

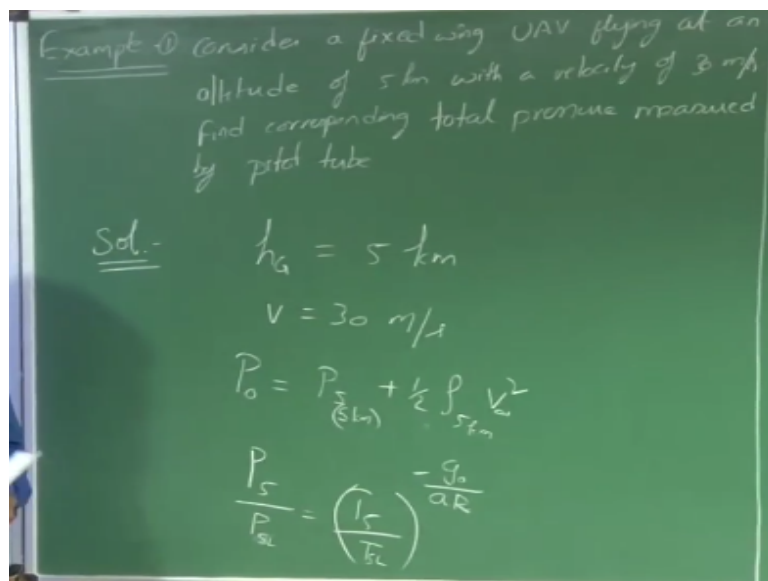
Design of Fixed Wing Unmanned Aerial Vehicles
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Lecture - 04

Examples, Pitot and Static Tube and Differential Pressure Sensor

Dear friends welcome back. In our previous example problems, there was a mistake, small mistake that we have made, so let us solve these example problems again right. I will quickly go through these examples again.

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First example is consider a fixed wing UAV flying at an altitude of 5 kilometers with a velocity of 30 meters per second. Find corresponding total pressure measured by the pitot tube? So the information that is given here is geometric altitude is 5 kilometers, so first we need to find the total pressure measured by the pitot tube. So we have velocity of the flight vehicle as 30 meters per second.

So how to find the total pressure? P_s at 5 kilometers that the static pressure at 5 kilometers + $\frac{1}{2}$ density at 5 kilometers and the corresponding velocity square right. So this is a dynamic pressure part and this is the static pressure. So at 5 kilometers we know the aircraft is flying in the gradient layer, you need to find the corresponding static pressure at 5 kilometers by using gradient layer equations.

So what we have is P at 5 kilometers/ P at sea level or P at sea level= T at 5 kilometers/ T at sea level- g_0/aR .

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$$\begin{aligned}
 & \therefore g_0 = 9.81 \text{ m/s}^2 \\
 & a = -6.5 \times 10^{-3} \text{ K/m} \\
 & R = 287 \text{ J/kg K}
 \end{aligned}$$

This is a relationship between pressure and temperature at different altitudes where g_0 is 9.81 meter per second square and a is -6.5×10^{-3} Kelvin per meter and R is 287 Joule per kg Kelvin right. Now if I want to find pressure, static pressure at 5 kilometers I need to know what is the corresponding temperature at 5 kilometers since we know the static pressure or static pressure at sea level and the corresponding temperature at sea level. So if I can find out T_5 , I will be able to calculate P_5 , P at 5 kilometers.

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$$\begin{aligned}
 a &= \frac{dT}{dh} \Rightarrow T_2 - T_1 = a(h_2 - h_1) \\
 h &= \frac{r h_g}{r + h_g}, \quad r = 6400 \text{ km} \\
 \Rightarrow h &= \frac{6400 \times 5}{6405} = 4.9761 \text{ km} \\
 T_5 &= T_{sl} + a(\Delta h) = 288.16 - 0.005 \times 4976.1 \\
 \Rightarrow T_5 &= 255.685 \text{ K}
 \end{aligned}$$

So to find out T_5 what I use is the definition of lapse rate, this implies $T_2 - T_1 = a \cdot h_2 - h_1$. So first we need to convert this geometric altitude to the geopotential altitude here. So

$h=r \cdot hG/r+hG$, so this r is the radius of earth, it has a small r where $r=6400$ kilometers right. This equals to $6400 \text{ kilometers} \cdot 5/6405$, so this is approximately equal to 4.9961 kilometers. See the difference is very, very less.

