**Introduction to Helicopter Aerodynamics and Dynamics**

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**Lecture No. # 05**

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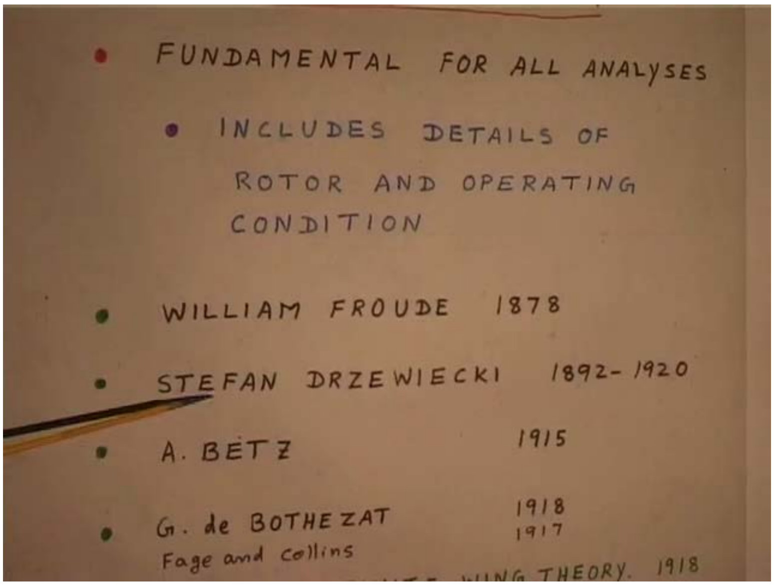


Blade element theory, because earlier we related the thrust, the inflow, the power, rotor disk area only induces using momentum theory, but that does not consider any detail. But now, we are going to look at the details of the rotor, rotor operating conditions everything, but that is the blade element theory, please understand this is the fundamental. We have to have this theory for all analysis of helicopters. And whether it is a fluid dynamics, whether it is aero elasticity, loads, stability everything, blade element theory is the fundamental. So, you have to know the blade element theory, but to give you a brief history, see it was proposed by basically started William Froude in 1878.

Because blade element theory essentially says that I am going to treat every cross section of the blade, because you know that the rotor blade is essentially an if you look at that this is a aerofoil shape. He said, I am going to look at the cross section of the blade, which is an aerofoil, because these are all please understand you know before the flight started, even the fixed wing, but then the real development was done by STEFAN

DRZEWIECKI or something it is 1892 to 1920. What he did was the propeller, these are all for propellers, all the propeller is going forward forward. It is not hovering; there is a velocity which is coming. And there is a omega R which is due to rotation velocity at any cross section. But he did not take into account the velocity due to the inflow or the induced velocity; he just took the propeller forward velocity and then the omega R.

And when he calculated the thrust the power etcetera it was not matching with the experiment, because please understand propeller experiments were lot of experiments were conducted earlier where power and thrust and he found that they were not matching. Because then he felt there is something wrong between theory and experiment, and the difference he thought it is due to the aspect ratio of the basically the propeller, because it is not a long propeller it is a finite propeller.

But he was not sure what is the aerodynamic characteristic, I must take into account at every section. So, when he makes some corrections for aspect ratio, because you know that there is a correction for aspect ratio in the finite wing theory. Similarly when the trends were, but still the results are not good, they are not matching with the experiment. Then several people attempted to see how to take the proper sectional characteristic, because you have two: one is the aspect ratio effect, another one is airfoil characteristic. These are the two things there is a something I have to make adjustment.

So, in 1915 and 1918 and also 17 that is Betz and Bothezat and then Fage and Collins these are the three groups of people, they said I have to take the induced velocity. But what induced velocity, I should take at the if I want to calculate the lift here, the propeller this is going this is the cross section omega R is there. So, you may take this is my omega R and there is a propeller is going axial velocity of the propeller which you may call it V plus some induced flow. This at we will take the induced flow from momentum theory or something like that. But still, but it is not aspect ratio effect there started using it. So, you have an aspect ratio effect was also brought, but you took the inflow also again there was a miss match.

So, they found that there is something wrong, in the sense wrong in the sense something inflow if I take it from momentum theory; if I take the aspect ratio they do not match again they give a trend. So, they try to adjust some empirical numbers that is why Fage and Collins, he took the aspect ratio as 6 fix the number and then he started correcting the inflow by some empirical factor. This is how it was going on to match the theory experiment purely thrust and power that is all nothing else, but later because Prandl finite wing theory that is a Prandl vertex.

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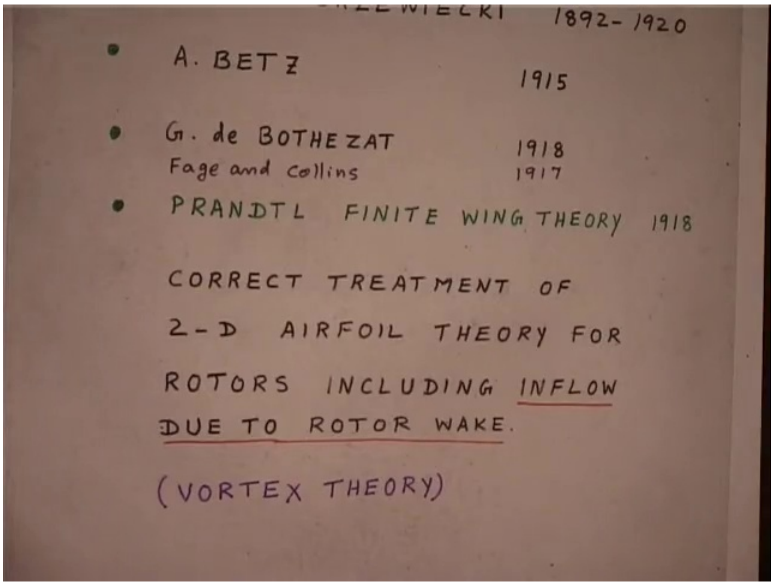


Now, lifting line theory that was in 1918 when he proposed then they said yes we have to this is a vortex model, because I should take lift is O U gamma and the gamma is the vortices. And you find out the strength of the gamma from actually the trailing edge, because you have to adjust lift you say O U gamma. And then you go back and then take a vortex finite vortex and then get this strength of the vortex. Then you use it then they said yes, we have to use vortex theory to get the proper value of inflow at the place where it is going at the rotor disk. That is why in the beginning the correct treatment of 2 D airfoil theory for rotors came from vortex theory. Because two dimensional there is a correction, because the vortex is given induced velocity in the that is what you get induced dragger in your fixed wing theory.

So, the vortex theory was dominating in the beginning of the rotors, because they said that is the correct treatment. So, everything went in vortex theory approach momentum theory was not really looked at. But subsequently then people realized yes, you can take momentum theory and then make, but even today if you want more précised treatment of induced flow at the rotor disk, induced flow I mean you all know that the flow normal to the rotor disk, if you want they get the correct value.

