Instability and Transition of Fluid Flows

Prof Tapan K Sengupta
Department of Aerospace Engineering
Indian Institute of Technology Kanpur

Module No # 01

Lecture No # 09

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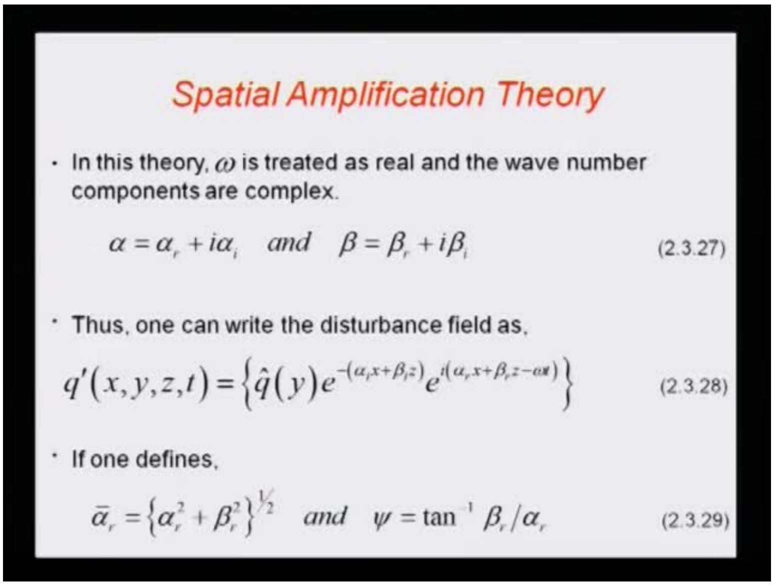

In the last class, actually we are discussing about the distinction between spatial and temporal amplification theory used in linearized stability analysis. So, we came up to the spatial theory where we said that, the wave number components are complex in the x and z direction that is, the stream wise in the cross flow direction, and as a consequence of that you notice that there is this part which is essentially an amplitude which is a function of y, x and z. This is the phase part and what we notice that because of this complex nature of wave number components, we can define in deduction along, which is the phase propagate and that is given by this alpha bar. Amplitude is given here and the direction is given by tan inverse of beta r by alpha r.

Then, what we did was basically defined the phase speed which would be then given by the circular frequency divided by that amplitude of resultant; a phase vector, the wave number vector. At the same time, having that imaginary part you can define a sort of a alpha i bar indicating the magnitude of total amplification or attenuation suffered in a particular direction that is given by psi bar which is nothing, but tan inverse of beta i by alpha i. So the compounded or the composite expression for the disturbance quantity as a function of space in time would be written like this.

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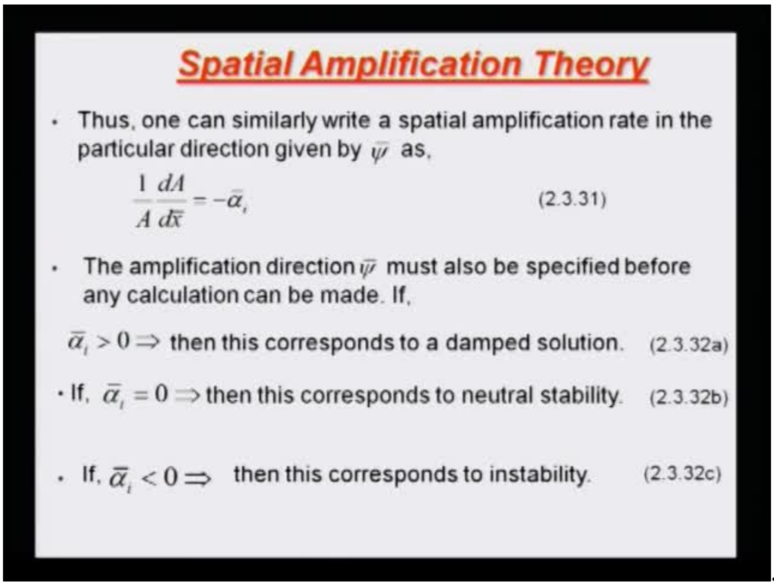