At lower altitudes, the geopotential and geometric altitudes are almost same right and now what is temperature T_2 , T at 5 kilometers= T at sea level+ $a \cdot \Delta h$ which is T at sea level is $288.16 \text{ Kelvin} - 0.0065 \cdot 4.9961 \cdot 10$ to the power of -3 correct. So this equals to that implies T at 5 kilometers is= 255.685 K right.

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The chalkboard shows the following calculations:

$$P_5 = (1.01325 \times 10^5) \left(\frac{255.685}{288.16} \right)^{-\left(\frac{9.81}{-0.0065 \times 287} \right)}$$

$$= 54.08 \text{ kPa}$$

$$P_{\text{at } 5 \text{ km}} = P_{5 \text{ km}} + \left(\frac{1}{2} \right) \left(\frac{P_5}{R T_5} \right) V_{\infty}^2$$

where $\rho_5 = \frac{P_5}{R T_5}$ (using eqn 4.16)

$$= 54.08 \times 10^3 + \frac{1}{2} \left(\frac{54.08 \times 10^3}{287 \times 255.685} \right) (10)^2$$

$$= 54411.638 \text{ Pa}$$

Now P at 5 kilometers is= P at sea level otherwise we know what is P at sea level $1.01325 \cdot 10$ to the power of 5 Pascals $\cdot 255.685 \text{ Kelvin} / 288.16 \text{ Kelvin}$ raised to the power of $-9.81 / -0.0065 \cdot 287$ right. What you get from here is 54.08 kilopascals correct. So now to find out the total pressure at the particular altitude I need to find corresponding density at that particular altitude right.

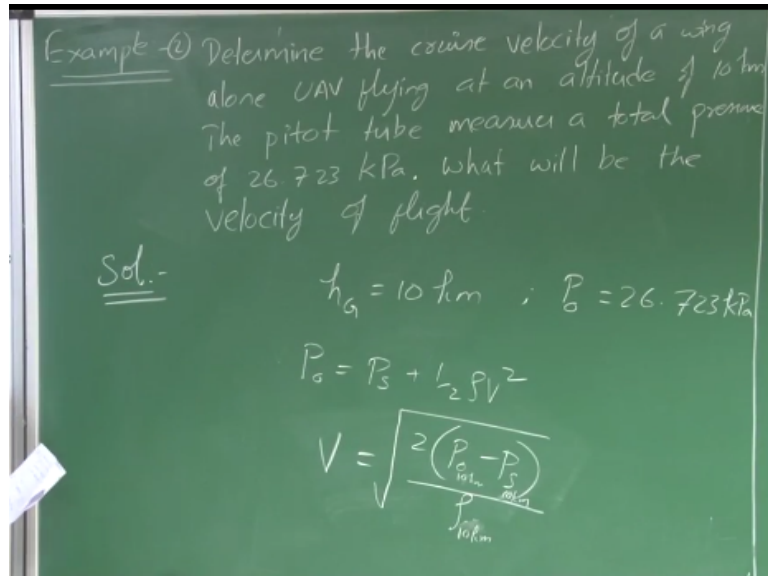
Total pressure at that particular altitude or 5 kilometers is= $\text{static pressure at } 5 \text{ kilometers} + 1/2$. What will be the density at that particular altitude? $P = \rho R T$ or you can use the gradient layer equation or P at 5 kilometers= $\text{density at } 5 \text{ kilometers} \cdot R \cdot \text{temperature at } 5 \text{ kilometers}$. You know these values right, you know P at 5 kilometers is 54.08 kilopascals, you know density at that altitude, no, I am sorry we have to find the density at that altitude.