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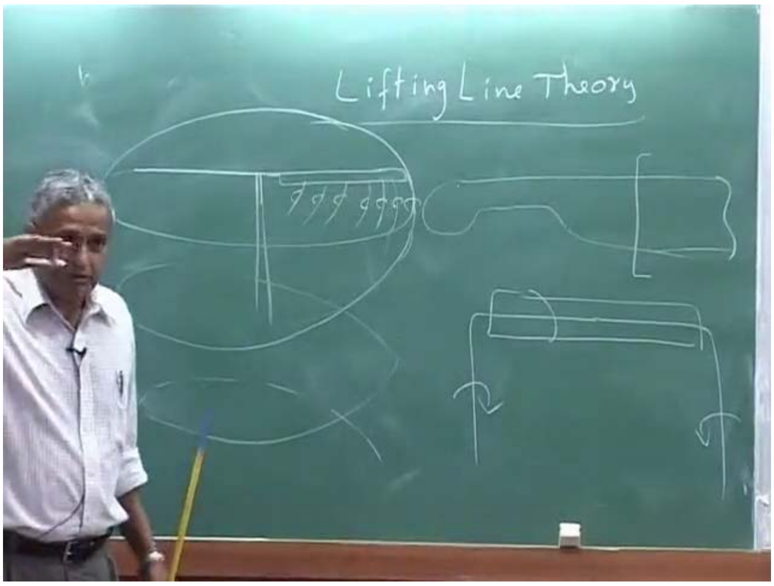


Because correct is something exact is different there is a vortex model, but that is more complex, because if you start looking at the vortex theory as such if I draw this as the rotor, it is a short, it is a rotor, it is rotating. The vortex is will be because as it goes you will find a vortex coming. And if the lift is varying along you will have a vortex sheet actually a sheet will come, because there is a lifting line theory. I hope you understand some of you may not know lifting line theory, and then the strength of the vortex that varies, how the lift variation and then what is the shape of because this is going to go in a in the rotor it will go in a helical shape.

Because in a finite wing case, you will say my vortex sheet is going like the horseshoe vortex, because this what do you say this is a horseshoe vortex in finite wing. The vortex swept back, but in the case of a rotor hovering rotor the vortex is piling below it, it will go in a helical path, because as it goes around it leaves and this one push down by the induced velocity by its own interaction.

Now, the structure of the vortex you have to make approximation, because if you experiment laws of observations were made on how the vertices look. And they found it goes in a some kind of an I will show later some pictures, it will like this. So, like that this will go vortex sheet and this what will happen initially, they will start sheet suddenly, they will all bunch up become a tip strong tip vortex. Because you know the strong tip vortex and how that is convicted down; that means, you have to take a structure of that, that is the wake structure. Now, that you have to know or you say if you have vortex to vortex interaction, if you consider then it is called the free wake theory.

But if you say I fix my structure of the vortex, it is going to be in a cylindrical vertical cylinder that is all no deformation. Then I call it prescribed wake either I prescribe the wake structure that is the relatively easy, but if I do not prescribe I say let it evolve by itself that is a free wake theory. But it is more complex even prescribed wake, pre wake both are complex. Now of course, people have developed their own computational course using vortex theory, today I am telling you vortex theory is there it is used.

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But the beauty is, if I consider the hovering rotor and then you assume that the vortex is in a cylinder and it is an infinite number of blades actuator disk; that means, it is just a lot of circular continuous vertices, it is easy for integration that was what was done in the earlier. Cylindrical wake structure and infinite number of blades means, there is no gap between one and the other please understand one blade gives a wake it will come down, then there is another blade that will also give; that means, there will be several helical wake. And there will be gap between those wakes, because you get one coming down then if you say another blade end, that will come and they will go intertwined.

But if you have infinite number it is almost like a continuous cylinder when you use a cylindrical wake like this. The induced flow calculated by this approach at the rotor disk as same as momentum theory uniform loading, uniform thing you take then you get the same result. So, what happens in momentum theory is not all that bad. So, we use in this course momentum theory please understand, momentum theory is even today in the research, it is used I will show you later. Now we use it, because if you want to have vortex theory then you need to sit under, it is like a pretty much computational more internship.

Whereas you will find even with a momentum theory, the result which you predict are reasonable good, pretty good I would say, but you have to make some corrections here etcetera with that your results will be good. And that is why momentum theory is still used in rotor analysis, this some people may use it that is a different point. It is a then school of school of thought and if you want to complicities, but it does not mean that everything you were able to predict. Because the wake structure unless it is correct and you do not know, because the viscosity will diffuse the wake how far you must go down.

So, there is on group of researchers working on wake, that type of prescribed wake free wake analysis, other group they said that I need an inflow. Because now, you see the research itself was directing towards whether modeling the inflow accurately, that is the research I do not do anything else except my focus is inflow modeling, how do I know what is the velocity at every point on the rotor disk. Even today it is a research later you will find how that itself gets complicated, another one is an I have a good theory pretty descent which is easy to incorporate in my aero-elastic loads responds analysis, because I need to do that also.

So, you see the research if I somebody focuses on only aerodynamic related problems, he will be focusing on only that inflow calculated, he may not get into rest of the things. But if you are working on aero-elasticity, loads, stability etcetera of the vehicle you need to know the inflow, you say I will calculate by momentum theory which is easy to incorporate. But of course, that vary somewhere, but it is not too bad that is what I am saying, it still gives a result which are quite close, but of course, there are always differences. Now, one can debate on whether one should use always this theory or the other theory, then you will find slowly as we proceed even that has certain draw backs.

Now, I just gave a brief introduction with the field how the research in inflow calculation is important particularly, but we will use not vortex theory, we will use simple momentum theory. But then we will complicate it not the uniform inflow that is what we did in the momentum theory, the flow over the disk is constant at every location that is uniform inflow. Later you will see hey that is too gross on approximation, let me have non-uniform inflow; that means, it is varying along the radial direction of course, if you want more complex in general various other situation you can say it varies everywhere. So, that is how the complexity in the momentum theory also went on.

At today, that is also there is one theory which is dynamic wake theory which is somewhat similar to this somewhat similar. So, please understand that model is what we use dynamic wake, we do not model this, we use the theory to get the time varying inflow at the rotor disk, please understand we use till now constant, it does not vary with time constant and uniform everywhere.

Now, you say initial development we learnt that there is a constant; it is uniform first theory that is what I am going to discuss. Now, do not think that is the end of it, because there is a lot more in the theory in the inflow modeling alone. So, when we go to the research level, we do a little bit more complex theory.

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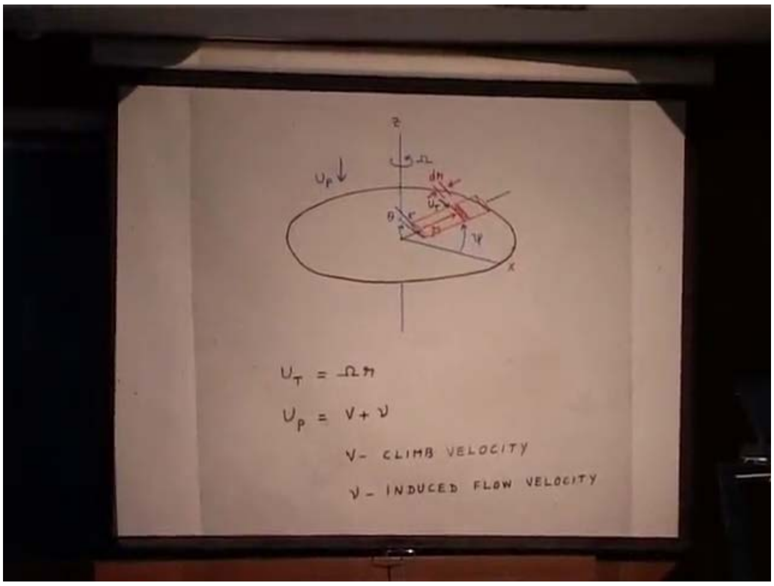


Now what is the blade element theory, this is a rotor blade which is rotating. So, I said the cross section is this I know what is the oncoming velocity, it is a 2D dimensional model that is all 2D airfoil model there is omega R. Let us first consider, hover and vertical flight in the sense the disk can go straight up down is a little, we will do that later vertical flight and up and down that has it is one complexities, we will do one by one slowly. Here I said that this is the rotor blade which is rotating with an angular velocity omega, and this is my frame X and Z and this is Y which is attached to the blade it keeps going.

