

## Introduction to Helicopter Aerodynamics and Dynamics.

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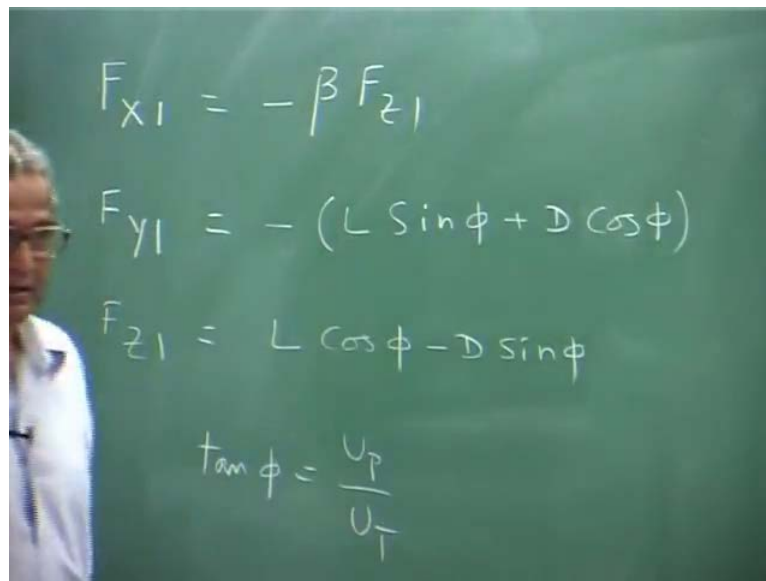
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Module No. # 01

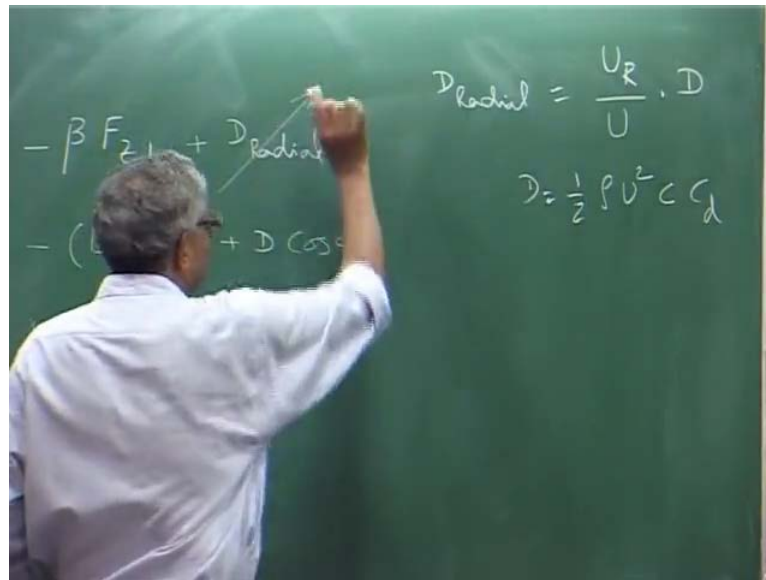
Lecture No. # 13

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$$F_{x1} = -\beta F_{z1}$$
$$F_{y1} = -(L \sin \phi + D \cos \phi)$$
$$F_{z1} = L \cos \phi - D \sin \phi$$
$$\tan \phi = \frac{U_P}{U_T}$$

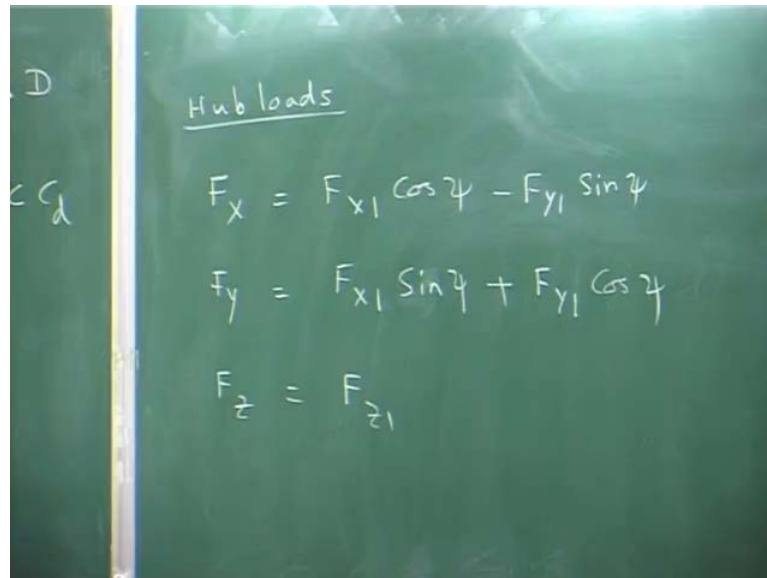
I thought just to give you a recap of what we did in the last class. I will just write them all the quantities. And then  $F_{z1}$  is  $L \dots$  These are our basic quantities and we know  $\tan \phi$  is  $U_P$  over  $U_T$ . And we had the expressions for lift, drag everything. Now, you can add radial drag to this term. That means, that is because of the flow along the span of the plate, but that is purely an approximation.

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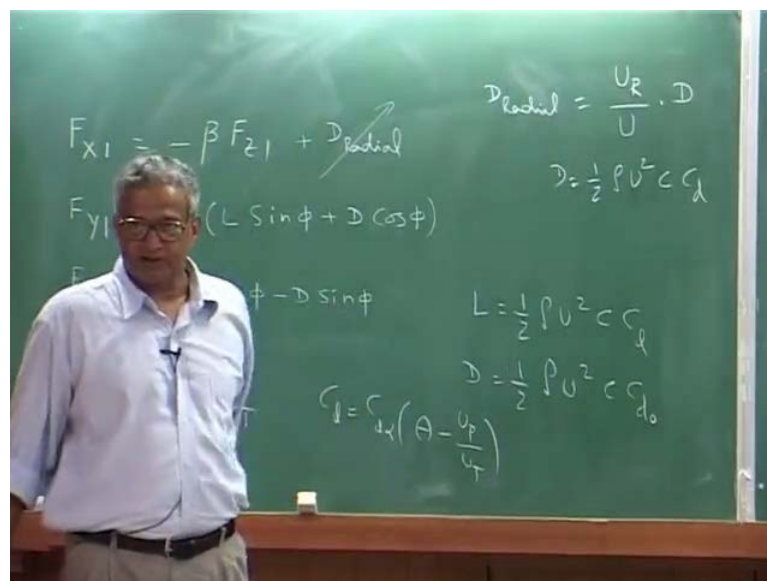
That approximation is given by... I will just give you, but we will neglect that term in this rest of the formulation. I will write it  $D_{radial}$ . This is drag due to like a skewed flow, flow past the aerofoil, actually along the blade. This is written just approximately, it is taken as  $U_{radial}$  over total resultant flow into drag on the blade. Drag on the blade, you know that  $D$  is given by half rho  $U^2 C_d$ . And this is just some fraction of the component, that is all. It is a very crude approximation which is done. Now, if you take that  $U$ , you can substitute. This is the distributed radial force, very crude approximation. But we will be neglecting it. I just want to say that how it is represented and then of course, we neglect it. Now, we throw this off. And we have to substitute for lift, drag expressions. And then we know the forces, transfer them to hub loads, which you already did last class.

