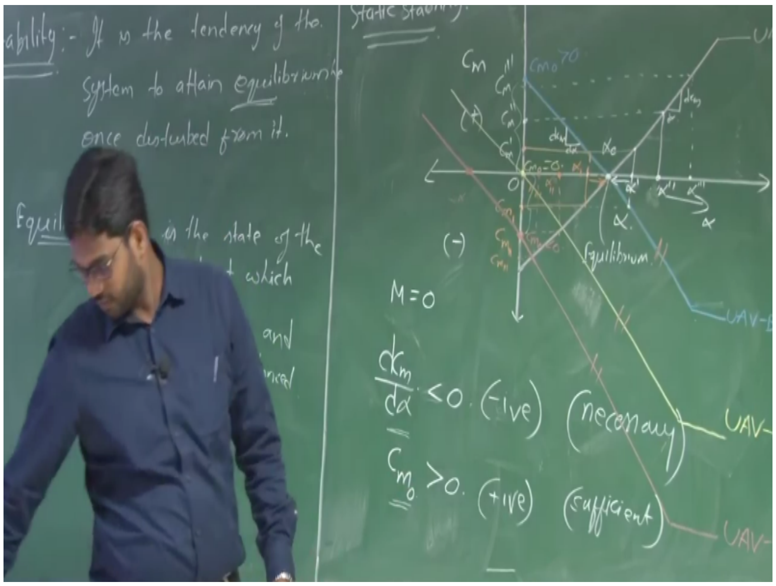
Design of Fixed Wing Unmanned Aerial Vehicles

Dr. Subrahmanyan Sadrela
Department of Aerospace and Aeronautical Engineering
Indian Institute of Technology – Kanpur

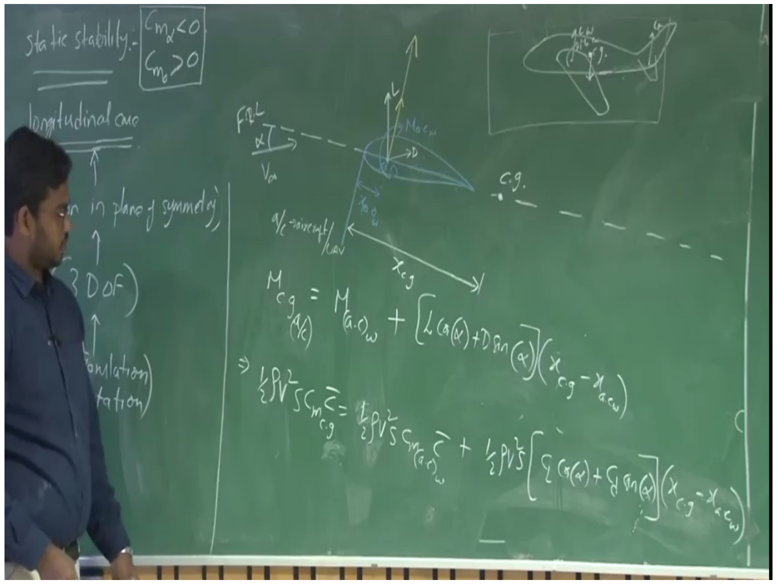
Lecture - 19
C.G. Location and Longitudinal Static Stability

(Refer Slide Time: 00:13)



The moments here the Cm we are talking about CG and this cm variation is angle of attack for static stability too and the Cm0 should be > 0, right. So what should be the CG location in order to have the Cm alpha 0 and Cm0 > 0; Cm alpha < 0 and Cm0 > 0, right. So let us look in to that.

(Refer Slide Time: 00:47)



So what is the condition criteria for Static Stability. Cm alpha has to be > 0; Cm0 should be > 0. So let us say this is your UVA; have a wing and tail. So when we are talking about longitudinal static stability. So let us talk about longitudinal static stability. So longitudinal case is the motion in this vertical plane, right. That means it can translate front and back; go up and down, right. I mean go up and down in the sense it can perform a loop in this blackboard in this;

plain, right. We have a plain of symmetric of this aircraft, right. So if we take a plane so which is cutting this aircraft exactly at this center line, right. So above this plane we have a symmetry, right. This plane of symmetry is known as; the motion along this plane of symmetry is a longitudinal motion, okay. So that means, so in this vertical plane it can do pitch up, pitch down that means a rotation, go front and back, move up and down.