Now, I look at a cross section which is at a distance r from the center of the disk or you can say hub that is r. So, your velocity is omega r simple, because please note this is only an introduction, and you go to actual blade it is not omega R (( )) you have so many other terms will start coming it. So, that is why when we start the course you will learn basics and then at this this is a small elemental cross section D R.

Because please note my omega R is changing depending on the radius, that is why that lower case r is used for running variable along the span of the blade. Now, I have omega R and I assume that this is the climb; you may put a subscript V C to denote it is a climb velocity. And nu is induced velocity you do not know that, but you say I know it somehow, you may get it from any theory that is where you can use momentum theory, you can use vortex theory etcetera.

And that is why I mark up which is perpendicular to the disk. So, now I have a disk and this blade is going in the disk please understand it is not coming out or anything like that, we will do later one by one, this is just a very, very simple theory. So, this is rotating and there is a normal flow. So, I have V C and nu now if this is kept at a pitch angle theta with respect to the disk on coming flow. Now, you can get the relative velocity as well as the angle of effective angle of attack you can say. So, this is the phi, phi is the change because of the normal flow.

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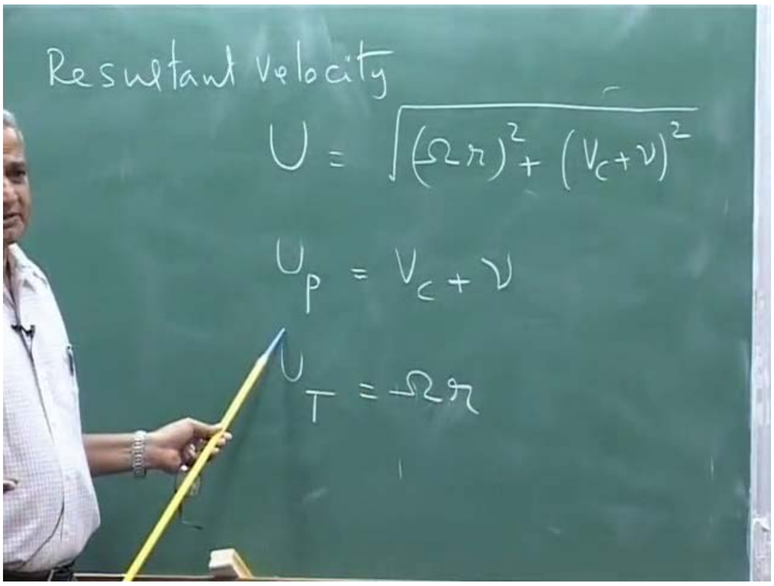


So, you will have your resultant velocity U is this is my resultant velocity, but for simplicity you know, because of every time we do not want to carry this, you use the symbol U P that is the velocity perpendicular to the disk this you call it as V C plus nu and U T which is omega r this is tangential velocity, this is the normal velocity instead of normal N you call it U P. So, this is U T this is U P this is a general symbol this is a standard notation that is used. So, that so, that it is ease of understanding U T is tangential U P is normal only everything with reference to the airfoil.

Now, immediately you go back what is my effective, because you know the tan phi is U P over U T tan phi is U P over U T. And now, I have to define sectional lift sectional drag that is all, and lift is normal to the I use simple 2 D airfoil theory lift is normal to the oncoming flow and drag is along the oncoming flow. So, now, you see and I define some psi just to denote, if the azimuth location this is for one blade, if I have N number of blades, I will use all the blades everything.

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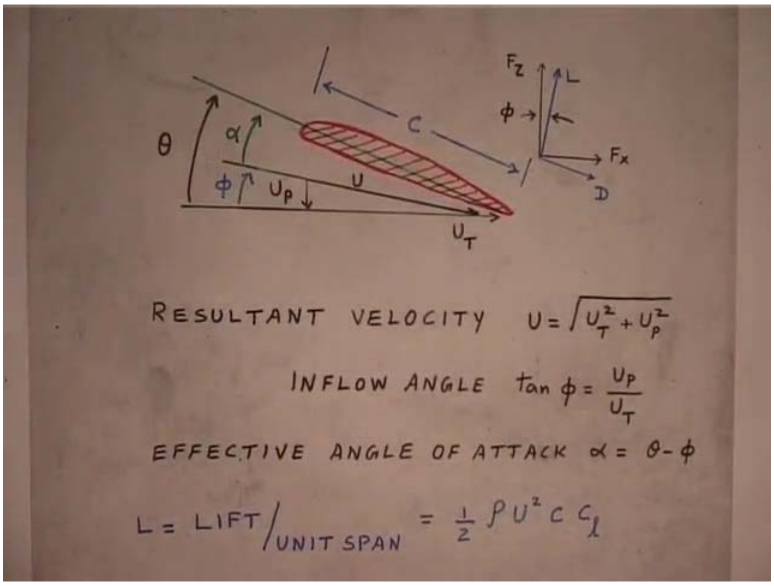


Now, I have got sectional lift and dragger this is for the please note very very simple model and this is what I have shown here. So, U P U T and this is the resultant U and theta is the pitch angle and alpha is the effective angle of attack. So, you see effective angle of attack is what I now you remember, why I used pilot input gives pitch change it does not change the angle of attack it will change indirectly. But he gives only pitch change in the sense he changes this angle, but the angle of attack changes depending on what is the U P, what is the U T etcetera.

I may assume now this induced velocity nu to be constant over the rotor disk that is it is not a function of r. It is a hovering theory, it is hovering or cline, it is not a function of radial, it is a constant then you see the phi changes, because of omega r. So, every cross section because the tangential velocity is changing. So, automatically my effective angle of attack which I call it alpha alpha is theta minus phi which you may call it theta minus tan inverse U P over U T and then I am going to make approximation straight away right here.

So, please understand write at the definition of angle of attack, you have tan coming in you make now an approximation, because in a text book form if I have to give you a closed form solution. Computationally you can always take tan inverse no problem, but you know that has R this is nothing, but what V client plus nu over omega r as r decreases this is very large; that means, phi is going to become more and more as you come near the root, and that means, the angle of attack whatever because this value will be classically increasing theta may be fixed, but this will keep on increasing.

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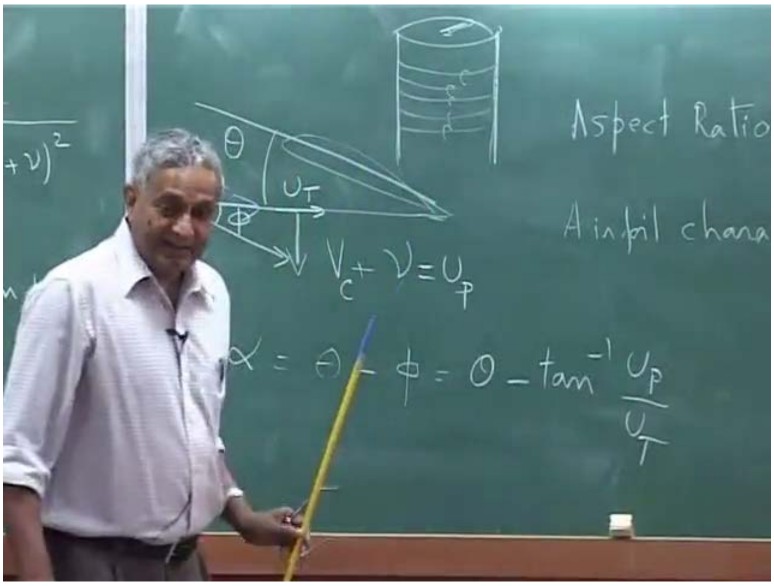


But then you were going to have problems, but usually what happens is your aerodynamic section starts not from the center of the hub. And another thing is the omega r; the dynamic pressure due to velocity omega r is very small near the root. Therefore, you make an error, but you say it is all right I accept that error. So, I represent this tan inverse, because I assume this angle is small.

