Introduction to Helicopter Aerodynamics and Dynamics

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Lecture No. # 23

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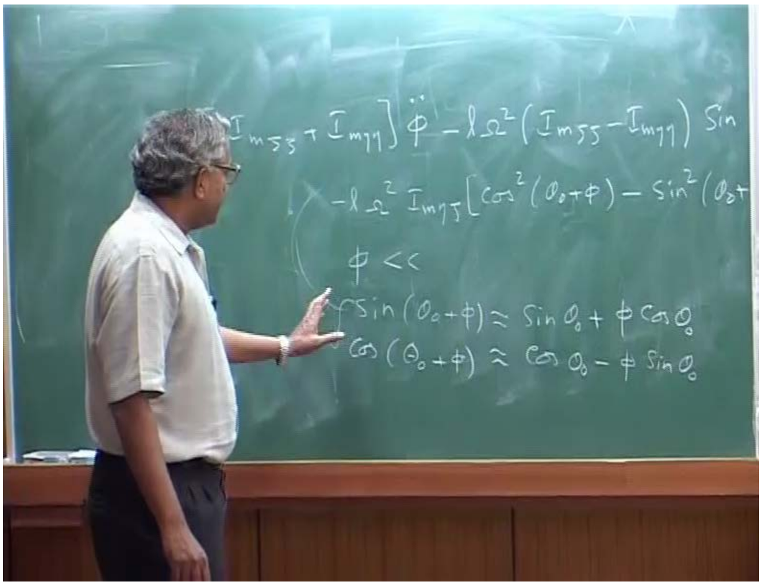

Last class we were discussing the torsion equation which I wrote; here I am just giving you a phi double dot minus l omega square I m… sine theta naught plus phi cosine theta naught… plus phi minus l omega square I m eta… xi cosine square minus plus M A;

Suppose in the, if we say phi is small; phi is small such that theta naught is not small, that is the pitch angle then, you can approximate sine theta naught plus phi cosine theta naught and then cosine as cosine theta naught minus phi and this you have to substitute. You can collect the terms as phi because otherwise, in this form the equation is non-linear even isolated torsional equation is a non-linear equation, but your pitch angle which you set that is theta 0 that can be large angle that is the finite input.

But, the phi is the elastic twist that you say that is the small angle. So, that is the assumption you are making here and then you substitute back then collect the terms of phi separately and then leave out all the other terms. If you write that that equation will be (( )) that is you substitute and then rearrange all the terms.

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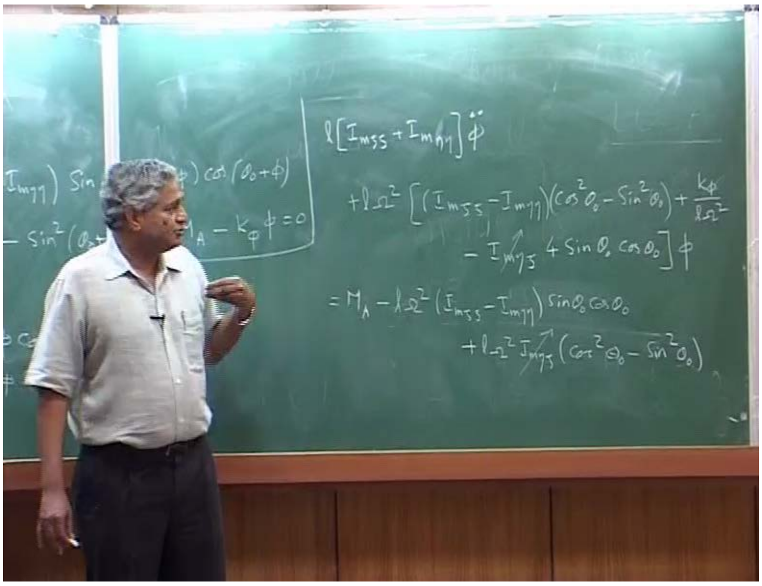

Then you will have, the equation will be l I m xi xi I m eta eta phi double dot plus l omega I m xi xi minus I m eta eta cosine square theta naught minus sine square theta naught plus k phi over l omega square minus I m eta xi 4 sin theta naught cosine theta naught phi equals aerodynamic moments minus l omega square I m xi xi minus I m eta eta sine theta naught cosine theta naught plus l omega square I m eta zeta cosine square theta naught minus sine square theta naught (Refer the above slide at the reference time for the equation). This is my equation; it is the linear differential equation because there is a phi double dot and there is a phi.

So, this like m s double dot plus k x equal some, but you look at the external loading and a is the aerodynamic loading there is one constant term. Please understand this constant term is there, which is a function of pitch angle. Now, you when you we wrote the phi square term, this one, yeah that see, always phi is small. Note that you make that assumption so that, the torsional deformation is small. So, pitch angle is large; large means it can be any (( )) 10 degrees, 15 degrees, 5 degrees, but phi is a small angle. So, the idea is this kind of tells you even if you do not have any dynamics; you keep the blade at the pitch angle theta naught and rotating theta, no aerodynamics in vacuum. This is a constant term which is the function of theta naught if theta naught is 0 this term is what this is 0. Only this will be there this is a cross product if it is a symmetric cross section then that will also go off, but if it is a non-symmetric cross section you will have some term.

But, usually looking its symmetric cross section of the aerofoil then you can always find what is the twist - elastic twist phi due to a pitch angle and that will be because this is usually a small number. This is, you can neglect this and even this quantity is larger than this quantity (Refer Slide Time: 08:00). You will find without dynamics in the sense without any epsilon, aerodynamic loads there is no phi double dot equilibrium torsional deformation of the blade. Your phi equal phi equilibrium equals this quantity divided by this quantity and that defines what is the torsion (( )).

