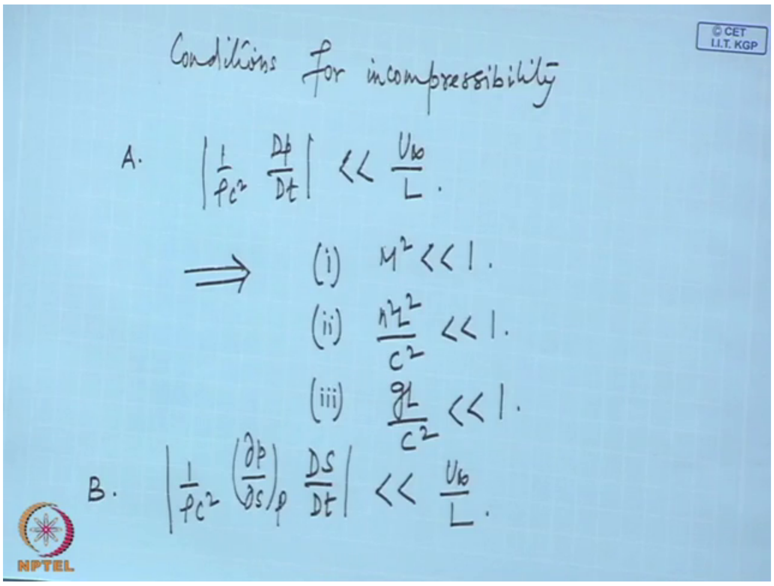
Introduction to Aerodynamics
Prof. K. P. Sinhamahapatra
Department of Aerospace Engineering
Indian Institute of Technology, Kharagpur

Lecture No.#28
Potential Flow

(Refer Slide Time: 00:39)



So, continuing with our discussion on the conditions for incompressibility. We have seen that the requirement, we show that the flow can be treated as incompressible or the velocity field can be treated as Solenoidal. If we have these two requirements that 1 by rho c square, d p, d t much less than u infinity by l and which we saw that will be satisfied, if m square is much less than 1 n square, l square by c square much less than 1 and assuming that the body force is only gravitational force that these three conditions together will give satisfy this condition. Also we had another requirement which says that what it was one by rho c square d p, d s at constant rho into d s, d t this also must be much less than u infinity by l. So, we will now see what of course, as you will see that these are not very important, but just let us complete.

(Refer Slide Time: 03:23)

$$\frac{1}{\rho c^2}\left(\frac{\partial p}{\partial s}\right)_\rho = -\frac{1}{\rho c^2}\left(\frac{\partial p}{\partial \rho}\right)_s\left(\frac{\partial \rho}{\partial s}\right)_p$$

$$= -\frac{1}{\rho}\frac{\left(\frac{\partial \rho}{\partial T}\right)_p}{\left(\frac{\partial s}{\partial T}\right)_p} = \frac{\beta T}{C_p}.$$

$$\frac{Ds}{Dt} = \frac{1}{T}\left\{\phi + \frac{1}{\rho}\frac{\partial}{\partial x_i}\left(k\frac{\partial T}{\partial x_i}\right)\right\}$$ – from energy equation.

Hence $$\left|\frac{1}{\rho c^2}\left(\frac{\partial p}{\partial s}\right)_\rho \frac{Ds}{Dt}\right| \ll \frac{U_\infty}{L}.$$

We can write that 1 by rho c square, d p, d s rho is equal to minus 1 by rho c square. I hope you remember this relation this is our thermodynamical identity. The product of three partial derivatives and this sorry this c square gets canceled here this d p, d rho at constant entropy that is what is c square. So, this c square gets cancel and this d rho d s, is written like this d rho, d t by d s, d t this is written as beta t by c p or this c p is specific heat at constant pressure these are thermodynamical relations and beta is conductivity. So, this is what this term can be written and then the remaining term what it was into d s, d t that d s, d t. If you look back your energy equation from energy equation d s, d t can be written as 1 by t into that phi mechanical dissipation plus 1 by rho d d, x I, of k this is obtained from energy equation.

So, this multiplied by this this will this means these into this the standard magnitude of these say if it is the fluid is water then this will have the general value is of the order of 10 to the power minus 10 or even smaller. So, you see that they are almost invariably satisfied it is very rare case when these are not satisfied is the very rare case when these conditions are not satisfied. So, even when the flow is not really incompressible the full flow is fully compressible still these conditions are satisfied both these quantities are always very small.