You know what is R and T_5 is derived here 255.685 , so using this what you can use is $\text{pressure at } 5 \text{ kilometers} / R \cdot T$ at $5 \text{ kilometers} \cdot \text{velocity square } V_{\infty}^2$ where density

at 5 kilometers = P at 5 kilometers / $R \cdot T$ at 5 kilometers using equation of state. This implies what is P at 5 kilometers is $54.08 \text{ kilopascals} \cdot 10^3 \cdot \frac{1}{2} \cdot 54.08 \cdot 10^3 \cdot \frac{1}{287} \cdot 255.685 \cdot 30^2$.

So the aircraft is moving at 30 meters per second, so the corresponding velocity square is 900. What you have from here is 54411.638 Pascal. Now let us look at example 2.

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Determine the cruise velocity of a wing alone UAV flying at an altitude of 10 kilometers. The pitot tube measures a total pressure of 26.723 kilopascals. What will be the velocity of flight? So we need to determine the velocity of flight at 10 kilometers altitude. So h_g is 10 kilometers and what we have is total pressure at 10 kilometers altitude is 26.723 kilopascal right, kilopascal okay.

Now if I need to find the velocity what I need is a differential pressure right. So $P_0 = P_s + \frac{1}{2} \rho V^2$ where s stands for static pressure here, so to find the velocity at that particular altitude we need to know the differential pressure which is total pressure - static pressure and the corresponding density at that particular altitude.

Now we need to find what is this static pressure at 5 kilometers since we know P_0 at 5 kilometers and density oh I am sorry 10 kilometers here, please make corrections 10 kilometers, density at 10 kilometers okay.

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$$h = \frac{h_G r}{r + h_G} = \frac{6400 \times 10}{6410} = 9.984 \text{ km}$$

$$T_{10 \text{ km}} = T_{SL} + a (\Delta h) = 288.16 - 0.0065 (9.984 \times 10^3)$$

$$T_{10 \text{ km}} = 223.26 \text{ K} - \left(\frac{g_0}{aR} \right) \left(\frac{9.81}{-0.065 \times 287} \right)$$

$$P_{10 \text{ km}} = P_{SL} \left(\frac{T_{10 \text{ km}}}{T_{SL}} \right)^{-\left(\frac{g_0}{aR} \right)} = (101325 \times 10^3) \left(\frac{223.26}{288.16} \right)^{-\left(\frac{9.81}{-0.065 \times 287} \right)}$$

$$= (101325 \times 10^3) \times 0.2619$$

$$P_{10 \text{ km}} = 26537 \text{ kPa}$$

Now first we need to convert h_G to h for this $hGr/r+hG$, this= $6400/6410$ which is 9.984 kilometers right. Now the corresponding temperature at that altitude is why because if I want to know what is the static pressure at 10 kilometers, in the previous example we witnessed we need to find temperature at that particular altitude right. So T at 5 kilometers from the definition of lapse rate is T at sea level, 10 kilometers is T at sea level+lapse rate times delta h .

This= $288.16-0.0065$ is a slope of this gradient layer, first gradient layer and the change in the altitude is geopotential altitude 9.984×10^3 meters. So temperature at 10 kilometer altitude is= 223.26 Kelvin. Now pressure at 10 kilometers altitude is by using gradient layer equation, we have pressure at sea level and temperature at 10 kilometers altitude, temperature at sea level raised to the power of $-g_0/aR$ right.

So $101.325 \text{ kilopascal} \times 223.26/288.16$ raised to the power of $9.81/-0.065 \times 287$ okay, so what we have from here is 101.325×10^3 this is the atmospheric pressure at sea level $\times 0.2619$, so this= 26.537 kilopascal. So pressure static pressure at 10 kilometers is 26.537 kilopascals. So to find the velocity we need density as well density at that particular altitude.