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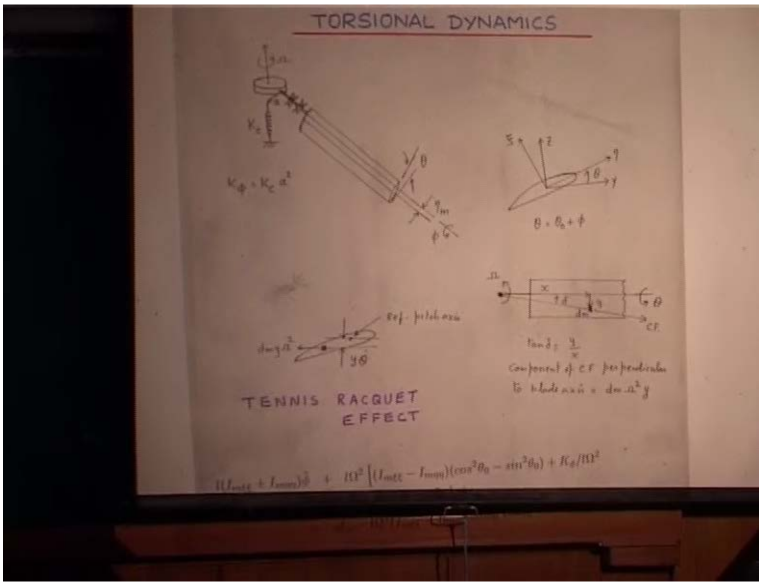

Suppose, if it is a hinge point there is no spring then you will find the answer will be phi, will be exactly equal to minus theta naught; that means, the blade will always come flap. In the sense, if you keep the blade like this and rotate it the blade will always come down it will be flat. This is because that is an actual equilibrium position; you see there is always a twisting moment (( )); twisting moment because of inertia load.

Now, you see the assignment which I gave will be rotating at the higher r p m; you can have in elastic twist of the blade due to control system, stiffness plus due to the blade elastic stiffness. That is the (( )) one can actually reverse calculate that should be the k phi assuming that this must be my pitch angle. So, usually what you set it, pilot makes you a command this is my pitch angle but, what the blade experiences is not the same just dynamics no aerodynamics comes that will also change angle of attack.

But, here because of the elastic twist you will find a loss in the angle of attack of the pitch angle that is due to the k phi k phi plays a major role. That is why the industries they calculate what is the control system stiffness. You will have swap plates and then there is a hydraulic jack at the below the swash plate above the control rods. You calculate the stiffness of that entire unit; do a fringe model some approximate calculation anything, but come up with the stiffness usually of course, control rods are very small.

So, they are by e a over l, but the swap plate will have its owns stiffness that nothing reached them so, that will give this. There will be a loss in pitch angle (( )) and now I will not write any expression for M A because, this is basically aerodynamics load. You can use quasi steady aerodynamic load or very simple instantaneous angle of attack. Take it, put it quasi static; these are all terminologies in the aero-elastic analysis. This is purely simple torsional dynamic set up from here. You can see if I m eta zeta is 0. What is my frequency - natural frequency of the blade in torsion like you have the flap frequency, you have the lag frequency now you have the torsion frequency. If you take the torsion frequency, omega r that is basically you have the stiffness term you have the massive term.

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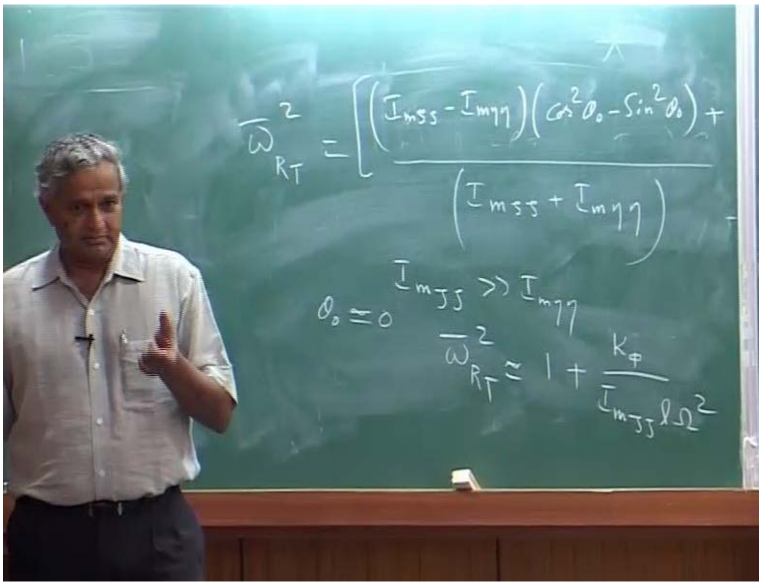

You will have omega bar that will be essentially this term divided by this term. So, you will have I m I m eta zeta cosine square theta minus plus k phi l omega square I am because l omega square this you make it non-dimensional. So, you will get another omega square will come all the l omega square will go off I have I m xi xi plus I m this is nothing, but the mass moment of inertia of the blade lower section per unit section that is all this is sum of this is the difference.

Now, you see when theta is 0 that is theta naught when you set the pitch angle of 0 this term will… and usually, this is much larger than because you need thin aerofoil - thin aerofoil this is what you can much smaller than I m xi in the inertia calculation. Therefore, you simplify this as theta naught is 0; you said then you will get a highly simplified expression.

This is this and this 1 plus I will put an approximation k phi over I m xi omega square this is my very simple form; that means, this is you can take it the blade torsional inertia because, I took this is a constant l into this will be torsional inertia of the blade about the (( )) and you are not (( )) with respect to omega square. This is like a simple torsional spring and this is due to inertia 1 plus something like you had flap motion where you always get 1 plus something here also this one will come.

