Introduction to Aerodynamics

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Lecture No. # 24

Exact Solution for Simple Problems (Contd.).

So, we will continue our discussion with exact solutions of the Navier stokes equation. As we said that for some simple cases and the simplest possible cases that we can see that is unidirectional flow. The flow is everywhere only in one direction and not only that in that direction. The flow is independent of distance along that direction, that is if the flow is taken to be in the x direction or whatever direction it is that is the x then, the flow quantities are independent of x. The flow is in the x direction, but the derivative in the x direction are 0. That is a simplest possible case and for which the Navier stokes solution can be solved exactly in some cases. One such example we have considered in the last class which is the classical Hagen Poiseuille flow and we will consider a similar example.

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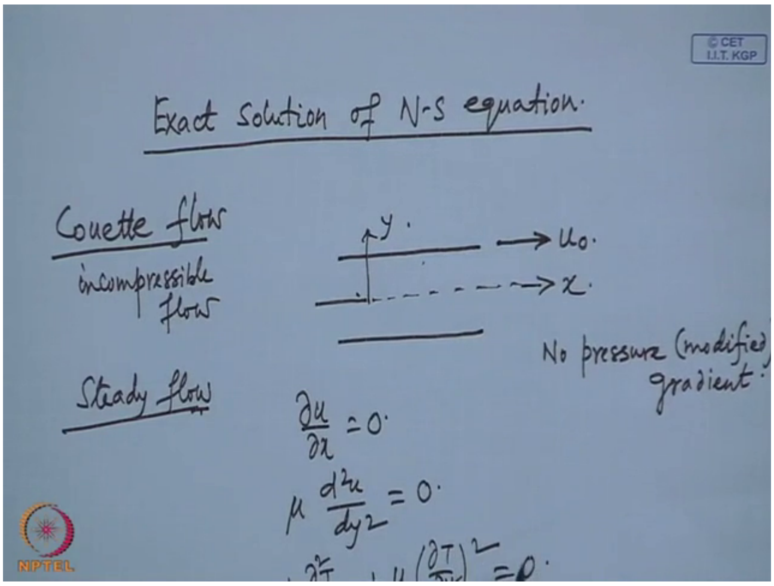

Again a very classical problem known as Couette flow problem, the classical Couette flow problem is like this that imagine you have two infinite flat plate and the fluid is flowing between those two plates. You have two infinite flat plates and the fluid is moving between those two plates. How the fluid is moving? The fluid is made to move by moving one of the plate in one particular direction.

Let this is the classical Couette flow, that is you have say two flat plate infinitely wide. So that there is no variation in that direction, we consider these this is the flow direction or at time t equal to 0 this is given a motion say u 0. The upper plate the upper plate is made to move with a uniform velocity of u 0. No pressure gradient is imposed in the general case but of course, if you want you can impose problem again solve the problem not being much of a difference.

Now, when the upper plate is moved made to move then there will be a shear stress is developed between these the interface. The upper plate and the fluid which is adjacent to it and this shear stress will make the fluid to move. So, this is basically a shear driven flow, the flow is due to the shear stress in addition to this of course, you can have pressure gradient if you want or even pressure gradient due to body forces. Like if you make these plates inclined then obviously there the difference is the gravitational or body force so, that will come as the modified pressure. So, all those maybe different condition of the flow the solution will be similar in all cases. This will say the x direction and this will take, let us say the y direction and the z direction is along the width of the plate in which in which the plate is infinite so that there is no variation in the z direction .

So, what will be the governing equation this for this problem we will consider steady flow that is the flow has been allowed to settle down. We are not considering the situation just when the plate has started moving you have seen that the plate is moving for some time. So, that finally, nothing is no more changed with time. When the flow starts of course, it will change with time, but then one time sometime will reach when there will be no further change with time, the problem will reach to a steady state. We will consider that the problem has reached to a steady state no more change in time.

So, for the steady flow what are the governing equation in this case. The mass conservation or continuity equation is simply this is of course, remember we are considering only incompressible flow no change in density. What will happen to the Navier stokes equation. We have seen that for these type of flow where the flow is unidirectional and the flow in the direction of motion is independent of that direction that is only u component of velocity is present and u is independent of x. The Navier stokes equation become d u, d t plus u d, u d, x.

