

Introduction to Helicopter Aerodynamics and Dynamics

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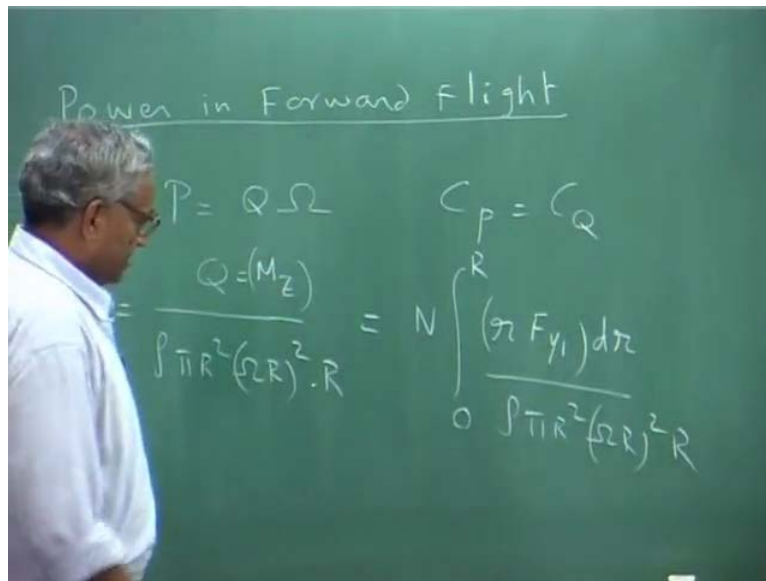
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Module No. # 01

Lecture No. # 14

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Now, this is very interesting, I would say, formulation for getting the power in forward flight which we saw because we know that power is, only the main rotor power; let us not bother about the tail rotor, main rotor power is Q times ω . ω is the rotor R P M. Now, we know that earlier C_P was equal to C_Q .

So, I will directly calculate torque coefficient for the main rotor. The torque coefficient is nothing, but the yaw moment torque divided by that non-dimensional quantity. So, we write our C_Q as Q which is actually M_Z , please note that. This is the Z component of the moment, last class I wrote that. So, ρ into R

In the last class, I wrote; let us take 0 to capital R , N is the number of blades we are taking mean value. I will not put that 0 to 2π and 1 by 2π ; it is assumed that we are doing that. Now this, we wrote $r F$, if you look back at your notes, last class I wrote this,

moment M_Z is this because the drag force into the radial distance and you again divide by ρ and we can non-dimensionalize.

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$$C_Q = \frac{N R^2 C}{\rho \pi R^2 (\omega R)^2 R C} \int_0^1 \frac{r}{R} F_{Y1} d\bar{r} \quad \bar{r} = \frac{r}{R}$$

$$= \frac{\sigma \cdot R}{\rho (\omega R)^2 R C} \int_0^1 \frac{r}{R} F_{Y1} d\bar{r}$$

$$F_{Y1} = -(L \sin \phi + D \cos \phi)$$

$$= -\frac{1}{2} \rho U^2 C [C_l \sin \phi + C_d \cos \phi]$$

So, I will write this, C_Q is $N R^2 C$, I am putting this. I will tell you how I got this. This quantity I divide by $\rho \pi R^2 (\omega R)^2 R C$. So, multiply by that goes there. So, this is r bar and this is also non-dimensionalize to $d r$ bar that is why I will get R^2 ; r bar is r over R and I multiply C just for and you know F_{Y1} which is given from the earlier expression that we will write it later. This is, you can now take $N C R$ over $\rho \pi R^2$ that is solidity ratio.

So, you will write σ into $1/R$ you will put ρ , cancel out this and you will have only this term. So, this is $\sigma \rho \omega R^2$ whole square chord in the denominator. Now, let us write our F_{Y1} . we had define earlier F_{Y1} , if you go back that is nothing, but the drag force look at your notes; it is essentially F_{Y1} . F_{Y1} is minus $L \sin \phi$ plus $D \cos \phi$ and you know lift, you have written as half $\rho U^2 C_l \sin \phi$ plus $C_d \cos \phi$.

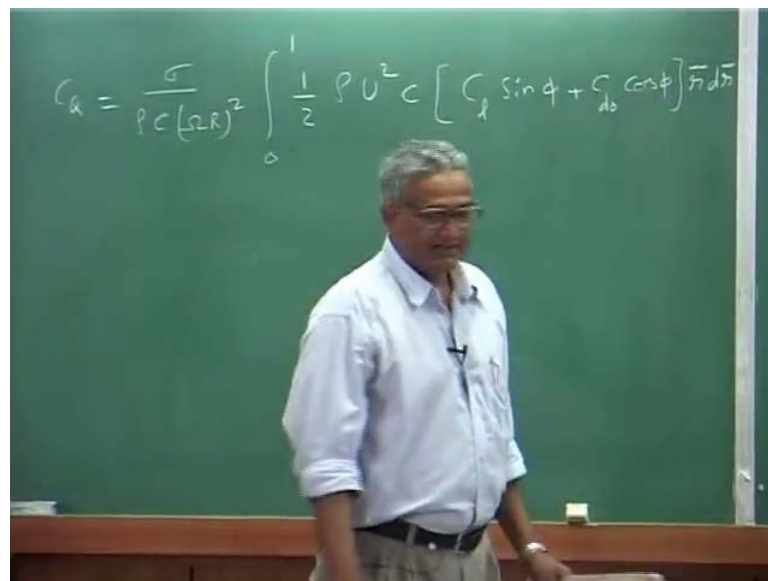
But what we will do is, we will substitute this term directly here, but the minus sign essentially represent that the torque is in the clock wise direction.

So, since we are calculating power, it does not matter, the sign will throw away. So, will not be carrying on this sign completely, sign does not matter. It is only to say that this is

the quantity because if the rotor is rotating counter clock wise, my torque is clock wise that is all.

So, I am getting the torque coefficient as a magnitude because that is basically the power coefficient. So, the minus sign- we are throwing it out. Now let us go back here and substitute this expression.

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So, you will get C Q which becomes essentially sigma rho C omega R whole square C l sin phi plus C d 0 cosine phi r bar d r bar, I put this here. Now, is the small jugglery we will do; we know F Y 1 is this similarly we have F Z 1 is essentially that l cosine phi minus d sin phi.

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$$C_Q = \frac{NR^2 c}{\rho^2 (\Omega R)^2 RC} \int_0^1 \pi F_{y1} d\bar{r} \quad \bar{r} = \frac{r}{R}$$

$$= \frac{\sigma \cdot R}{\rho (\Omega R)^2 RC} \int_0^1 \bar{r} F_{y1} d\bar{r}$$

$$F_{y1} = -(L \sin \phi + D \cos \phi)$$

$$= -\frac{1}{2} P U^2 c [C_l \sin \phi + C_{d0} \cos \phi]$$

$$F_{z1} = L \cos \phi - D \sin \phi = \frac{1}{2} P U^2 c [C_l \cos \phi - C_{d0} \sin \phi]$$

You are going to replace the l by F_{z1} in other words you substitute here that half rho U square C, this will be $C_l \cos \phi$ minus $C_{d0} \sin \phi$ write C_l in terms of the rest of the quantities you will get C_l .