So these are the three motions in the longitudinal plane. So this motion about plane of symmetry, right. How many motions? 3 DoF, 3 degrees of freedom. So 2 translations, one rotation. Okay. Now let us assume I cut this aircraft at this plane of symmetry and say the aircraft has a CG located at some location, at some point here with respect to the leading edge, right. This is your CG.

Now when you are talking about longitudinal stability which means what are the forces in this longitudinal plane? One is a weight other one is the lift from the wing and lift from the tail. And say you have a pitching moment about the aerodynamic center of the wing. And generally tail is a symmetric aerofoil. So the purpose of this tail is to create a moment rather than generating the force, generating a greater force.

So it is like creating a moment by generating a small force about the CG. Now let us assume this as Fuselage Reference Line okay. So in some cases you also encounter Iw that is inclination of wing with respect to this Fuselage Reference Line; what is your wing sitting angle I of w, right. Say this is your v infinite and say this particular angle is your; say if I define angle of attack with respect to Fuselage Reference Line this is your alpha with respect to F.R.L.

So the total angle of attack at the wing will be I of w + alpha. Okay. Now for this case what we assume is this F.R.L and wing chord are along the same time. So I am representing this whole wing as by means of this aerofoil which is a root chord aerofoil, right this is your root chord. So the root chord, so this is your root chord; it is a straight line joining a leading edge and the trailing edge of this root chord.

The root chord and this fuselage reference line are alight with each other. They both coincide. So whatever the angle of attack that you see here with respect to alpha, so that Iw you can incorporate at anytime. So say this is your aerodynamic center of the wing. This is measure parallel to this fuselage reference line, x ac of wing, this is your aerodynamic center, right. This is your x ac of wing. Say, this is your CG, location of your CG. So this is x cg with respect to this leading edge, right.

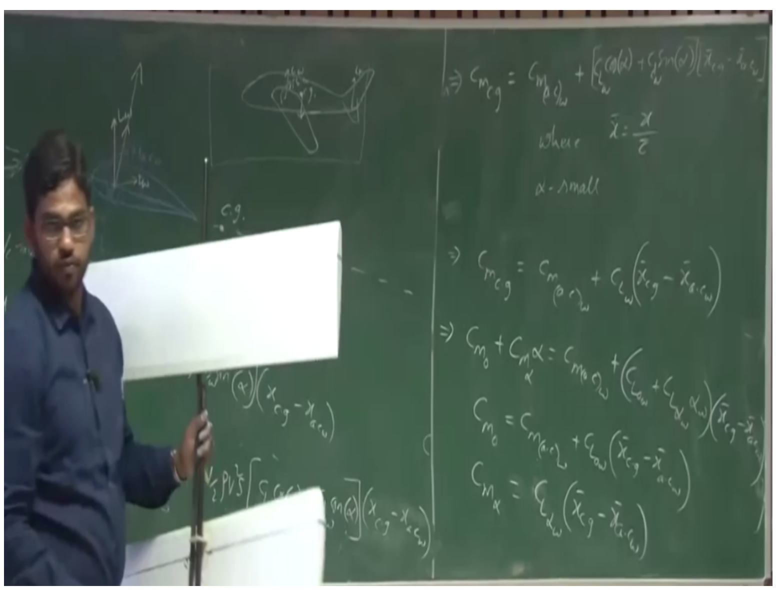
So now let us not consider the tail. Let us say; this is a wing alone right. It is a wing alone system, we need to know how it fly. So the wing alone can also fly. See what should be the CG condition to get this Cm alpha negative and Cm0 positive to make it statically stable. So when I say there is a flow there is a lift acting perpendicular to v infinite and there is a drag, right. And also about aerodynamic center we have something called moment.

Moment about aerodynamic center of the wing. Do you accept this? Now if I write moment about CG of the aircraft, see moment about CG when I say it is about the entire aircraft, not exclusively writing this, otherwise you can say moment about a/c is an aircraft; there a/c represent aircraft or UAV, okay. This equal to; so this is a moment about CG, right. So there is a moment about aerodynamic center of wing, moment about ac of wing, right as it is.