So, please understand I assume phi is small, I write it U P over U T tan phi is phi, tan phi is phi. When phi is small U P over U T, but I am violating this rule as I come closer to the hub, but I still say it is all right, I accept it. Because I know that dynamic pressure is very low. So, the lift is not very large. So, I may usually you will find the aerodynamics section of the rotor blade starts about 20, 25 percent away from the center, because of the geometric, because you have to have you have a attachment everything. So, it will be around 20 percent 20 25 percent and the error you make in that is not allowed that is why you make this approximation first theta minus U P.

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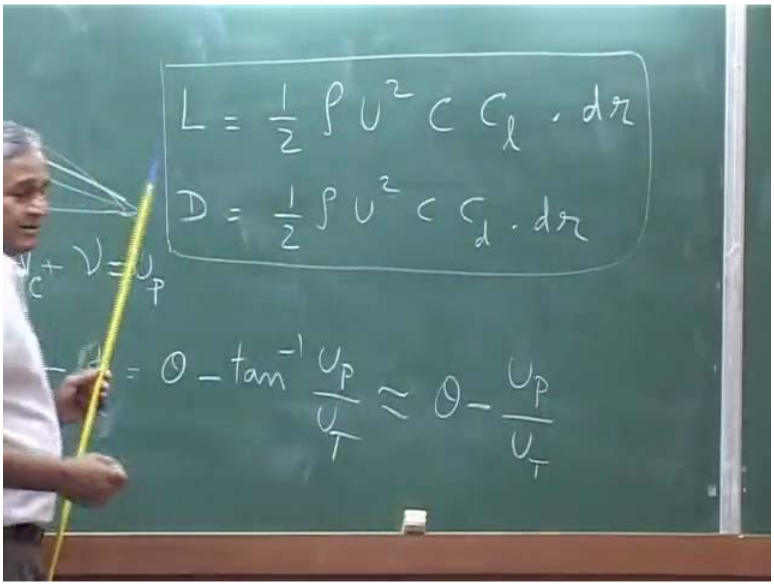


Now, you know angle of attack and I can write the lift per unit span straight from your aerodynamics, lift per unit span is lift per unit span please understand, otherwise over a small element you put a D R, you will have half rho U square card and C L. This is lift and the drag will be if you want to put you can put A D R half this is per unit span this is the lift C D, this is my at any small element.

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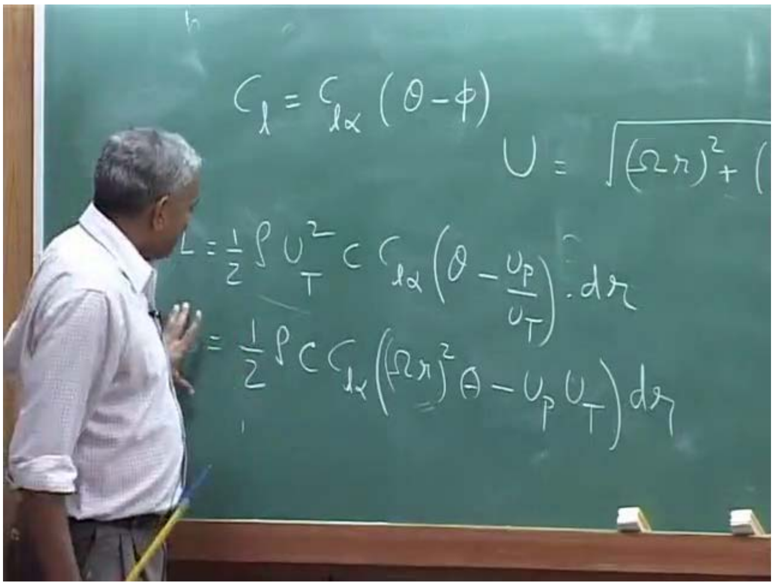


Now, if I want to get the total lifts I need to want integrate that is all. Now, I make more approximation the more approximation is I am going to call now, U I say omega R is because when phi is small basically omega R is large. So, when omega R is large this is a small quantity. So, I which is basically UT so, I represent my U as UT resultant velocity as U T, but that does not been I change the angle of attack also I keep alpha as this. Now I know directly I go back, I write my lift as half rho UT square this is an approximation I am making. And see now what is my C L, C L this is from your elementary, C L alpha lift curve slope tan phi angle of attack. Now, if I substitute here this values C L I will get C L alpha into theta is the which input minus phi is U P over U T into d r.

This is the approximate expression now you see this is much easier if I write it I take the half rho CUT square I take it inside. So, you will have C L alpha U T square is omega r theta minus U P U T all right. And UP you know v plus nu U T is o mega r now it is easy for me to integrate along the radius on the other hand if I put divided by omega R and every other factor I do not take it you know it is going to become a messy stuff that is what is done first. Now this is my lift for an elemental length if I want the total I go back. So, I erase this part.

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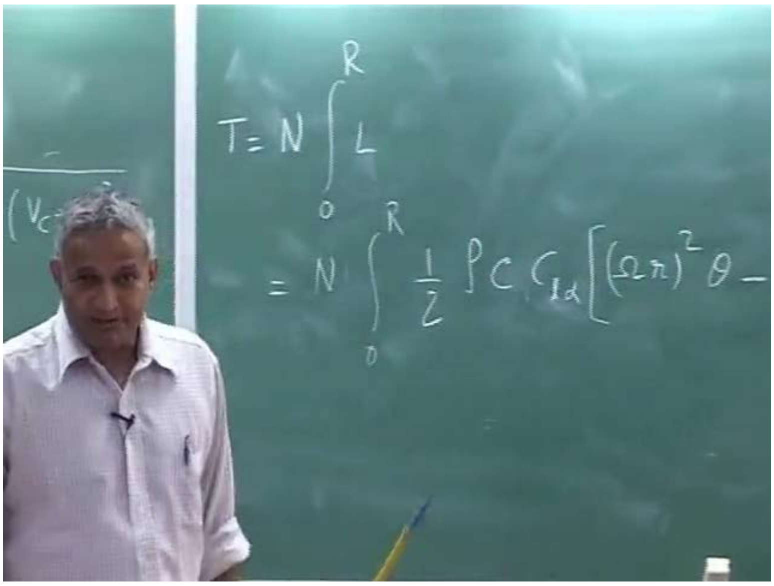

Total thrust total thrust is total lift that acts on all the blades in the rotor system; that means, I assume every blade in hover behaves the same way; that means, I will simple put number of blades N all right. And then I have to integrate from this expression I will put 0 to capital R the entire expression you write this you will have the full expression which is maybe I will write this then I will do the non-dimensionalization because that is important.

So, you will have this is N 0 to R half rho C C L alpha omega r whole square theta minus U P S V climb plus nu and U T is omega r into d r this is my thrust rotor thrust. Now, you surely we non-dimensionalize the quantity, we do not carry this whole thing when we non-dimensionalize we use I erase this part here velocity quantities are normalized with respect to tips speed and you use omega r.