The d u, d t is 0 u d, u d, x is also now 0 equal to minus d p, d x plus mu into d 2 u d, x 2 plus d 2 u, d y 2 plus d t2, u d, z 2.

For this case the way we have taken d p, d x is zero. We are considering no pressure gradient by pressure, we are meaning modified pressure.

Student: (( ))

If you want to consider some pressure gradient or if in some problem you need to consider the pressure gradient here then of course, you can add that. So, you have the pressure gradient what will happen to the Navier stokes equation all the terms are 0 except mu d 2 u d y 2. So, this equation simply becomes mu d 2 u d y 2 is 0.

If you want to solve for temperature then, that also can be solved as what k d, t d, x sorry d t, d y, d t, d y square d 2 t d, y square plus mu sorry yes. It becomes c.

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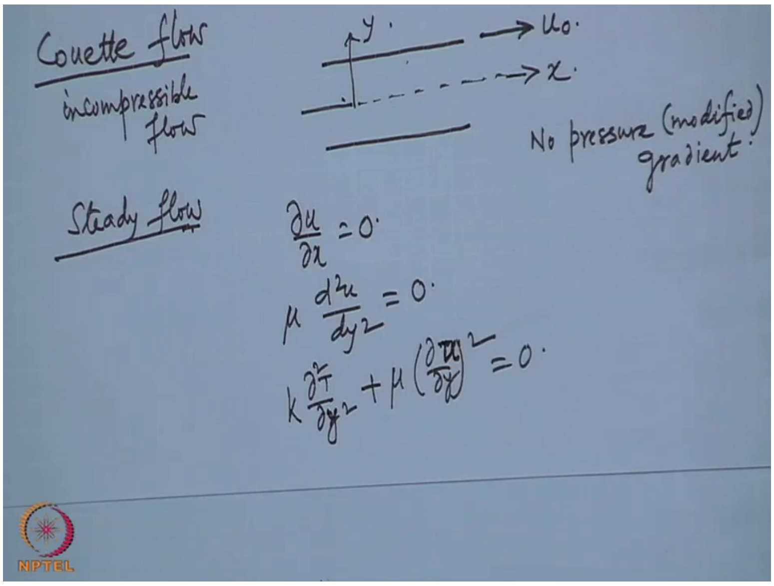

Sorry sorry this is the energy equation in case you need to solve for the temperature field this will be the form of the energy equation in this case [FL]. Now the solution of this equation is basically its trivial the velocity field is basically a trivial.

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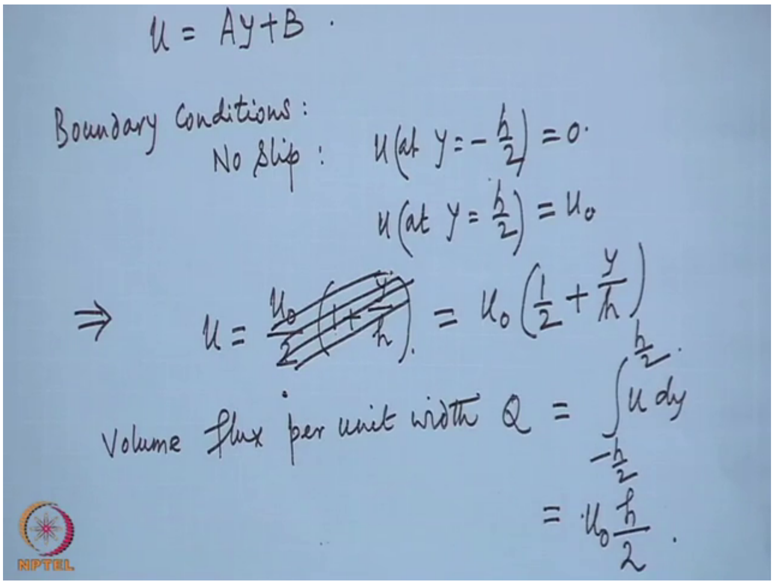

What is the solution?

Student: (( ))

Second derivative is 0.

Student: (( )).

That means the velocity profile is linear velocity variation with is linear u equal to.

Student: a y plus b.

A y plus b the constants are to be determined using the boundary condition in this case.