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$$P_{10km} = \rho_{10km} R T_{10km}$$

$$\Rightarrow \rho_{10km} = \frac{26.537 \times 10^3}{287 \times 223.26}$$

$$\Rightarrow \rho_{10km} = 0.414 \text{ kg/m}^3$$

$$\rho_{sl} = 1.225 \text{ kg/m}^3$$

$$V_{\infty} = \sqrt{\frac{2(26.723 - 26.537) \times 10^3}{0.414}}$$

$$V_{\infty} = 29.972 \text{ m/s} \approx 30 \text{ m/s}$$

So by using the equation of state, P at 10 kilometers = rho at 10 kilometers R*T at 10 kilometers. What we have here is density at 10 kilometers is = what is P at 10 kilometers is 26.537*10 power 3/287*223.26 Kelvin. So this implies density at 10 kilometers is 0.414 kg per meter cube. So density at sea level is 1.225 kg per meter cube right, 1.225 kg per meter cube. See the density is decreased by more than half.

Now the velocity of flight at 10 kilometers where the total pressure measured by pitot tube is 26.723 kilopascals-static pressure at that altitude is 26.537 kilopascals/density at that particular altitude which is approximately 30 meters per second. So this is the velocity at which this wing alone UAV is flying right at 10 kilometers altitude. So let us revisit example number 3.

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Example 3 Consider the differential pressure measured by pressure sensor of a UAV, cruising at 30 m/s, is 409.05 Pa. Find the corresponding altitude of flight.

Consider the differential pressure measured by the pressure sensor of a UAV cruising at 30 meters per second is 0.4 kilopascals. Find the corresponding altitude of flight?

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Sol. - $(P_0 - P_s)_h = 409.05 \text{ Pa}$
 $V = 30 \text{ m/s}$
 $\Rightarrow V_h = \sqrt{\frac{2(P_0 - P_s)_h}{\rho_h}}$
 $\Rightarrow \rho_h = \left(\frac{V_h}{2}\right)^{-1} 2(P_0 - P_s)_h$

So what we have here is $P_0 - P_s$ at a particular altitude h is 409.05 Pascal and we know the corresponding velocity of flight is 30 meters per second. Say by some means you got to know some other instruments you got to know what is the velocity of this flight right. Now we have to find the corresponding altitude of this flight. So we know the velocity is root over twice the differential pressure velocity at that particular altitude say h is twice the differential pressure at that particular altitude/density at that particular altitude.

The whole thing is under root. Yeah, since we know velocity and the differential pressure we can find the density of that particular altitude. In that case, density at that particular h is = velocity square yeah $-1 * 2$ times $P_0 - P_s$ at that particular altitude okay.

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$$\rho_h = \frac{2 \times 409.05}{30 \times 30} = 0.909 \text{ kg/m}^3$$

$$\left(\frac{\rho_h}{\rho_{SL}} \right) = \left(\frac{T_h}{T_{SL}} \right)^{\left(\frac{-g_0}{aR} - 1 \right)}$$

$$\left(\frac{0.909}{1.225} \right) = \left(\frac{T_h}{288.15} \right)^{\left(\frac{-9.81}{-0.0065 \times 287} - 1 \right)}$$

$$\Rightarrow T_h = 268.624 \text{ K}$$

Now density at height h is = 2 times 409.05 pascals/30*30, this = 0.909 kg per meter cube. So this is density at h right. Now since most of this flights are in gradient layer, we assume that for this current course we assume that the flight happens in a gradient layer. So for this particular density at h and density at sea level, we can relate this to the corresponding temperature because if I need to find the altitude what I need to know?

See the only way is like I have the definition of lapse rate where I have dT/dh. If I know this dT, I can find this dh since the slope for this first layer is constant right, the slope for any of this gradient layers is constant right. Now this is T at particular altitude/T at sea level raised to $-g_0/aR-1$ okay. This = $0.909/1.225 = \text{temperature at that particular altitude/sea level temperature}$ 6 Kelvin which is $-9.81/-0.0065 \times 287 - 1$. This implies T_h , T at the particular altitude is 268.624 Kelvin.