So, this is your amplitude a and this is a phase part, and then we did talk about that we have two directions one is x tilde that is along that resultant phase direction alpha r bar, and x bar along the amplitude of decay direction that is given by alpha i bar and that is what you see in that equation 30 there. And as a consequence you could find that quantity; the net amplification in that direction psi bar is once again given by this relative rate at which the amplitude changes, and what we note that if alpha i is positive we have damped solution.

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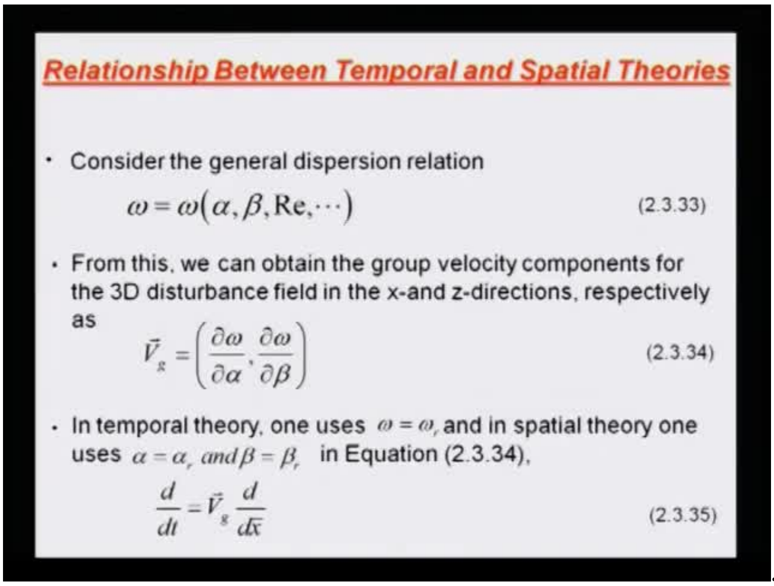

If alpha i is 0 then we have the neutral solution, and if alpha i is negative then we have case of spatial amplification. Now, what happens is, basically what we need to do is figure out how these two theories relate to each other. That comes about by considering the general dispersion relation which tells the circular frequency is a function of the wave numbers apart from any other parameters that you may have, and once you have the dispersion relation you can calculate the group velocity given. By this, it is implicitly understood that, this wave numbers that appear in this expression alpha and beta are nothing but the real part. If alpha and beta are complex or if you are looking at the temporal theory then, alpha and beta are necessarily real but then, you will be taking omega as the real part of that which is omega r.

Now, I told you historically that people found solving the temporal problem easier for two dimensional problems although there is no much of a difference between spatial and temporal theory. But, suppose you are looking at a three dimensional flow, and then what you can do is you can get away from this complexities of defining that psi and psi bar. So what you do each we performing temporal theory calculation having obtain the temporal rates etcetera you can convert that into a spatial rate through a relation like this, and this relation was originally adopted by Schlichting in his 2D calculations. And then, as we see today, that there is a basis for this equation 35 that comes out from the fact that this complex circular frequency are in general, functions of the complex wave number components.