What are the boundary conditions? We can apply the no slip on the two walls, on the two walls that, there will be no relative velocity between the fluid and the solid at the interface. So, at y equal to minus h by 2 that is the lower wall u is 0 and y equal to plus h by 2. If we consider the distance h this distance let us take h. So, the boundary conditions are no slip boundary condition u at y equal to minus, then what happens to u? what is u then? U equal to u, equal to u 0 by 2. (How you will find the volume flux?

Student: (( ))

Volume flux is flow velocity multiplied by the sum area.

Student: (( ))

What will be that area here?

Student: Z x is an infinite.

Yeah z x is an infinite so, in case of infinite dimension in one size all the quantity with respect to that dimension is taken as unit basis. So, in this case volume flux per unit width (you can complete this one how much it will come I have not calculated how much is the value coming.

Student: (( ))

Yes.

Student: (( ))

Can you show that sheet. Oh you cannot see it. Is there u is different? It is not in there. U is… U is u naught into y by h plus infinity inside the brackets y by h plus 1 by 2.

Student: Y by h.

Plus 1 by 2.

Student: Plus 1 by 2.

Yeah in the brackets.

I mean say that this is not u 0 by 2.

Student: Yes sir. Sir you can change that is omega h.

U unknown to h by h by. Oh [FL] if we that means here it is implies 2 h.

Student: (( ))

No we have found out this a and b and this expression.

Student: Sir, this expression is not coming in the values of a and b the expression is u naught and then inside the brackets y by h plus n by 2.

You mean to say this will be.

Student: U naught into.

Into.

Student: Y by h.

Y by h.

Student: Plus 1 by 2.

Plus 1 by 2 oh, this is [FL] this half is inside. Yeah. Oh [FL]

Might be. Oh [FL] might be that.

Student: (( )).

Oh [FL].

Student: (( ))

I took the width as two h i took the width as 2 h so, it has it is fine let it be.

Student: (( )).

Yeah in my calculation I took that 2 h. So, these were y equal to minus h and y equal to plus h so, how much is this q, q is coming.

Student: U naught h by 2.

U naught.

Student: H by 2.

U naught h by 2, this expression also would have come simpler. If we would have taken 2 h it would have become only u naught h anyway, the shear stress how much is the shear stress which is causing this flow wall shear stress.

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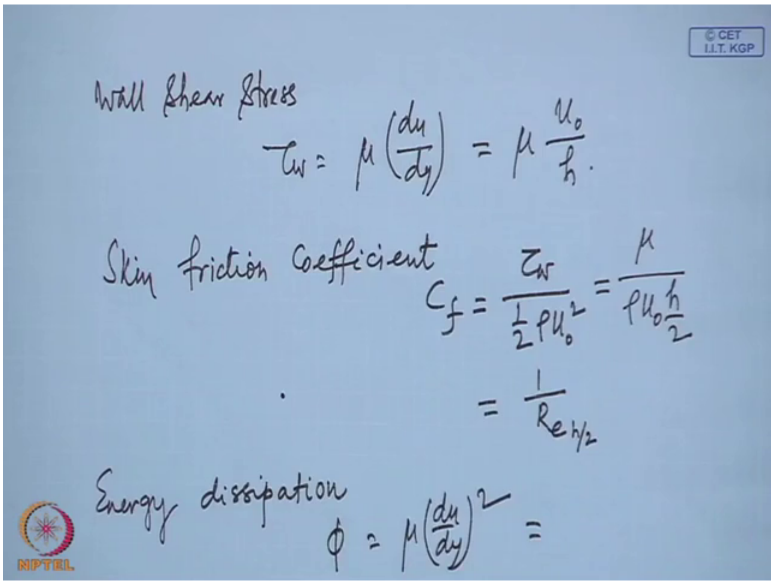

You can find it in any of the plate in this you can see will come as mu.

Student: (( )).

U 0 by h and the skin friction coefficient (how much is the skin friction coefficient mu by rho u 0 h by 2 or a Reynolds number based on half width or half depth Reynolds number based on half depth.

Student: (( )) sir, in the definition of skin friction coefficient.

Yes.

Student: The denominator that term that is u naught or u bar.

See this

Student: U naught (( )) right

In the last problem in the Hagen- Poiseuille flow problem that was defined as the average of the velocity in this case it is taken as the.

