solid oval then of course, there is no flow or even if it is a hollow oval there is no flow inside.

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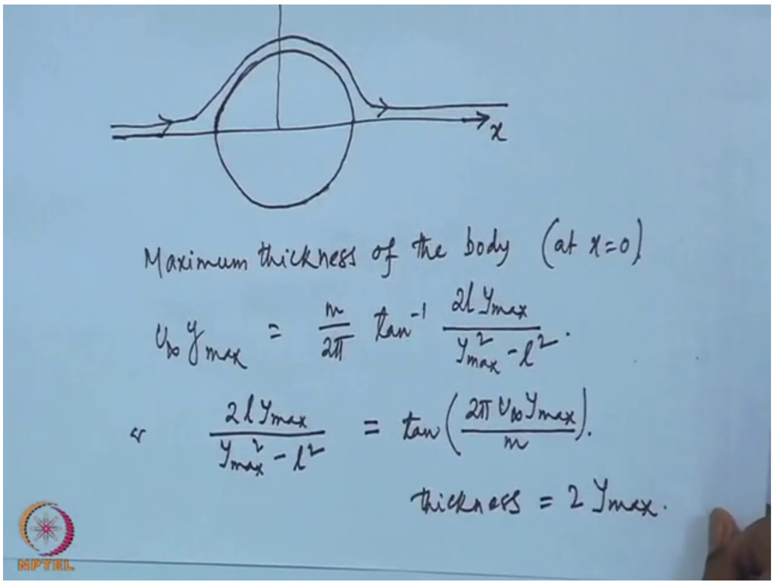

We are not interested in it. We are not interested anything in it, this is the closed body. Let us say this is the oval and I have represented only 1 stream line just to show similarly, you can plot as many stream line as want for different constant of psi.

They are now very easy to plot 2 is a value of psi, a constant value of psi or go on increasing psi in an uniform rate. You can plot as many stream line as you like and we can see what the flow looks like. What is the maximum thickness of this body? You can see it from here that the body is both symmetric with respect to x and y axis and the maximum thickness is of course, at y axis that is at x equal to 0.

So,what will be the maximum thickness of the body? How much? phi and y when I take x equal to 0 u infinity y the actual thickness will be 2 times the y max. Where will be the stagnation points? Stagnation points means points where the velocity as 0 flow velocity is 0.

Though we can get get an easier formula if we substitute x equal to zero in the complete expression of scaled portions so we can get another ..

So, this is these are the formula which will be used to find eventually m value of source strength. Remember, in a practical problem let us say that you are given an oval and you have to find the flow, you know this much that the flow over an oval is superposition of a uniform stream plus of source and sink pair of equal strength but, for a given oval you have to find what would be the source and strength. That is what is unknown to you for the practical problem and that source strength can be obtained say from this formulae or something like this where the actual thickness will be known to you. Find the stagnation points? It is very easy to say from here that the stagnation points will be here, here and here.

These will be the 2 stagnation points. Of course, you can the velocity, write the expression for velocity and from there also you can find it but, sometime from the problem itself you can straight away say that these will be what it is ,that these will be the stagnation point these and these.

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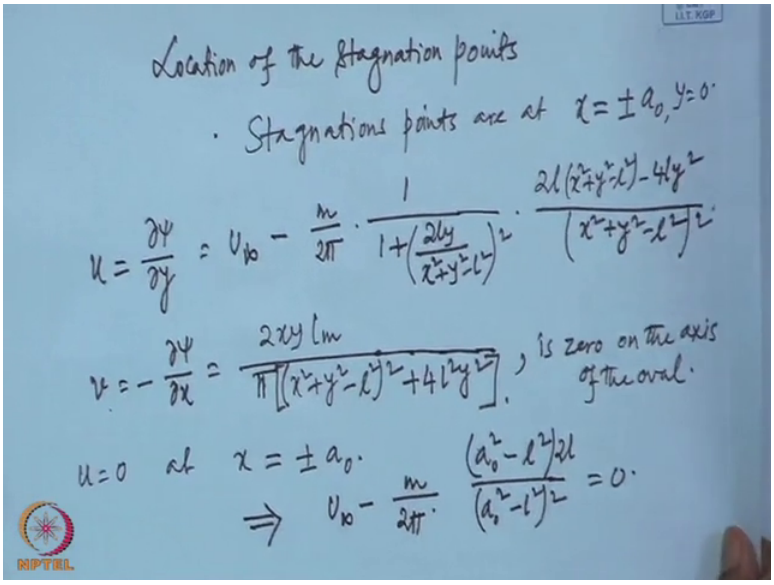

So, locate those stagnation point. So, this part I am writing without finding the velocity that the stagnation points are at x equal to say plus minus a 0 and y equal to 0. Remember, this is not the location of source. So, x that stagnation point will not be at x equal to minus l and plus l eventually this a 0 will be larger than the source will be somewhere here say perhaps and sink will be somewhere here again. And this will be required to find that l. Look again, if you have if you have the oval with you, you know what is this a 0, that a 0 is known to you. What is not known to you is l. What should be the separation of source and sink that will give this flow.