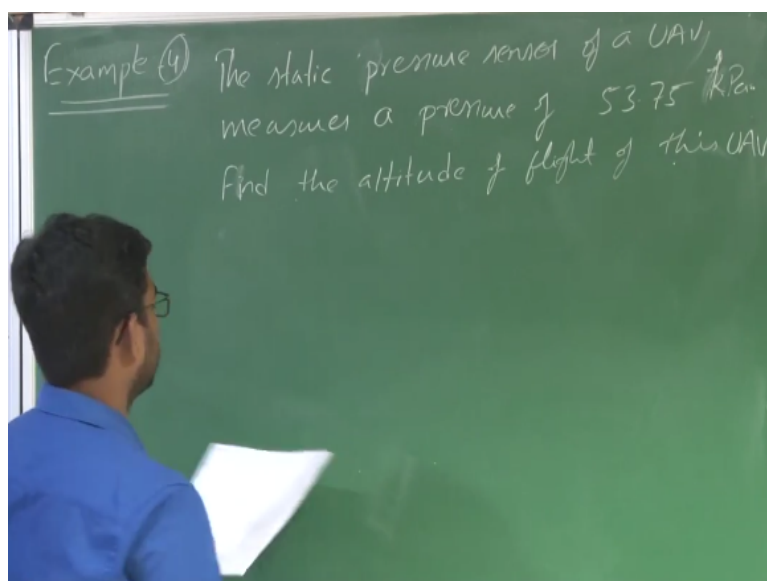
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$$\begin{aligned}
 a &= \frac{dT}{dh} \\
 \Rightarrow dh &= \frac{dT}{a} \\
 \Rightarrow h &= h - 0 = \frac{T_h - T_{SL}}{a} \\
 \Rightarrow h &= \frac{268.624 - 288.16}{-6.5 \times 10^{-3}} \\
 h &= 3.005 \text{ km} \\
 h_g &= \frac{h \left(\frac{r}{r-h} \right)}{\frac{r-h}{r-h}} = \frac{(6400) \times 3.005}{(6400 - 3.005)} \\
 h_g &= 3.0064 \text{ km}
 \end{aligned}$$

Now I know what is the temperature at this particular altitude h right by using definition of lapse rate, $a=dT/dh$ and what we need is $dh=dT/a$, dh is h because with reference to sea level I can write dh as h here, $h_2-h_1=h-0$ that is T at altitude $-T$ at sea level $/a$. This implies $h=268.624-288.16$ is the sea level temperature $/-6.5 \times 10^{-3}$ Kelvin per meter.

This equals to 3.005 kilometers is what you have the h here is 3.005 kilometers. Now the corresponding geometric altitude is $h \times \text{radius of earth} / (r-h)$, this $=6400 \times 3.005 / (6400 - 3.005)$, h_g is 3.0064 kilometers. So you are approximately flying at 3006 meters right. So the altitude of flight is approximately 3 kilometers, so the difference between geometric and geopotential altitude is very, very less here. Let us solve this final example again.

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The static pressure sensor of a UAV measures a pressure of that means this is a static pressure that is measured by this UAV which is approximately 53.75 kilopascals. So we need to find the corresponding altitude of flight of this UAV right?

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The image shows a green chalkboard with handwritten mathematical equations. At the top, it states $P_{Sh} = 53.75 \times 10^3 \text{ Pa}$. Below this, it shows the equation $\left(\frac{P_{Sh}}{P_{SL}}\right) = \left(\frac{T_h}{T_{SL}}\right)^{-\left(\frac{g_0}{aR}\right)}$. This is followed by an arrow pointing to $\Rightarrow \left(\frac{T_h}{T_{SL}}\right) = \left(\frac{P_{Sh}}{P_{SL}}\right)^{-\left(\frac{aR}{g_0}\right)}$. Finally, another arrow points to $\Rightarrow T_h = T_{SL} \times \left(\frac{P_{Sh}}{P_{SL}}\right)^{-\left(\frac{aR}{g_0}\right)}$.

That is straight forward right but this is in fact the practical way to find out the altitude by using the static pressure sensor right. You have the static pressure sensor and that particular sensor measures the static pressure at that particular altitude. Now you need to find out the corresponding altitude of flight right. So what I have is P_s static pressure at h is $= 53.75 \times 10^3$ Pascal right. So what I need to find is altitude, so if I have to find the altitude I need to know what is the corresponding temperature at that altitude.