Student: (( ))

Maximum velocity u 0, u 0 happens to be the maximum velocity not taken as the average velocity. So, this velocity or what like in the definition of this Reynolds number. We will see that quite often we will choose different type of velocity, different type of length depending upon which is the most characteristics of the particular problem for this problem. The velocity which is the characteristic of this problem is. The velocity at which the plate is moved, because that difference the problem. So, that is the most important velocity here so, that is what is taken here and in that case the average velocity was taken as the characteristic velocity, because there was no particular velocity which was say more defining the problem.

So, it was taken the average velocity. In many other cases we will see that the velocity changes from point to point in that case, if you want to take some velocity for some purpose which velocity we take like say a Reynolds number which is taken as some density into rho into a velocity into some distance divided by the coefficient of viscosity. Now, what velocity and what distance, what length we take think about a act of line. We want to say what is the Reynolds number of that aircraft. We usually do not go on saying that Reynolds number at this point is this much at this point is this much. We usually says one Reynolds number for the flight obviously then, we have to take one particular velocity one particular length for an aircraft usually, these are.

So, the flight speed of course, as you know that flight speed has only one value, but the flow velocity on the entire aircraft will have different value at different point at each and every point the value will be different, but those values are not to define the Reynolds number only the flight speed the that is what is taken as the characteristic velocity of this problem and as far as the length is concerned usually it is taken as the mean aerodynamic chord. So, there are different situation in which this length and velocity and other things will be taken, So, in this problem this is half rho u 0 square in the Hagen- Poiseuille flow problem it is half rho u r square.

How much energy is lost that is dissipation? How much energy is lost? This equation as we have talked earlier is the work done by the shear force against that steadying motion and it is one way energy transfer part of the mechanical or useful energy is lost in that process that is what is. Well the last problem also we found out what is the dissipation, here also we have the dissipation term mu d u, d y square the term which is present in the energy equation this of course, very easy to find there is nothing. Once you get the value you can substitute in that energy equation and solve for t, you can see that the solution of t will become quite simple. And let us say we will not go for finding the temperature that you can do there is nothing in it.

Once you get this you can put it in that equation k d 2, d y square equal to this much and solve for t and again that t will contain two unknown coefficient which again you can find by satisfying the boundary condition. Boundary condition in this case may be quite different well usual boundary condition is that you can say that the lower plate is fixed at this temperature upper plate is fixed at this temperature anyway we will not do that. Let us say modify this problem a little bit. Think about we do not have the upper plate we do not have the upper plate and the lower plate is now inclined and the fluid is coming down, because of the body force which is a potential force. Can you solve the problem then, assuming a steady state solution. Let's tell me what will be the change in the equation.

Student: (( ))

For this classical Couette flow problem we had the momentum equation or the Navier stokes equation become simply mu d t u, d y 2 equal to 0.

Student: (( ))

Will there be a change in it.

Student: There will be a term for the pressure gradient.

There will be a term for the pressure gradient and can you say what will be that pressure gradient let us say the body force is the gravitational force.

Student: Rho g x is 0 rho g x is 0.

Rho g x

Is components of the rho g minus grad p plus rho g plus u square mu plus square b that will be equal to 0. That Laplacian is simply will come as d t u, d y 2.

Student: (( ))

Of course we are taking x is along the inclined plane and y is normal to it.

Student: (( ))

Because whatever, is the direction of the flow that is what is x to us.

Student: (( ))

For this problem

Student: (( ))

See the plate is inclined of course; the flow will be along that inclined plate.

Student: (( ))

And that will be the x, because our basic assumption is that the flow in that direction and we will have no gradient in that direction it is, Let us hope you can solve this problem you can find. What will be the appropriate pressure gradient in the x direction? In this case the pressure gradient is just, because of the gravitational force that is modified pressure only.

Student: (( ))

No actual pressure gradient ok.

We will move to a third problem. This third problem of course, we will later on come back to it again, but this time we will consider, because it has a very special importance to us subsequently and would like to have this consideration here let us consider we have a flat plate and again for simplicity consider this is infinite in one direction. Again let us say it is infinite in the z direction and x is along the plate and. So, y is normal to it, lets say the fluid this plate is emerged in a fluid then suddenly the plate is move with a speed u 0 then, what will happen to the fluid? Is quite obvious that the fluid which is adjacent to the plate we will experience a shear stress and we will start moving with the plate.

