Introduction to Aerodynamics

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Lecture No. # 14

Kinematics of Fluid Motion-Velocity without expansion & vorticity.

So, we have from purely kinematic consideration have already established that in general the fluid velocity is coming from three different contribution, one is an isotropic expansion, one is rigid body rotation and the third is sum together of uniform velocity plus a steering motion without change in volume. So, we have now discussed to how to find the velocity field, if the rate of expansion is specified everywhere. If the vorticity is specified everywhere so what left is how to find the velocity which is associated with a pure steering motion without change in volume.

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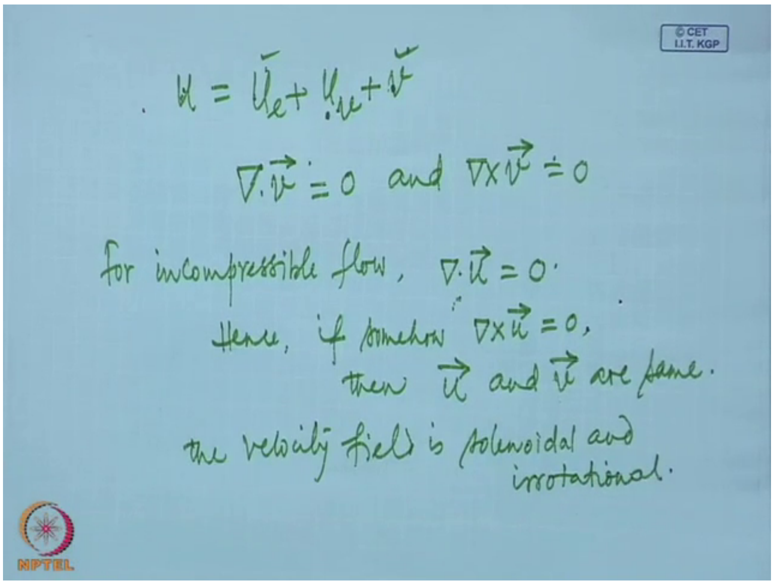

if you remember we mathematically wrote that velocity field in generally is u is u e plus u v where u is the part that gives the rate of expansion, which we can obtain if the rate of expansion is specified everywhere, u v is the velocity contribution coming from the rotation rigid body rotation. And we have seen that this also can be found if the vorticity is specified everywhere, and the last part which is associated with a pure steering motion without any change in volume. This part by definition that this part is now is not associated with no expansion, associated with no vorticity. That is there are there is no expansion associated with it there is no vorticity associated with it or other way that both divergence of v equal to zero and curl of v equal to zero.

So, this is the part of the velocity which is without any expansion without any rotation. And now we will try to say something how to find it, but before doing that see one thing in particular. If the flow is incompressible if the flow is incompressible then this total velocity or the actual velocity u, that itself satisfies divergence of u equal to zero. You know that for incompressible flow and we also know that for incompressible flow there is no rate of expansion. That is what it means this equation divergence of u equal to zero that is this u v component is not present in incompressible flow it will not be there.

Now if it so happens I am saying if it happens that the in particular cases the incompressible flow is such that, its vorticity is also zero. Then what happens? Then this u and v become same.

And in that case the velocity field is simply solenoidal and irrotational both are satisfied, divergence of u is zero curl of u is zero. At this stage of course, we would not be able to tell it definitely, but I will just tell you that you might have a question that, what is utility? Because in aerodynamics we are mostly concerned with gases and gases are highly compressible. So why we are interested in incompressible or a solenoidal velocity field? However it is not true, in many practical cases we will see later on that under certain conditions which are quite common, under certain conditions all flows can be treated as incompressible. And the velocity field to be solenoidal, and there are many common situation in all practical problems where the most part of the flow field is irrotational.

So even though its looks that these are too highly simplistic the flow field is solenoidal and irrotational. But they are not uncommon there are many practical situation where this are quite acceptable, that the flow field is almost solenoidal as well as irrotational. And it is then highly important to discuss about these type of flow. In the first look or first glance it may appear that is not a real situation at least in aerodynamics where you are mostly concerned with gas. But later on we will show that that is not so even in flow of air or flow of any other gases there are many situations where the flow field can be considered almost incompressible and the velocity field to be solenoidal as well as irrotational. Now since we will work again not in terms of u but in terms of v that is considering is one of those three contribution.