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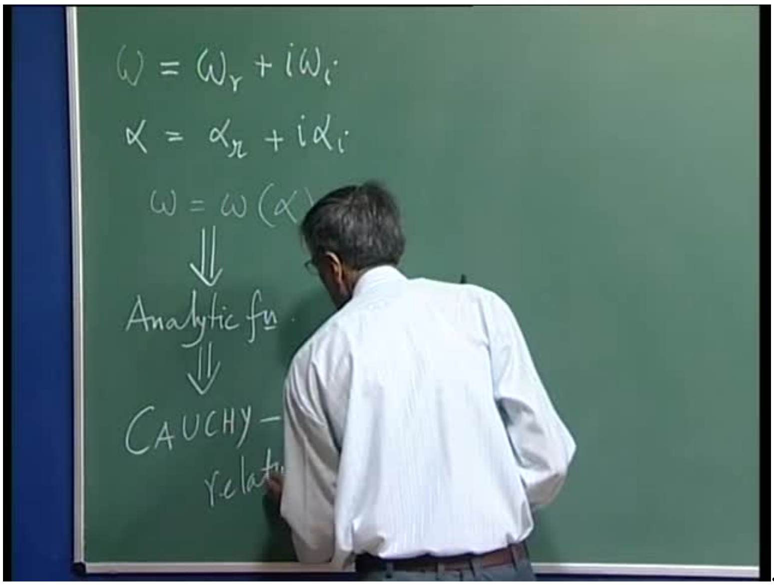

So, what we could do if we write omega as omega r plus i omega I and write one of the wave number in the x direction also as similarly like this, then what we could do, basically we could write omega as a function of alpha. If I write this omega as function of alpha, that immediately gives me the following relationship between the partial derivatives of the real and imaginary part of omega with respect to this wave number components. How does it come about, that comes about because you say that this relation is analytical. What does it mean that you have well defined derivatives as we where you can perform on that another consequence of that, if this is analytic function you can actually get what it means. Taught to you earlier also is the Cauchy-Riemann relation and that Cauchy-Riemann relation is what is given here.

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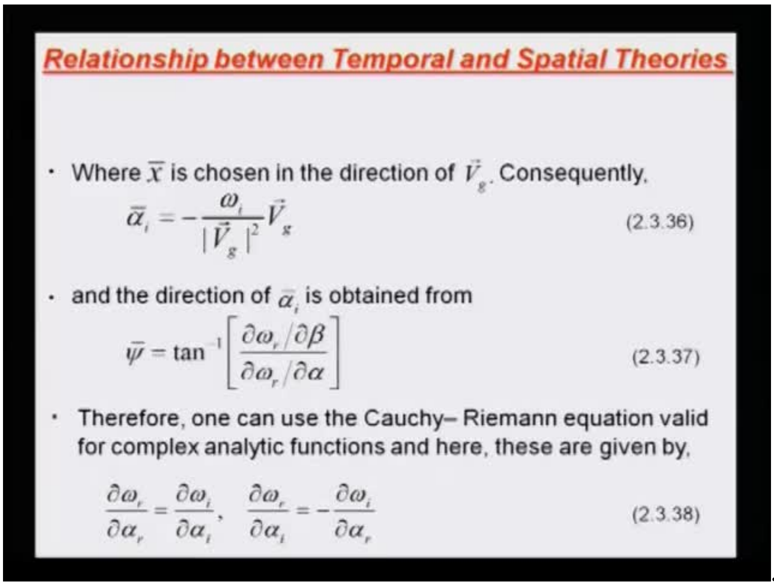

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Now, let us try to identify the meaning of the terms. Look at the first of the two Cauchy-Riemann relations, what is this quantity; the left hand side quantity that we have del omega by del alpha r is nothing but the x component of the group velocity, that is how we have define and that is what you are seeing here in the denominator that is, the V g x a group velocity component. And the right hand side what we are noticing is a kind of a variation of the temporal growth rate with respect to the spatial growth rate.