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$$C_Q = \frac{\sigma}{\rho c (\Omega R)^2} \int_0^1 \frac{1}{2} P U^2 c [C_l \sin \phi + C_{d0} \cos \phi] \pi d\bar{r}$$

$$C_l \sin \phi = \frac{F_{z1}}{\frac{1}{2} P U^2 c} \frac{\sin \phi}{\cos \phi} + C_{d0} \frac{\sin^2 \phi}{\cos \phi}$$

$$C_Q = \frac{\sigma}{c} \int_0^1 \frac{1}{2} P U^2 c \left[\frac{F_{z1}}{\frac{1}{2} P U^2 c} \frac{\sin \phi}{\cos \phi} + C_{d0} \frac{\sin^2 \phi}{\cos \phi} + C_{d0} \cos \phi \right] \pi d\bar{r}$$

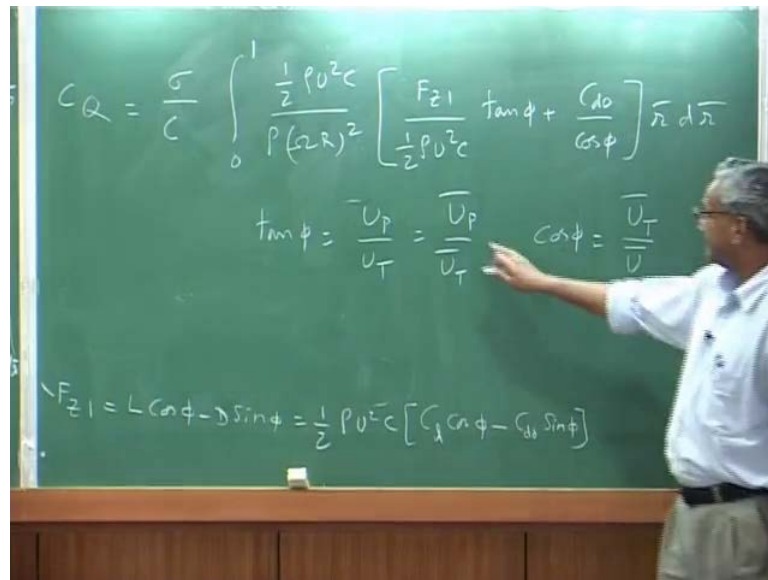
Let me put it C_l into $\sin \phi$. I am multiplying by so, divide by cosine phi. So, you will have F_{z1} over half rho square C, that is this term into $\sin \phi \cos \phi$ plus $C_{d0} \sin^2 \phi$ over, please note this I am getting it from this equation; $\sin \phi$ is just I am multiplying because I need to substitute this right here. This is just a jugglery that is all

mathematically. I am not making any approximation here that is why this is a little tricky derivation.

Now, what happens is when I substitute this expression here I will put, what will happen is C Q will become sigma, let me write the whole thing sigma over; let us write C 0 to 1 half rho U square C is there divided by rho omega R whole square.

Is it correct? Yeah, rho omega R square half rho and I am going to substitute for C l sin phi this entire expression F Z 1 over half U square C sin phi over cos phi plus; this is C d naught sin square phi by cosine phi plus C d naught cosine phi into d r bar sorry r bar d r bar. Now, if I club these 2 terms this will become C d not over cosine phi.

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So, now let us see. So, your C q will become sigma over C half rho U square C over half rho U square C this is tan phi plus C d 0 cosine and you know from our flow tan phi is U P over U T or you may also write it as U P bar over U T bar and cosine phi will be over phi; this is the total velocity U bar. You can directly take this substitute here and it will be kept in one particular form, after that what is done let me erase this part because this not required.

Now I will just leave it at that stage and get back to another expression which is if I put all of them because you know that this term gets cancelled with this term.

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$$C_Q = \frac{\sigma}{C} \int_0^1 \frac{1}{P(2R)^2} \left[\frac{F_{z1}}{U_T} \frac{U_P}{U_T} + \frac{1}{2} \rho U^2 C \frac{U}{U_T} C_{d0} \right] 2\pi R dr$$

$$\tan \phi = \frac{U_P}{U_T} = \frac{\bar{U}_P}{\bar{U}_T} \quad \cos \phi = \frac{\bar{U}_T}{U}$$

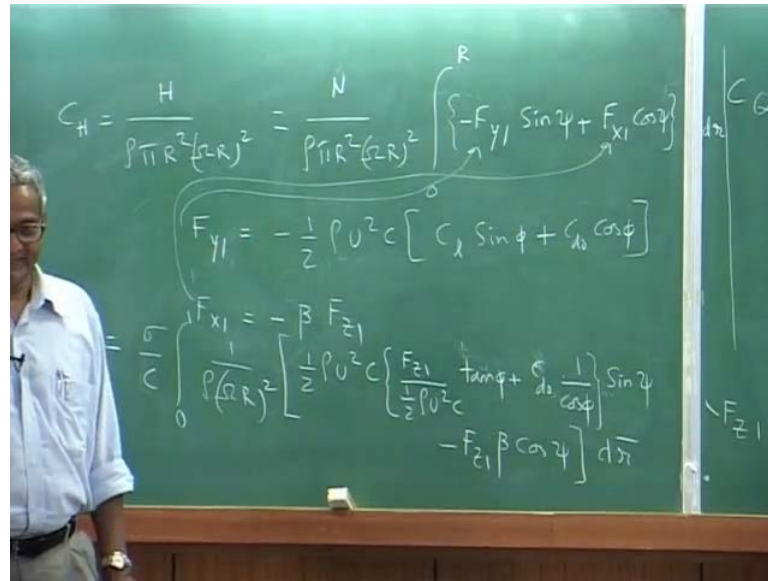
$$F_{z1} = L \cos \phi - D \sin \phi = \frac{1}{2} \rho U^2 c [C_L \cos \phi - C_{D0} \sin \phi]$$

You will have 1 over rho, so, maybe I will write this, that is better, you can put it U P over U T or you can put the bar also; there is nothing wrong and then here, this will become half rho U square C and U bar over U T bar C d 0 r bar d r bar.

This is, you can put a bar it does not make a difference. Finally, it will get nicely cancelled and it will come into a nice clean form. Now, this is the torque if you see it has 2 components, 1 is due to the profile drag C d naught and another 1 is due to the thrust part.

Similarly, we will formulate another expression in terms of that horizontal force which is C H horizontal force. C H is basically H over pi R square omega R whole square. This is again number of blades you go back look at the whole thing it will be 0 to 1.

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Sorry 0 to R F Y 1 with a minus sign sin psi plus F X 1 into d r you know F Y 1 F X 1 please understand because F X 1 is F Y 1 we wrote earlier, that is minus half rho U square C C 1 sin phi plus C d 0 cosine phi.

And your F X 1 if you look at it F X 1 that is minus beta F Z 1 earlier, what you do is you take these 2 expressions put them here and again you do the non-dimensionalize that r bar 1 r will come, now you will get the C H expression sigma over C 0 to 1 because I am just saying you put this here and this also there put there what you will have after N C pi R you write it in terms of sigma, that is solidity ratio N C R over pi R square.