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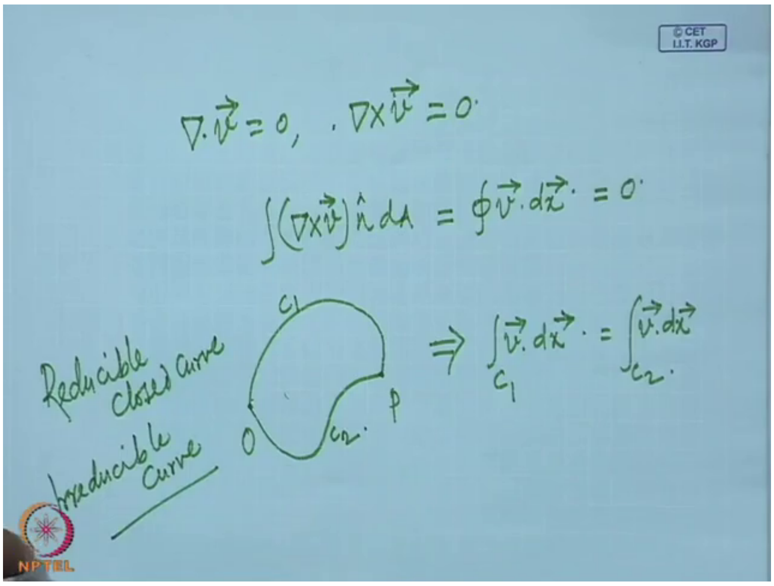

Since, we have divergence of v equal to zero as well as curl of v equal to zero we can show that over any open surface think about any open surface or let us say curl of v n d A where this area A area is completely merged within the fluid it is within the fluid, then this integration will be what?

Student: (( ))

And this is zero curl of v is zero everywhere. Now think about any two curve or rather let us say consider two any two points within the fluid. And then consider two curves that joins these two points and let us say that these two curves form a reducible closed curve like say these two points say o and p and say one curve and other curve. Then what this equation gives? Agreed provided that c1 and c2 forms a reducible close curve, if the curve once again to remind you what is this reducible closed curve? A curve will be closed called reducible if it can be shrunked to a point without going out of the domain.

So, in this case any curve within a fluid called can be called reducible, if this curve can be shrunked to a point without going out of the fluid. And we can see that in this case if the interior is just fluid we can very easily shrunk it to a point just by deforming. Think about say aircraft wing we have a curve round that aircraft wing is it reducible or not is the curve reducible or not? Its reducible. Let us say we have this I have this pen in my hand think about a curve round this pen, is that curve reducible?

Student: No.

Yes or no.

Student: No.

Why cannot we shrink it to a point without going out of the domain?

Student: (( ))

So, this is everything is the domain here, from here to infinity in all direction that is the domain. Can we shrink the curve to a point? You can the curve is here let it slide here and then shrink, finish so same thing if it is a aircraft wing or say an aircraft a curve rounding it, you can slide it. So, that it goes out of the wing and then shrink it. So, if there is a three dimensional object within a domain in a three dimensional domain we have a three dimensional object, one then all curves are reducible. We can simply slide it and do it we cannot do it in case of two dimensional, by our definition a two dimensional say wing means, a wing which is infinite in this direction. So, you cannot slide it out of the wing we slide it, it is still there the wing is still there.

So, in a two dimensional case we cannot have all reducible curve. But in the if there is a three dimensional body within the domain which is actually our interest in aerodynamics we will have the aircraft or say wing and then the atmosphere. The fluid air see in our case the fluid domain is the entire atmosphere and there is another the wing is a boundary of that fluid the other boundary perhaps will consider somewhere. So, we are not going out of the boundary. But if it is two dimensional then we cannot do it we have to go out of the boundary. If we want to shrink it we have to go out of the boundary however there might be some other curves which is not enclosing this object that can be shrunk.

So, there are different curves which can be shrunk which cannot be shrunk, but in case of this other type of domain wing within the infinite atmosphere in that case all curves can be shrunk all curves are reducible curve. But if it is two dimensional wing then all curves are not reducible curve there are some curves which are not reducible there are some other curves which are reducible. Any curve that loops the wing is not reducible but, any curve that does not loop the wing that is reducible. So, we have these two concept reducible closed curve and irreducible.

What it will be? What this will be? Apply divergence theorem What is this?

Student: (( ))

We want to express it as a Surface integral. So, what it is?

Student: (( ))

What is the surface of this volume? That is some of those two surface a one and a two that is the boundary surface for this volume v, so the surface integral now will have two terms, one for surface A1 the other for surface A2 and lets into is here so it will be what phi.

Student: V

V dot.

Student: (( ))

Into

Student: (( ))

D A over a two or we can write d A2, it does not matter minus phi v dot n one the two has different sign, because the with respect to the fluid domain the two normal has different direction. One is a outward one is the other is inward. We have taken two outward normal with respect to the boundaries but with respect to the fluid one has become inward the other has outward so that is why the two signs are different.

Now we see a special situation think that this v dot into is a normal component on the surface A2 normal to the surface A2, and v dot n one similarly normal to the surface A1 and if both of them are zero that is both boundary surface have no normal velocity, both boundary surface have no normal velocity. Then what will happen? Then v is zero if both the boundary surfaces have no normal velocity then this v is zero or other way that solenoidal irrotational velocity is not possible. If both the boundaries are stationary, if both the boundaries had rigid and has no normal velocity then the fluid contained between these two rigid boundaries we have no solenoidal irrotational velocity field.