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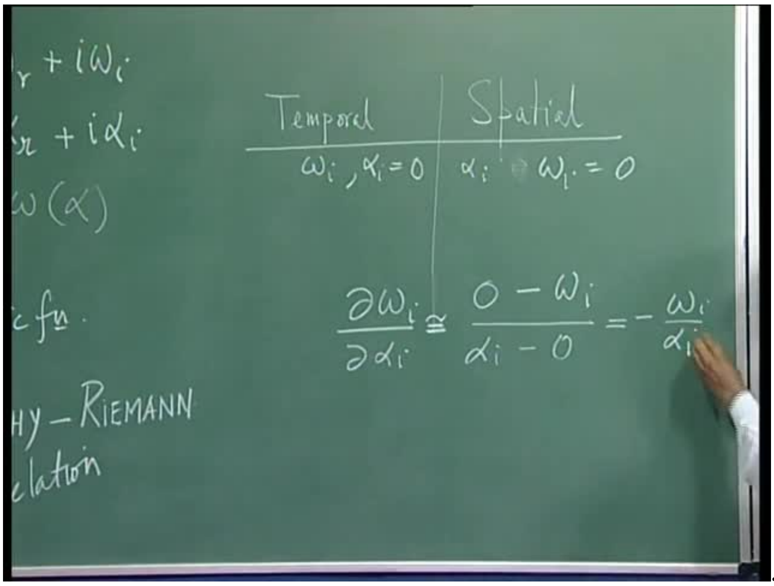

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Now if I draw this table here, that if I have a temporal theory is a v a spatial theory, then what are we getting for a temporal theory we have omega i and what about a corresponding spatial growth rate temporal theory is zero. So that is that and if I adopt instead a spatial theory, then what happens I have alpha i and the corresponding alpha i and omega i should be equal to zero. So if I look at it this way then let me try to evaluate this partial derivative del omega i del alpha I. So it is like suppose i am going from temporal to spatial theory then this is my final condition, this is my initial condition.

So, if I look at it and if I can try to write it approximately, what is omega final; omega final is zero, what is omega i initial; it is the omega itself and then I have the final is alpha i initial is zero. So what I get is nothing but omega i alpha I, well of course this is an approximation of course this is an approximation for the derivative. This we can leave with it because we are dealing with linear theory, so the growth rates in space or in time are going to be small quantities and I can afford to take this kind of a first order approach in defining that. So that is precisely what we do is, basically write this quantity.

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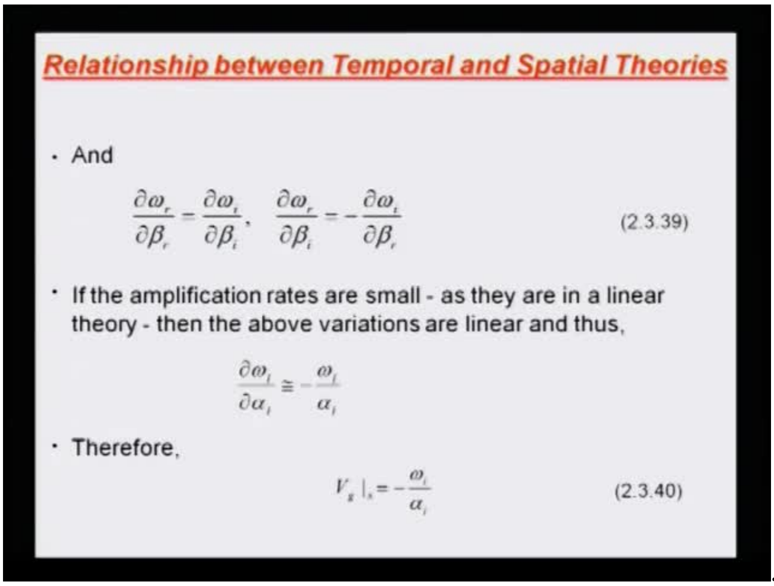

So, del omega i by del alpha i is equal to this, so that itself was V g x and this is an expression for V g x. So V g x simply turns out to be minus omega i by alpha. What about V g y,V g y you can do the same exercise.