You will get one over rho omega R whole square half rho U square C open up a bracket, I am writing F Z 1 over half rho U square C tan phi plus C d0 into one over cosine phi sin psi minus F Z 1 beta cosine psi d r bar.

So, please note that what I have done I have substituted this expression there after that for C 1 I substituted from here in terms of F Z 1 that is why it becomes C d not one over cosine phi you follow and this is in terms of F Z 1; it is a 2 step. So, I am writing my C H, erase this part also. So, this is my entire expression for C H.

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$$C_H = \frac{\sigma}{C} \int_0^1 \left[\frac{1}{\rho(\Omega R)^2} \left\{ \frac{U_P}{U_T} \sin\psi - \beta \cos\psi \right\} F_{Z1} + \frac{1}{\rho(\Omega R)^2} \frac{1}{2} \rho U^2 \frac{U}{U_T} C_{d0} \sin\psi \right] dR$$

$$C_Q = C_{QI} + C_{Q0} + \mu C_{H0} + \mu C_{HI} - \mu C_H$$

Now, comes you can substitute here simplify let me simplify the whole expression here. This become C_H is σ over C \int_0^1 over $\rho \omega R$ whole square. What you do is, you take the F_Z terms you will have this is $\tan \psi$ is U_P over U_T ; this will cancel with this. So, F_Z 1 U_P over U_T $\sin \psi$ minus F_Z 1 β cosine ψ .

So, you will have 2 terms. I am going to write only the U_P over U_T . This is this, $\sin \psi$ this is minus β cosine ψ , you take it F_Z 1 . Maybe this bracket is not necessary, I do not think I need this; I am substituting for C_I that will add with the other C_{d0} F_Z 1 you will have F_Z 1 .

See F_Y 1 has a C_{d0} that will combine leaving you in this similar fashion, identical to this. After that you will have plus 1 over $\rho \omega R$ that is this term we have to take this term.

Half ρU square you will have U bar by U_T bar is a cosine ψ and you will have a chord, then there is a C_{d0} and $\sin \psi$ and this entire thing is bracketed dR , is because this is that term.

I am taking half ρU square C_{d0} U square $C \cos \psi$ that is given U_T by U that is what is coming here and then there is a C_{d0} there is a $\sin \psi$ that is what here C_{d0} $\sin \psi$ and this is the remaining expression.

Now you have written 2 expressions, 1 for C Q and 1 for C H you know C H is the force coefficient longitudinal force coefficient C Q is the torque coefficient. Now only little more, I erase this part because now some algebra is done.

You write your C Q has 2 components 1 due to induced, another due to profile I is induced zero is profile. Now, what you do is similarly, you have C H also induced that is, F Z and there is a profile C you add and subtract mu times C H0 plus mu times C HI minus basically mu C H because C H is this entire thing.

I am writing this term as C H0 sorry C HI, this term as C H0. Same thing here, this term as C Q0 sorry C QI that is C Q0 and then you add and you subtract because basically C H is C H not plus C HI. It is kept in this fashion now, what is done is they calculate these 2 you add these two terms that is C QI plus mu C H, you understand.

Then C Q naught mu C H naught, like that you combine the terms. When you combine if you write C QI plus mu C HI, let me go back and then possibly may be I will use this.

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So, we will have C QI plus mu C HI, this is sigma over C 1 rho omega R whole square r bar U P U T or you can write U P bar by U T bar.

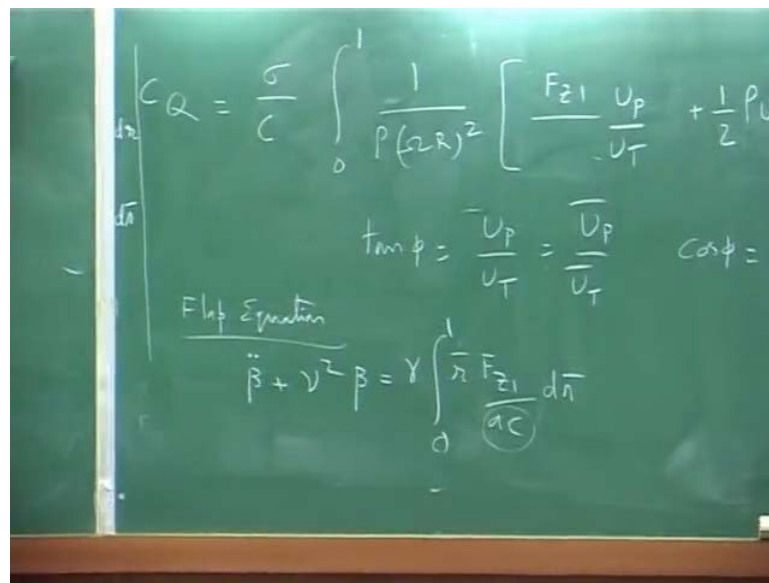
So, you will have r bar, U P bar over U T bar does not matter bar you can keep it then plus I have to take this term that is U P bar over U T bar sin, what is there mu is there. So, I will put a mu then minus mu beta cosine psi d and F Z 1 is there sorry F Z 1 d r bar.

Now, you look at this term, if you go back to your notes you will see U_T was defined as $\omega R \bar{U}_T$ which we wrote as $\bar{r} + \mu$. Similarly, U_P as $\omega R \bar{U}_P$ which is $\omega R \lambda + \bar{r} \beta \dot{\psi} + \mu \beta \cos \psi$. What you do is, you substitute here because U_T is common what is $\bar{U}_T \bar{r} + \mu \sin \psi$.

Because between these 2 terms this term is nothing, but \bar{U}_P is here $\bar{U}_P \mu \beta \sin \psi$ that will go off. So, leaving behind your $C_{QI} + \mu C_{HI}$ is nothing, but $\frac{\sigma}{C} \frac{1}{\rho \omega R} \lambda^2 + \bar{r} \beta \dot{\psi} F_{Z1} d\bar{r}$.

Now, here something is done; that is, if you take this term, there is a force $R \beta \dot{\psi}$ is the velocity of that point and which can be written as $R \beta$ divided by $\dot{\psi}$ because you can convert $d\beta$ by $d\psi$ and this, if you do averaging over one revolution the work done by this force because this is the periodic motion you are assuming, but here there is a nice explanation, but for that you need to know the blade flap equation, which I will be deriving later.

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Usually the flap equation I am just writing it here for you. This is the flap equation; it is a general flap equation. Let us assume that it is a rigid blade, whatever we made some hinge of said everything, flap equation which we will be deriving later.

I am just writing it here some into some gamma which is again 1 more number $\bar{r} F Z$
 1 over a C 1 gamma is some non-dimensional number $\bar{r} F Z 1 d r$ bar into a is the lift
 curve slope chord these are all you can take it as constants you can take it out this is my
 flap equation. 1 way of writing it is what people do is this is also 0 to 1 $\bar{r} F Z 1 d r$ bar
 $\bar{r} F Z 1 d r$ bar.