Plus y+; because the lift; so I should take the component of these forces acting perpendicular to this fuselage reference line, right. So what is the perpendicular to this fuselage reference line? Say this is your fuselage perpendicular to fuselage reference line. So the component of L along this perpendicular and D along this perpendicular which is L cos alpha + D sin alpha * this is a total force at a multiplied by the distance between this ac and CG which is x cg-x ac of the wing.

So this is x cg-x ac of the wing, right. This I can express in a non-dimensional form Cm about Cg aircraft or Cm cg = 1/2 rho v square S*Cm ac of wing * C bar, sorry please make it correction it is a; I miss C bar here. So 1/2 rho v square C bar * Cm cg = 1/2 rho v square S*Cm ac of wing * C bar, right+1/2 rho v square S*CL cos alpha + CD sin alpha * x cg – x ac of wing, right. Now what is Cm cg?

(Refer Slide Time: 10:54)



Cm cg = Cm ac of wing. So by the way the moment so the reference area for this moment is a wing area, wing planform area. And the dynamic pressure is a dynamic pressure faced by the wing here, right the reference dynamic pressure; 1/2 rho v square S*Cm cg this is because we are talking about moment due to aerodynamic forces about cg = moment about moment coefficient about the aerodynamic center + CL cos alpha + CD sin alpha * the distance x bar cg – x bar ac of wing where x bar = x/c bar, okay.

We are talking stability about this equilibrium, right. So this alpha is small, we assume the alpha is small or say initially it is trimmed. Now we write the moment equation about this change in alpha which is delta alpha, right. So what is the change in the force, how this change in the force is doing, right, he is going to. He is going to affect this stability of the system. See we talk about perturbation about some equilibrium right.

So let us say CL is the list generated due to a perturb delta alpha. Initially the moment is 0 that is a equilibrium. Now say alpha is a perturb alpha itself, whatever the alpha that you. So assuming alpha a small what you have is CL cos alpha is 1, when alpha is 1 inside alpha is alpha; CD*alpha is further small compare to CL so you can neglect this particular term. So what you have is Cm cg = Cm ac of wing + CL*x bar Cg-CL of wing here CL of wing Cg of wing.

So I am sorry I forgot to mention this. This is CL of wing and Cd of wing, CL of wing and Cd, x bar cg – x bar ac of wing, right. So can I express this entire pitching moment of the aircraft as Cm0 of the aircraft or Cm0 about Cg or Cm alpha about cg*alpha. This is equals to Cm of Cm about aerodynamic center of wing + what is the CL of wing? CL0 of wing + CL alpha of wing * alpha of wing * x bar cg – x bar ac of wing.

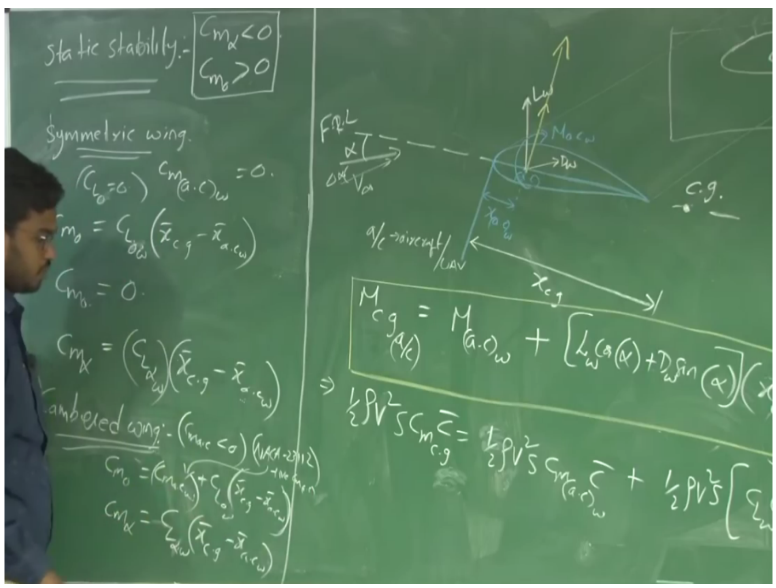
So where the Cm0 of this aircraft is Cm ac of wing+CL0 of wing * x bar cg-x bar ac of wing. That comparing the constant and coefficient here, right. Cm alpha of aircraft it CL alpha of the wing * x bar cg – x bar ac of wing. Am I correct? See, we ultimately need what is a Cm alpha and Cm0 of the aircraft, so we are trying to arrive at the condition. So for a static stable system let us say if this wing alone has to be static stable;

So you can assume it is a big wing, right the same wing what we have here, right and the cg before that. Understand? You assume there is a wing on either side and the cg along this axis. The wing is coming out in this direction, right. Now what should be the cg location to make have a stable flat? So this is a wing alone configuration so this Cm0 this depends upon the Cm ac of the wing+CL0 of the wing * x bar cg – x bar ac.