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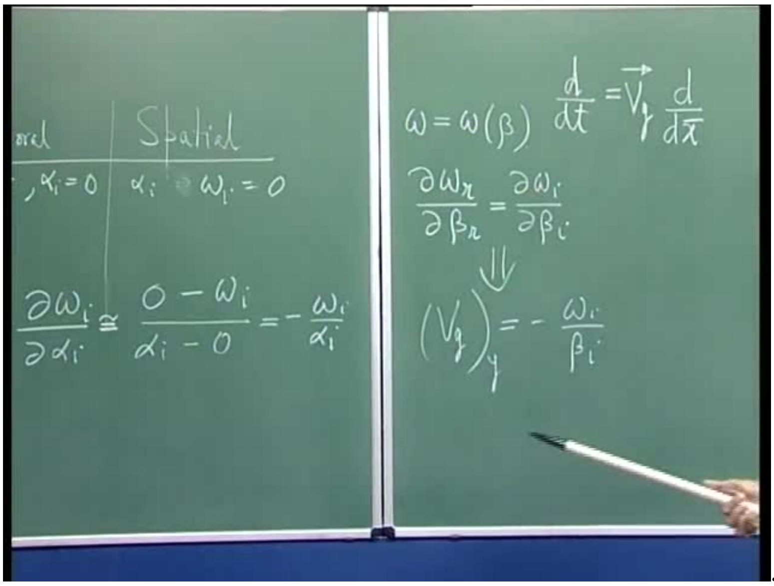

Now, instead of writing omega as a function of alpha you write it as a function of beta, and once again the first of the c r relation; the Cauchy-Riemann relation could be given by this and going by the same logic this will give you the y component which could be simply nothing but minus, that is what is written down in transparency there. So basically, then you have the V g x and V g y component. So the growth rate in the spatial theory gives you what this and that should be now what should we do dimensionally if I multiplied by what should I get that should be giving me the temporal

So, basically this is what one would like to do if you have a three dimensional instability problem, you adopt a temporal theory, you work out the components; the temporal growth rate omega r omega i. And then you try to estimate the group velocities through this type of approximation and then converted into a spatial (( )).

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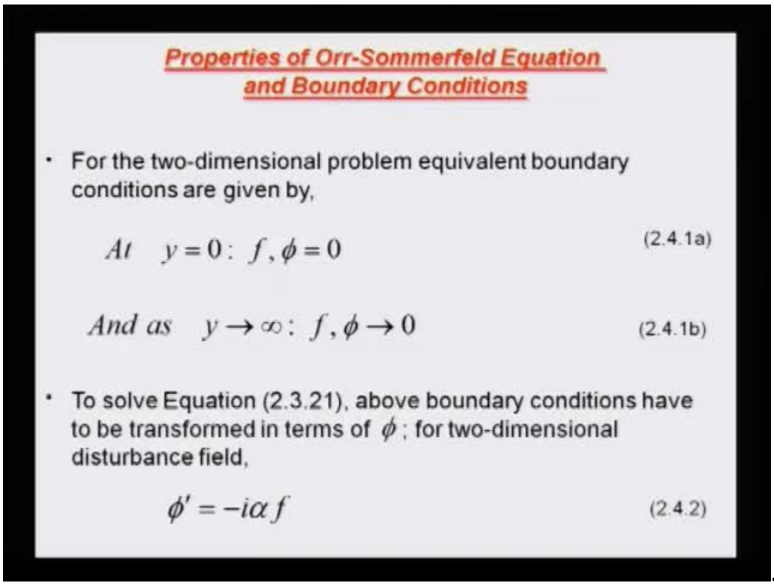

So this is what it is, we will come back to three dimensional instability problems much later, but let us for the time being now talk about the integrity of solving a stability problem. We begin by the simplest possible place which is also quite as we discuss, the two dimensional problems are more relevant in many situation in 3D. So let us look at 2D problem which are also the Orr-Sommerfeld equation that we have written down earlier. What was it, that was written in terms of the phi and we had a fourth order ordinary differential equation. So to solve that forth order o d e we require 4 boundary conditions, so let us try to explore what those boundary conditions are and what those boundary conditions imply.