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So, our hub loads are... But only you have to integrate. But before that I am just giving the  $F_x$  is  $F_{x1} \cos \psi - F_{y1} \sin \psi$ . And  $F_y$  is  $F_{x1} \sin \psi + F_{y1} \cos \psi$ . And your  $F_z$  will be  $F_{z1}$ . Now, we have written the expressions in different forms. The reason I am writing this again is just for clarity. Because we wrote  $L = \frac{1}{2} \rho U^2 C_l$ . And  $D = \frac{1}{2} \rho U^2 C_d$ .

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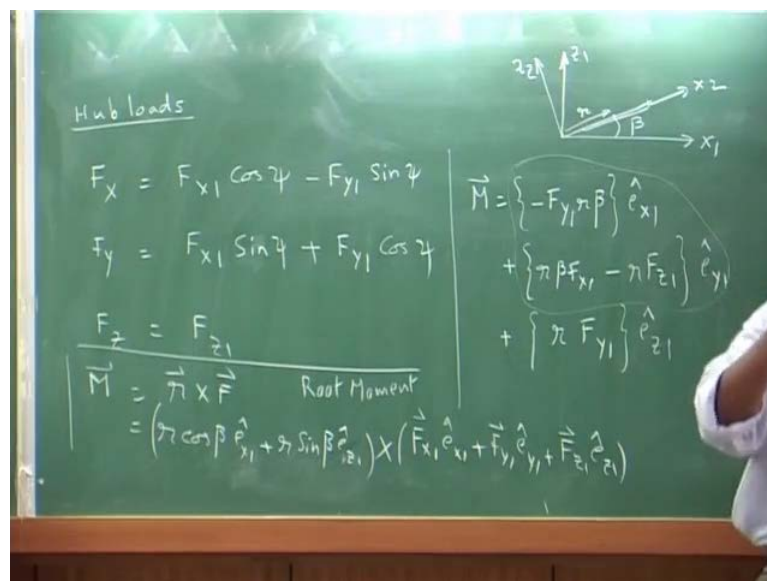


And  $C_l$  in one form, we wrote as  $C_l$  is  $C_{l0} \left( \theta - \frac{U_p}{U_T} \right)$ , which is  $U_p$  over  $U_T$ . And then,  $U_p$  we approximated  $U_T$ . Because why I am writing everything is, when I

display the expressions, in some of them we make the approximation. Only in one, that is a little interesting derivation. That is particularly in getting the torque.

Now, these are distributed forces. You have to integrate over the blade and then sum it up over all the blades at a particular time. Then you are doing the mean value, but this is only forces. These forces are acting on the typical section which we have converted into hub loads. But when you transfer them to the hub, they will also give hub moment.

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So, you have to take the moment also into account. But when we do the moment expression, I will write moment in hub coordinate directly. When I write that means, we had our blade. This is our X 1, Z 1 and this the blade, this is X 2, this is Z 2 and this angle was beta. This force is acting at a location which is r. Now, when I want to take the moment, you can write this is moment is r cross F. Now, the r you can write r cosine beta e X 1 plus r sin beta e X e Z 1 cross F X 1 Y 1 Z 1. They are here X 1, Y 1, Z 1. You will have F X 1 e X 1 plus F Y 1 e Y 1 plus F Z 1 e Z 1. This is my moment expression.

Now, you can make approximation that beta is small, so cos beta is one, sin beta is beta. Then make the cross product and I will write the expression for the moment here in vectorial form. And then later I will... This is e X 1. Because you just have to take the cross product, because y is zero. So, Y Z is zero and this is r beta F Y 1 plus r beta F X 1 minus r F Z 1. This is along e Y 1 direction. Please understand. These are all in X 1 Y 1

direction, still they are rotating, it is not actually the hub loads. It is still at the root of the blade. Then I have to convert. And then plus you will have  $r F_Y 1 e Z 1$ . These are my root moments. May be I will put a line here, this is root moment.

Now, for simplicity because later I will bring out. Instead of writing in these two quantities, I will replace them by another term. That we will come to that later. Because when we model the blade, how we model the blade? That will come. That is only a moment along X Y directions.

Let us first take only  $e Z 1$ .  $e Z 1$  because this is the drag force and this is my position. So, that is going to give a torque. And you know that  $e Z 1$  is same as  $e Z$ . So, my torque acting on the shaft is basically  $r F_Y 1$ . And you can take this quantity, calculate torque at the hub due to all the blades. Because all the blades will sum it up. There is no cosines and sins because all of them are in the Z direction.

Now, using this I will represent all the quantities because there is only algebra now. I do not want to again spend time on algebra. So, if you have any questions you can ask me because if this is clear, we got the sectional load and transfer to the hub and then we also got the hub moment. Now, we are going to get all the hub loads, but non-dimensionalise  $\rho \pi R^2 \omega R$  for forces, moment is  $\rho \pi R^2 \omega R$  into R. This is the moment, non-dimensional.