If this has to be positive but let us look at what is Cm ac of wing. If I take a symmetric wing the bottom one is a cambered one the top one is a symmetric wing, right. It is a symmetric wing you can see. So for a symmetric wing the moment about aerodynamic center of wing is 0, right. For a cambered wing it is negative. This is a cambered wing, this is a cambered aerofoil and this is a symmetric aerofoil.

So this is negative, so this the moment about this aerodynamic center is always negative quantity, here it is 0, right.

(Refer Slide Time: 16:43)



So say if you have a symmetric wing what you have Cm about aerodynamic center of wing is 0, right. So in that case what happens? Say this is your st-1 stability, right and tails sizing will perform from there. This is your st-2; st-3 say stability equation 3; this is your st-4; this is your stability equation 2;

Or say this is your stability equation 2, st-2, right. st-2; st-3; st-4, okay. From st-3 and st-4 what you have is Cm0 = Cm ac 0 for a symmetric wing, so CL of wing * x bar; sorry CL0 of the wing

* x bar cg – x bar ac of wing. But for a symmetric wing CL0 = 0, right. Lift coefficient at 0 angle of attack is also 0 there is no lift at 0 alpha. So Cm0 is 0 for a symmetric profile.

So for an aircraft scene or previous case when we are talking about Cm versus alpha variation we witnessed like Cm0 is 0; when that can happen when you have a symmetric wing. All right. But to trim it at a positive angle of attack you need to produce a continuous negative elevator defection that gives you a positive moment, to trim it at positive angle that is a, let us say if you are trimming elevator at a particular delta I mean deflection;

If you are trimming the elevator that means you are giving constant deflection that means increasing the drag there induced drag at the tail, right so the overall drag is also increasing that is a penalty that you need to pay if we are going for a symmetric aerofoil. So Cm0 is 0, what is Cm alpha? CL alpha of wing * x bar cg – x bar ac of wing. So this quantity is not 0 because this slope lift is positive. So this particular quantity has to be 0.

That means the aerodynamic center should be > cg. When it can be greater than? That means the cg has to be ahead of this aerodynamic center then you will have Cm alpha negative. But again you cannot fly at positive angle of attack. So this wing which is symmetric with a symmetric profile; so in one case I try to wing it to the aerodynamic center cg to the aerodynamic center. So in that case it is continuously dipping, dipping down.

So when I make it bit ahead and give certain angle of attack during the flight, right that means I am throwing it horizontal but orienting at a particular angle then it will gild to certain extent after that it will try to, yeah. Because as an angle of attack changes you have negative moment here with the increasing angle of attack. Understand? So for an acrobatic aircraft you need to produce equal force on either sides.

So let us say if you performed roll right so this the aerofoil in this orientation should also produce equal lift. I mean lift equal to the weight that means so if; for an acrobatic aircraft or a 3D aircraft you need to use a symmetric aerofoil here, symmetric profile that produces equal force on either side. So if the ac is ahead of cg you have Cm alpha negative, right for a wing alone configuration to be stable the aerodynamic center ahead of the cg.