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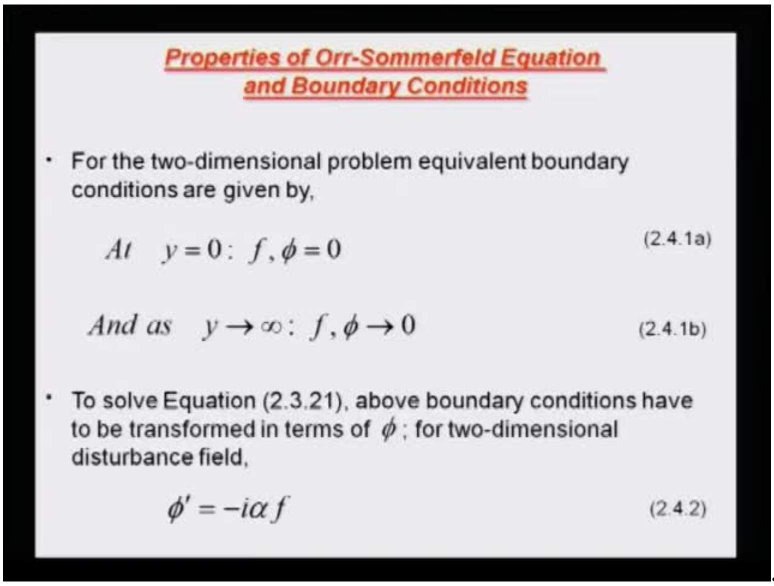

Now this is given that at the wall, suppose I am talking about flow past a flat surface or surface of mild curvature, then i can define my x axis along the surface. So that would correspond to y equal to zero, and on that surface you have zero disturbance velocity. Please do understand that we have some problem here, because I am emphasizing time and again that in nature you do not get something out of nothing, that means you cannot have a homogenous differential equation and homogenous boundary condition, and you get a nontrivial response. However, unfortunately that is how all of you have been talked about this so called Eigenvalue problem that, you take a homogenous equation and force a homogenous boundary condition. That is what we have doing here, but I will show it to you what is the connection between a trivial and nontrivial boundary condition. Essentially, if I have a flow and that is unstable, and if i want to see it is manifestation I must have some kind of excitation.

So, what I would show it to you very shortly that I will provide some kind of excitation at the wall, and I will show that essentially those instability waves are getting created because of that. And I told you very clearly you got to appreciate the fact that instability theory was compounded in the early part of twentieth century, but people had to verify it experimentally and that took roughly about 40 years. And how did that come about it is not by doing some arbitrary experiments, there the experiment was done in the following scenario that you basically removed all possible trace of background disturbances and then you excite the flow with a determinacy disturbance.

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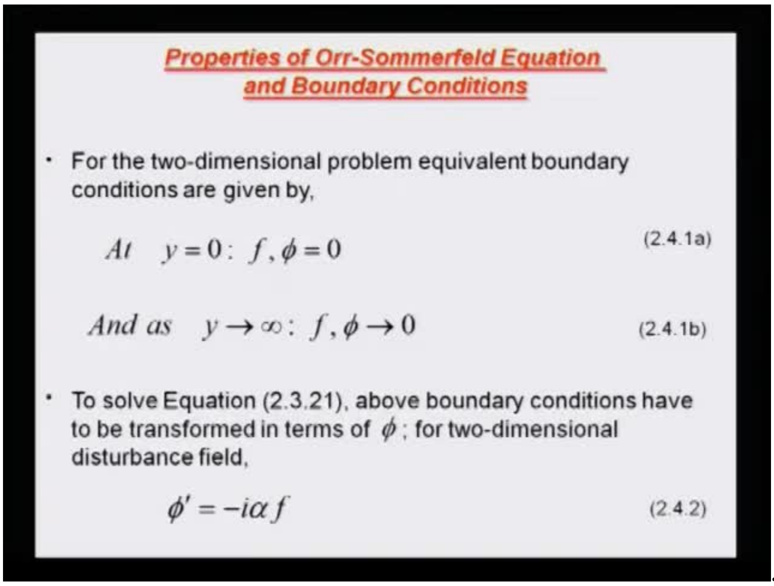