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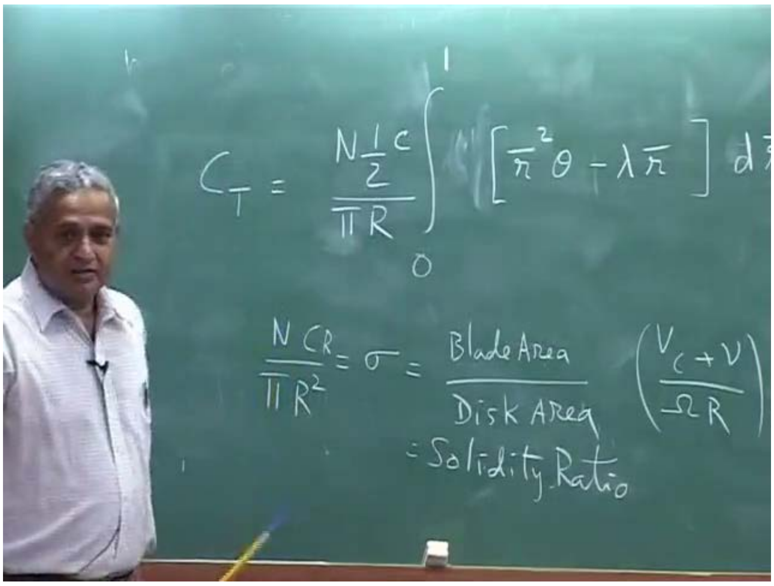


So, you see CT is thrust coefficient is thrust divided by rho pi r square, this is the area of the rotor disk and omega this is my C T. Now I divide this entire quantity by this, so now, please understand one thing density card card can vary along the blade, but if my today most of the rotor blades actually they have a constant card except near the root.

Otherwise your card can vary that is what your small model card is varying. So, you cannot use the same expression you have to put that integration, but normally what is done is we take some constant card etcetera and then we calculate. Now when I do this I get a quantity I will just briefly describe I will have N 0 to R. So, I am going to divide divided by rho pi R square omega capital R whole square this is what I am dividing when I divide let me write the expression here C T I will have N there is a half factor outside any way rho will cancel out.

Now omega R whole square you take it here, when you take it here this will be omega square will go up R bar square we write it as I will say integral leave it is essentially 0 to 1 it will become. Because pi R square is there, there is a C, there is a d r. So, one of the R you take it to this that will come R bar. So, I will make this bracket here this will be d r bar r bar means r over r. So, integral is 0 to L right.

Now, inside quantity let me first write it that is r bar square theta minus I am going to call this symbol this V C plus nu over omega capital R by a new symbol which is lambda, lambda is inflow. Please understand this includes climb and induced velocity later I may split this lambda C and then nu some lambda I, but right now I am calling it as lambda because this itself we can write it as lambda C plus lambda I; that means, sorry C is climb I is induced.

You can split that is later right, now we will take it as lambda and one omega R one another omega lambda r bar. So, this is gone then you are left with pi R. So, I can have here there is a C and because there is a one of the R has gone to r bar. Now, if you assume it is a constant card then you can take out the constant card means C also outside. So, I will take the C outside.

Now, this quantity I am going to call it as sigma which is essentially blade area over disk, because if I multiply R and square this is C R is card into span that is a blade area N is number of blades. So, because please note all blades are identical. So, you will have sigma blade area over disk area this is called solidity solidity ratio.

Now, I can write this entire quantity as sigma over 2 and then there is a C L alpha I have to add up. So, C L alpha is there sorry I forgot to put that C L alpha, because this is there. And C L alpha, you may call it C L alpha or some time we call it a by the symbol C L alpha or a both mean the same. Now, you had left with completely the solidity ratio is important, usually blade area divided by disk area. And this is an important parameter like your thrust co-efficient, now we have introduced one more new quantity which is the solidity ratio of the rotor, because that is that plays a key role.

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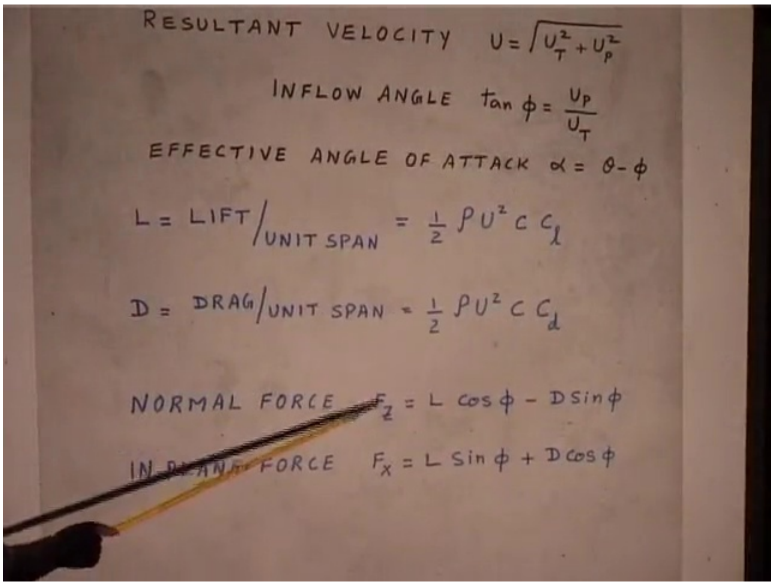


As we go along you will see now, this is what my expression for C T. Now, this is the lift it adds please understand now I go back here a little, we make a we made some approximation what is by lift, lift is normal to the resultant flow right and drag is along the resultant flow, and I said in my hub co-ordinate system X is in the plane of the hub. So, that is normal to that I should take if I want actually the force normal to the rotor disk or the hub I must resolve these two components L and D along vertical and this X direction that is the precise. Now, I go and do that I have written here normal force f g is L cosine phi minus D sin phi that is a subtract, because this is the resultant is coming here this is omega R. So, you have your L your right I think I should this is L this is D and this angle is phi, this angle is phi, this is we said X, this is we said Z. So, L cos phi minus D sin phi so, I miss drag drag also into account. And then in-plane force is L sin phi plus D cos phi now I make another approximation that because my phi is small and the drag is usually for aerofoil is very small it is you can take it at the most .01.

Whereas lift C L is C L alpha that is 2 phi, so he says my drag is very small therefore, F Z please understands F Z I say is L directly. now that is what I have used only L in defining the thrush coefficient; that means, basically this is normal to the rotor disk that is all, this is approximation I make this. But in the case of a pin-plane force I say F X, I cannot neglect lift lift large phi is small.

So, I am gone to write it as L phi plus d, because phi is small. So, cosine phi I take it as 1, sin phi I call at as phi whereas, when I go here D is also small phi is also small. So, product is much smaller. So, I neglect that whereas, when I come here I do not do that. So, this is the now you understand right at the beginning I make all sorts of approximations.