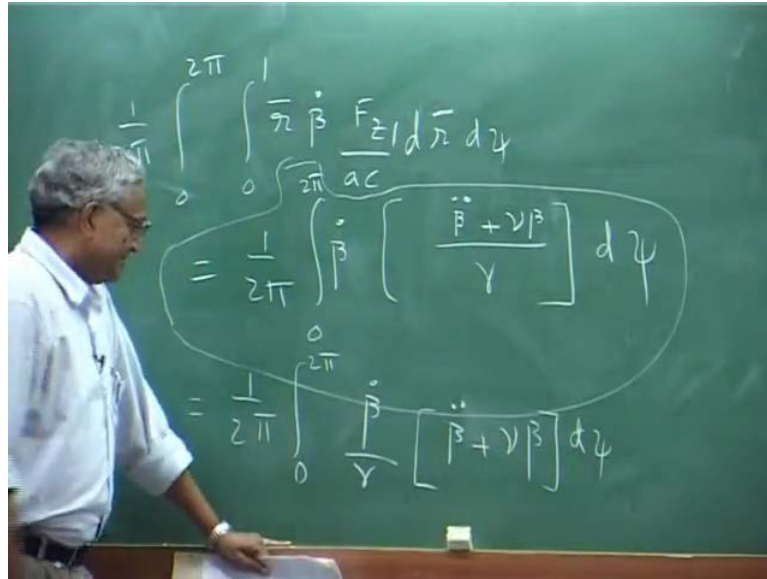
So, this ν square is a frequency term you can substitute this term only for this and then
 you integrate; that means, everything is converted into basically β double dot into β
 dot and then ν square β dot something will come and integral of course, that we have
 to do over 1 cycle.

When you do 1 cycle, what happens is the aerodynamic force this is basically the
 aerodynamic force this particular side is the inertia term. The aerodynamic force because
 dynamical equations what you do is external force and centrifugal force is also there that
 is why this is the inertia term is it.

When you do that since you assume periodic motion you will find that the total work
 done by this force in going around this is 0 therefore, the proof if you want you have to
 substitute this here and then get it. I will show that that quantity has to be identically 0 .

So, the C_{QI} plus μC_{HI} will simply become $\lambda F Z 1 d r$ bar. I will first because
 the lack of space, see if I do this part I do not know where to write. Let be erase this
 because I will just show that because this is a very interesting thing that is all because
 these are all practically valid. I am writing this; I am taking average integral 0 to 1 β
 dot $F Z$ over a $c d r$ bar $d \psi$ this is what I need $\bar{r} \beta$ dot $F Z d r$ bar.

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And a c is because when you take the non-dimensional it becomes that factor. This is nothing, but this is equal to- you go to this equation, this will be 2 pi. I am taking beta dot 0 to 2 pi then I am doing 0 to 1 this is r bar F Z 1 a c d r bar. I will just take a gamma and divide. So, I will put this is beta double dot nu beta sorry nu beta over gamma here what some d there is no d psi is there 0 to, no 0 to 1.

I am sorry because there is no 0 to 1 because this is 0 to 2 pi that 0 to 1 r bar F Z 1 a c d r bar, that is written by this, replace by this. So, you have essentially one over 2 pi 0 to 2 pi beta dot gamma is another constant which we will define later beta double dot plus nu beta d psi.

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$$\frac{1}{2\pi} \int_0^{2\pi} \frac{1}{2\gamma} \frac{d}{d\psi} \left[\dot{\beta}^2 + \nu^2 \beta^2 \right] d\psi$$

$$\underline{\underline{\dot{\beta}^2 + \nu^2 \beta^2}} \Big|_0^{2\pi} = 0$$

$$= \frac{1}{2\pi} \int_0^{2\pi} \frac{1}{\gamma} \left[\ddot{\beta} + \nu \ddot{\beta} \right] d\psi$$

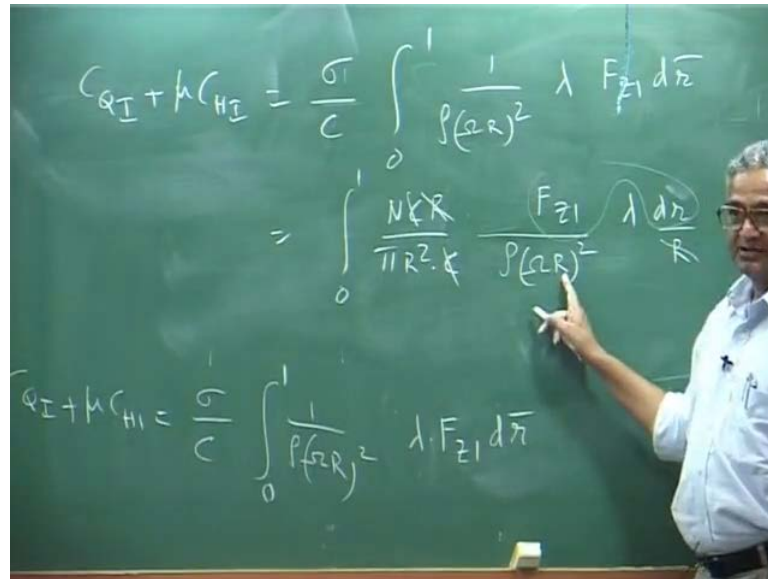
This particular term, maybe I will erase here again I will write it, this particular term can be written as d by d, because dot is nothing, but d by d psi non-dimensional time derivative this will be plus, so, this is square nu square beta square d psi and since this will go off, that mean differential of some quantity which I am integrating from 0 to 2 pi and this quantity this is a velocity square.

This is position deformation square. This is like a kinetic energy and this is like a strain energy position square. We will later know that this is what it is and it is 0 to 2 pi means I will get what 0 to 2 pi and it is periodic motion. When it is periodic motion, that means, what you will start you come back to the same point therefore, this entire thing will be 0.

So, it is essentially the work done by the term r bar beta dot is identically 0 term. Therefore, your C QI mu C H it will become a very neat simple expression.

There is a factor 2 here you can take it. There is a 2 take sorry because twice you can put a 2, factor 2 because this is a d by d square. So, 2 beta dot beta double dot, so, the factor 2 cancels out.

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So, essentially I can now write my $C_{QI} + \mu C_{HI}$ in this fashion. $\lambda F_{z1} d\bar{r}$ is $C_{QI} + \mu C_{HI}$, now if you look at this particular term $C_{QI} + \mu C_{HI}$, which is $\frac{\sigma}{c} \lambda F_{z1} d\bar{r}$, let us take this, σ is what $\frac{N c R}{\pi R^2 c}$ you can take it like that let me put it this way \int_0^1 .