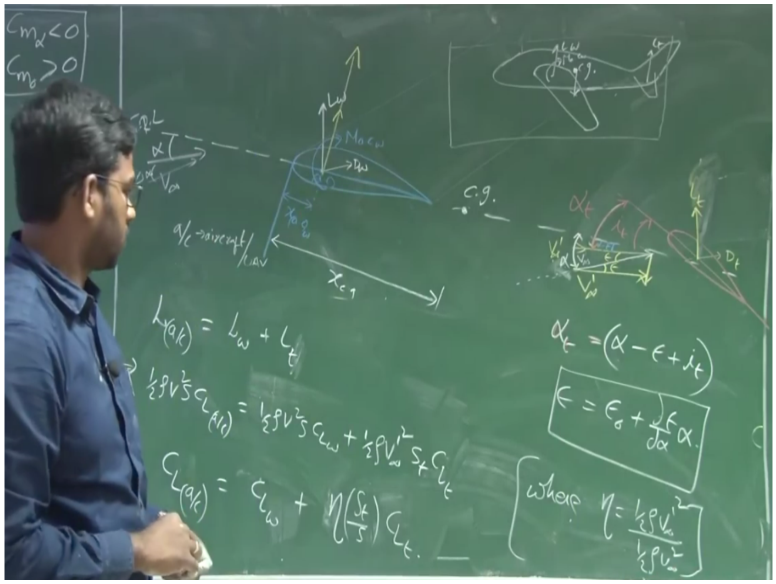
So in other case where this Cm; so in the cambered aerofoil case where the Cm about aerodynamic center is negative a symmetric cambered wing. Cambered wing, Cm0 Cm ac of wing + CL0 * x of wing * x bar cg – x bar ac, right. And Cm alpha = - sorry CL0 of wing Cl alpha of wing * multiplied by ac of wing. So for a cambered wing if Cm alpha to be negative I need to have;

Since it is a wing alone configuration irrespective whether it is a cambered or not I need to have aerodynamic center ahead at the cg ahead of the aerodynamic center to make it negative. So this particular quantity is negative. CL0 is positive for a cambered aerofoil, so this particular quantity is negative and Cm ac of wing is negative for a symmetric aerofoil. So Cm ac is negative for cambered aerofoil positively cambered aerofoil.

So this Cm0 is becoming negative that means you will not be able to trim at positive angle of attack. So for a wing alone UAV; in the introductory lecture we witnessed a flightiness of wing along UAV right. So for that UAV if you want to design such UAV you need to use a reflex aerofoil and this static margin or the; so in this case like the distance between cg and ac you should true such a way that this Cm0 will be positive, is a marginally stable marginally ahead.

And this quantity should be higher than the product of this quantity. Okay. So otherwise you have to hold elevator up continuously. Got it right? So for a wing along configuration to make it stable you need to use a reflex aerofoil where the Cm ac is positive in that case. So one such reflex aerofoil is NACA 23112, you can note it down. So NACA 23112: NACA 23112 for reflex for positive Cm ac okay for positive Cm ac.

(Refer Slide Time: 24:06)



Now let us now add a tail, tail to it. See it is a symmetric aerofoil; this is your chord of the tail, so this angle is known tail setting angle I of; the angle made by this chord of the tail with respect to fuselage reference line is known as tail setting. Now again this is your v infinite here. This is your v infinite, v infinite free stream velocity. But we also studied about this; there is a wing three-dimensional wing.

You have a downwash affect and upwash affect right. So induced downwash, you will have what is a induced downwash here at the tail because of the presence of the wing, okay. So because of which there is a downwash at the tail that reduces the angle of attack. What is downwash, this is the downwash there is a downwash flow. So the resultant vector will be, so this will be the resultant vector v infinity, so v infinity prime is the actual velocity here.

So the resultant vector at this tail is; so V infinity is an actual flight velocity and V infinity prime is a resultant velocity at the tail because of its downwash. So the angle induced by this downwash is epsilon, right. So this; now it become bit flatter the V infinity prime will make a lesser angle compare to V infinity. These two are parallel and this angle is epsilon. So ideally this is your alpha t with respect to V infinity fuselage reference line this is your alpha.

Say this is your V infinity right. This is your alpha. This angle is your alpha. Now what is the angle of attack for the tail, this is your total angle of attack for the tail, angle of attack is angle made by the chord of this tail with respect to the free stream velocity resultant free stream velocity here that is V infinity prime. This is your alpha t, right. So what is alpha t? See this i of t+this angle is a angle; there is a resultant angle here; which is alpha – epsilon?