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NON DIMENSIONAL QUANTITIES

$$\text{Thrust: } C_T = \frac{T}{\rho \pi R^2 (\Omega R)^2}$$

$$\text{Longitudinal in-plane force: } C_H = \frac{H}{\rho \pi R^2 (\Omega R)^2}$$

$$\text{Lateral in-plane force: } C_V = \frac{Y}{\rho \pi R^2 (\Omega R)^2}$$

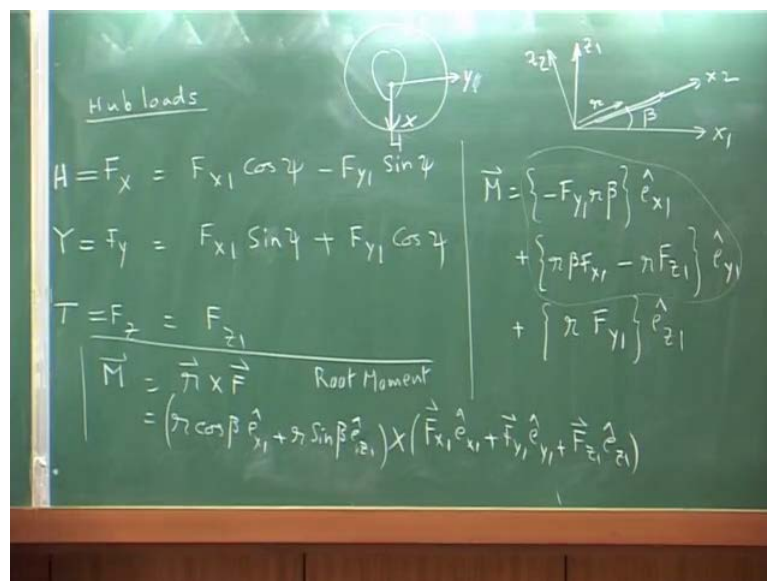
$$\text{Roll moment: } C_{M_x} = \frac{M_x}{\rho \pi R^2 (\Omega R)^2 R}$$

$$\text{Pitch moment: } C_{M_y} = \frac{M_y}{\rho \pi R^2 (\Omega R)^2 R}$$

$$\text{Yaw moment (Torque): } C_Q = \frac{Q}{\rho \pi R^2 (\Omega R)^2 R}$$

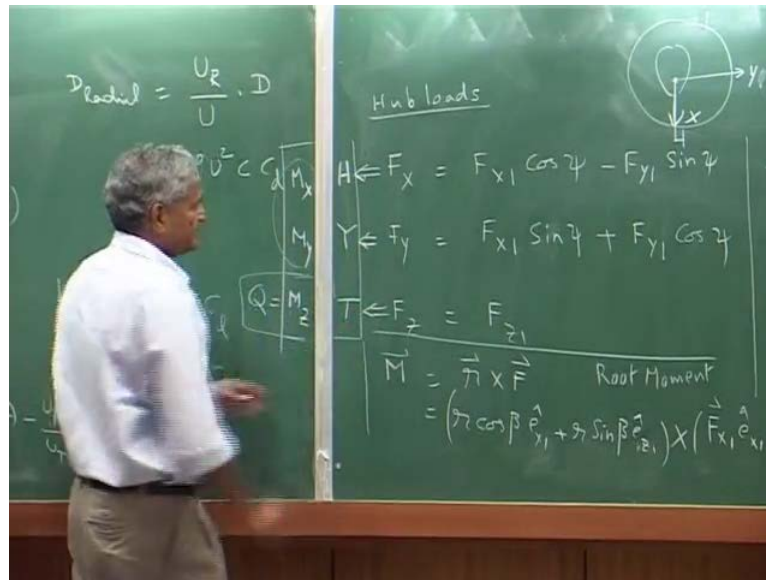
Now, I have to write thrust is  $F_z$ . And when I am using the hub, I always call this load as thrust. And  $F_y$ , this is my side force. And this I call it  $H$ , which is the longitudinal force. This is the symbol convention we use  $H, Y, T$ ; that means,  $H$  is  $F_x$  which is this. But please remember this  $H$  is towards the tail acting on the hub. Because you remember the coordinate system,  $X_1$ .  $X$  is you know that this is a rotor disk, so your  $X$  is this and this is  $Y$  and  $Z$  is up and here is your tail.  $H$  is the longitudinal force, this is lateral force, this is the thrust.

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But please remember all of them are acting at the hub in the hub coordinate system, hub fixed coordinate system.  $T$  is perpendicular to the hub,  $H$  and  $Y$  are in the plane of the hub. If you tilt the hub, it is tilt  $T$  is in this direction. That is why you have to follow very systematically the coordinate system and finally, you define these are my forces. Moment when you go I will write only the pitching and rolling moment, which are actually  $M_x$  will be the roll moment,  $M_y$  will be the pitch moment and  $M_z$  is basically yaw moment or you call it the torque, due to the only the main rotor system. That is all.

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So, you have correspondingly  $M_x$ . Maybe I will write here,  $M_x$ ,  $M_y$  and  $M_z$ . Sometimes this is written as  $Q$  capital  $Q$  also, this is also used. So, this is the roll moment, pitch moment, torque acting at the hub center. But please remember this is one section. When I want to get these loads, you have to integrate along the blade, you have to add all the blades and then take the mean value. So, the entire thing is done before you write these six expressions. Of the six, right now I will show only four. These two I will keep it pending. Because one way is you can do this way, another way is there is some easier way. These all elements. When you get this, you have to integrate everything, integrate along the blade, add all the blades and finally, write that as the expression. That is why I used the word correspondingly means this side I should not put an equality. This is basically, you can take it like this. They lead to these quantities.

Now, imagine I have to first get these expressions, which I get from here. And then I will substitute for  $L$  and  $D$ , then I put them here, then I will do an integral over the span of one blade, then put summation. Is it right? So, all these steps I just mentioned in words because this you have to do by algebra. This is where the time, number one. Number two, people may make mistakes. Usually because this is not very long, but still it is reasonably long to make mistakes. So, what I will show you is, I do not want to go through these derivations, I am only giving words. These days you can do in computation, no problem. Just take  $L$  and  $D$ , substitute that every section, then put it here, integrate, then use an integration scheme and then get it. But what is done is to get a closed form



expression. Please understand, closed form, that is only number. Here I will provide a closed form expression and that expression you are actually doing simplifications.

The simplification is I neglect in writing F Z, I throw away the drag term. And I will take only 1. cosine phi phi is small, I will say F Z is 1. You follow? These are the assumptions I make in trying to get a closed form expression. Now, I will show those closed form expressions.

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The image shows handwritten mathematical derivations on a piece of paper. At the top, the title "THRUST COEFFICIENT" is underlined in red. Below it, the thrust coefficient is defined as  $C_T = \frac{T}{\rho \pi R^2 (\Omega R)^2} = N \int_0^R F_{Z_1} d\bar{r} \frac{1}{\rho \pi R^2 (\Omega R)^2}$ . This is followed by a more detailed expression:  $C_T = \frac{\sigma a}{2} \int_0^1 \left[ (\bar{r}^2 + 2\mu\bar{r}\sin\psi + \mu^2\sin^2\psi)\theta - \{ \bar{r}\lambda + \bar{r}^2\dot{\beta} + \bar{r}\beta\mu\cos\psi + \lambda\mu\sin\psi + \bar{r}\dot{\beta}\mu\sin\psi + \beta\mu^2\sin\psi\cos\psi \} \right] d\bar{r}$ . The final result is  $C_T = \frac{\sigma a}{2} [\text{Function of azimuth } \psi]$ . Below this, the title "LATERAL IN-PLANE FORCE" is underlined in red. The lateral force coefficient is given as  $C_Y = \frac{Y}{\rho \pi R^2 (\Omega R)^2} = N \int_0^R \frac{(F_{Y_1}\cos\psi + F_{X_1}\sin\psi)}{\rho \pi R^2 (\Omega R)^2} d\bar{r}$ .