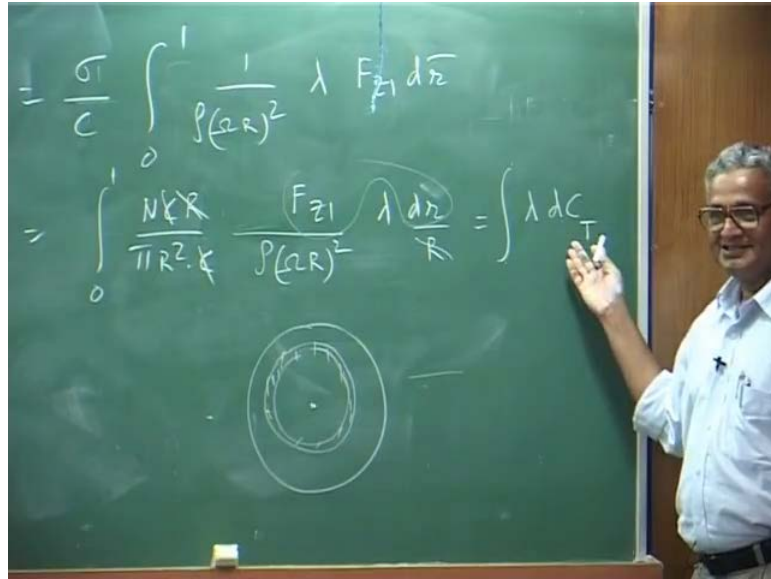
I will take the c also inside. So, $\frac{N c R}{\pi R^2 c}$ into c $\rho \Omega R$ whole square and you have $F_{z1} \lambda d\bar{r}$; $d\bar{r}$ is $d r$ over r .

Now, $N c$ you cancel out, this r you cancel out. You will have $n F_{z1}$, $F_{z1} d r$ is what this is the force per small and F_{z1} is force per unit length into $d r$ is that force, divided by $\rho \pi R^2 \Omega R^2$.

This is basically non-dimensional which we use for thrust; N is the number of blades. So, this particular term is written as $\lambda d C_T$ differential, this is the differential force only small elemental force.

This is done, you have to do an average in over the 2π , that is why the averaging is always included, but this is over the full disk because you have to do the full disk because if average is over the entire disk $\frac{1}{2\pi}$ it is always there.

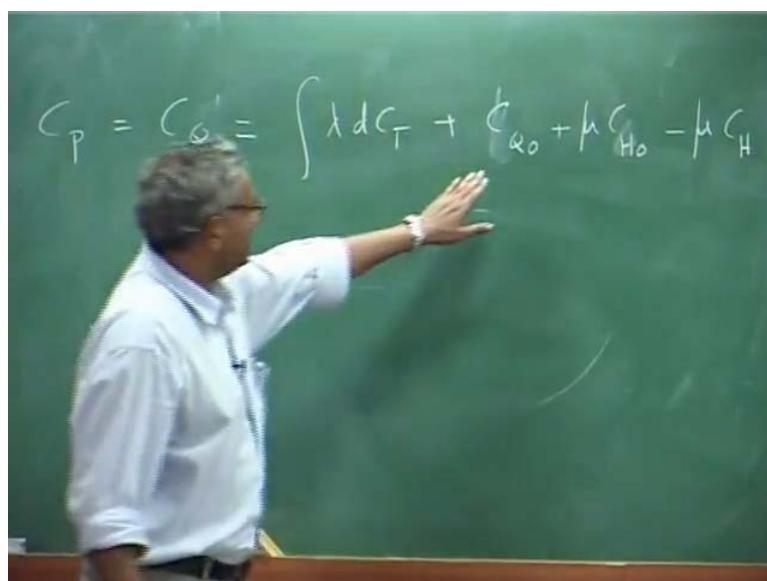
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So, dC_T because we mentioned earlier C_T at every time it is varying, but when you do the averaging you get the mean value of the thrust and this is basically the average value of the thrust, but differential element and lambda is the inflow to the disk at that location.

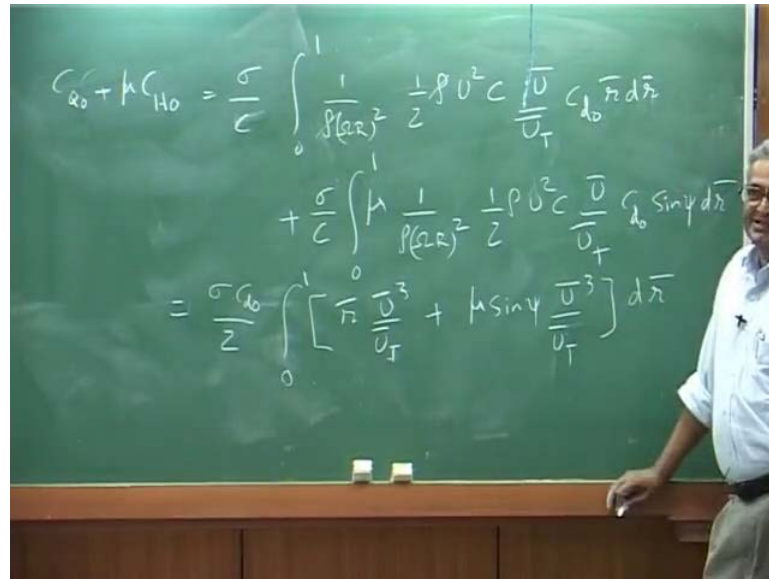
So, it is like the entire disk, you can split it into small rings. It is like you take the whole disk make them small like we did thrust and you know lambda it is like a weighted thrust, but later if lambda is constant this will come out and this is nothing, but thrust coefficient that is what we will use after reduction.

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Now, if we go back to our formulation of C Q the complete because we said the power coefficient C P is nothing, but C Q which we got it as 1 of the terms lambda d C T this is the mean this is an average value plus you have sorry C Q0 plus mu C H0 minus mu it is there. This term is still there because this is only C QI mu C HI let us now take only C Q0 mu C H0 term now. So, again I go back erase and write the term.

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So, you will have C Q0 plus this is sigma over C 0 to 1 omega R square half rho U square C you got U bar over U T bar C d r bar C d0 sorry this is C Q0. Then, plus maybe I can put a plus sigma over C, I will write again 0 to 1 you have a factor mu is there again this will be multiplied by 1 over rho omega R whole square half rho U square C U bar by U T bar C d0 sin psi d r bar.

This is the C H naught term. Now, what you do is you combine both of them. Here you will have mu sin psi and you can now make it in a different form that is I will write it sigma you take out C d as you know it is a constant. I am taking is C d naught sigma C d naught and there is a half is there. So, I am taking that half outside 0 to 1. You know that rho and rho will cancel.

This is U bar, this is U bar because U square over omega R square is U bar square. So, this will become U bar Q bar. So, I am going to write it as r bar U bar cube over U T bar plus, here also mu sin psi again U bar cube over U T bar.

Now, you know that that is \bar{U} cube only. So, this entire term will become, I erase completely here and write it as $\frac{\sigma C d_0}{2} \int_0^1 \bar{U}^3 d\bar{r}$. Now, what is that we are going to put for \bar{U} is the resultant flow now I am saying my inflow is very small.