Anyway so what is required that to have some solenoidal irrotational velocity flow field at least part of the boundary must have some normal velocity. Part of the boundary must have some normal velocity otherwise no solenoidal irrotational velocity field is possible. Now just to check uniqueness let us consider that we have two velocity field v one and v two. And to corresponding potential field phi one and phi two since the equation is linear then the difference of these two solution is also a solution. So, if we have two solution say v1 and v2 and corresponding potential as phi one and phi two then v1 minus v2 with associated potential phi1 minus phi2 will be a another solution. So, let us take then that lets take I do not know two solutions v1 associated with phi1 and v2 associated with phi2

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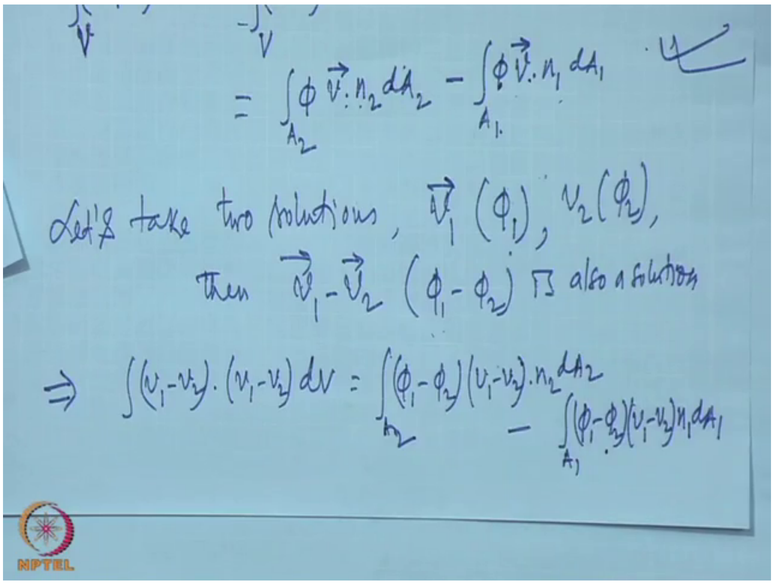

Then we can write this that v1 minus v2 dot v1 minus v2 integrated over the volume v is phi1 minus phi2 into v1 minus v2 dot n2 d A2 minus we have substituted phi1 minus phi2 and v1 minus v2 in this equation in this equation we have substituted this. Now if a solution is unique, if the solution is unique then phi1 minus phi2 and v1 minus v2 must be zero the solution will be unique if v1 and v2 are same phi1 and phi2 are same, and how will that be possible?

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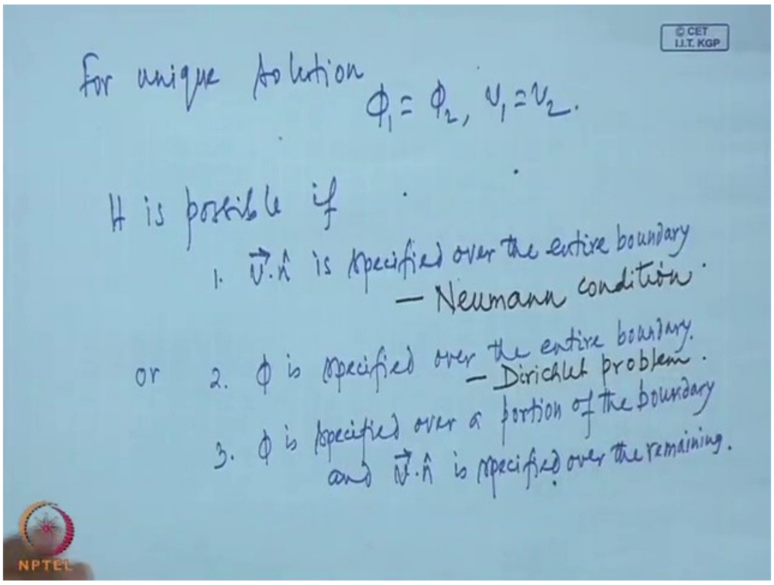

From here can you tell that will be possible if v dot n is specified if v dot n is fixed, alternatively if phi is fixed on the boundary. If v dot n on the boundary or phi on the boundary is fixed, either of the two or a combination of the two. That on some part of the boundary v dot is n is specified on the remaining part phi is specified then also it is possible.

So, in a problem where v dot n is specified the problems are called neumann problem or this see this is a condition on the boundary. So, they are called boundary condition, this particular type of boundary condition where the normal component of the gradient of the unknown function is specified. In this case of course, it is the normal velocity but this might be any general problem laplacian phi. It might be any general mathematical problem rather. This is very common equation in almost all branches of science laplacian phi in many cases it comes of course, the phi has different meaning in different situation and in all cases this is this is general. This is mathematics here is no fluid mechanics or anything.

So, these are general so if the normal component of the gradient is specified on the boundary that type of boundary conditions are called neumann boundary condition and then the problem is called neumann problem. So, this is neumann condition or neumann problem in a situation where the unknown function itself is specified over the boundary that is called Dirichlet problem and the last one is of course, a mixed. So, when these conditions on the boundary will be specified the solution will be unique so only or only those solutions which will satisfy these conditions they are the actual solution.