But, this you have to be very careful; how it comes? it comes because of this term and then and sometimes people make mistakes (( )) because these are all very small quantities in a coupled (( )) torsion equation. Of course, but usually you know control system has a sufficiently large disturbance because you do not want the control platform is soft because, whenever you give an input it should fake fully go immediately and there should not be any phase neutrons between this and that. So, usually try to design that this is stiff control rod stiffness, but we take an actual blade. You have a control rod stiffness you have the elastic g j (( )) of the blade both will contribute to that those equations there you have write the elastic blade here.

We assume rigid blade, why do you place the frequency of torsion? If I flap is around 1.09 it is a 4 0 3 like that lead lag is 0.25 or 0.7 and the non-dimensional torsion you keep it far away far away means usually in the around, but 3.5 is low torsional stiffness.

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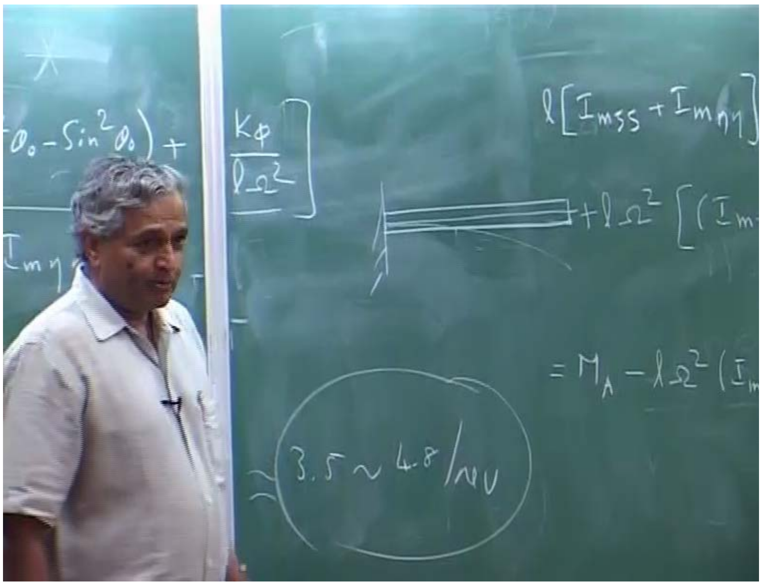

So, this is around 3.5 to 4.8 something like that around this range it will be 4.54 3.5 is very low torsionally it soft blade, but you see this frequency is far away from flap and lag where you know I want to bring this and I have problem; this is mainly dominated (( )) by control system.

So, if we want to analyze the blade dynamics you need to know the control system stiffness. So, that has to be calculated separately by a stress analysis that you have to get some estimate of the torsion stiffness and I will not go into the details of this particular term. I wrote the tennis racquet quick that is basically this; see even if k phi is 0 (( )) it is like a centrally hinged flap.

So, torsion also we will have,, if you disturb it will alternate like this as it goes around once per revolution that is all it is like a fight. See, that is why when you analyze torsion it is not a line it is actually a 2-D about the axis to blade tangent oscillate like this. that is why that is with this we kind of pretty much analyzed; this is of course, you know this is the non-rotating torsion frequency because the spring inertia that is all.

This is non-rotating non-rotating plus 1, that is what I wrote the, but omega non-rotating torsion plus 1 omega t. now, this all for isolated system but, if you ask what about the damping in torsion lead lag, we discuss the damping is very small and duplicate external damper is provided. If torsion the main damping comes because of aerodynamics usually it is about 10 percent. That is why there is no damper provided in torsion. Flap is heavily damped lead lag you put and external damper torsion it also damped out isolate torsion, but further you need to know unsteady aerodynamics not the steady then only you will get the phi dot like (( )) term. You need to know the unsteady aerodynamics how do you get pitching moment (( )) that a several theory.

That theory is in the aero-elasticity fixed wing theory. Fixed wing theory, is it applicable to rotary-wing? We use it; applicability is something different, but, that is the theory available and it is used but, with the correction for the inflow that is all.

Sir fixed wing and rotary wing other than the actual forces is acting to what would be the (( ))

Torsion is pretty much there is no rotation that is all this is free externally only this there is no rotation this will not be there.

(( )) a rotating blade and a fixed blade fixed blade there is that bending torsion problem what you want in the structure one course that is it.

Even we will be adding axial load. Yeah axial load, but I has made it as a rigid blade. Please understand in all my assumptions, in all these things I said my blade is rigid but, this question is, what we have a centrifugal force what you will do? That is an axial deformation with that it is there, but, then you have to include the axial frequency axial mode. Now, treatment of axial degree of freedom itself has several approaches.

How would if done is one is you make it as a rigid blade approximation that is it why is it valid the reason is axial stiffness is always much higher it is e a than bending and torsions therefore, solve it then in the bending problem you have an axial because it is a combined if you want to treat axial motion centrifugal force you have to bring it to a effect if you make it as a rigid blade it is taken care because you are not bother about that only thing is what is the effect of the centrifugal load in flap motion you consider lead lag motion you consider torsional motion you consider.

But if you say, no I want to include elastic deformation in axial direction, then, you write the axial equation. Then there are certain approximations made which is usually what normally make in traditional bending problems. What you do you draw a line reference line and then it deforms you see the length of that line does not change any material above that this is what you will not you take cantilever beam then you take a line. Then, you say what this deforms, what is the length of this line, is it the same or different or what (( )). You make an assumption that the reference we call neutral axis. Neutral axis does not extend, that is basically called extensionality condition. If you impose that condition then you can sum; so, that is also done. Suppose, you say, no I do not want to do also then, you know that problem. You include axial mode in your formulation that it is really missing it will (( )) complicate. The intense of aerodynamics, what is the difference in (( )) between a static wing and rotary wing aerodynamic (( ))?

Aerodynamic static wing and rotary-wing plenty of (( )) first of all through velocity is varying across the span you will have radial flow.

Yeah and you have a inflow everything is that flow is in order. Inflow can be accounted how to what.