So, I have non-dimensionalized the quantities. C T is thrust divided by rho pi R square omega R square. You can see here, N is the number of blades. I am assuming that all the blades are doing the same motion etcetera. So, thrust is a function of azimuth, that is what I told you earlier. Similarly, when you go to lateral, that is the in-plane force, F Y 1 cosine psi F X 1 sin psi. This is basically this quantity, F Y side force, lateral force. And again you will have all the blades.



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LATERAL IN-PLANE FORCE

$$C_Y = \frac{Y}{\rho \pi R^2 (\Omega R)^2} = N \int_0^R \frac{(F_{Y1} \cos \psi + F_{X1} \sin \psi)}{\rho \pi R^2 (\Omega R)^2} d\bar{r}$$

$$C_Y = \frac{\sigma a}{2} \int_0^1 \left[ -(\bar{r} + \mu \sin \psi)^2 \frac{C_d}{a} \cos \psi \right. \\ \left. - \left\{ (\bar{r} + \mu \sin \psi) \theta - (\lambda + \bar{r} \beta + \beta \mu \cos \psi) \right\} \right. \\ \left. \left\{ (\lambda + \bar{r} \beta + \beta \mu \cos \psi) \cos \psi + (\bar{r} + \mu \sin \psi) \beta \sin \psi \right\} \right] d\bar{r}$$

$$C_Y = C_{Y0} + C_{Y1}$$

And you are substituting, I have actually gotten this expression after substituting for  $F_{Y1}$  and  $F_{X1}$ , collect the terms. And I actually split the terms into two parts, one due to  $C_d$ ,  $C_d$  naught actually, here I used the symbol  $C_d$ . One due to only profile drag. And other terms we call them as induced drag term, which is independent of  $C_d$ .

So, you collect  $C_d$  term and the remaining term basically comes from the lift, that is the induced drag term. Same way I am representing my longitudinal force here again  $F_{X1} \cos \psi$  minus  $F_{Y1} \sin \psi$ . I again substitute all the expressions, get it, derive. This also I split into  $C_d$  naught and then remaining terms.

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LONGITUDINAL IN-PLANE FORCE

$$C_H = \frac{H}{\rho \pi R^2 (\Omega R)^2} = N \int_0^R \frac{(-F_{Y1} \sin \psi + F_{X1} \cos \psi)}{\rho \pi R^2 (\Omega R)^2} dr$$

$$C_H = \frac{\sigma a}{2} \int_0^1 \left[ (\bar{r} + \mu \sin \psi)^2 \frac{C_d}{a} \sin \psi \right. \\ \left. + \{ (\bar{r} + \mu \sin \psi) \theta - (\lambda + \bar{r} \beta + \beta \mu \cos \psi) \} \right. \\ \left. \{ (\lambda + \bar{r} \beta + \beta \mu \cos \psi) \sin \psi - (\bar{r} + \mu \sin \psi) \beta \cos \psi \} \right] d\bar{r}$$

$$C_H = \frac{\sigma a}{2} [\text{profile drag term} + \text{induced drag term}]$$

$$C_H = C_{H_0} + C_{H_i}$$

TORQUE (OR POWER) COEFFICIENT

Now, you see here there is a theta, beta, all the beta dot. And you have to substitute for... I told you earlier assume that theta is theta naught 1 c cosine psi plus 1 sin psi. And beta is beta naught plus 1 c cosine psi plus beta 1 s sine psi. So, these are... I am not substituted yet. I will be doing later.

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Just to give you a glimpse of how they look. And here the torque. I put a minus sign here because this is basically just to make it plus, that is nothing else. Because if the rotor blade is rotating in the counterclockwise direction, the drag force is actually you know

that the drag force is minus Y 1 direction. So, minus Y 1 direction in this. This is the blade, so this is my Y 1. So, minus Y 1 is this; that means, my drag force is like this. If I take a cross product, my torque is clockwise.

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TORQUE (OR POWER) COEFFICIENT

$$C_Q = \frac{Q}{\rho \pi R^2 (\Omega R)^2} = N \int_0^R r \{-F_{y_1}\} dr \frac{1}{\rho \pi R^2 (\Omega R)^2 R}$$

$$C_Q = \frac{\sigma a}{2} \int_0^1 \bar{r} \left[ (\lambda + \bar{r}\beta + \beta\mu\cos\psi)(\bar{r} + \mu\sin\psi)\theta - (\lambda + \bar{r}\beta + \beta\mu\cos\psi)^2 + (\bar{r} + \mu\sin\psi)^2 \frac{C_d}{a} \right] d\bar{r}$$

$$Q = \frac{\sigma a}{2} [\text{Induced drag term} + \text{profile drag term}]$$

$$C_Q = C_{Q_i} + C_{Q_0}$$

$$C_P = C_Q$$

But to get a positive, just to get a positive number, that is why I will put a minus sign. So, that clockwise moment is negative. But put one more minus sign, I will get everything positive. That is how I get the torque. Now, you see these are my expressions. But once I get torque coefficient, I know torque coefficient is power coefficient. So, C P equals C Q. So, directly for... But I have not taken the tail rotor. Please understand. This is a main rotor, power main rotor torque.

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AVERAGING OVER AZIMUTH  $\Rightarrow \frac{1}{2\pi} \int_0^{2\pi} ( ) d\psi$

Mean Thrust Coefficient  $C_T$

$$C_T = \frac{\sigma a}{2} \left[ \frac{\theta_0}{3} \left( 1 + \frac{3}{2} \mu^2 \right) + \frac{\theta_{tw}}{4} (1 + \mu^2) + \frac{\mu}{2} \theta_{1s} - \frac{\lambda}{2} \right]$$

In-plane Force Coefficient  $C_H$

$$C_H = \frac{\sigma a}{2} \left[ \theta_0 \left\{ \frac{\lambda \mu}{2} - \frac{\beta_{1c}}{3} \right\} + \theta_{tw} \left\{ \frac{\lambda \mu}{4} - \frac{\beta_{1c}}{4} \right\} \right. \\ \left. + \theta_{1s} \left\{ \frac{\lambda}{4} - \frac{\mu \beta_{1c}}{4} \right\} - \theta_{1c} \frac{\beta_0}{6} \right. \\ \left. + \frac{3}{4} \lambda \beta_{1c} + \beta_{1s} \frac{\beta_0}{6} \right. \\ \left. + \frac{\mu}{4} \left( \beta_0^2 + \beta_{1c}^2 \right) \right] + \frac{\sigma a}{2} \left\{ \frac{\mu}{2} \frac{C_{Dn}}{a} \right\}$$

In-plane Force Coefficient  $C_Y$

Now, since these are all functions of time, I have integrated, substituted that theta naught 1 c 1 s. Because please remember all these loads. I call it loads, they vary every instant. Is not that they are fixed. Your thrust force is varying, side force is varying, longitudinal torque moment. All these quantities every instant they are changing. That means, what is my thrust value? You have to take only mean value. What is my longitudinal force? Mean value of the longitudinal force. So, that is why when we get these loads, we take average values.