So from the gradient layer equations, P at altitude/ P at sea level static pressure at that altitude/static pressure at sea level= $\text{temperature at that altitude}/\text{temperature at the sea level}$ raised to the power of $-g_0/aR$ right. So this implies what I have is T_h/T at sea level is $= P$ at altitude and static pressure at sea level raised to the power of $-1/g_0$ or $-aR/g_0$.

This implies the temperature at that particular altitude is static pressure at that altitude, it is the ratio of static pressure at that altitude at sea level, static pressure raised to the power of $-aR/g_0$.

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$$T_h = (288.16) \left(\frac{53.75}{101.325} \right)^{5.256}$$

$$\Rightarrow T_h = 255.378 \text{ K}$$

$$\Delta h = \frac{T_h - T_{sl}}{-a} = \frac{255.378 - 288.16}{-0.0065}$$

$$\Rightarrow h = 5.043 \text{ km}$$

$$h_g = \frac{r \times h}{(r - h)} = \frac{6400 \times 5.043}{(6400 - 5.043)}$$

$$h_g = 5.0469 \text{ km}$$

So the temperature at this altitude is=288.16 Kelvin times temperature pressure at that altitude 53.75/101.325, so both are in kilopascal 1/5.25 right. So this=temperature at that altitude is=so 255.378 kilopascals sorry Kelvin. So you have the temperature, now by using the definition of lapse rate delta, $h=T_2-T_1$ or T_h-T at sea level/ $-a$ this= $255.378-288.16/-0.0065$.

This implies h is=5.043 kilometers right. So we have geometric altitude. Now convert this to corresponding geopotential altitude right, $r \cdot h / r - h$, say $6400 \cdot 5.043 / 6400 - 5.043 = 5.0469$ kilometers. This is the altitude at which this UAV is flying right when it is measuring the static pressure of 53.75 kilopascals.

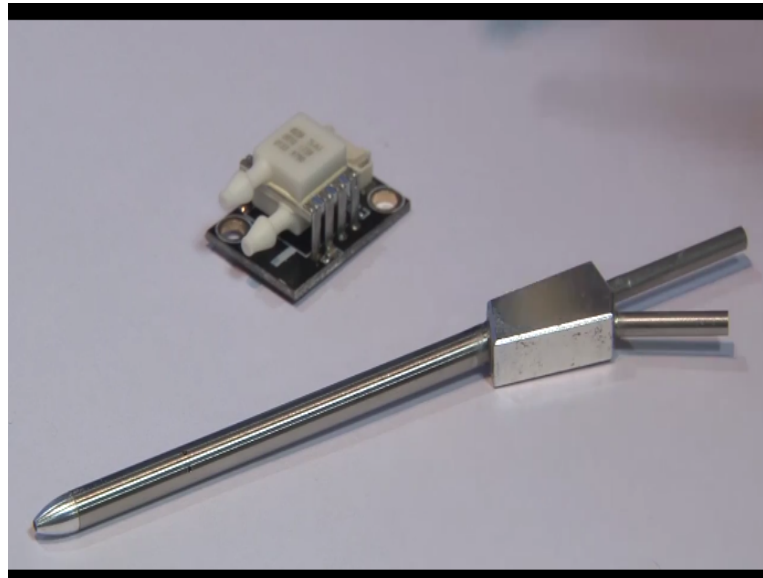
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$$h_g - h = 5.0469 - 5.043$$

$$= 3.9 \text{ m}$$

Now what is the difference between geometric and geopotential altitude? 5.0469-5.043, this is hardly 3.0 meters. So this is the difference between geometric and geopotential altitude right. Now here we have static pressure right, so let us look at some of the sensors that we have right now that can measure the total pressure and static pressure.