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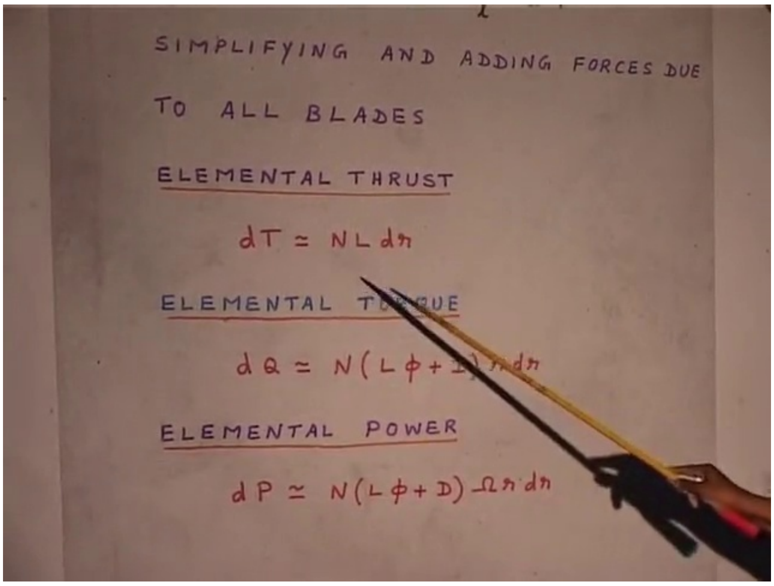


That is what I given here I put it and elemental thrust, because this just to indicate to you what all we have done L d r, L is lift for units pan, this is d r here I modifies slightly and then N is number of blades D T is actually thrust which is F Z this is per unit area. So, I multiply by d r is it clear. And then when I go for torque, because I am not interested in the drag force I want to see what is the torque because drag force I want just integrated I will integrate and then get what is the torque required to rotate, but later if I want hub hub loads I need to get the drag also I will do that later. But right now I am calculating only the torque is L phi, because I approximated affects L phi plus D and it is acting at a distance or from the center. So, the drag force into distance is elemental torque and the power into omega, because torque into omega is power. Now I have all the three quantities elemental thrust, elemental torque, elemental power I only shown you this particular thing.

If I integrate this, before I let me write the expression C T is here I put what sigma, now C L alpha what type of aerofoil you are having at every section where it is a constant it can vary we are we said card is a constant, but my aerofoil can change, but we make again no I am going to have the same aerofoil through out. Now, C L alpha also comes out which I call it as using the symbol a because most of the helicopter literature they use the symbol a lift curve slope C L alpha is a sigma a over to 0 to 1 r bar square theta minus lambda r bar into d r bar. This is my C T which is the non-dimensional actually the first expression d T is there right if I integrate the d T over r I will this is the thrust that is normalized I get this expression.

Now how my theta is varying along the blade I said pilot gives an input that is the pilot input he gives at the root, but the blade itself can be twisted; that means, there can have a pre-twisting the blade geometric twist; that means, what initially pilot gives and then what is the angle and as it goes what is the pitch angle at every cross section of the blade that you have to take it.

Now, here only there are if it is constant, constant means there is not twist zero twist, zero twist means you can directly take it if you integrate which very simple. Assume zero twist theta is constant everywhere this is very simple this is what C T becomes sigma a over 2, theta over 3 minus lambda over two that is all very simple expression. But you do not know lambda that is the different thing, but if it is hover you will see how we get it from if you have at twist built in the blade then you have to take that into account. Usually rotor blades they do with a linear twist, linear twist means the angle theta varies linearly from root to tip with a reduction.

So, you can write it in a form where you have a twist then you can integrate fully now why do you give twist that is another question. So, you can have, but then the proof for twist if you give you vary your inflow also, there is a little different because we still have not obtained this value please understand if you use momentum theory assuming hover I am just going a little only with this I am discussing I am not gone to be other two terms only this term zero twist I am considering hover, then lambda becomes lambda I, that is all induced. Because there is no climb then I will have my C T is sigma a over 2 theta over 3 minus lambda I over 2. But momentum theory hover gave me what lambda is root of right sorry lambda I, I should use the symbol lambda I because this is for hover. Now blade element theory is relating pitch angle, induced flow, solidity and aerofoil characteristic. Because all these are there number of blades actually this is are non dimensional quantity if you want then operating condition is there angle of attack everything is there. Now, I can substitute for lambda I here.

And then I can get theta directly what should be the pitch angle I must give if I want to have this much C T then once I get the once I know the C T, I can get the inflow. On the other hand in a experiment if you give a pitch angle I must know what should be the C T; that means, I have to do iteration, because first I assume the value of theta then I do not know this value I am take it as 0 you follow.

Then I put the value of theta I get a C T when I get the C T, I go back and substitute here, then I get the lambda I put it back here, then I get the new value of C T go back do the iteration, that is in a wind tunnel test if you want correlate hey I have given this much pitch angle what is my thrust because these test are done lot of.

So, you see only these two equations you are using only these two now in helicopters you understand that you cannot live with only momentum theory or you cannot leave only with blade element theory you can replace the momentum theory by some other theory vortex theory, but this is essential blade element theory is essential. So, you always have two theories either you say I take prescribed wake or free wake any analysis and then I get the inflow then use that inflow put it here get the thrust then go back again to see whether that thrust gives me that inflow.

Now you see the loop of the calculation in just a very simple problem, if I give you a weight of the helicopter and the rotor radius some C L alpha aerofoil shape etcetera calculates what should be the pitch angle, you should be able to do it.

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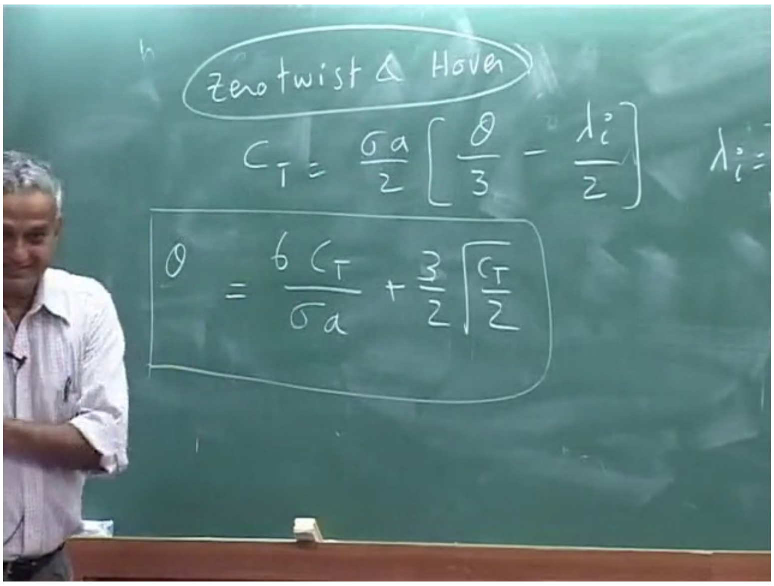


Now I will just compile this staff and then write it here later we will see the twist part right. Now untwisted zero twist so, I have C T and zero twist and hover please understand because I am doing both C T becomes sigma a over 2 theta over 3 minus lambda I over 2 and lambda i from momentum theory is root of C T over 2. Now, I just want to this is for practical purposes you substitute this here and then collect the terms of C T on one side just in this expression if you do that you will get what, 2 C T over sigma a this is theta over 3 equals this will go here plus one over 2.

So, I take the 3 please understand I am taking the 3 here at 6, Zero twist hover if I know the thrust coefficient please understand if I know the thrust coefficient; that means, I know C T thrust coefficient I know, because that is the weight of the helicopter I now you see I want really thrust coefficient I equate to sometimes people say weight coefficient.