So, these two conditions are almost invariably there it is stated that this condition for incompressibility will be violated. If you are interested when your length scale velocity product of this u infinity l is less than even point 1 product of u infinity l is less than even point 1 and you are considering a very high temperature flow. In that situation this is this will be violated, but see those are hardly any practical case at least for us.

So, we will never encounter any situation, where these two conditions will be violated. So, based on these two conditions rather we will be able to say that all flows are incompressible however, we have seen that there are three more conditions. So, out of these five conditions we now have five conditions. The three coming from that d p, d t term and two coming from this terms out of these five conditions, these two are almost invariably satisfied. The condition related body force g l by c square that is al almost in invariably satisfied. As we have seen that when you are interested to in the scale of the atmosphere itself only then that term will be important, we would not be able to consider the flow incompressible. If we are considering the whole atmosphere, but if we are interested only a small portion of the atmosphere that term will never opposes in our approximation of incompressible flow.

So, since these three conditions are almost invariably satisfied what remains are basically the other two conditions that m square much less than 1 and n square l, square by c square is much less than 1 and in case the flow is almost steady or nearly steady or fully steady then that condition n square l, square by c square is also not important. So, for steady flow the only important condition is that Mach number is small square of Mach number is much less than 1. So, for steady flow the only important condition that is required for the flow to be treated as incompressible is that, the square of Mach number is much less than 1, but it never gives much less than 1 means, how much it never say purely that that is square of the Mach number should be less than point 1 or point 2 or point 5 nothing, but since it say that the if the square of Mach number is much less than 1.

Then the flow can be approximated incompressible meaning as your square of Mach number become smaller and smaller your approximation will be better and better, If the square of Mach number is a point 1 your approximation of incompressible flow will be better than if it were at say point 4. If m square is point 4 and you are using an incompressible flow and for m square is point 1 and you are using incompressible flow it is expected that, when the m square is point 1 your approximation is better your solution is more accurate, but it never specifies that m square has to be this much it says m square in much less than 1. So, much less than 1 is of course, your interpretation there is of course, a standard thumb rule it often says if Mach number is less than point 3 Mach number is less than point 3, then the flow is approximately incompressible and given that in most cases. If your Mach number is of the order of point 3, the pressure change involved in that situation is at the most of about 5 to 10 percent of the absolute pressure Mach number point 3 say in case of air is approximately as speed of nearly 100 meter per second of near a flow speed of nearly 100 per second of Mach number of point three is.

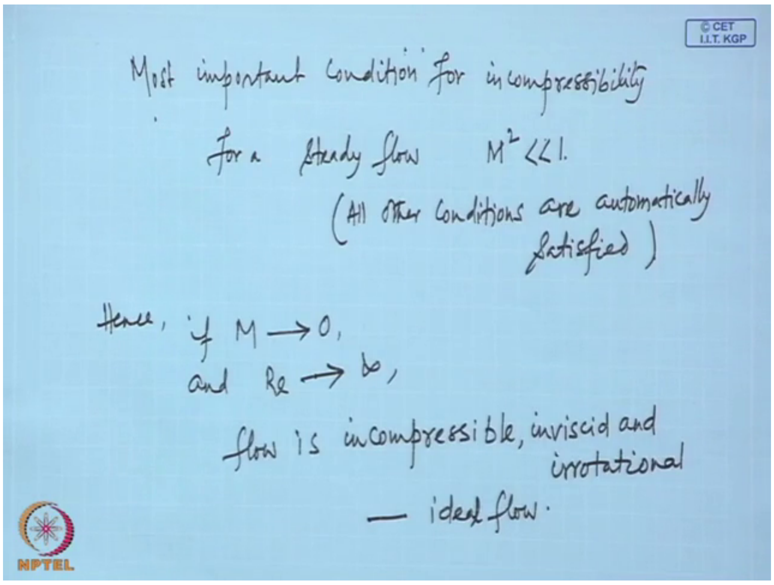
So, if you want to have a flow at Mach number hundred. The pressure difference that you really need is of the order 5 to 10 percent of the atmospheric pressure or absolute pressure. So, that is a very small change. So, when the flow Mach number is point 3 this is taken as a thumb rule, but do not take it for granted that in all situation. If Mach number is above point 3, the flow will be very good approximation for incompressible flow not necessarily there are many situation even when at Mach point 3 the flow is quite compressible particularly. If you remember we talked about I think in the very beginning about high speed flow a high lift devices that during landing and take up the aircraft leading edge and the trailing edge can be deflected with respect to the remaining part of the wing. If you remember which are called high lift devices to produce high lift during time of landing, takeoff and landing and with those high lift devices deflected a flow at Mach point 2 or point 3 is highly compressible not incompressible. So, even though it is taken at Mach number point as a thumb rule for incompressible flow limit for incompressible flow, but do not take it to be gospel this is this depends on situation to situation .