That is your doubt how do you take phi dot?

Yes. Is that, yes, phi dot…

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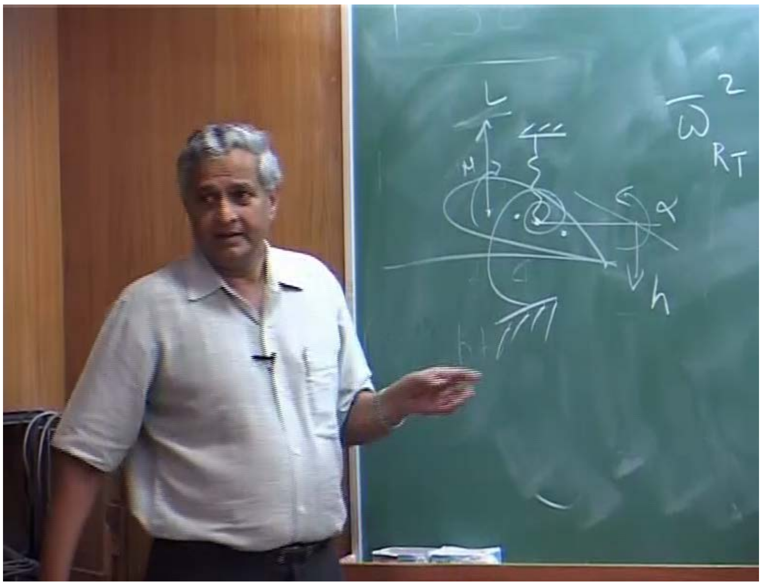

If you give because, that is not taught to you. Phi dot is suppose you have to take the theory, there is in the unsteady aerodynamic theory. You take an aerofoil; this is a quarter dot. You take the half point and you take some a elastic axis this is the most simplest model and then mass center. So, you take this point can come down and it can rotate this is the elastic axis. So, this is the 2-D therefore, now the theory associated with a 2-D is the (( )) theory in the subsonic in cumbersome and that theory takes care of both the h and alpha h is the skewing motion alpha is the pitching motion. So, you will get the lift and moment expressions are given about sometimes what (( )) and these expressions will be a function of alpha - alpha dot, alpha double dot, h - h dot, h double dot, h triple dot.

So, you find everything and this theory is the vortex model, this is the unsteady aerodynamic theory, this is not torque; suppose, this is what I am telling now if you (( )) want to get into aero-elastic problems some rotary-wing you start teaching. This you teach, make your all approximations and go and do it. That is, you start with fixed wing aero-elasticity is that, that is actually unsteady. Aerodynamic theory of a 2-D aerofoil alright, then you learned then from there, how do you incorporate that in your rotor lead? You can solve the (( )) problem this was the 1937 or something like that theta theory but, subsequently it was modified by (( )) and there is a Louis theory, but still theta theory is because it is experimentally who want to be valid for this.

Now, you can have start c f d, but c f d how to because this gives a form of the equation which you can handle the stability of the problem c f d is a numerical calculation. So, it is only time here you were analyzing in the frequency domain and get the stability directly. So, there is a lot of difference. How do you see the c f d for a stability problem? Then, you will have time domain integration and then (( )) the damping still it may be a question I have not seen any publication in that they try to use for loads, but not for stability.

These are differences because this is going more deeper into the modeling and then predicting what is happening as we have shown here into first level, I will call it, this is a helicopter dynamics is the first level, but you learn isolated effect. You see what each blade does in flap motion, lead lag motion, torsion motion and then you know the frequency you keep; what range they are. These are all isolated motion to understand what influences the frequency. Now, you have learnt up to this next what because, that is a where you built the subject.

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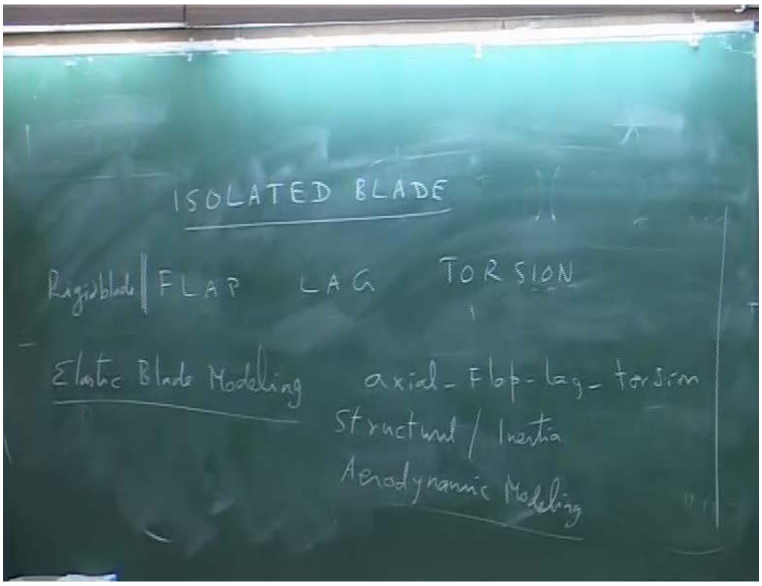

I will see after having analyzed the individual dynamics of the rotor blade, what does the various topics how people can go because isolated blade here you have flap lag torsion? Now, when you did this, what you have learnt is rigid blade, rigid blade, rigid blade. This is the assumption you have made. Suppose, you say, no I do not make this blade; I want to take it is as an elastic blade then, you have to go. That means, model in itself will lead to different direction that is elastic blade. These suppose I think one of you were using that how do you model elastic blade? Whatever he learnt he will give this by itself is separate study. Modeling an elastic blade elastic rotor blade with all complexities please understand you will have it is a twisted blade it is flexible in bending flap bending lag bending torsion axial. So, you will have axial-flap-lag-torsion problem complete all of that. One is structural model then you have to have that aerodynamic, sorry, the inertia model then it comes aerodynamic. Aerodynamics I leave it as it is purely blade modeling is only a large structural and inertia it is a structural inertia only this. Actually, people spend their entire some of the people research career only on modeling structural.