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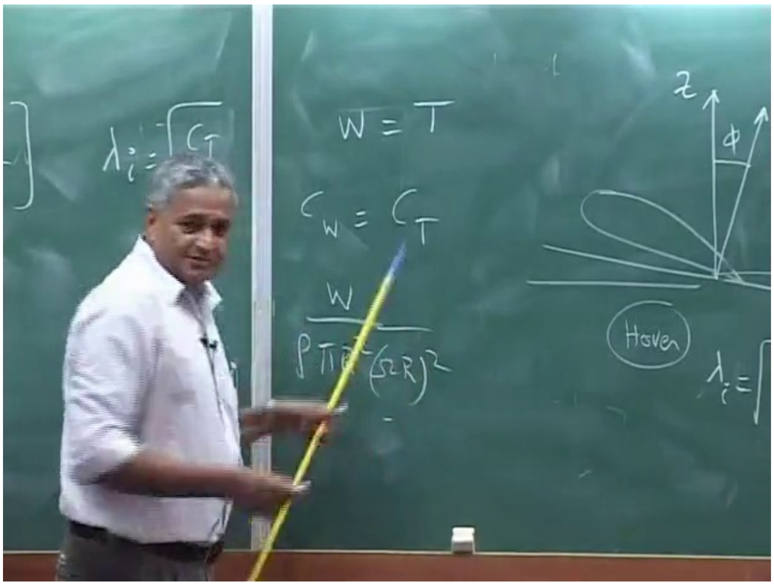


Basically weight is supported by the thrust you say thrust coefficient is equal to C T equal to C W please understand. So, I use this approximation again here if you are given the weight of the helicopter if it has to hover W you say that is t that is all weight is equal to the thrust. So, you will have C W is C T weight coefficient thrust coefficient weight coefficient is nothing but W by rho again pi R square omega R whole square which is same as if you are given a helicopter with the radius and with the rotor angular velocity you can get the C T. Now you see what should be my pitch angle I must provide for hover you substitute the value.

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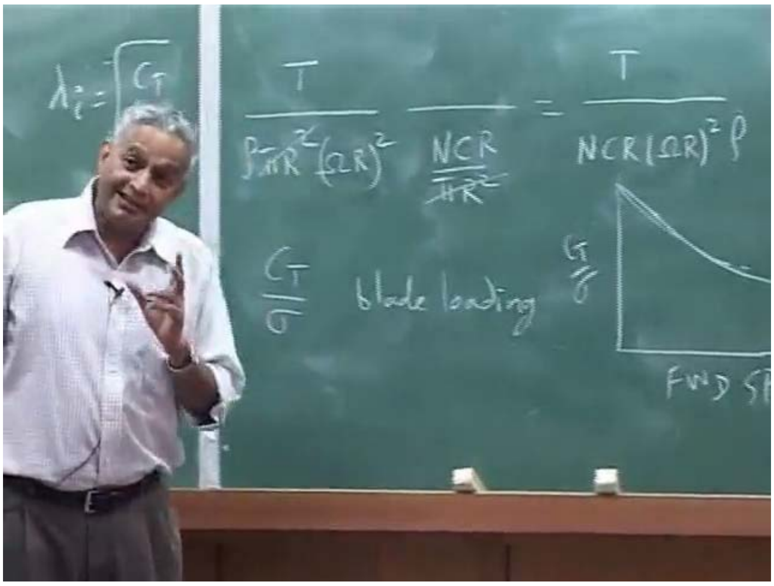

Here this quantity you know that a is the lift curve slope is theoretical value is 2 pi theoretical value is 2 pi this is almost close to 6. So, you not deserved now you see C T by sigma this is the mean angle this the term due to variation in the because of the inflow. Because of the inflow there is a change C T over sigma now please understand why this parameter becomes important thrust coefficient over sigma this gives you mean angle approximately straight short.

Because sigma takes N C over pi R C T is thrust coefficient, now you will immediately say hey what should be the angle I must provide for hover just mean value just you you take its if you negated it is all right that is why in practical situations helicopters they always want C T over sigma that is the value Because that tells you what is angle you must provide suppose if this value become for the aerofoil 15 degrees then it is your nearing stall of the blade. So, it gives you they do not go you immediately see if I increase by solidity ratio I reduce the angle for hover.

That means my aerofoil is good, but if I reduce my solidity I am actually increasing my angle of operation, but you may stall. So, these are very important you have to operate and if you are near the stall angle if you want to climb up, because this is hover if you want to I have increased my thrust increase my thrust I increase my collective, but what will happened is you will not be able to lift the blade will stall. So, this is very, very important C T over sigma has a parameter. This is also called blade loading why it is called blade loading I will briefly describe, because C T is let me erase this why it is called blade loading thrust divided by rho pi R square omega R whole square this is C T sigma is N this is sigma.

Now, this factor you can reduce it in a slightly different form and which will give you approximately in a different form of expression that is I will writing it in this fashion because if you take it as N C R pi R square what happens this will become pi R square will go this will become thrust divided by N number of blades C R of course, omega R whole square and rho this is basically for non dimensional staff, but N C R tells you the blade area.

So this is also called C T over sigma is called blade loading see you saw disk loading disk loading was T over A then you saw power loading now this is blade loading it also tells you how much each blade lifts how it is shared by how the load is shared by the blades. So, you see two non-dimensional parameters giving you the pitch angle of operation of a blade in hover that is the you know critical thing.

Now, I want you to note down because this particular quantity you may call it blade loading you may call it just mean angle of attack whatever it may be mean angle of attack actually mean pitch angle you may call it I thing I would better mean pitch angle would be a better mean pitch angle required for operation. How this value please understand this is just for you to know we are talking about hover, how this value vary with forward speed will limits because we know it is related to pitch angle pitch angle means if you keep on increasing your C T I am giving you one rotor everything dimension everything is fixed operating condition is fixed omega R is also known.

But you keep on increasing the weight of the helicopter what you will do you have to keep increasing your blade angle up to some blade angle you will lift, after that blade will stall; that means, you cannot lift more weight; that means, that set the stall limit in the sense the limit the rotor can lift beyond which that rotor cannot lift because you have exceed that. Now this is true for even forward flight, but in forward flight also you have to lift the weight in addition to overcoming drag etcetera. Now industry requires this C T over sigma it is not a constant with forward speed please understand this varies with forward speed, this is the what I gave you as a project, because this is a very tricky stuff. Limit of the rotor I design a rotor how much you can take it, because this very, very important if you are making you cannot just like that take a rotor put any weight they have to make sure what is my operating condition and then I have a envelop well set. So, C T over sigma plays a very very important role in the helicopters that is why I am saying and this quantity is not when as a blade loading.

You tend to get an idea that you know the weight of the helicopter everything is known; that means, its constant you will say please understand tends to give you a feeling that is the constant; that means, irrespective of the flight condition it is a constant. Normally using this quantity you define a limit with respect to I will just draw the diagram, but I will not C T over sigma this is speed forward speed, hover yeah I know that mean how much this quantity in hover you can go for a given rotor please understand this is drawn for a given rotor, given rotor means sigma is fixed right. These quantities are fixed even rotor omega R I is also fixed, but this you keep on increasing up to a point where blade will the angle you will reach that the blade will stall and the same think can happen at every forward speed, because every forward speed you will find I can keep on increasing the weight till the point where my blade will stall; that means, that becomes your stall limit.

It will something like this it may actually come down down down; that means, the limit you can go it is drawn in C T over sigma curve, but these are actually I had a lot discussion with about this particular thing finally, we figure out this is that key this becomes important, but how do you generate this curve theoretically that will now we are going to do it.

But we will give because this requires very detail if you want actual helicopter thing, but in a preliminary thing we can do simplistically. So, this is just for you to know C T by sigma is also used for drawing a limit of a rotor stall limit you may say, because beyond which if you go blade will stall you cannot rotor will stall you may call it rotor may stall or blade may stall.