So, the average, how it is done is, you have summed up over all the blades, but then you integrate. In one revolution, how the loads change? Take the mean value and the mean value is same for all the blades. So, just multiply by number of blades. So, that is what is done in getting the loads.

So, please understand there are several steps before you arrive at these expressions. So, it is not very simplistic like what we have done in hover, it is much more involved. But you made assumptions that you know these quantities. Even though you do not know the number, you say I know that, I assume that. Similarly, beta naught 1 c 1 s, you have assumed, well this is how my flap is going to be. But please note I am neglected all the higher harmonics. If you include, that they will also come and sit here, but then these expressions will be very long. Numerically you can do that. That is why this becomes algebraically cumbersome, really cumbersome. And just to give an idea of what is the

minimum one can get an equation which you can have a look at it, that is this. With all these assumptions this is made. And you make small angle assumption, phi also you make small and you get C T and twist is added. You can add a twist also, plus theta twist R over R. So, this is my thrust, mean value of the thrust. Please understand. And then this is my in-plane force coefficient in the longitudinal direction C H. If you look at here, sigma a is lift for slope. Sigma contains number of the blades, that is lag number. You will see lambda, mu, theta naught, beta 1 c and then you will have beta 1 s and 1 c square, beta naught square so many other terms. And then there is one term with the C d naught, which is the profile drag term.

So, one term with profile drag, rest of the other due to induced drag. That is all. So, you will have all these terms. And there all small values, but finally, you have to use them. And now similarly I have written the in-plane this long and you can write the torque.

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$$+ \frac{\mu}{4} (\beta_0^2 + \beta_{1c}^2) + \frac{\sigma a}{2} \left\{ \frac{\mu C_{D0}}{2} \right\}$$

In-plane Force Coefficient  $C_Y$

$$\begin{aligned}
 C_Y = & -\frac{\sigma a}{2} \left[ \theta_0 \left\{ \frac{3}{4} \mu \beta_0 + \frac{\beta_{1s}}{3} \left( 1 + \frac{3}{2} \mu^2 \right) \right\} \right. \\
 & + \theta_{tw} \left\{ \frac{\mu \beta_0}{2} + \frac{\beta_{1s}}{4} (1 + \mu^2) \right\} \\
 & + \theta_{1c} \left\{ \frac{\lambda}{4} + \frac{1}{4} \mu \beta_{1c} \right\} \\
 & + \theta_{1s} \left\{ \frac{\beta_0}{6} (1 + 3\mu^2) + \frac{1}{2} \mu \beta_{1s} \right\} \\
 & - \frac{3}{2} \lambda \mu \beta_0 - \beta_0 \beta_{1c} \left( \frac{1}{6} - \mu^2 \right) \\
 & \left. - \frac{3}{4} \lambda \beta_{1s} - \frac{\mu}{4} \beta_{1c} \beta_{1s} \right]
 \end{aligned}$$

Now, this is just for, you cannot make out anything from here I am telling. You here only I am telling you what was done. If you look at any book, if they have written all these things, that means where did they come from, that is all. Because from here you just make out anything. Here it is all some mu beta naught, simply whether plus sign is there, minus sign is there. If there is an error, sorry you cannot make out. This is just for you to tell the procedure and this is how they look like, just to give you a glimpse of how these loads will look like, expressions for the loads, closed form. And here you have integrated

full, zero to tip. All those assumptions you have made. Now, the question is how does the pilot balance? Because if you want to hover or forward fly, all the forces must be balanced and the moment about the center of mass is zero. Force also zero, moment is zero.

The tail rotor, not only it gives a torque, it also gives a side force. So, the side force has to be balanced. Now, what is the attitude of the helicopter? Please understand. Because these are all hub loads. You have to transfer the hub load to the C g. And C g you have C g coordinate system. And transfer the tail load to the C g. Since you have asked, that is what next in the another two, three lectures I will bring in those loads. Then I will say this is how your full expression will be. And that is the assignment which I will give, you people have to write a code, get it and I know you may make. It is an iterative procedure, it is not just a straight forward, that is why unless you solve the problem yourself you will not be able to appreciate the complexity involved in the helicopter dynamics, aerodynamics or whatever it may be.

I am saying this is the simplest problem, this is not the complicated problem because there are more complicated problems. But simplest in terms of, we have reduced everything, made lot of assumptions and then finally, say take this and then try to do some simple trim calculation.

So, this is only the load due to main rotor. That is all. Now, if you want tail rotor, you can use the same expression. Please understand. Except tail rotor does not have  $1 c$  and  $1 s$ , only collective. A tail rotor can have twist also. That is all. But only thing is that tail rotor will be vertical and it is flying like this. Main rotor is horizontal, it is flying like this. That is all. So, you can use the same expressions for tail rotor also. But if you want, usually what we do is you neglect all the other loads, tail rotor thrust only you take. But please understand, tail rotor will rotate at a different rpm, radius is different, everything is different. So, you have to use those quantities in getting the tail rotor force, that is they are non-dimensionalized with respect to tail rotor quantities. If you want to take all the forces, no problem take everything, then it becomes a little bit more complex.