Now, let us see then summarize, we have first of all seen that if the flow Reynolds number is very high then the viscous terms in the governing equation can be neglected or at based it can be neglected. For the most part of the flow except a very thin region near the body except that everywhere else the viscous forces can be neglected and we have also see that if the viscous forces are absent then the circulation around a closed material curve will remain independent and if the circulation was initially 0 it remains 0 and when this viscous effects are absent a any vorticity that is produced that is also confined within that thin narrow region near the body outside that region there will be no vorticity in other way when the flow is at every high. Reynolds number we approximate the flow to be inviscid as well as irrotational except a very thin region near the body surface which we call the boundary layer and outside this outside this boundary layer the flow field inviscid irrotational.

So, at high relends number the flow can be approximated to be inviscid and irrotational. We can forget about that boundary layer a small part or whatever solution we obtain we call that this is the solution outside. The boundary layer either we completely forget about the boundary layer or we say the solution that you are obtaining it is valid outside the boundary layer also. We find that there are certain conditions, where the flow of a compressible fluid can be considered as incompressible flow and out of that in particular. If the flow is steady flow then the only important condition is the Mach number is small Mach number is small.

So, if the mach number is approaching to 0 and Reynolds number is approaching to infinity then, we can consider the flow to be inviscid incompressible irrotational in the limiting case of Mach number approaching to 0 and Reynolds number approaching to infinity the flow can be consider as inviscid incompressible irrotational.

(Refer Slide Time: 20:57)



So, let us summarize that most important condition for steady incompressible flow these flows are of course, called ideal flows by the way, what we mean that Mach number approaching to 0 do not take it this way that the velocity is approaching to 0 no see the Mach number is a ration u by a where a is the speed of sound. So, when can it become 0? It is much less than.

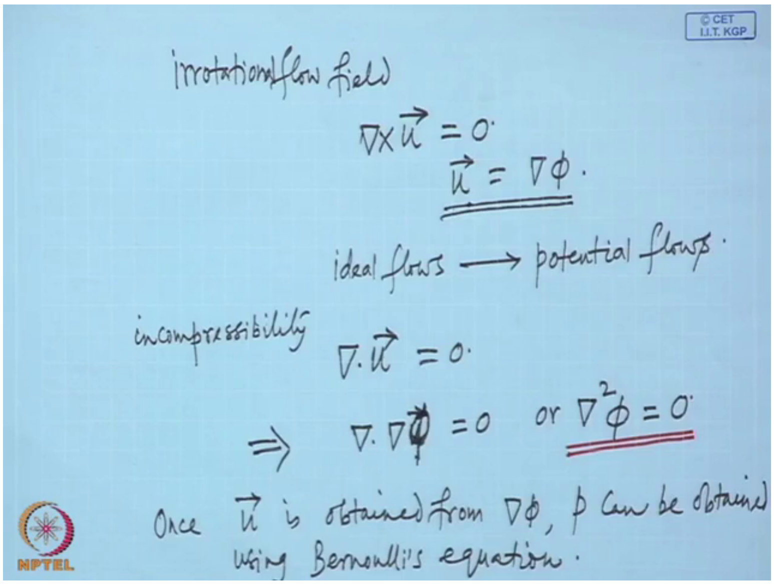
If a approaches infinity or u approaches 0. So, here when we say that the Mach number is 0, we never take it or think it this way that u is 0 or u is approaching to 0 it is the other way a is approaching to infinity which is actually the case see what is d p, d rho speed of sound d p, d rho that is square of the speed of sound. So, in an compressible flow where density is independent of pressure what is d p, d rho? Is infinite, there is no change in density due to change in pressure that is by definition and incompressible flow no change in density due to change in pressure.

So, what then d p, d rho is infinite. So, the speed of sound is infinite what is the meaning of that the speed of sound is infinite in an incompressible fluid? You speak here any one at infinite distance will listen you instantaneously would not even take some time. What you speaking you are disturbing there when you speak you disturbed there of course, the disturbance can be done by many thing even this is also a disturbance. If a body moves through it a ball travels through air that is also disturbing there. So, any disturbance it will reach infinite instantaneously that is what it is when if the speed of speed of propagation becomes infinite then information reaches instantaneously.