Now, as time progresses this shear stress or the effect of the shear stress penetrate deep within the fluid and after sometime a layer of fluid will move along with the plate is quite obvious that as time progresses. The thickness of the fluid which is moving with the plate will increase, if we consider that empty this is also another problem. Basic problem is again same that the fluid velocity is only in one particular direction in which the plate is moving and in that direction there will be no gradient of the fluid.

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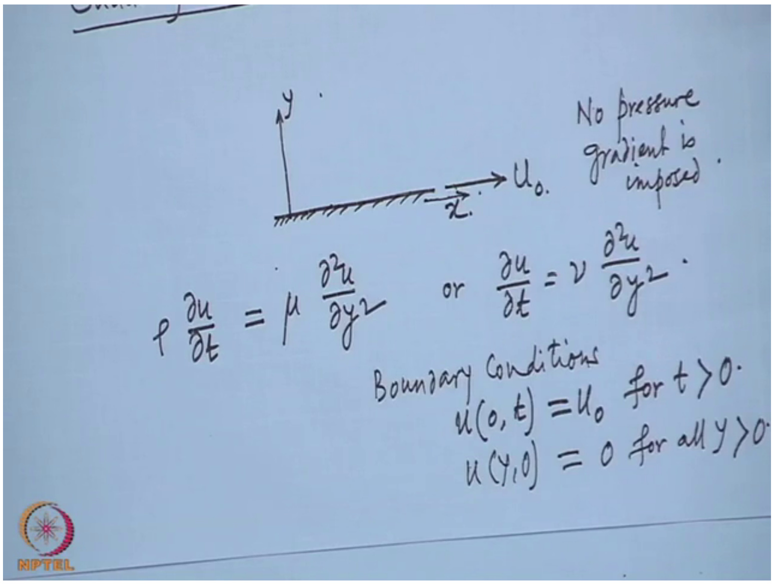

So, as suddenly moved plate suddenly moved plate as before we are considering unidirectional incompressible flow. So, unless we mention now we will always consider incompressible flow, let say this is the plate moved suddenly at this is the x direction as well. What will be the governing equation of this case continuity equation? We need not write that is simply d u, d x equal to 0. So, there is no point in writing it and the momentum equation is rho into d u, d t. Once again we will consider no pressure gradient is imposed no pressure gradient is imposed. What is the boundary condition what are the boundary condition.

Student: Equal to 0 u equal to (( )).

Hmm

Student: U I equal to 0 equal to u naught.

For time greater than 0 for time greater than 0 u y say write in mathematical form we are calling that is the time t equal to 0. When the plate is moved suddenly at t equal to 0 the plate is given a certain motion and also this is the initial condition you need an initial condition also of course, y the lower half is of no interest to us. We can modified this problem little bit that is this u or we can divide this equation by u 0 divide this equation by u 0 and made this boundary condition as 1 instead of 1 and u by u 0 we can treat as the variable.

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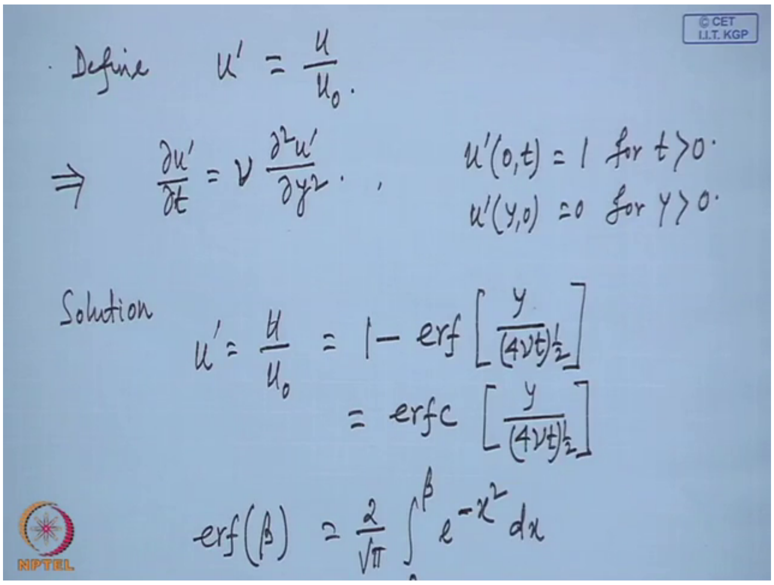

Let us do that, let us call it u prime then the equation will become and the boundary condition will become the other of course, same what is the solution of this equation have you solved this equation. I think perhaps you have solved this equation in your mathematics class.