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$$C_{d0} + \mu C_{H0} = \frac{\sigma C d_0}{2} \int_0^1 [\bar{r} + \mu \sin \psi]^3 d\bar{r} = \frac{\sigma C d_0}{8} [1 + 3\mu^2]$$

mean

$$\bar{U} = \bar{U}_T + \frac{\sigma}{C} \int_0^1 \mu \frac{1}{\rho(\bar{r}R)^2} \frac{1}{2} \rho U_T^2 \frac{\bar{U}}{U_T} C_{d0} \sin \psi d\bar{r}$$

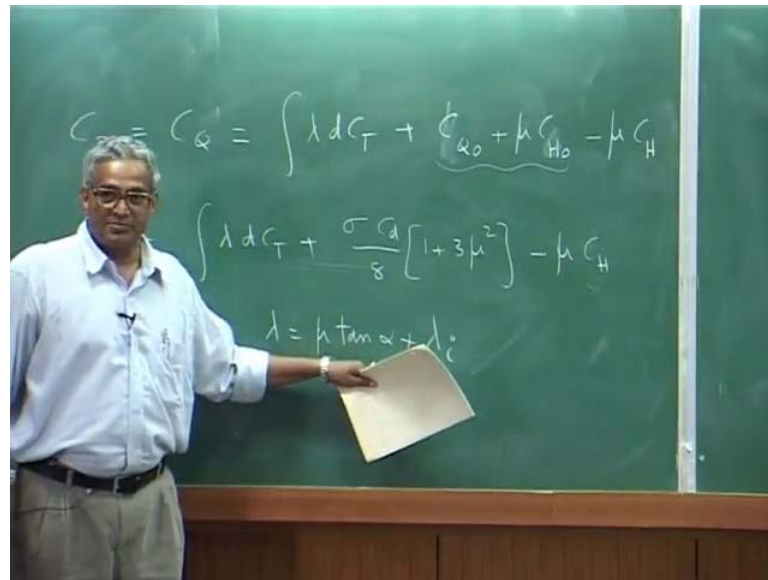
$$= \frac{\sigma C d_0}{2} \int_0^1 \left[\bar{r} \frac{\bar{U}^3}{U_T^3} + \mu \sin \psi \frac{\bar{U}^3}{U_T^2} \right] d\bar{r}$$

So, \bar{U} is almost \bar{U}_T then \bar{U}_T if I say \bar{U} is \bar{U}_T that is nothing, but what whole cube $d\bar{r}$.

If you integrate and then take the mean value, mean value means 0 to 2π you do it. If you do the mean value you will get $\frac{\sigma C d_0}{8} [1 + 3 \text{mean value}]$ this is mean because you have to take the cube because \bar{r} cube this will be or 4, 1 by 4 that 4 only that become 4 that 4 into that become 8 and then anything which has a $\sin \psi$ integral $\sin \psi$ over the 0 to 2π that value is 0 then $\sin 2\psi$ or cosine 2ψ that is also 0.

So, only thing is when you go to square term you will write $\sin^2 \psi$ is $1 - \cos 2\psi$ something like that then you do the mean value, you will find this is the quantity $\frac{\sigma C d_0}{8} 3 \mu^2$ exactly it will come.

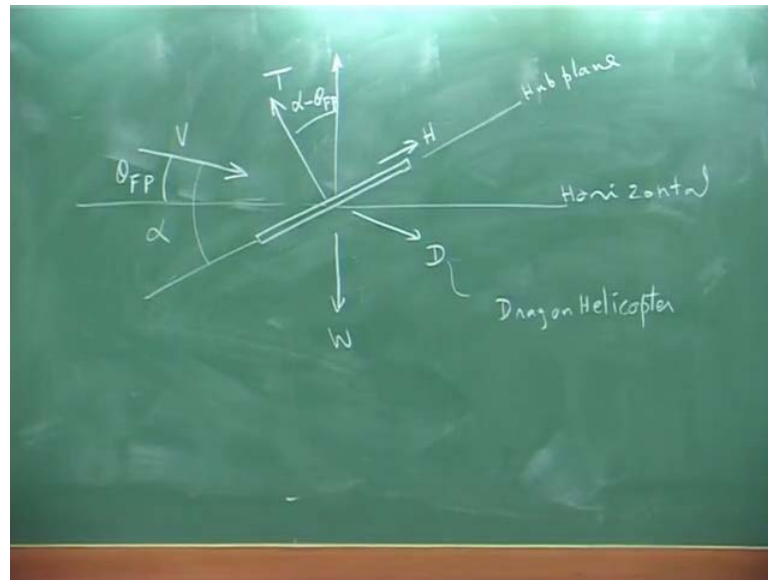
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So, you will find your C_p or C_q is integral λdC_T plus σC_d over 8 1 plus $3\mu^2$ minus μC_H , but now let us go back and then do some more adjustment and you see that expression will become, because μC_H is still hanging there. So, what is done is, now, what I am going to draw is related to flight dynamics. It is related to flight.

What is my λ ? λ is μ , go back to your notes λ_i . This is the induced this is due to the forward speed in the hub $\mu \tan \alpha$ also plus λ_i ; you look back your notes.

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Let us now draw a diagram, this is an interesting diagram. This is my hub which is inclined at an angle. So, I will say this is my horizontal, this is my vertical and this is the velocity, which is coming in this angle and it is horizontal. For now, this angle is alpha because we know as per our definition this is the hub. So, I will say hub plane.

So, this is my velocity, oncoming velocity with respect to hub plane $V \cos \alpha$ that is how I define my mu. Now, my thrust is given normal to the hub and this my H force. H is, because the helicopter is flying like this, is the H and weight of the helicopter and that is the drag force on the helicopter, weight of the helicopter is down and the drag force is along V.

So, I am putting this is drag on the helicopter. Now, I am going to call this as theta FP flight path angle. This is the flight path and this is alpha minus theta FP because the tilt is only that much.

Now, let us write our equilibrium equation. Equilibrium means because thrust and H force they are generated by the rotor system whereas, the weight and the drag they are coming because of the fuselage getting dragged.

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The image shows a chalkboard with the following equations written in white chalk:

$$T \cos(\alpha - \theta_{FP}) + H \sin(\alpha - \theta_{FP}) = W + D \sin \theta_{FP} \Rightarrow$$
$$T \sin(\alpha - \theta_{FP}) - H \cos(\alpha - \theta_{FP}) = D \cos \theta_{FP}$$
$$T(\alpha - \theta_{FP}) \approx H + D \leftarrow$$
$$\alpha - \theta_{FP} \approx \frac{H}{T} + \frac{D}{T}$$
$$\alpha = \theta_{FP} + \frac{C_H}{C_T} + \frac{D}{W}$$

So, let us write the equilibrium in the vertical direction $T \cos \theta_{FP}$ plus $H \sin$. This is equal to W plus $D \sin \theta_{FP}$ and then in the horizontal direction you will have $T \sin \alpha$ minus θ_{FP} minus $H \cos \alpha$ minus θ_{FP} equals $D \cos \theta_{FP}$, this is from equilibrium.

Now, I am going to make lot of assumptions. The assumption is, these angles are small. Now, what I am going to make is, these angles are small and thrust force is large H is small when I make that from this equation drag is also small this is also small. So, I am writing here T is approximately double from this equation.