So, that can be obtained from here. This will give a relationship between a 0 and l and since a 0 is known to you the oval will be known to you. So, what will be a 0? Find the velocity components? Find the velocity components? What is the velocity component u? Anything you want you can use either phi or psi say we are working in terms of psi so it would be easier to find it in terms of psi d psi d y u infinity then differentiate that tangent function. How much is that? 1 by 1 plus square.

Oh, this is square this and what about the other component? What will be the other component, give me that final expression what is v ?

Student: 2 x y l m.

2

Student: X y l m.

2 x y.

Student: L m

L n.

Student: L m.

L m.

Student: Upon pi.

In the brackets x square plus y square minus l square whole square plus 4. Plus a separate term or on the denominator itself. In the denominator plus 4 l square y square.

Student: 4

L square y square.

Student: L square y square.

Pi is also in the bracket.

Pi I have written you must have said this. This see it is not very important I could have mentioned here at least for this purpose, as far as finding the stagnation point is concerned. You can see that this v component is always 0 on the x axis as long as y is 0 v is 0.

So, for stagnation point what you need is we need to make u 0, that is all. v is 0 at all point on the x axis. So, to find stagnation point finding v was not necessary you could have straightway said that the u will be 0 on the x axis, at all point on the x axis. Then at the stagnation point that is when x is plus minus a 0, u is 0. So, find out what is this? and this gives.

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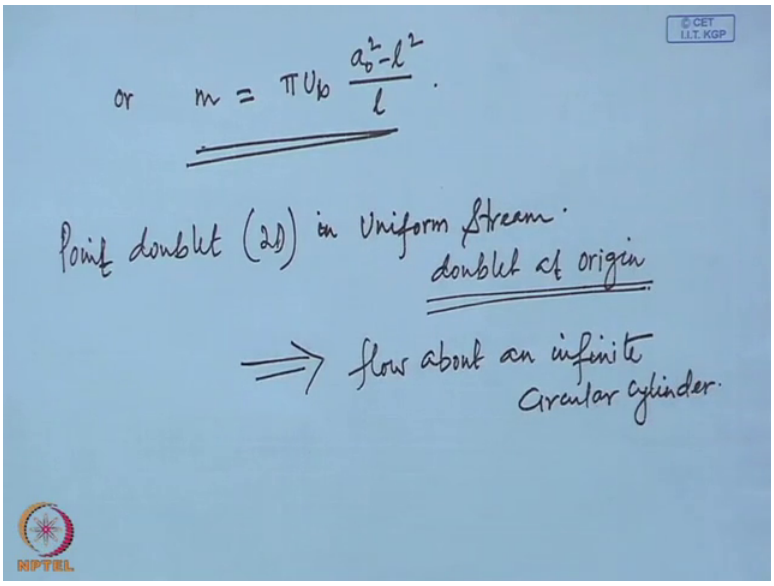

So, if we take now these two relations. These and that earlier thickness formula that we had. Using these 2 we can find what are m and l. That is the source strength and the separation between the source and sink to generate a practical oval or real oval. You can find the pressure distribution at any point or pressure at any point or in particular the pressure distribution on the surface of oval by the simple application of Bernoulli's equation. We can use the Bernoulli's equation at 1 point far away from the body that is at infinity and at 1 point on the body surface or anywhere in the flow field. This flow is irrotational flow.

So, we can apply the Bernoulli's equation at any point. The 2 points need not be on a streamline. You can apply it anywhere. So, if we apply it between a point at infinity where the pressure is p infinity and the free stream speed u infinity so that Bernoulli's equation at infinity represents p infinity plus half rho u infinity square and another point on the body surface p plus half rho say q square. If q is the velocity on the body surface which of course, you can find the velocity on the body surface from these general relation for u and v satisfying x and y for the body. Of course, as you can see that the expression for u and v are not very short or very huge handy expression, they are quite unwieldy.

So, we will not try to write those equations but, this is what it can be done straight away. If you have numbers then you can get a very small number and can use it but, they are not that convenient to handle in an analytical fashion so we will not do it now. Can you say what type of force this rank in oval will experience because of these pressure distribution? What about the pressure distribution will be even though we are not writing the expression can you say what type of pressure or force this rank in oval experience because of this flow?