So, in an incompressible fluid or incompressible flow any disturbance will reach everywhere instantaneously, because the speed of information propagation that is what the speed of sound speed of sound is a wave it carries the information. So, that is speed of information propagation is infinite and consequently any disturbance will reach infinite distance instantaneously would not taken a time however, if the fluid is not incompressible that is if d p, d rho is not infinite then, it will take some time it may reach it may not reach what it actually happens anyway to we are not much interested at this stage in that part of the flow.

So, we have see that under what condition we can approximate the flow to be inviscid incompressible irrotational which is ideal flow and we know already that if the flow is irrotational, if the flow field is irrotational that is curl of u is 0 or irrotational flow field.

(Refer Slide Time: 26:50)



Irrotational flow field means that curl of u is 0 and what does it mean that u is gradient of phi this phi is velocity potential please, sometime we are using phi as dissipation and some we are using as velocity potential, but I think the context if you look to the context it will be clear. If we are talking a energy equation only then we are using phi as potential I mean dissipation otherwise phi is potential and you see that simplification instead of the three component of the velocity vector we need only one scalar function of position of time of course, in general.

So, instead of three unknowns we now a only one unknown phi, once that phi known the three component of velocity are known from this relation nothing else and remember in this case earlier also we considered irrotational flow, but in that time we considered the irrotational flow as part of contribution there are three contribution. We talk talked about one isotropic expansion one rigid body rotation and one no expansion, no rotation, but that has only one contribution, but in this case it is no longer only one part of the contribution to the velocity. In this case this is the complete velocity this is the complete velocity there is no isotropic expansion or no rigid body rotation associated with it the entire velocity field is irrotational.

So, this is what we get from this and irrotational that is why these ideal flows are also called potential flows. Earlier when we discussed about it the flow was not potential only one component or part of the flow was potential, but there are other parts also, but now we do not have. We have only potential part and that incompressibility imply that velocity field is solenoidal or divergence free the velocity field is solenoidal or divergence free no rate of expansion, no expansion and if we substitute it here sorry divergence gradient of phi divergence gradient is laplacian a partial differential equation to find phi.

So, this alone is sufficient to find phi we do not need anything else. So, this replaces the entire set of governing equations and this becomes the governing equation for ideal flow or potential flows. Once we find the solution for phi we can find the velocity field the conservation equations of course, the conservations equations are here within it, but we do not need the conservation equations separately this alone will give us everything. Once we find the velocity field we need to find pressure which can be obtained by satisfying the integral form of the momentum equation that is the so, called Bernoulli's equation.

So, once u is obtained for phi (and the solution is complete this governing equation is linear. So, we see that the simplification that we are looking for that simplification is obtained, we said them reason of the two reason of difficulty of this solution of the Navier stokes equations or Euler's equation is the inherent non-linearity of the equations Euler's equation or Navier stokes equation are non-linear and in general it is not possible to find solution for those equations this equation is linear. So, that difficulty is avoided.

However, this does not mean that we have the problem has become linear, because the problem is not only the governing equation. The complete description of the problem needs a governing equation and the boundary conditions. The boundary conditions can still be non non-linear or either in general they are in genera,l they are the conditions are also non-linear.

So, for a complete solution of course, the boundary conditions also need to be linearized that of course, we will postponed now we will talk about this linear equation this. Already we have mentioned that there are many solutions for this laplacian of phi equal to 0 unknown and we talked about some solutions earlier, the solutions are in general called harmonic. Functions laplacian of phi is an elliptic partial differential equation which needs that phi within the domain of description is continuous, there cannot be any discontinuity there cannot be any discrete jump phi has to be very smooth and continuous function, that is the requirement of all elliptic partial differential equation all this we have already mentioned we talked about some solutions also.

So, all those solutions are now valid and for the entire flow field they will now give the entire flow field not a part of the flow field, it will give the complete flow field also we earlier discussed about different condition. Under what condition we can get a unique solution for this problem? We have seen that if it is a singly connected domain then to get unique solution, we must know the boundary condition in the form of either this phi is given or specified on the boundary or the normal gradient of phi is specified on the boundary or a combination there of in a doubly connected domain. We have also seen that to get unique in a doubly connected domain this phi is not a singly connected sorry phi is not a single valued function rater phi is a multi-valued function. If it makes a complete turn around and irreducible curve in a doubly connected domain then for every turn its value increases by a cyclic constant the cyclic constant.