When you say blade modeling you have to say what material. I said material composite material, how do you go, how do you model it, how do you get it, etcetera, this people have spent their whole career on. This is one direction; another one aerodynamic modeling is required if you want to solve aero-elastic problem. Traditionally I will put it, traditionally people use a relatively simplified aero-elastic aerodynamic model of course, simplified in the sense quasi steady with the dynamic stall with a some dynamic way etcetera all because that what we have used. Now, today c f d is entering into that it will have worked on that c f d problems some of them and they trying to relate, but it is have some approximation still it is not fully advanced to the level that you can do everything.

Now, that is it is a aerodynamic model. So, you can get into aerodynamic modeling purely when I say aerodynamic modeling, what is that I do? In the rotary wing specifically I have to take care of the wing which is a structural means the blade gives you can have lifting line theory, but here when you go steady unsteady (( )) good unsteady aerodynamic model and then the wake that is the inflow which is generated at the rotor disk. We always use constant inflow in some uniform, but in hover only we had the very good forward flight is only constant inflow.

Now, here the inflow constant it can with varying time then above how do you model that. So, inflow modeling itself has gone through very interesting, I would say development that is an interesting developing and today is that (( )) we have certain models which were developing. That is a dynamic inflow, dynamic wake etcetera these are all global theories still I am telling it is not a c f d per say but still, it is able to give you the variation of inflow.

When there is a variation in the rotor blade loading, the whole concept started like that if there is a change in load inflow will change when the inflow changes, load changes. how do you relate these two? This is one line of problems we will have developed the time on modeling that another one is unsteady. Aerodynamic model for getting lift drag moment etcetera, one is please note one is getting only the inflow another one is getting lift drag moment on sectional.

So, if you want to include stall include stall models develop stall models 2-D and then use it in 3-D. So, one can get into this also, but when you talk about aero-elastic you need to have all. So, will you make (( ))? If you do steady aero-elasticity then may get rid of this and then we have good model in everything and then use it for aero-elastic problems.

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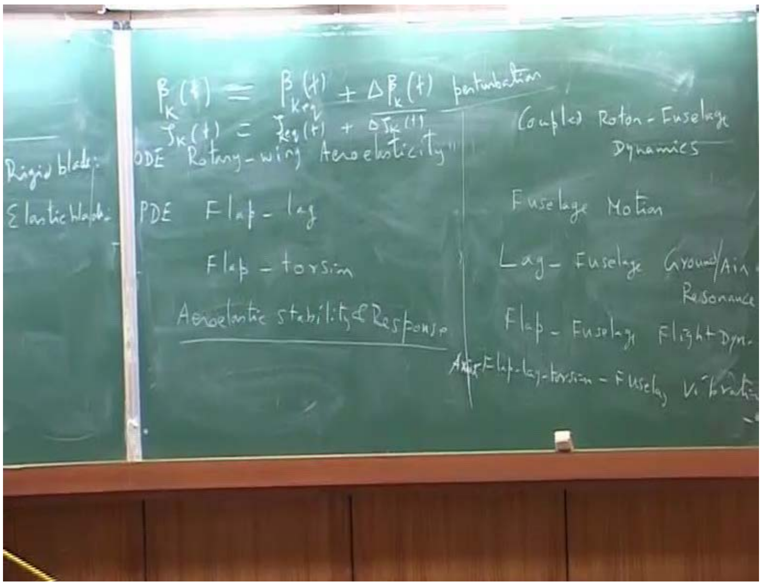

You can get into rotary-wing, aero-elasticity this is one line when I talk about rotary wing aero-elasticity. I have flap lag torsion; initially, development if you see that was not done with elastic blade rigid blade, but put flap lag combine then put flap torsion. So, flap lag problem, flap-lag flap-torsion problem, flap-lag then flap-torsion these two problems are treated initially that is one line because that you wanted to understand when the blade will go unstable etcetera.

Subsequently, because this particular problem flaps torsion do slight bending torsional problem of a fiction. Only thing is this is the rotary wing, the aero-elastic problem in bending torsion for fixed wing. Here it is flap-torsion subsequently they said that nothing we should have everything. So, flap-lag torsion problems but, these results were generated with various coupling various things.

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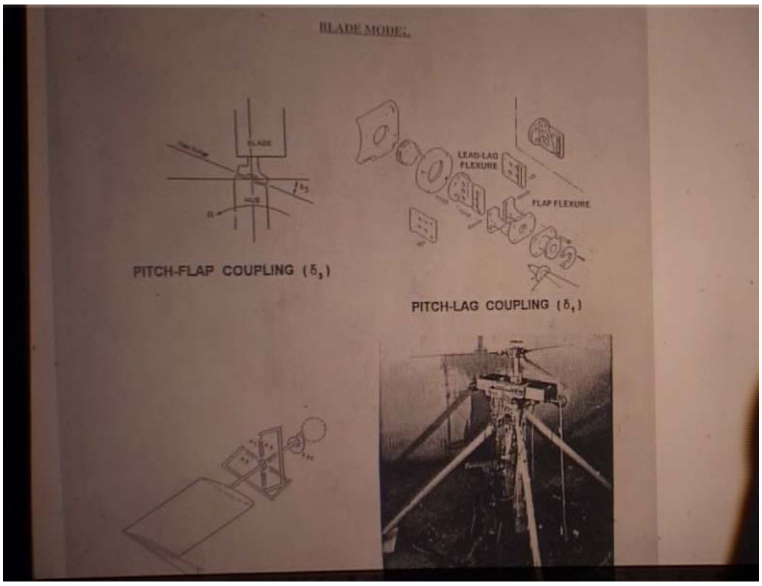

I will show one diagram not (( )) experimental set up because it was a essentially see this was a model - model rotor. So, they put because that model was built to represent the theoretical assumption please understand that is why you see the other diagram not that they made a model (( )) and the you start modeling is actually whatever theoretical assumption you make we corporate in the experimental thing.