We have to assume constant anyways. As I assume both equal to some constant. Both equal to some constant ok. You can, but why should you assume both equal to constant.

Student: (( ))

They are equal, but they need not be constant each of them need not be a constant, why should they be constant? I mean in general in some particular case it maybe that is a different matter, but in general you cannot assume them to be constant. I mean what is that logic behind it that you are assuming them constant.

All the derivatives are with respect to different variables. The solution it take it I think you have solved it in mathematic class this is also called e r f c. This e r f stands for error function and I think you know what is the error function say error function of any particular quantity let say beta equal to what 2 by root pi into 0 to beta, e to the power minus x square d x that is what is error function. So, this is the solution and this e r f c is complementary error function. See it is very easy to understand the solution will depend on y by root nu t. This way see this equation what are the parameter in this equation u prime t, nu and y, u prime is the dependent variable the others are either t and y are independent variable and the nu is the parameter in this equation u prime is a non-dimensional parameter.

So, obviously it will be function of a non-dimensional function of the remaining parameter and if there are y, nu and t the possible non dimensional variable is y by root nu t. The non-dimensional combination coming from y nu and t is y by root nu t. So, it is quite easy to understand that it will be a the solution will be function of y by root nu t or u prime will be a function of y by root nu t as it. So, happens it is function of y by 2 root nu t what will be the wall shear stress.

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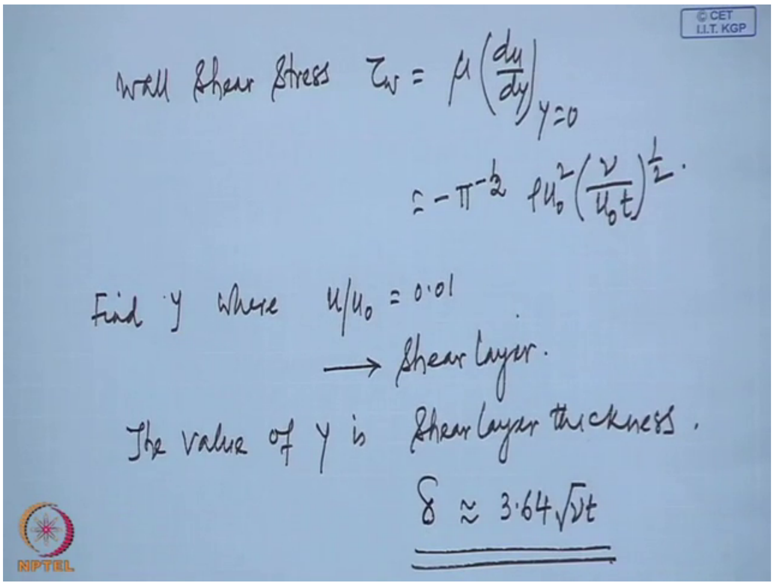

This is very important wall shear stress tau w is mu, d u, d y at y equal to 0, how much is that this will be see we see what happen. If we look to the wall shear stress. This wall shear stress decreases as t to the power minus half, can you say why the wall shear stress decreasing with time as time increasing more fluid is started moving so, why that makes ah.

Relative velocity of wall and that way in to this near to the wall. In terms of say let say this is think it this way this is a two dimensional flow. What is the wave vorticity in this case and vorticity is of course, in the z direction, but forget about the direction what the magnitude of the vorticity is or how much is the Vorticity? d v, d x minus d u, d y. It has only one component of curl of u for a two dimensional flow is what is simply d v, d x minus d u, d y, k z direction k in a two dimensional case the curl has only one component, which is normal to the plane of plane. See if you have two dimensional motion is in x y plane then the Vorticity is in the z direction and its value is d v, d x minus d u d y.

Now, in this problem there is no v so, d u d y itself is the Vorticity. So, what we are calling velocity gradient or shear stress that is also can be called as the Vorticity. Now, we know that Vorticity what happen that Vorticity first of all, it can convict with the flow it is diffused by viscous action and it might have some redistribution due to stretching and deformation effect looking to that forget about that stretching or deformation in two dimensional case that stretching or deformation is 0 in two dimensional case that stretching or deformation term cannot be present so, that is 0. So, what remains is that Vorticity can either be convicted and can we defused by viscous action.