Now, we can relate with circulation that every complete turn it makes around and around. A irreducible curve the value of phi increases by a constant which we called cyclic constant. Which can be said the same thing as the circulation in a doubly connected domain and in such a doubly connected domain. We saw that in addition to the boundary conditions, the boundary conditions that phi is described on the specified on the boundary or normal gradient of phi is specified on the boundaries or a combination there of is specified on the boundaries in addition to these boundary condition. We also need to know the value of that cyclic constant unless we know the value of the cyclic constant we cannot get a unique solution for phi in a doubly connected domain. So, all these of course, now valid for the complete velocity field or complete incompressible flow about any arbitrary body whatever the body is.

So first of all if we need the solution of this ideal flow or potential flow or incompressible inviscid irrotational flow about anybody then, if the domain about anybody means. We are interested in the flow outside the body the flow is outside the body and the domain of the flow is, then we can say an infinite domain flow avoid an aircraft or say aircraft wing is, what it is flow that is over the aircraft not within the aircraft and then there is no nothing else in the boundary the other boundary at infinity. So, in this case the boundary is at one boundary is it at infinity and one on the body surface, if this is a singly connected domain of course, it is in this case then we can obtain the solution unique solution. If we know the boundary conditions either in terms of phi known on the surface of the aircraft as well as at infinity and or the normal gradient of phi known on the surface of the boundary surface of the aircraft as well as on the infinity.

However, if instead of a complete aircraft we have only a single airfoil a cross section of the wing. Which actually means a wing of infinite length then the domain is two dimensional then the domain is two dimensional and sorry domain is doubly connected this I think we discussed earlier. We will talk once again and then in addition to the boundary condition that is either the potential at and on the airfoil wing surface or the normal gradient of potential on the wing surface is required in addition. We need to know that cyclic constant otherwise we cannot get a unique solution.

See in a two dimensional wing, if we consider two dimensional wing, the wing is infinitely long. Now, think about any curve which loops this wing can you reduce it to point without going out of the domain you cannot, but think about the complete aircraft any curve that loops the aircraft can very easily be reduced to or shrink to a point without ever going out of the domain think. How it is possible? This time you think last time I told. So, this time you should tell me we think a curve which is looping the aircraft.

Now, going out of the domain, the domain is the surface of the aircraft the complete surface of the aircraft and thinks about a sphere of infinite radius within that that is our domain we cannot you cannot go out of it. So, that means if you go inside the aircraft you are going out of the domain it is very easy slide the curve take it out of the aircraft in that process. If you think my curve was not as big as the aircraft then you stretch it make, it longer make, it bigger and then go out of the aircraft. Once you go out of the aircraft you just shrink it the aircraft only a small portion of the complete that infinite sphere know.

Sir in cross sectional also we can do shrink here.

How can you this wing is infinitely long, how can you go out you any wing any curve which is looping it. How can you go out of it? Which side then you have to go through the wing, but within the wing the inner part of the wing is also out of the domain. However so, this problem for laplacian phi already there are many known solutions.

However, remember that even though the equation is quite simple linear equation and let us say for a given situation. The boundary condition is also not that complex, but still in general you will not get a direct solution. It is usually not possible to get a direct solution, the usual practice is it, is not that that you have given the equation and given the boundary condition, then you are doing something and then finding the solution that is in general not possible. In some cases of course, it is possible find what is the solution of Laplacian phi and then satisfy the boundary condition and go it is rather other way take some solution of phi has many solutions known we discussed like say one by r and. So, they are all solution of this Laplacian phi equal to 0 of the Laplace equation take some of this solution, because the equation is linear solution of some of the two solutions is also a solution.

Then just consider a certain combination and then try to satisfy the boundary condition there say as an example let us say that phi 1 is one solution phi 2 is another solution phi 3 is another solution. So, you can construct a solution which is 1 phi 1 plus a 2 ph,i 2 plus a 3 phi 3 and so on. And then you try to satisfy your condition and try to get the values of a 1, a 2, a 3 that is the usual approach of solving this laplacian equation of course, we will solve few cases and we will see, how we are getting some flow before we try to solve flow avoid an airfoil or wing or aircraft. We know that 1 by r phi equal to 1 by r gives a solution due to source remember. We said that is a distribution of rate of expansion with singularity at the point of the source that is, if we distribute a rate of expansion everywhere 0, but at one point it is almost infinite that is what is a point source equivalent to now your first question will come.