So, what they did was they put a… it is actually nice kind of combination of flap and lag flexure at the same location because, you already say spring model and the spring model was putting and there is no torsion blade of course, there it a little bit stiff. In the sense, I am not like a rigid blade, but then they have pitch-lag coupling they also introduced pitch-flap coupling everything in this model.

I am just studying only the stability of that and of course, ground resonance problem also we will talk later that again coupling. So, this is analyzing aero-elastic sometimes you may require stall please understand because stall may be required in this stall modeling then stall modeling you have to bring in even here because, even aerodynamics may not be good. So, it is like a valid term structural model was up to the minimum check all this couplings effect of aerodynamic modeling that was this, but this is isolated blade you are only solving blades problem, but in a helicopter it is a rotor. That means, I have to go for coupled rotor fuselage dynamics. That means, fuselage is shaking blade is moving you have to study this problem this is the actual problem, but in this you have like you have aero-elastic stability and response here I will call it here aero-elastic stability and response only these two yes. How do the flap motion?

Experimental measurement of flap motion means what exactly you would have to think in (( )). They usually have some string hinge in the root of the blade. So, that will give you what is the bending moment flap bending moment, lead lag bending moment and may be torsional moment. Torsional moment normally goes to the pitch link load.

So, pitch link load they measure what is the load on pitch link. So, flap link they put a string hinge where they will measure the root bending moment. Please understand, it is not that entire motion every section they will do, but recently means may be about 10 years 1 u h 60 helicopter that U S army a experimental program was conducted on which they had some 220 ports some. So, many (( )) meters I do not know I have the figure on the blade both (( )) span wise pressure codes (( )) meters and they measured the in-flight load; that is only one test and that is what people have been using it for predicting their models.

Even yesterday, there was a seminar; of course, it was a (( )) seminar and (( )) because that data they are not supplying to everybody it is a flight test data and they did it for certain monomers and then they are trying to predict what theory, how could it be experimental even now that research papers skip (( )) because that has a brought certain problems. Before that the response was not properly predicted, those things were identified now. See, with other models we explain certain phenomenon, with other models even though we do not have that data.

If you change your aerodynamic model from simple to a more sophisticated, so we have five different models. So, we said if you use the most simplest model what will be the response; as you complicate the model how the response really changes because, there was some phase shift phase shift was where the minimum some load happens on advancing side something like that. They found out one of the measurement and theoretical was actually shown somewhere else.

So, they want to know what is really happening; that means, I still have not understood, but those problems have been not solved this is only the way otherwise there is something called a tracking and balancing tracking means you want to track the tip path because, all the blades must do same motion. Because, if all of them are identical right no blade is, no 2 blades are identical. There is always some reflection and they have a criteria by which they will bring the mass and the first moment between some limit after that they put the blade they rotate it they have color code.

Earlier they put some chalk and then they bring a cloth; they will know where it hits each color chalk. Then, they will know this blade is high that blade is low then they change it. Another way is, they also have a vibration machine inside because, if blades are not tracking same position you will have a one per revolution vibration. They will try to minimize that and that is the way. Otherwise, you can put some other wireless system and then measure. These are all measurement systems; what industry does? They will take and then earlier only what is that the slip ring data - will come to the slip ring.

Now, you can have wireless also, but that you can have; but then, technology has to be implemented on real life things and then take the data. That is what I think if you want add anything you can add, that is the vibration machine. If you tell which blade to adjust that is all. So, blade stiffness is what blade stiffness is e I; e i is flap but, e i is the variable it is not the uniform see the aerodynamic section to almost near the tip it is a constant value when you come near the root the value will change.

So, it is essentially a variable property beam; it is not the same value it is not uniform beam it is an nonuniform beam with property varying near the root section drastically down varies they change very large small range.

Sir this spring which we are modeling for flap the spring constant would be (( )) lower than the

No, spring constant these you trying to estimate spring constant estimate please understand estimate the spring constant from frequency that is all and the hinge offset that is how.

We measure that spring constant with respect to?

What is the question of measurement of spring constant? There is no measurement.

sir.

You measure only frequency

Yeah.

From the frequency because you know that suppose you take flap flap is 1 plus what M m x c z e over I b right plus k beta over right this is omega rotating flap square suppose if you say non-rotating blade flap frequency you can call it omega N R flap that is only this quantity this you can estimate because in a hingeless blade it is very difficult to know where exactly is that there is no point.

So, it is like from the frequency you try to estimate what is this and what is e if you assume this you can get this equation is only one suppose if you have non-rotating frequency then you can say this is my k and then once I know that then I can go and get there is another way or else they will say if my k beta is 0 what is my hinge offset; that means, that will be an equivalent articulated blade this is the what is the hinge offset estimating (( )) of orders less than.

No estimate (( )) dimensionwise you have to find dimensionwise this is Newton meter per radians that is what e i is what Newton meter square where what is the connection no connection is you can you see one is the different unit that is the different unit you can directly put it there of course, that affects this you are looking a numerical value.

My question was, is the thing we have to take the most of the bending which is (( )) so that, different (( )) and blade hinge? The blade see please understand not that your question is blade also will bend.

Yes.