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POWER COEFFICIENT

$$C_P = \int \lambda_{id} C_T + \lambda_c C_T + \mu C_H + \mu \frac{D}{W} C_T - \mu C_H + (C_{Q_0} + \mu C_{H_0})$$
$$C_P = C_{P_i} + C_{P_c} + C_{P_p} + C_{P_0}$$

$C_{P_i}$  - INDUCED POWER

$C_{P_c}$  - CLIMB POWER

$C_{P_p}$  - PARASITE DRAG POWER

$C_{P_0}$  - BLADE PROFILE DRAG POWER

That is why you say let me throw away rest of the terms, only take tail rotor thrust because that is good enough, which is reasonably a good approximation. You know that I am just using, you know torque coefficient C Q. C Q is C P power coefficient, both are same. One is you directly use the earlier expression. Because this is the torque, you can substitute F Y, I have substituted here and then used. One way is you can calculate everything by this approach, which is fine. And that expression I will give you. Another way is you have torque, main rotor torque I say convert to power, because main rotor power. Now, I split that term. Basically what I will be doing is, this F Y 1 because this is the torque r F Y 1. And this F Y 1, I have here. What I will do is this is L sin phi D cos phi. And I will replace this L in terms of F Z 1. Because you know L from this expression. I will put F Z here. I will write everything in terms of F Z. And the drag. That is the lift and the drag term. So, this jugglery is done. But when I do that, I do not make any assumptions, I will just take the entire expression as it is. It is not that I throw away the drag term and then I will take F Z is only lift, then I cannot get this type of form. And then I will add sum because the forward flight you have the longitudinal force. So, the longitudinal force, I will take and force into velocity. I will take velocity is forward flight, that is again another term. So, that is what I will do. mu into C H is again a power term. And I will subtract the same mu into C H. This I will do because if time permits I can start today, but it will not be done within one short lecture. It takes time because lot of back and forth substitution. And then you will find a very beautiful



expression. Some other terms will drop out, some will combine to give you essentially because then there is you have to take the flight condition. You make lot of approximation there also. That is why I said to come to this form is a little tricky, really tricky, you cannot. If I give you derivative, you will not be able to derive it, I am telling you, unless you understand all this.

The power is split into four sub components of the main rotor only. One of them call it  $C_{Pi}$  which is induced power. Induced power is basically you want to lift the helicopter. It supports the weight of the helicopter. Then you have climb, that is the climb power. Then you call it parasite drag because you are dragging the helicopter. So, the helicopter will have its own drag force. And that drag force, a power is required. And that is what is called the parasite power. And the last expression is profile. This is purely  $C_d$  naught. Please understand. So,  $C_d$  naught is profile drag of the blade, sectional drag, but this is drag of the fuselage which you are taking.

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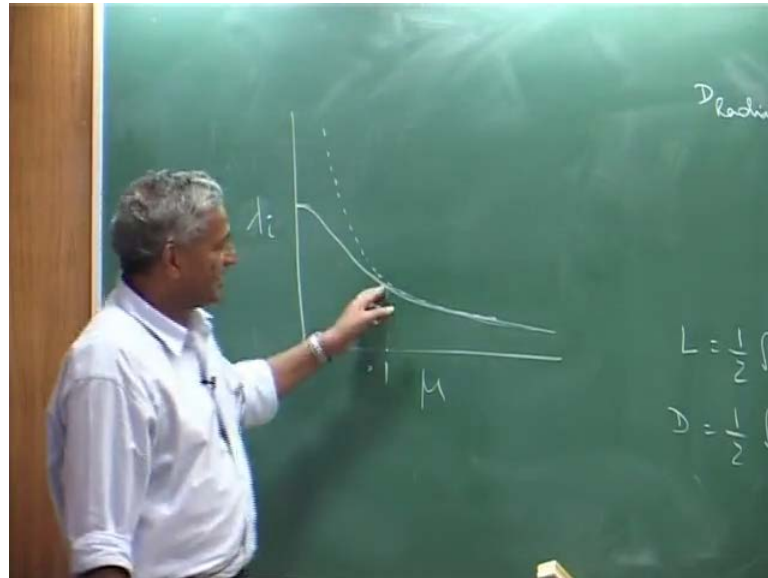
$C_{Pi}$  - INDUCED POWER  
 $C_{Pc}$  - CLIMB POWER  
 $C_{Pp}$  - PARASITE DRAG POWER  
 $C_{Po}$  - BLADE PROFILE DRAG POWER

$$C_P \cong \frac{\kappa - 2}{2\mu} + \frac{\sigma C_d}{8} [1 + 4.6\mu^2] + \frac{1}{2}\mu^3 \frac{f}{A} + \lambda_e C_T$$

INDUCED                  PROFILE DRAG                  PARASITE DRAG                  CLIMB

So, you have split the power into four sub components. And each sub component, you have a very nice expression. That expression is what is very interesting. That is given here.  $C_P$  of course, I put an approximation symbol. Because if you remember, we had seen the inflow curve in forward flight.

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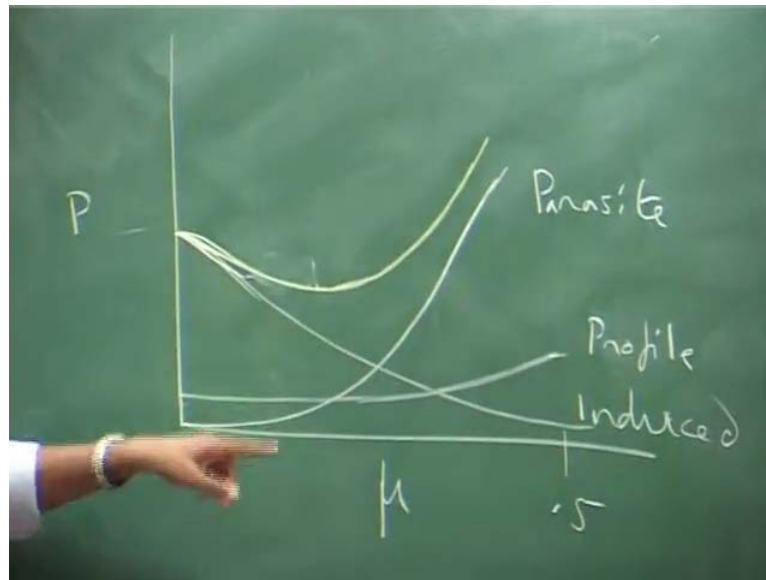


I will just briefly draw. I will put it  $\lambda_i$  and  $\mu$ . It was coming down. It is around say somewhere around 0.1  $\mu$ , it starts deviating. Look back your... That is you can take high speed condition for  $\mu$  greater than 0.1. Assume the high speed condition and you can write your  $\lambda_i$  as  $C_T$  by  $2\mu$ .