Let us see if we distribute rate of expansion, but this is incompressible flow there is no rate of expansion. How can we have even at rate of expansion even at one point is not possible. So, how can we use source yes we can put the source inside the aircraft or inside that body, no problem, because inside the body is not part of the flow. So, even if there is you imagine there is source or there is an infinite rate of expansion, it hardly affect the flow there is no expansion as far as the flow field is concerned.

Similarly, if we need to consider a vorticity here also we can put the vorticity with the understanding that this vorticity will inherently be within the body. So, that actually there is no rotation is created within the body within the flow. A point vortex is what an infinite amount of rotation at that point outside that point there is no rotation, but it produces a circular type of velocity field circular streamlines. We found that gamma by 2 phi r become the velocity field for an infinite line vortex a point vortex in two dimension is equally at actually at infinite line vortex. So, in infinite line vortex gives velocity at a distance r from that line vortex of the amount of gamma by 2 phi r where, gamma is the circulation about that line vortex, but no rotation the rotation is or the vorticity is confined to that line or to that point only.

Now, if we can make it this way that the point is no longer within the flow field it is there, but it is not within the flow field then the flow is not rotational there is no vorticity in it, but the effect of vorticity is still there. There is no rigid body rotation there is no vorticity in it, but that a velocity field tangential velocity field that is present. So, that is the way we can use those solutions once again to obtain the solution of now for complete flow field. How can you (( )) those are the basic solutions how many (( )).

Those we will decide those we will decide, we will see how we are, how we are deciding depending upon it try it this way. What is the what would be the potential function for a uniform in the x direction, which would be the potential function for a uniform stream along x direction straight forward u infinity x. The potential function for a uniform flow in the x direction is u infinity x or other way u infinity x represents a uniform flow in the x direction. Consider another point source consider another point source for simplicity, let us consider it is two dimensional instead of three dimension, let us consider a two dimension and a two dimensional point source of course, is equivalent to an infinite line source in the third direction see. If we consider a two dimension is the x and y dimension then a point source is an infinite line in the z direction, but in the cross section of course, we are seeing only one point.

Let us say at the origin of that axis system we have a point source at the origin. We have a point source of strength earlier, we are taking the strength the point source as m. So, let us take once again as m so, let us take once again as m and what is the potential due to a point source. If you remember for a two dimensional point source we did it earlier point source point source for a two dimensional sorry potential function for a two dimensional point source. What is the potential function for a two dimensional point source or perhaps, we have mentioned there as a line source infinite line source. One by r one by r. One by r is actually for three dimension or a three dimensional point source or a genuine point source is one by r rather m by 4 pi r.

Student: (( ))

Ok for two dimensional case infinite line m by two pi log r.

Student: Log (( )).

Hmm

Student: (( )).

Now, m by 2 pi log r is a solution of two dimensional laplacian equation two dimensional laplacian equation u infinity x is also a solution of two dimensional laplacian equation. Sum of the two will then give solution of this Laplace equation, because the Laplace equation is a linear equation. So, we can super impose solution u infinity x is a solution m by 2 pi log r is also another solution. We combine the two solution combine and then see what it is please do it find out the velocity field find the velocity field u infinity x and m by 2 pi log r (better still write the stream function write the stream function (what is psi the stream function.

Student: Some function into u into (( )).

Huh.

Student: A divided by infinity (( )).

Infinity.

U infinity u infinity what is psi for this that uniform flow along x stream function for uniform flow along x u infinity y. The stream function for uniform stream along x is u infinity y see some of these things are is better to remember every day of course, you can get all these by starting from the definitions that u equal to d phi, d x equal to d psi, d y using the definition of phi and psi you can find phi and psi alright, but for some of these basic solutions it is easier to remember instead of trying to derive everyday and when it is so, easy to remember potential function for stream uniform stream along x is u infinity x stream function for uniform stream along y is u infinity y and for point source in two dimension. Can you say what will be the streamlines for a point source streamlines for a point source.

Student: (( )).

Point source streamlines are circular.

Student: (( )) radial approach.

Radial.

Radial just rays then what will be the stream function you know constant stream function represent a streamline that also we have done earlier psi equal to constant represents a streamline. So, what will be this stream function? When the streamlines are just radial lines what are the equation of radial lines think r theta coordinate system what are the equation for radial lines all radial lines are theta equal to constant one radius is given by one particular value of theta. So, the stream function will contain only theta and that m by 2 pi then of course, there is a question of sign whether plus or minus. So, what we have taken it should be consistent that is all use any sign consistently anyway we will continue it next time.