Now, from this equation T into α minus θ_{FP} this is all this is cosine. So, these angles I assume small. So, this is H plus D . I am just taking H that side. This is this equation. Now, I will write α minus θ_{FP} , it is H over T plus D over T or in other words α is θ_{FP} plus H is non-dimensional I am putting C_H over C_T . Here it is, I am writing it T is W . So, I am putting D over W just an algebra modification. Now, here I will write it α is small.

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$$C_p = C_d = \int \lambda dC_T + \underbrace{C_{\omega_0} + \mu C_{H_0}} - \mu C_H$$

$$= \int \lambda dC_T + \frac{\sigma C_d}{8} [1 + 3\mu^2] - \mu C_H$$

$$\lambda = \mu \tan \alpha + \lambda_i$$

$$= \mu \alpha + \lambda_i = \mu \left(\theta_{FP} + \frac{C_H}{C_T} + \frac{D}{W} \right) + \lambda_i$$

So, mu alpha plus lambda i approximately for alpha I am going to write that expression which is mu times theta FP plus C H over C T plus D over W plus lambda i. Now, I am defining my climb velocity, what is V climb velocity of climb that is theta sorry not theta V sin that is a climb velocity which I call it V C non-dimensional if I want lambda climb that is I call it lambda C which is this divided by over, V over omega R, I am writing it as mu theta FP is approximation.

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$$V_{climb} = V \sin \theta_{FP}$$

$$\lambda_{climb} = \lambda_C = \frac{V \sin \theta_{FP}}{\omega R} \approx \mu \theta_{FP}$$

All right now, let us go back here and our equation becomes, so, maybe it is not necessary, now I can erase this part. Our lambda, I defined as mu theta FP, mu theta FP is basically non-dimensional lambda C.

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$$\lambda = \lambda_c + \lambda_i + \frac{\mu C_H}{C_T} + \frac{\mu D}{W}$$

$$C_P = C_Q = \lambda_i C_T + \lambda_c C_T + \mu C_H + \frac{\mu D}{W} C_T - \mu C_H + \frac{\sigma C_{A0}}{4} [1 + 3\mu^2]$$

So, that will become lambda C, there is a lambda I and then plus mu C H over C T plus mu D over W. Now, I go back and put it here, if I am assuming lambda, if I say it is a constant over the disk this is nothing, but C P will become lambda C T and I have the expression here.

So, I will write C P which is equal to C Q which is equal to lambda C T and lambda I am writing all this things. So, I will have lambda i C T climb velocity suppose if you do not want to do all those things, no I do not want to substitute I will keep climb velocity because I can substitute for lambda this entire expression.

I may say lambda i may be varying, but rest of them is all constant for the vehicle. So, I can directly substitute here in which case I can put lambda i D C T separately, but lambda C T that is lambda C T this is a constant then I will have mu C H that is anyway coefficient. So, C T and C T will go of plus you will have mu C H then this will be that is again mu D over W and you will have a C T.

So, I will write that expression mu plus mu D over W C T minus you still have that mu C H term. So, minus mu C H so, you find minus mu C T and plus mu C H will cancel out

and leaving behind just a 1 2 3 and then where is that? this is another term that is plus sigma C d naught over 8 1 plus 3 mu square. So, I have 4 terms 1 2 3 and 4.

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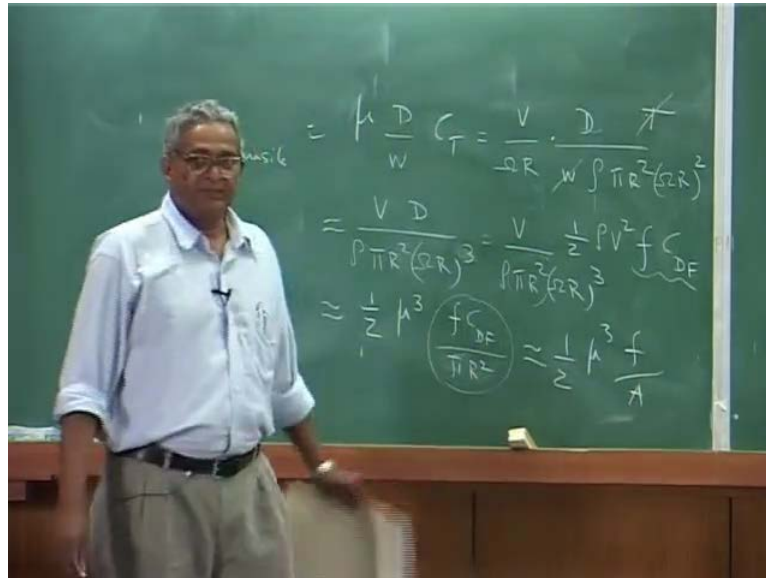


So, I call my power, now, I write everything in power expression, power is C Pi induced power due to induced then C Pc power for climb and then plus, this term I write fourth term which is C P this is due to the drag of the helicopter. So, I call it parasite track C P may be parasite plus C P profile.

You have power for 4 terms and for each 1 of them you have the expression if lambda is constant you know you can get a lambda i at high speed the approximation is C T by 2 mu for mu greater than 0.1 non uniform in flow is there.

So, put a some factor. So, you will get C T square over 2 mu that is the induced power climb power lambda C T and then this power you will now write it in a different form.

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See, how do you write the fuselage dragger? Fuselage drag you were writing C P parasite as μD over $W C T$ μ is you take it approximately V over ωR , drag force is drag force divided by W and thrust there approximately. So, you will have thrust is this is $C T$. So, T and W $\rho \pi R^2 \omega R$ whole square. This is approximately because thrust is W . So, you will have V drag $\rho \pi R^2 \omega R$ whole cube.

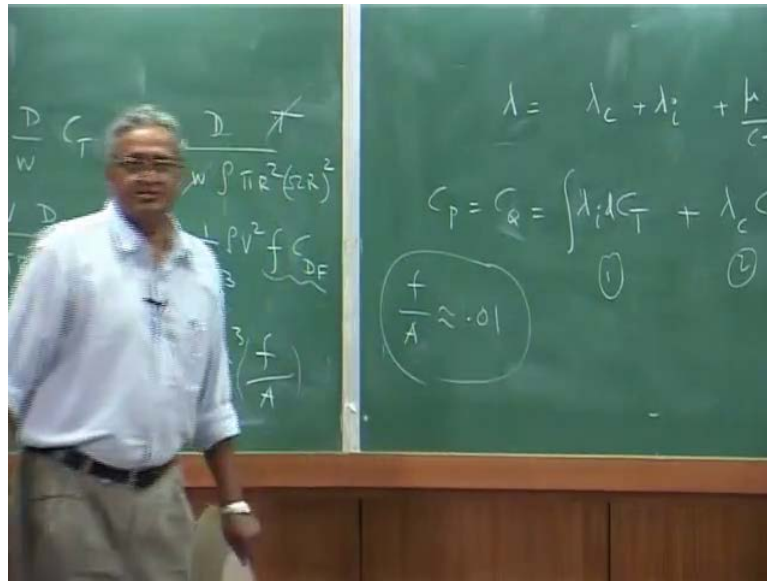
Drag force on fuselage you normally write it as ω sorry whole cube there whole cube drag force you write it as $\frac{1}{2} \rho V^2$ some frontal area please understand, I am using a word frontal area of somebody in to a $C D$ fuselage some drag coefficient.