If you go back your notes, you will find that  $\lambda_i$  at high speed. You can write it as  $C_T$  by  $2\mu$  and there is factor you add for non-uniformity. Then your induced power is some non-dimensional correction factor,  $C_T$  square over  $2\mu$ . But you cannot use this when  $\mu$  is zero. Please understand. This is for  $\mu$  at least more than 0.1. Then you will have profile drag,  $\sigma C_d$  over  $81$  plus, I have put  $4.6\mu^2$ , that is again some reverse flow etcetera, if you include  $4.6$ . Otherwise it will become  $3$ , when you derive it will become  $3 + 3\mu^2$ , that is  $\sigma C_d$ .  $C_d$  is the drag or  $C_{d0}$  you can take it. And then there is another term  $1$  over  $2\mu q f$  over  $A$ .  $A$  is the area of the rotor disk,  $f$  is fuselage frontal area. So, that is normally obtained from wind tunnel, frontal area given in terms of drag. And this is  $\mu^3$ . And then of course, you have a climb,  $\lambda C C_T$ , climb.

So, you see this forward flight power required... Tail rotor power is separate which you have to add, that is all. So, this is a very neat expression for power. And this curve if you plot, because you know that this is a thrust square, anyway  $C_T$  is fixed. So, you see as you increase your forward speed, this term will actually come down.

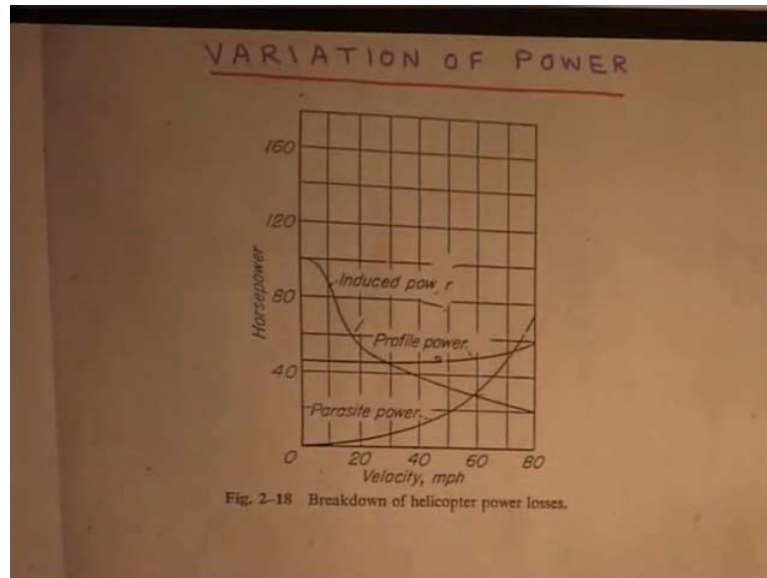
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Now, you see if you look at the power graph. This is the rotor power, this is forward speed. Induced power will go like this. This is induced because induced will keep on decreasing with forward speed, because with  $\mu$ . I am taking starting from hover even though it is not valid. That expression is not valid at hover. I am taking induced power decreases. But if you look at the profile power, this is  $\mu$  square. But please understand  $\mu$  all of them are in the range of maximum 0.5, do not take it 1, 2, 3, 4, 5. Please understand. Here all  $\mu$  is in a very narrow range, 0.5 is very large, actually 0.35, 0.4, that is it. That is why do not try to put 1, 2, 3, 4, 5 like that, you will take it 0.1, 0.2, 0.3, 0.4, that is it.

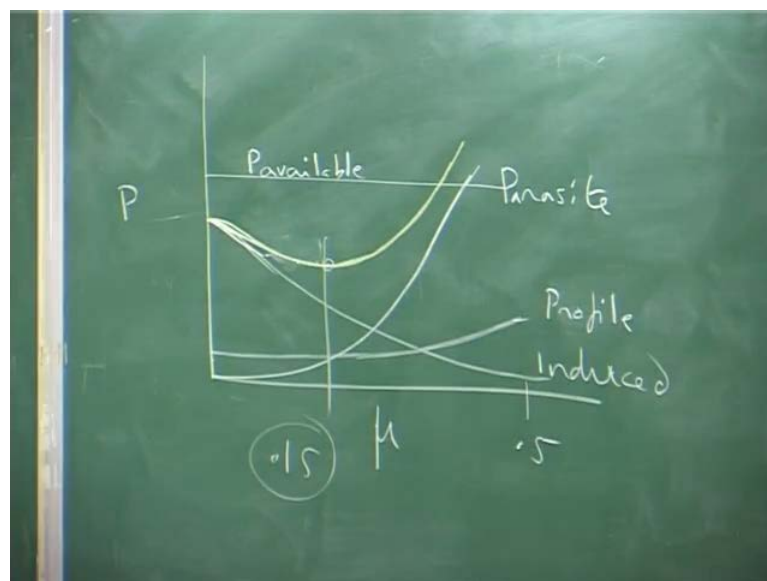
Then your  $C_d$  curve may go like this, may be profile drag may go, because it is increasing with  $\mu$  slightly, but the square. But if you look at this term, parasite drag, that is  $\mu$  cube. This will start like a cubic. So, this is induced. May be I will write it here separately. This is induced, this is profile, this is parasite. Climb, you do not bother about that. Now, if you add all of them. I think I should take it like this. If you add all of them, the curve will look like this. It will go something like this. Maybe I should put that dip a little down. Because you have to add everything. This is the rotor power in forward flight.

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So, this is the parasite profile induced. Now, you add all of them, they will go like this. So, this is my power curve. Let us take, we will just describe a few of the power curve. That means, you see you fly at low power somewhere here. Power required to fly the helicopter is minimum at some forward speed, but hover it requires more power. And as you go to forward speed, the power actually decreases that is because this power decreases, induced power decreases drastically. But the increase of profile power is not too much. This will start picking up only after some speed, the parasite power.

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So, you have a minimum power at some particular fly. Usually it is in the range of, this may be somewhere around 0.15 something like that, around 0.15, 0.1 to 0.2. It will be around that. Sometimes 0.11, 0.12, it is in that zone, minimum power forward flight. And then for climb. Please understand. What is the power available on the helicopter? That you may draw some line. This is P available at any time assuming it is sea level. But as you go high altitude, power available will come down. Power required because of induced flow is more, power required will go up. Here you will have this much excess power for climb, if you go here it is very less. But if you go to high speed, the power required climb is less.

Now, you see when you want to do. Let us go back to the autorotation. You know in autorotation, you are descending. And basically autorotation, the power required by the rotor is supplied by the air in descending. That means the descend kinetic energy is converted into rotor rotation kinetic energy. If the required power for the helicopter is less, then your descent velocity also will be less because you do not have to descend much faster.

So, if you auto rotate. You do not auto rotate in hover. You can do, but the descend velocity will be much larger. Whereas, if you auto rotate at the minimum power, the descend velocity will also will be there. Of course, he is moving forward. Please understand. He will come forward, but he will auto rotate, but his descend velocity will be less. But when he comes near the ground, usually what he will do is he will increase the collective so that he goes up. It is like a breaking. He will try to go up and then he will come. So, autorotation descend will be minimum when the rotor power is required is minimum.