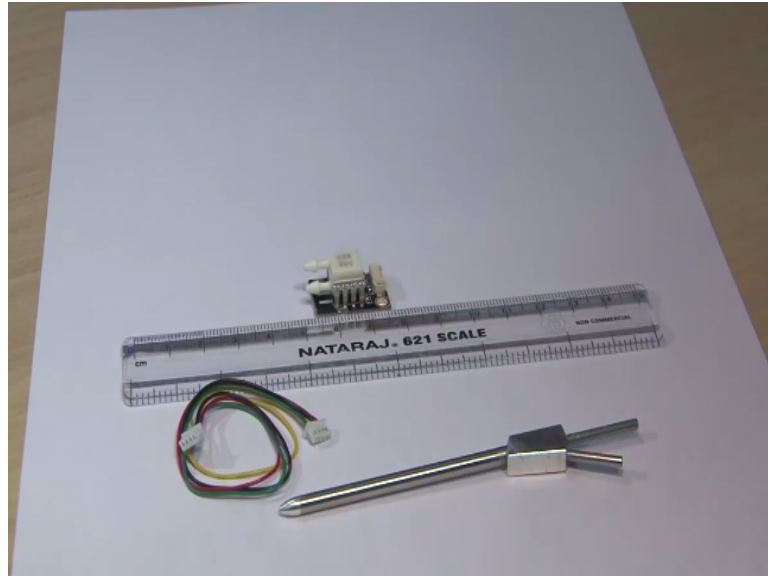
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Let us look at this pitot-static tube, see the holes that you can see on this periphery are meant for static pressure and you can see one hole which is along the longitudinal axis of this tube that is meant for total pressure right. So these holes are along the circumference of this outer tube right. You can consider there are two coaxial tubes, one with the closed mouth, the other with the open mouth here right.

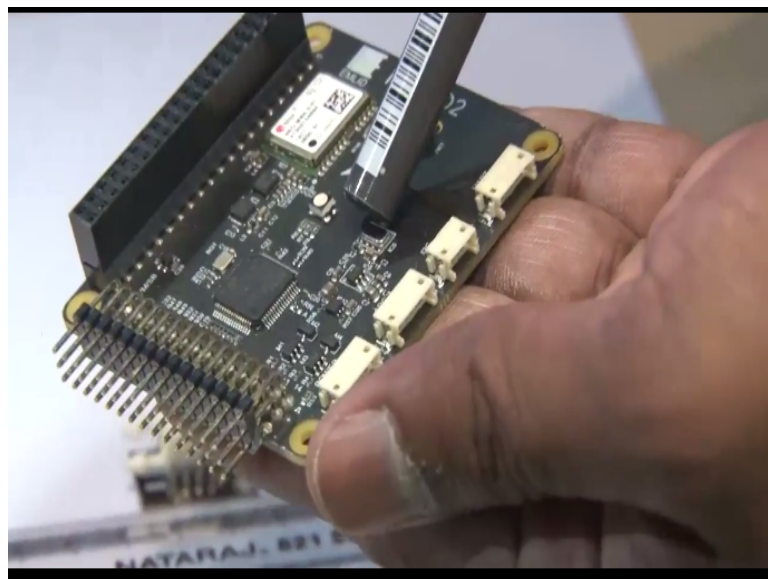
So this one, this particular outlet is for static pressure and the one which is straight here along with the tube along the longitudinal axis of this tube is for total pressure. So let us look at this pitot-static tube and the corresponding pitot-static sensor pressure sensor, it is a differential pressure sensor right. So the bottom inlet is meant for static pressure to measure the static pressure and the top one is for the total pressure. So the output from here what you get is a differential pressure.

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There is a differential pressure sensor, you can see the scale of this sensor is about a centimeter long right less than a centimeter and the total sensor size is about 2 centimeters right. This is the differential pressure sensor and the size of this sensor is about 2 centimeters here including the PCB which helps you to acquire the data right and it weighs hardly 15 grams right.

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And we have another sort of sensor that is mounted on this Navio2 Autopilot. See what you have here is a static pressure sensor. So for this static pressure sensor, you will not be able to connect any tube from the static port rather wherever you mount this you need to allow this particular board to interact with the atmosphere right. You cannot make it isolated system. You cannot keep this isolated from the atmosphere. All you need to do is make a small slit, so that the air in that cabin interacts with the surrounding atmosphere right.