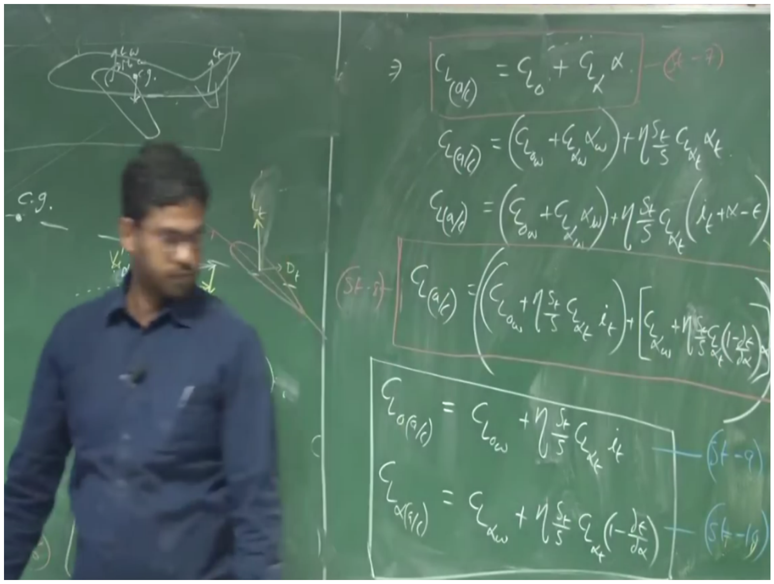
This is alpha–epsilon. So alpha at the t is alpha – epsilon +i of t. So this is an alpha at the wing, okay. Where epsilon you can express this as the downwash, the epsilon by dou alpha*alpha, this we already discussed. So during this discussion we neglect epsilon naught will be which is very, very small, right. And we know how to get epsilon/dou alpha from within aerofoil theory; we discussed in our previous lecture.

Now what will be the; so the angle of attack effective angle of attack with respect to fuselage reference line is induced here, right. So ideally this will be your lift that is generated at the tail, say which is perpendicular to this yellow line. This is your free stream at the tail. This is your lift at the tail. And say if you consider the magnitude it will be less than the wing, right and you have drag at the tail, right.

Now let us look at the moment equation about cg. Or say what is the total lift of this aircraft; lift of the aircraft = lift of wing + lift of tail, right. Simple right? There are two forces upward force acting here; this a lift from the wing and lift from tail the summation of work, a principle of superposition. So this is 1/2 rho v square S*CL of aircraft = 1/2 rho v square S*CL of wing+1/2 rho V infinity prime square S of t because tail area is different compare to wing area.

S of t * CL of tail, okay. So if I express the non-dimensional lift coefficient of the entire aircraft this equals to CL of wing+1/2 rho or say Eta St/S*CL of tail where, Eta=1/2 rho V infinity prime square/1/2 rho V infinity square. So it is a tail efficiency, tail efficiency factor.

(Refer Slide Time: 29:56)



So CL of the aircraft can be expressed as CL0 of the aircraft + CL alpha * alpha of t he entire aircraft, right which is equal to CL of wing can be expressed CL0 of wing + CL alpha of wing * alpha of wing + Eta St/S*CL alpha of t*alpha of t.

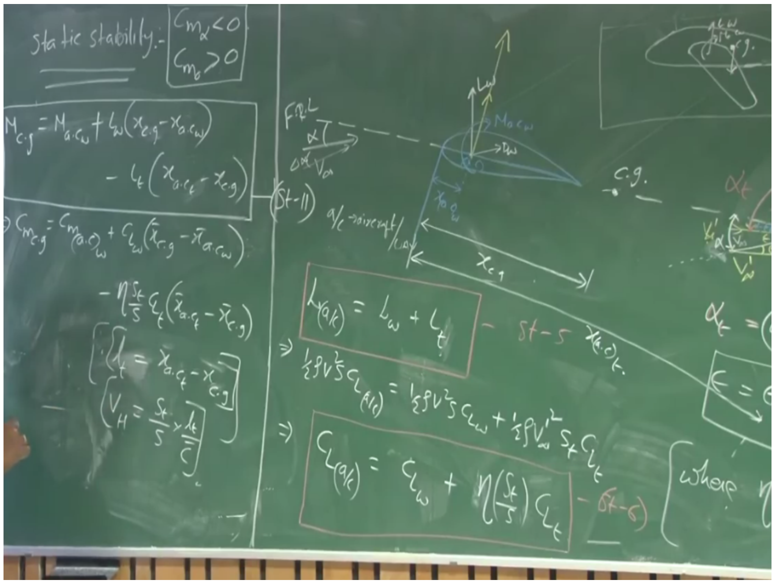
Because CL0 of tail is 0 because we are using a symmetric tail. So CL0 of wing + CL alpha of wing * alpha of wing + Eta, you consider this as your; so stability equation 5. So this is your stability equation 6. So this is your stability equation 7; is equals to Eta * St/S*CL alpha of tail. What is alpha of tail? i of t alpha-epsilon. This I can express with as i of t +; so with the help of this, okay.