So, initially when the motion started this Vorticity was confined very close to the wall very close to the wall. Now, as more and more amount of fluid is starting moving the viscous action is actually that is why the motion is starting that the viscous actions is defusing the Vorticity to other part say that is a natural diffusion phenomena. If there is high concentration of any quantity then the diffusion try to make it even over sudden region. So, if there is a Vorticity highly concentrated Vorticity near some region then the diffusion will try to spread it out over certain area. So, that there is no high concentration in any one region, but a smooth or milder concentration over a larger region. That is what always if process of diffusion and because of that process this Vorticity is being diffused to part and consequently the fluid which is started moving along with it is also coming under that Vorticity or rotation rotational motion.

So, this is one example where we are seeing that Vorticity which has been created near the plate is being diffused over time to other part of the fluid due to viscous action. The convection of course, always there, because in this case both are continuous process convention and diffusion you are not stopping it. We are not changing its velocity or anything so, the convection and diffusion both are continuous process and. So, wall shear stress decreases, because the Vorticity is being diffused to other part by viscous action and if we find out that how. What is the thickness of the layer of the fluid that is moving with the plate and that part.

Where the region over, which the fluid is moving is called in this case shear layer. Shear layer because there is shear present on it and the shear force course has started this motion. So, it is a shear layer and the thickness of the shear layer is defined. The thickness at which the flow velocity falls to say one percent of the u 0 then, you find the thickness of that region where find the y where u is say 1 percent of u 0.

So, find y where or say u by u 0 equal to 0 point 0 1 this will this is called the shear layer and is this is general anywhere. There is some interface this interface maybe even two fluid. So, like you know when air moves over water it is common case all the time happening air moves over water that is also an interface between air and water and again that interface will behave something like this flat plate and again a shear layer will form. Shear layer both in the air part as well as in the water part and you know, because of the motion of air or a wind over sea. The see sea surface particularly up to certain depth acquires some special type of velocity and that velocity profile is like a spiral which is called Ekman Spiral .

So, anyway what I was telling that whenever, there is an interface and then there is a motion in the interface as shear layer will develop a shear layer will develop. So, the shear layer can be bounded to a solid boundary is not essential to be bounded to a solid boundary it will form when there is no solid boundary. Just as in case of air water interface it can even form in air interface I tell you an example, where there is an air interface think about the jet that is the burnt gas coming out from an engine nozzle, maybe a rocket nozzle, may be a jet engine nozzle that is air a very small amount of hydrocarbon which does not change, it to change it from air that is still air it comes out and mixes with the atmosphere,

The atmospheric air is more or less at rest this air coming from the jet is at very high speed. So, that interface is again an interface this air and this air which is coming out as a jet at a high speed is different from the air which is at rest and the surface between the divides these two air is again an interface. So, again in that interface there will shear layer develop? So, whenever there is some interface a shear layer will always develop and the shear layer mean the flow is rotational there has Vorticity in it and the thickness over which this Vorticity spreads. Of course, it spreads to be precise theoretically it into infinite, but it is taken that by which this distance something like this depending upon.

If it is in this case we are calling it one percent, because we are moving the plate. So, gradually the Velocity is decreasing. So, where it will become 1 percent that is what is thickness. If we it take it the other way that is the plate is fixed and the fluid is moving suddenly then of course, at the interface the plate is stationary at that point there will be no velocity and then gradually it will increase. In that case we will call it where the velocity has reached 99 percent where the velocity has reach 99 percent. So, depending upon the situation either 0 percent 99 percent that part is usually defined as the shear layer thickness.

So, this is the shear layer thickness this value of y is called shear layer thickness and I want you to find out this in for this particular problem I will give you the value will be the shear layer thickness. The value will be approximately we will call this as delta the shear layer thickness is near about 3 point 64 root of nu t. So, it is increasing with t to the power half this result. We would like to use sometime later so, I will expect you to remember at least this much that the shear layer thickness increases as a rate.

Student: (( ))

Square root of time.

Student: When a both grad phi (( )).

That result will be important for us for a sometime later any way try to find the shear layer thickness and if you do not get it we will do it.