But (( )) the second mode third mode first mode majority of the deformation if you say this is the hub majority of the deformation will happen like this is the first mode usually you will find the this is the if it is the hingeless blade, but you can also get something here

(( )), but it is a hingeless blade if it is a articulated blade then you have to calculate because the root hinge moment is 0 then the blade may have some other point bend moment will be higher, but that is usually not going you also make a measurement you put a spring in there and make measurement only for because that test I told you they measured and 50 percent 55 percent something like that bending moment that 50 percent and then try to correlate how your theoretical value matches with the flight test. But, number wise I do not see your question really is it a numerical value you are saying not a numerical value then It is a b blade stiffness is say E i blade stiffness is e i.

Yes.

See these are all equivalent model equivalent model that is all if your actual analysis if you want to do aero-elastic load then directly put the e i you do not go to this model see this model gives an idea of what is my flap frequency what is my hinge offset that is all not this k beta you adjust it to match that.

If you want to represent your blade as a rigid blade with a root string then you go and then put this; that means what you are doing elastic blade you are representing by a rigid blade with root spring. So, the modeling if you want you may use this and then you estimate the value beta and e that estimation (( )). The industriously root is going to where we (( )).

Always, everywhere the root only it takes only the root see if you really no no blade middle point (( )) root only have all the loads see one is the centrifugal load centrifugal load is of the order of 100 1000 Newton phenomenally large. But the flap load will not be large; see, if you take flap shear load, if it is a 4000 kg helicopter it is 40,000 newton; for 4 blades that means, per blade it is 10,000. You support the weight of the helicopter by 4 blades, 4 shear load 10,000. But, centrifugal load in order of 100, 1000, but that and then in flap load is useless because, that will give you the fatigue because it will oscillate all the vibratory load comes here. Non inverse the flap because it is a bending problem load wise may be magnitude may not be very large, but this cyclic load vibratory part this is the one which is the critical mean value and then vibration mean is usually not tremendously large because you of you take what 4000 kg helicopter means and 10000 Newton for four blades that is all that is the mean value in the shear is it clear. Now, I will go back to this because, I am giving only the type of problems this requires now fuselage motion. So, you will have fuselage motion in addition to flap-lag torsion; that means, hub is going to move this problem is more complicated.

But all flight dynamics problems come under this because you want to (( )) you want to do something then you need to have a model for the in addition there are certain problems where you face instability that is I will just list them out one is lag and fuselage I will put it only lag fuselage you will have air ground and air resonance ground air resonance and then flap this is a gross approximation flap and fuselage this is flight dynamic usually that is what in that.

But if you want vibration then, you have to have flap lag torsion; flap lag torsion axial everything you can put axial also and then fuselage this is vibration now you see there are different types of problems you want to you can say the most sophisticated analysis is coupled flap lag torsion axial fuselage motion and fuselage also elastic modes of the fuselage flag is the motion, but this not perfectly that it becomes it is really today it is not done its high complex and then if you want to take transmission tail rotor fuselage aerodynamic interaction, that is a problem is really very complicated, but how can the industry is very demand go ahead and make a helicopters you have to make simplifications the simplification ground resonance when they analyze they take only lead lag motion of the blade and the fuselage as a rigid body.

You understand and then solve this problem ground resonance problem; so, that means, the modeling that now changed only I take lag and fuselage motion suppose you say I want flight dynamics flap motion and fuselage motion because the flap controls your loads hub loads your monomer everything.

So, you included plus there is also one more what is the frequency range you are interested usually the rigid body dynamics the frequency is low in the sense it is less than one two or may be at the most three thirds not all hertz whereas, this will be about 3 5 hertz fibers this problem you will have from 20 hertz 40 hertz everything all the frequency will be there now you see depending on the frequency range of interest your modeling also changes, but technically if you have a complete model you can solve all problems, but then when you solve the problem what approach you will use.

So, that mathematical technique becomes very important if you want analyze stability then you have to know how do I analyze stability of the problem of the system if I want response control response how do I solve the control response problem.

Now, these are now going much deeper into the helicopter problems. So, depending on the problem you address that issue. So, you find the modeling will change which of these I am going to do (( )). These are to a large extent, these are all research problems even though the flap is, but if you want to add all of them and then vibration and then flight dynamics and control characteristics everything today models are getting developed everywhere people are improving their fundamental understanding based on some flight test or experimental data improve keep improving, but to have a simple understanding let us take just flap-lag aero-elastic stability.

That means, you are having two motions of the blade it is an isolated blade if you want to go to rotor now please understand this is one blade analysis this you have to take all the blades and all the blades they are all kept at different different azimuth locations you have to take everything into account and there are certain techniques associated with treating this problem how do it (( )). We have developed equation for one blade, but I am putting four blades or three blades two anything as the number of blades you have to have. So, many equations then you have fuselage motion then there is something like let we not look at individual blade I will start looking at this as a rotar disk I do not worry about individual blade I see how the disk is moving

Now, again in the flap motion how the disk will move in the lead lag motion similarly that is where from a rotating frame to a non-rotating frame some transformations only if you make if you want to analyze certain stability problems otherwise it becomes (( )) complicated of course, they do some approximations. We all said that the flap motion is like this if it is cone in forward tilt it is like suppose if this itself is oscillating; that means, they there is an equilibrium position of the flap about that it is oscillating.

So, now you see my flap motion itself is not a fixed value it is going to be if I say beta k which is a function of time I am going to write it actually not (( )) approximation I put it equal to I will say there is a beta k equilibrium which can be a function of time plus this can also we please note that this can be also be a function of time this can also be a function of time this is a perturbation. If I want, if you just (( )) the... So, this is the first thing you looks that a learning perturbation analysis and your analyzing either rotor fuselage stability or rotary wing aero-elasticity stability you have to know perturbation analysis. So, what is the perturbation analysis you have developed your flap-lag equations as you know that inertia load aerodynamic load and root spring model.

We have developed a coupled flap-lag equations, but those equations should be non-linear then how you solve stability equilibrium what you do is you take those equations those equations may have time dependent coefficients everything itself non-linear complete my c equation.