CL0 of wing + Eta St/S*CL alpha t*i of t + CL alpha of wing + Eta St/S*CL alpha of tail*1- dou epsilon/dou alpha*alpha.1-dou epsilon/dou alpha*alpha. Eta St/S*CL alpha of tail; 1-dou epsilon/dou alpha/alpha where, I am taking alpha; say substituting what is epsilon here by assuming epsilon naught the 0 here then substitute what is epsilon, take alpha common what you are going to get is; and separating the constant in the coefficient of alpha, right.

So this is CL of the entire aircraft. Consider this as stability equation 8, right. Okay. Now comparing St-7 and St-8 what you have is CL0 of the entire aircraft = CL0 of wing + Eta St/S*CL alpha tail * i of t, right. And CL alpha of the entire aircraft = CL alpha of V + Eta

St/S*CL alpha of tail*1-dou epsilon/dou alpha. These are two equations St-9 and St-10; Stability 9 and 10.

(Refer Slide Time: 33:57)



Now let us look at moment about cg because of the addition of tail how the pitching moment is changing. So moment about cg for this wing and tail combination is; again even in this case what we do is, see further the angle is very small here, so we need to resolve this Lt perpendicular to this V infinity right. I mean while taking the moments we need to resolve perpendicular to this fuselage reference line.

In this particular distance, this particular aerodynamic center distance; measure parallel to this fuselage reference line is x ac of tail. So this is your aerodynamic center of tail. This corresponding distance is x ac of tail here. Okay. I forgot to mention it, please make a note of it. The moment about cg is moment about aerodynamic center of wing; so let us consider only the lift components and cos alpha in either case change in the alpha is small;

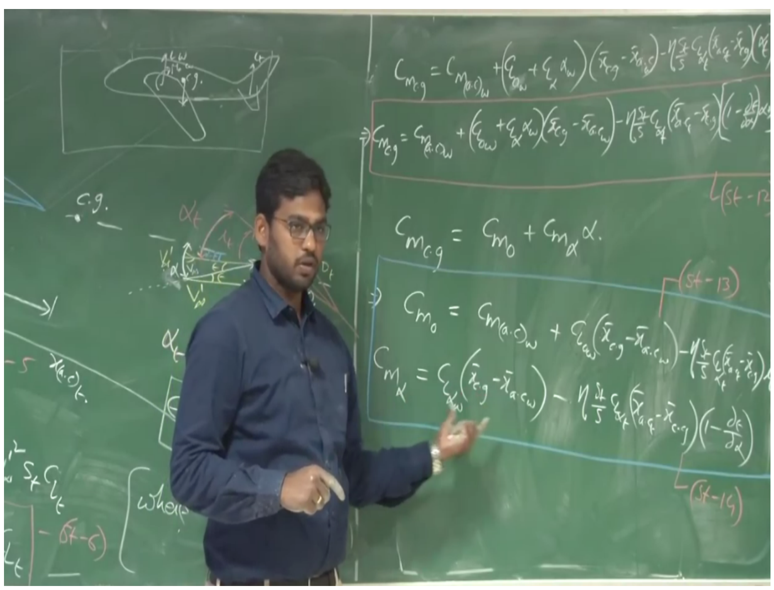
So let us consider only the lift directly so that is how the end result will also have the same component like the cos alpha will be considered as 1 and lift will be the corresponding contribute of a pitching moment. So Cm ac L of w; sorry + L of wing * x ac of wing x cg – x ac of wing – L of t*x ac of tail-x cg. Am I correct? Why because about cg this lift of tail is giving you a negative moment.

So L of t cos alpha cos of epsilon-alpha or alpha – epsilon which is considering it as small quantity so it will be L of t cos alpha. And D, Cd* this epsilon; alpha-epsilon is further small so we neglected the other term here, drag contribution. So L of wing * x cg-x ac – L of t*x ac -; or x ac of tail-x cg here. So if you non-dimensional this, this is Cm about cg = Cm about Cm about ac of wing + CL of wing * x bar cg – x bar ac of wing -;

So this you consider this equation as St-11 or 10; St-11, so consider this as St-11 this equation stability level. This is like Eta St/S*CL of tail x bar ac of tail-x bar cg. So this St/S*x ac of t. So generally, L of t length of tail is considered as the distance between aerodynamic center of tail – x cg, right. So the tail volume ratio is now defined as St/S*L of t/c bar, right this is considered as tail volume ratio. Please make a note of it.