Now, if you cancel out because V over ωR one of the ωR you will get it because you know that V over ωR this is V cube ωR cube that will be, I am writing it as approximately μ cube, but there is a factor half is there half μ cube $f C D$ over πR^2 square.

But how they usually represent this as, assume that $C D F$ is 1; find out that equivalent area frontal area, you follow what I am saying. So, normally internal they will put the fuselage, they will get the drag force then find the equivalent f for $C D$ equals 1.

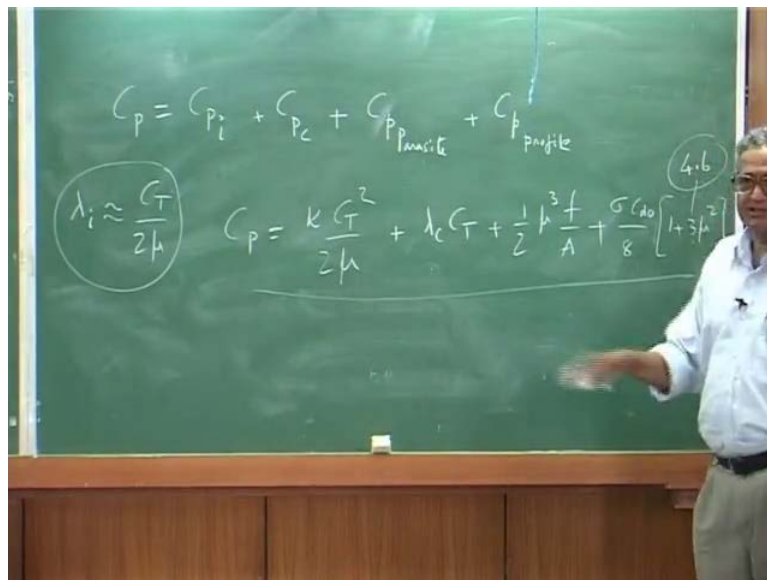
Since this is anonymous, this entire factor they take it as simply half, I think μ cube f over A that is all. A is the rotor area, usually this factor f over A is of the order 0.01, it may be slightly more 0.01213 something like that f over A .

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That means, this is the fuselage frontal area over the rotor disk area. Now, you have the expressions for all of them if you combine everything.

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So, you have C_p this term plus sigma, that is half mu cube f over A plus sigma C_{d0} over 8 $1 + 3\mu^2$ this is the power. But please understand there is another little bit more involved derivation.

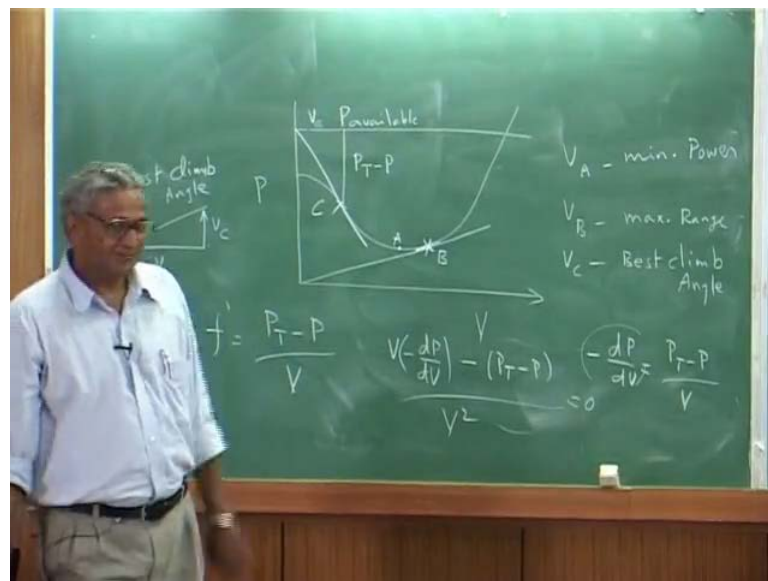
If you want to take radial flow and then reverse flow and because of the radial flow that is the skewed flow what happens to the drag it will take all those this factors 3 is

modified to 4.6 that is all, but I will not go into the details of that derivation. Again you make those assumptions because this is skewed flow it is like a flow over a swept wing and then try to get the equivalent value.

After that you do the same integration take an average and if you come to instead of 3 it becomes 4.6 that is again some approximation now you have your complete power. Now, you see that is why the fuselage drag is actually cubic to the forward speed.

So, as you increase the speed that power will tremendously go up. So, the power curve that day we drew even if it is not climbing. So, you will have your (C) . So, you can use this expression instead of getting C Q for power expression you can directly use this for your computation or anything because this is slightly better in the sense it gives a clean.

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So, it goes like this, this is power versus velocity. There is a minimum and we drew for range, maximizing the range this is the velocity, this is the minimum, this is the slope P over V where ever it meets, that velocity is for maximum range power available may be here. There is also another point which gives me an optimum climb angle what angle what velocity if you fly I will be climbing, but if you want to clear something the distance to at height. So, that is again the power available to P that is drawn like this, this point you will know. The slope this is P T minus P this is V climb.

So, slope of that is basically the tangent of that curve. See, if you draw that is that comes from this expression you take a function which is $P_T - P$ you want this to be what good glide on that because you want to have minimum power, but not 0 velocity.

You want to have a certain angle of climb, this is good such that this is V_c this is V . So, the power available is directly related to how much climb velocity you can have, but you may say that here I have lot of power, but your velocity here is large. So, the angle will be, that is why they call it as a nice word that is a glide angle or something like that glide angle not glide angle but best climb angle.

If you want to maximize that is what will you do $V_{d_u} - u_{d_v}$ when you do that you will have $V_{d_u} - P$ by d_v because power is a function of velocity power available is fixed, then $v_{d_u} - u_{d_v}$ that is into $P_T - P$ over this you said it 0.

So, you will get d_p by d_v minus sign is actually $P_T - P$ over V . So, the slope of this curve because of the minus sign automatically this is a negative slope. So, this is $P_T - P$ this is V . I will put it instead of V_c it is V . So, this is the best climb angle this is the best range, this is the minimum power velocity.

So, at this velocity if you fly you will cover more distance. If you go this way you have a longer flight time. So, you can call it may be C, I think A B. So, V_A min power long endurance this is V_B max range and V_C what is the best climb angle, just power versus the velocity curve that is all.

So, anything they define it like that best climb angle that is all. So, I say because if you look at it you may if you have, but please understand do not use that expression when μ is 0. For μ is 0 take the derivation what we have earlier used even though because we substituted λ I which is for μ greater than 0.1 you may take it even for 0.07 or something like that approximately.

So, you have to be careful when you are using this expression, that is why climbing when you say what that $\mu = 0$ is going to be have infinite power when you go to hover you take the hover part. Now, there is 1 term which is left which is the moment of the hub moments in the sense in the pitch moment roll moment, I think that I will do in the next class.