Rotor power required is minimum, somewhere around 0.1. It is around 0.15 to somewhere in that zone, mu advance ratio. But you see this power curve, this raises very sharply. You just cannot fly beyond that speed because the power required will be tremendous. You will not be able to do it.

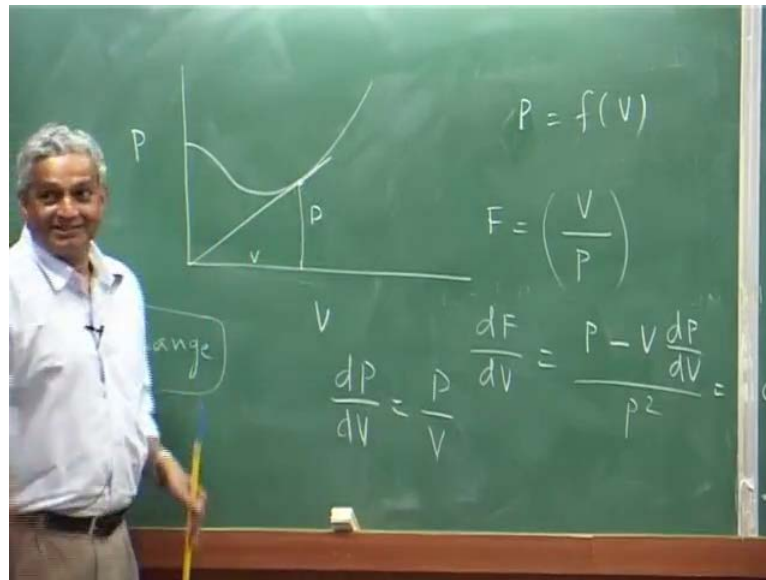
So, you will find this curve will shoot up like this. The power required will be much more, that is number one. And your vibratory loads also will increase very high. Sometimes, usually you want the helicopter to fly without any problem. You have designed for some forward speed. You have the power, you put everything. But when

you are trying to fly forward speed, your vibration starts increasing. Even though I may have enough power in my engine to take you to a higher forward speed, but my vibration is so high I cannot even go to that speed. So, the restriction comes from not because of lack of power in the engine, the restrictions comes from vibration in the helicopter.

So, these are real design challenges. The best design is one where your flight envelope or the max speed is restricted because of my power is not that, so I am not able to fly. You follow? On the other hand, it is not like that, you may have power, engine can give. But you say I cannot go my forward speed is restricted because I have lot of vibrations. You follow what I am talking? And that is happens in almost majority of the helicopters. Because you know that we are all talking about mean value, they have never mentioned anything about the vibratory loads, because we took the average. But the vibratory loads are the ones which contribute. So, helicopter essentially is a vibrating machine. And unless you go sit there and see for yourself what that vibration is, you know you will not believe it. It is phenomenal. You have to experience it, if you get a chance.

But not the normal flight. If you fly somewhere here, you will say beautiful nice flight you had. But you go here, then only you will start seeing the vibrations, not here. Of course, here you may say a little bit, but that is a different thing. But here it is most of the commercial flights, low power required you can fly nicely etcetera. But when you want to go here, that is why in the military helicopter they have to sometimes go on a high speed. That time the vibration will be phenomenal. So, you will be restrict. Now, with this curve I will give you one another interesting result which can be...

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So, we take this power versus curve. What is the speed for max range? If you want to maximize the range, what speed should I fly? That means because the fuel consumption is related to power. So, how do we decide? Because if you fly high speed you may say well I will go fast, but you may burn lot of fuel. So, you will not have left with fuel to go. So, what is the speed for max range? So, what you do is, simple, you take the function f. Because power is a function of what? Power is a function of this curve. What is flight speed for max range?

So, you take a function. Let us say that function F is V by P. I want to basically maxima minima. If I maximize this, that means velocity is maximum, power is. I am minimizing the power, I am increasing my velocity. So, what you do is, you take this d F by d velocity. This will be V d U minus U by can put it, but this is 0. That means d P by, numerator d P over d V is... d P by d V is slope of this curve. Slope of the curve is basically P over V. So, you draw a line, this point. If you fly at this speed. Because this is what? This is P over V, P over V slope of that. Is it clear?

Now, you do not fly at minimum speed, you actually fly at a higher speed. So, if you want endurance, that is long time you want to float around, then you use minimum power. But if you want to increase the range, you do this. Then there is one more problem in this which I will come to that later. Is it clear?



Because this much, this is for forward flight, power required range. And another one is a climb also, there is something related to climb. That I will ask you later. Now, there are few directions the course will proceed. Like what we derived in the hover; thrust, power. Here of course, it is not just thrust alone, we had thrust, side force, forward force, power further moments are there, which we will get them. But in between we have used  $\beta$  naught,  $\beta_1 c$ ,  $\beta_1 s$ . That means, the flap motion of the blade. How are you going to get it? Because then you need to know the blade dynamics. How my rotor blade is going to respond? Because that is very essential, which means you study only the dynamics of the blade in forward flight, you do not bother about anything else. How my blade will vibrate for a given pitch input? Given pitch input is pilot gives,  $\theta$  naught,  $\theta_1 c$   $\theta_1 s$ .

You assume that pilot is giving this much value and then you have to write the equation of motion for the blade. Now, for the simple case we will take only the flapping motion. Because we have you included only the flap dynamics. Because we did not include lead lag or torsion. So, how do you get the flap dynamics of the blade? So, you have to proceed in the blade dynamics separately. Please understand, this is the one direction. So, that is required for even getting the loads. Because if you want to get the load, because you I have put just  $\beta$  naught  $\beta_1 c$   $\beta_1 s$ . Who is going to give me those values? Similarly,  $\theta$  naught,  $\theta_1 c$ ,  $\theta_1 s$  and then  $\lambda$ . All these quantities are only symbols now. How do we get those values? This is one aspect. And when you restrict the blade motion, because we said that the blade is free to do flap, lead lag, torsion everything. It is a long beam it has vibration, that means you are analyzing only the blade dynamics.

So, for simplicity in at least getting the hub loads, we need to have a highly simplified model. Because I am not going to analyze the blade real complicated, because please understand complicated analysis is one problem. So, that will take you in a different direction completely. You will not be then bothered about any of these things. I am only analyzing the blade. But here I want loads, but at least give me a simple model. So, what we will do is, we will make a highly simplified blade model to describe certain quantities. Later we will take, but at the slight complications in the blade model itself. That will come later, because if you want to use this you need to have those things.