But I am not going to use this Lt or VH here in this case. So we will see why we are not going to use.

(Refer Slide Time: 38:03)



Cm cg = Cm ac of wing + CL of wing which is CL0 of wing + CL alpha * alpha of wing * the moment term corresponding moment term which is x cg x bar cg – x bar ac – Eta because lift from the tail is contributes toward negative pitching moment; Eta St/S*CL alpha of tail*x bar ac of tail – x bar cg * multiplied alpha of tail, right.

This implies Cm about cg = Cm about ac of wing+CL0 of wing + CL alpha * alpha of wing * x bar cg – x bar ac of wing, right. –Eta St/S*CL alpha of tail*x bar ac of tail-x bar cg*alpha of tail is 1-dou epsilon/dou alpha*alpha+ i of t. So consider this as; this equation as stability. So we know the pitching moment coefficient about cg=Cm0 about cg+Cm alpha * alpha, right; of the entire aircraft this is cg about for the entire aircraft, right.

Now comparing these two equations what we have is Cm0 of the aircraft is Cm ac of wing+CL0 of wing * x bar cg – x bar ac of wing, right –Eta St/S CL alpha of tail* x bar cg ac of tail – x bar cg*i of t. Similarly, Cm alpha of the aircraft = CL alpha of the wing*x bar cg-x bar ac of wing-the tail contribution. Eta St/S CL alpha of tail * x bar cg-x bar ac of tail – x bar cg multiplied by 1-dou epsilon/dou alpha.

So these are the stability equations, statics stability equations for wing and tail combination. So consider them as St 13 and 14. This is stability 13 and this is stability 14. Now, let us look at this equation the Cm about aerodynamic center of the wing is negative, right. CL0 is positive and in this case if the cg is behind the ac this is positive. So this contributes towards a positive term the bigger positive term in fact because Cm ac is far weaker than this, right.

And Eta- Eta St/S*CL alpha of tail; if the aerodynamic; see this is also a positive; these are all positive St/S so the only thing is negative here is i of t; if you have i of t negative it becomes positive, so this becomes positive term. If i of t is 0 let us say, so you should have this Cm0 from the wing alone, Cm0 required Cm0; that y intercept in that curve Cm versus alpha need to be produced from the positive y intercept only from this terms, these 2 terms.

This negative, right for a cambered aerofoil; you need to design your distance between; you need to locate your cg of the aircraft or the UAV in such a way that this particular term is positive enough right, big enough to overcome this negative quantity if i of t is 0; at 0 tail setting angle. Now what happens in this case Cm alpha? Say, if the cg is behind the aerodynamic center this is positive, CL alpha of wing is positive so this particular term is positive.

So I need to have enough distance x ac of tail – x cg; and the corresponding tail volume ratio here which is multiplied by St/S. If this momentum multiplied by non-dimensional momentum multiplied by St/S is the tail volume ratio. So you should have enough tail volume ratio to give a Cm alpha negative. So the typical values of this tail efficiency factor is around 0.9, 0.95 right. So you need to locate your cg to make this conditions satisfy.

So we have the conditions for stable and unstable, right. So do you have something called in between these two a limiting conditions for these? We have something called Neutrally Stable System. For example, if I take this tape, two sided tap right. so if I take this roll and start throwing it on this table, right. So this is one equilibrium; let us say now it is at rest; now if I apply small force, okay it is gone down, okay, it has become.

So what happened is; so apply a small force it will go and stop at a different location. So this is a tape, if I roll it down, so yes. It reached different equilibrium point, right. So it altogether attends the new equilibrium point, right. So it is neither unstable nor the stable. It has achieved a new equilibrium from point, we cannot comment about stability about this particular point. So this is the limiting condition for t his stability; table and unstable system called Neutral Stable System.

Where if you just apply; see this is also a stable case here, if I apply a force it is attaining a new equilibrium here, right. So this particular condition is known as neutrally stable condition.