Any guess? Any guess? See, without doing anything or without using any of these expression see to it this way that to the flow which is coming along the x axis of the oval, say this flow will see this is a symmetric body to the flow this body will appear just a symmetric body. Symmetric with respect to x axis as well as in this case it is symmetric with respect to y axis also that means the body has a symmetry both top and bottom as well as front and rear.

So, the flow will also be symmetric. In this case the fl[ow]- with respect to the flow the body is symmetric so the flow is symmetric, the velocity filed, pressure field everything will be symmetric. A perfect symmetric, both front and rear that is the front half of the body this front half of the body and the rear half of the body velocity distribution and pressure distribution is perfectly symmetric again the top half of the body and bottom half of the body, again velocity distribution and pressure distribution will be symmetric.

So, now let us consider this top and bottom. If the pressure is symmetric with respect to this top and bottom so what will be the net force in that direction or in the y direction in this case? 0 again it has a front and rear symmetry again the force in the x direction will be zero so this pressure distribution will not create any force not impose any force on the body neither a lift force nor a drag force.

How the pressure is symmetric in rear and front side in uniform string is coming from inside only?

Uniform stream is coming in from one side only but, the body shape and this inviscid irrotational incompressible nature of flow is making the flow perfectly symmetric both with respect to top and bottom the velocity symmetric that you have no doubt about that. That part you are convinced that the velocity is symmetric. If the velocity is symmetric, why should the pressure will not be symmetric because it has to satisfy the condition p infinity half rho infinity square which is happens to be a constant. is p plus half rho v q square, if the q is symmetric, p has to be symmetric is a constant of course, as I said that the writing an expression for p is quite unveiled expression.

So, that is what we are not doing of course, it can be done but, it will be a very lengthy very complex equation. So, this is little absurd know that if there is a oval and there is a flow then there is no force acting on it (( )). Perhaps lift can be expected, that there is no lift force acting but, no drag is also acting that is little difficult to believe because that type of situation never occurs, there is no lift or practically no lift that is the quite common situation but, no drag that never happens but, it gives that we will now move to a different situation but, which is very closed to it, that is we will now bring the source and sink close together.

So, close that they forms what we have called earlier a doublet that is the separation between source and sink approaching to 0 and simultaneously the strength of this source and sink is increasing so that the product remains fixed number that m l or m into 2 l that product as a l approaches zero and a m approaches infinity that product still remains the same which we call the doublet strength mu.

So, that 2 l m will be mu in those expression that 2 l m will replace a mu and what we will get is flow due to a uniform a point doublet in a uniform stream and since both source and sink are approaching each other they will approach when they reach to that origin. We kept the origin between mid way between the source and sink.

So, now we are approaching them so in the limiting case the doublet will be placed at the origin. So, the resulting flow will now be in limiting case flow due to a point doublet placed at origin in a uniform stream. So, that is what we will next consider. Again the point doublet we are meaning two d, so not a three d point or not actually point a doublet, it is a infinite line doublet. (But, before we start writing these potential and stream function, of course, by now you can do it easily the approach is same we will bring something to your notice look the first problem that we consider a point source in a uniform stream, the resulting body is closed at one end the end near to the source the body has a thickness and it is open at the other end.

Now, see that when we placed a point sink on the other side it is it closes on that other side also the body has still a thickness. This gives a general idea or a modelling philosophy that if we want to close a body, if we want to have a close body then we must have source in combination not only source or a sink, we must have source and sink combination such that the total strength is 0, total source and sink strength is 0 ,also this body will have a thickness.

So, if we want to have a thick closed body any body which has certain thickness and the body is closed then we must have a source sink combination such that the total strength is 0, this is a general principle a combination of source and sink. I mean number of sources and number of sinks such that the total strength is 0 then all streamlines which immanent from the source will end at terminate at the sink and there will be one streamline which will enclose all these source and sink there will be one streamline which will enclose all these source and sink which will represent a body this. This body will have a thickness it can also be not a discrete source and sink but, a continuous distribution of source and sink we will call it continuous distribution of source only such that its total strength is 0 sink we will just mention as a differently sink. We will just mention 1 source a continuous distribution of source.

So, that its total strength is 0 meaning that source is made up of source and sink both this will be useful later on. So, when you have a point doublet, doublet also has a net source strength 0. So, here also we are or we should have or we will have again a closed body and what body we will get that is also I think quite easy to guess. That what will be that limiting form of oval when the tail approaches 0, it will be a circle and if the construction is circle, the bodies are cylinder infinite cylinders. So a point doublet in uniform stream will represent flow past a infinite circular cylinder.