If it is a rigid blade approximation those equations are ordinary differential equations a rigid blade ODE (( )). ODE of course, non-linear time varying everything you can add of the I will tell you one by one. If it is a elastic blade this is p d e there will be partial differential equations elastic blade. Now, what are the various other additional characteristics if you say, I am just giving you only the description?

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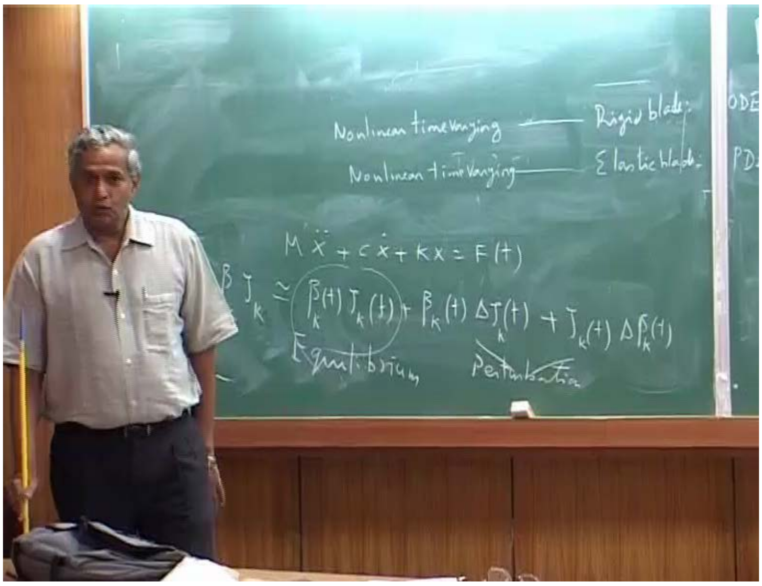

This ODE will be non-linear time varying coefficients this also non-linear time varying. So, you have everything you can analyze you look at the equation you will not know what is what I am talking about in the sense the coefficients of you all know that there is very simple in the ODE may know that these are the x plus k x f of t right, but you know these are time varying; that means, they are also functions of time right in whole only the time varying is gone, but in forward flight time varying will appear.

If a, if you solve non time varying simple case its always hover problem in the earlier then people are just over case, but then if a time varying system comes how we analyze stability even a simple case how do I solve stability problem in a time varying situations systems. So, that part I will cover in the last week that is not really helicopter (( )), but that is not usually talked in any other course a time varying systems stability problem you will develop from systematic things that will take above 2-Days you will add that there is something which is totally different from what you learnt from any of the course.

Now, PDE ODE; how do you… saw which you were if you are taking elastic blade; that means, you have to have a very systematic approach for addressing response stability problems. The PDE problems are converted into ODE unless you have studied vibration. Vibration means not spring mass, it is the theory of vibration or continuous system vibration. You will not know how do I convert this to this; that is you have to learn that something separately.

On the other hand, you will say I am starting directly rigid blade that the rigid blade I assume then I will get the directly ODE equation straight away I go to ODE now in the ODE how do I solve. So, the first thing is all the degrees of freedom that degree of freedom is flap degree of freedom is lag if you are solving flap-lag problem each degree of freedom that is an equilibrium there is a perturbation. So, you always a flap equilibrium perturbation and in the lag equilibrium and perturbation.

This is for isolated blade I am writing. So, isolated blade means that k will only doubly index it need not be even there because there is no 1 or 2 3 (( )) you are giving at only one blade. So, the k does not come into picture on the other hand if you go to this you will have k index has to be there. Then how you treat the class of problems how do you treat this class of problems. So, isolated blade the index scale has no meaning only one side that what you do is you have your equation of motion that is the non-linear ODE it is time varying you substitute this is the equation and then the product of perturbation neglect it that is if you have a term like this beta zeta is normally with the… you write it as beta k plus and then plus theta k t delta beta k.

The product of these perturbation; you throw it out you say it is a higher order term I do not cancel that; then, you collect all the terms which do not have the perturbation quantities. Write them, you collect all the terms which do not have the perturbation quantities. Write that separately and all the terms which contain the perturbation quantities (( )) except both of them equal to independently to 0; then you will get 2 sets of equations. So, what you started with one set of equation which is the flap-lag non-linear, everything you now have 2 sets of equations. One set of equation is the equilibrium that is the term which contains this; that is called equilibrium equation and these are perturbation, you will have like this. If you first solve the equilibrium equation to get data and zeta, first of this can be a differential equation. But, you solve once you solve that, then you take that than varying here; then you solve the perturbation equation first (( )).

This is how the helicopter problems are solved. Stability is analyzed and the perturbation equilibrium is analyzed in the actual non-linear set up and I think this (( )) because you cannot solve any one problem. That is because you go into the flap-lag stability I have to give a fully equation; you did the full equations, then you do it.

The nature of equations, please understand, the nature of the equation can be in this non-linear time varying ODE flaps stability linear time varying coefficients. Now, if you take the case of hover in condition then what will happen is the ODE will become non-linear algebraic equations. That will be this will not come; so, this will go up.

So, you will have a non-linear algebraic equation; so, flap-lag in hover to solve that. Once you know what is the flap equation what is the lag equation you will substitute here, you will have the linear stability equation constant coefficients and you solve for eigen values (( )). I do not know that the matrix (( )) solve for the eigen values and you will get instability results. This is the general procedure and this is consistently (( )) whether you do numerically all this calculation by hand, this is the procedure.

If it becomes time varying coefficients, how do you solve the stability? That part will (( )) one approximation where you convert this time varying into another frame that is called a multi-blade coordinate. That is not a exact transformation, but it is the approximation. Convert the time varying coefficient to constant coefficient; then you (( )) another one is you applied (( )) theory. That part I will teach in the last week I think. I leave you, if you have any questions you can ask (( )). With this pretty much your course, I am stopping.