So, it may come and may hit like this if you say you see if I go I am stalling, because I crease my C T by sigma gone up, pitch angle is gone up, blade stalling same thing will happened at every forward speed, but how do I get this curve in forward speed with all complexities that is the key. And they make sure that when they design the helicopter your has below here you do not go anywhere near because you know because you never know at some operation you may stall and if you stall loads will go up and actually I do not know whether you people know there are couple actions that which came in the video and other things.

The pilot was doing a turn and usually you go and climb, but when he started for climbing he was actually going down climbing mean what you increase your collective you increase the collective, but he was going down which means what blade has gone into stall. So, these are very critical things in Manuel suddenly you may stall and then you thing that because the pilot is that is why it is very, very tricky of course, he walked out from the accident in one case in one case it was fatale. So, these are very important parameters in the helicopter field. Now let us go to the next factor which is the basically the torque part and I will I will also give you a little bit of this twist before I go to the torque part I will give the twist just directly from here.

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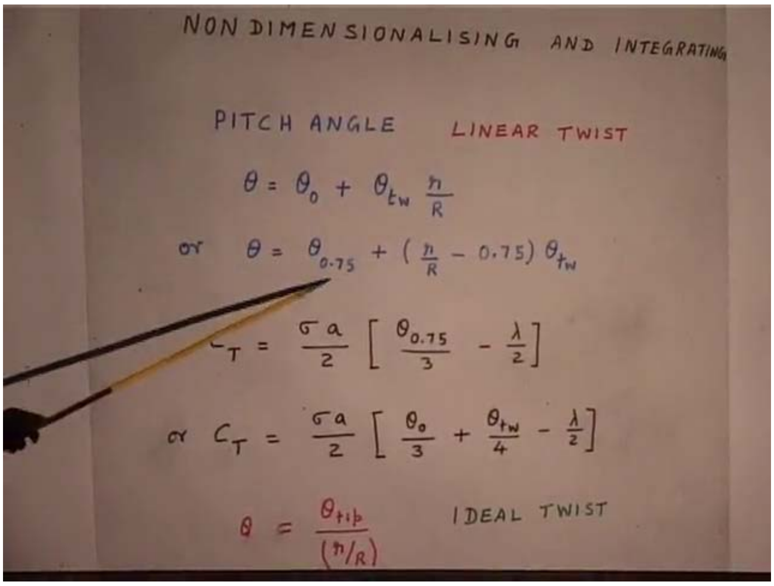


I am now going to write it just for if I have a linear twist, linear twist please understand here I use 0 twist if I have a linear twist theta naught that is what is the value theta twist into r over R this can be written in terms of the angle at .75 R. 75 R if you take it in this fashion please understand if you take it in this form and put it in the integral and then integrate you will get the expression like this which is I use lambda it exactly same you follow please understand.

If I take .75 R as my reference angle I am representing theta equal to theta .75 into R over R minus .75 theta twist this fine this expression is this is basically same as this I am putting it in a different form. If I use this expression in my thrust and integrate where this you can do yourself because you do it as a exercise if you take this expression integrate you will get this expression which is essentially sigma a by 2 theta by 3 minus lambda by 2 now you see theta represents the angle at 75 percent R.

So, why 75 percent is so, critical you know most of the time we say because that is where the lift also becomes high around that region because more near the root lift is small then as you go towards end the lift will rise up and then of course, the tip it will drop. We will come to that tip etcetera later right now or if you directly use this expression you will have this because you see if you take the 3 out theta naught plus 3 by 4 theta twists that is nothing but .75 angle at .75. So, the 75 percent of the blade is always they will say what the angle theta is?

Now, there is something called a ideal twist I will just mention this and then we will close that is if my pitch please understand r over R is in the denominator tip angle I said as I come in board it will go like this ideal twist, linear twist is straight line say it will be that if theta and linear twist may be like this R over R this may be one.

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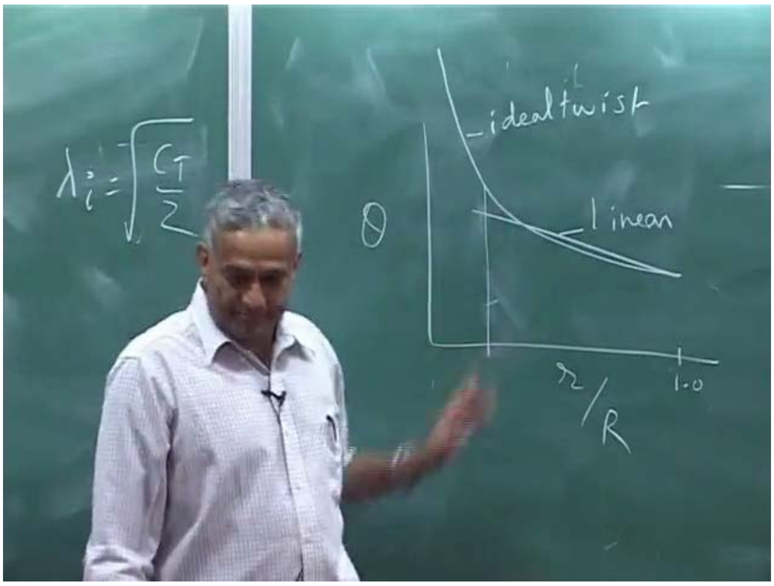

But ideal twist this is ideal, this is linear you cannot make it because you know I will take infinity twist infinity has no meaning, but up to some point to radius because near the root it is all not an aerofoil section. So, you can cut it out here or somewhere place then you says I can do something like that I can manufacture. So, you see this I will give you an exercise which you can try I can have an ideal twist.

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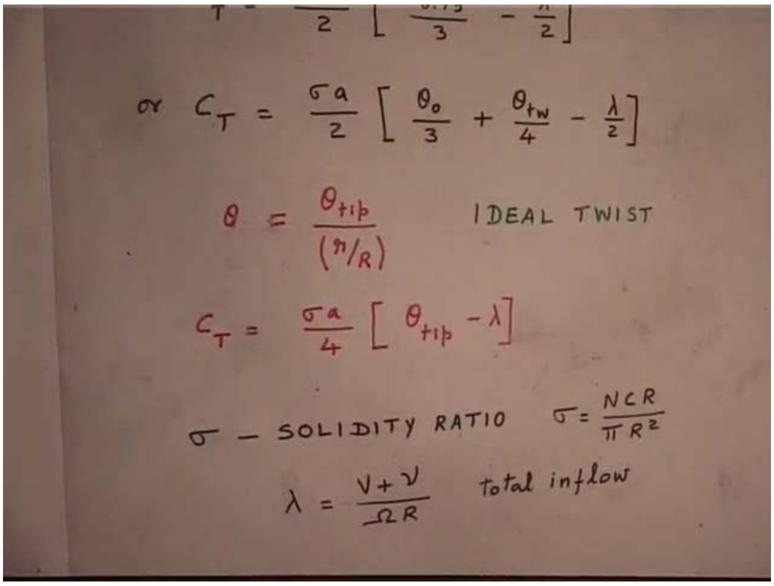


And if I use this my C T will be this expression; sigma a over 4 theta tip minus 4, expression is different. Now you see depending on the twist, the expression for now why ideal twist is in this form, but why do I call idea, is another question. The reason is I call it, if I use this twist my inflow will be uniform I can prove that using the another again that theory we have to come, whereas in all these things I assume that inflow is constant everywhere, please understand I assumed inflow is constant, but I do not know whether it is constant or not.

That means I said lambda I equal to root of C T over 2 I assumed it, but it not true; that means, I am making even this writer very simple expression, I am making a error, I assume this is constant, but I do not know whether it is really constant or not; this is a problem. You can use that, but you make errors. Now you say how do I later we will do, now the torque part we will do it in the next class.