So basically, even though we write this; please pardon me for pointing it out time and again that, this is a very fancy way of doing mathematics part of it but this does not add in understanding the physics of the problem. None the less if we are creating a disturbance inside the boundary layer and if we go far away outside the boundary layer then of course we would demand those disturbance velocities would decay, that is what we have been doing. So that relates to the Fourier-Laplace amplitude of the U component, and the V component going to zero as y goes to infinity.

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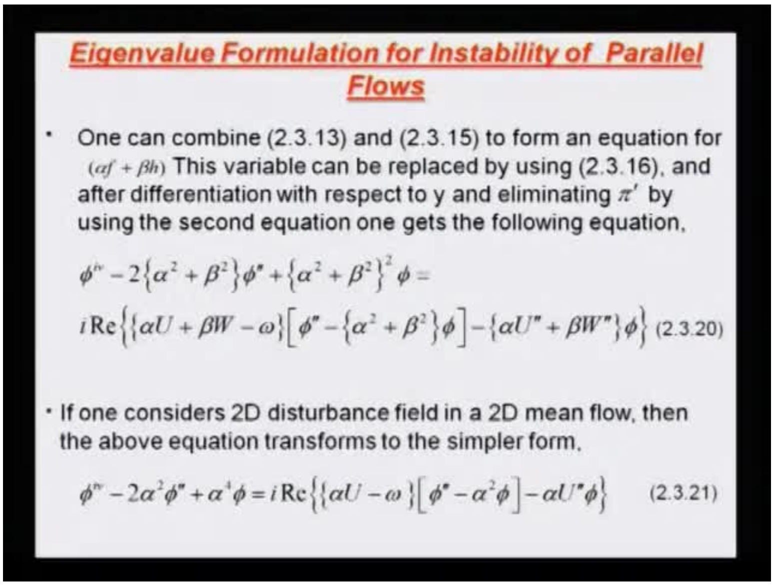

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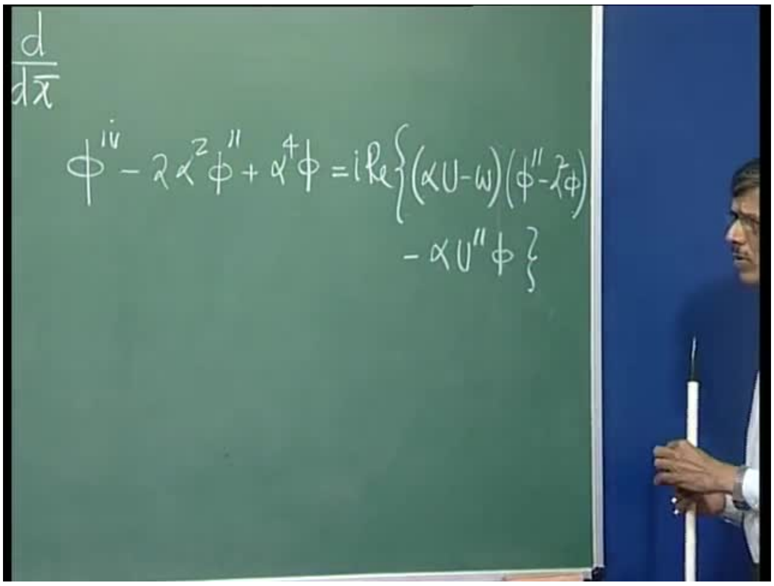

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Now what is this 2.3.21, that is hunted back so we are basically writing down Orr-Sommerfeld equation which this phi 4 minus 2 alpha square. We are writing it for two dimensional disturbance, and what we get is the other term this is your Orr-Sommerfeld equation that we would just keep it here with us and then we will talk about this.

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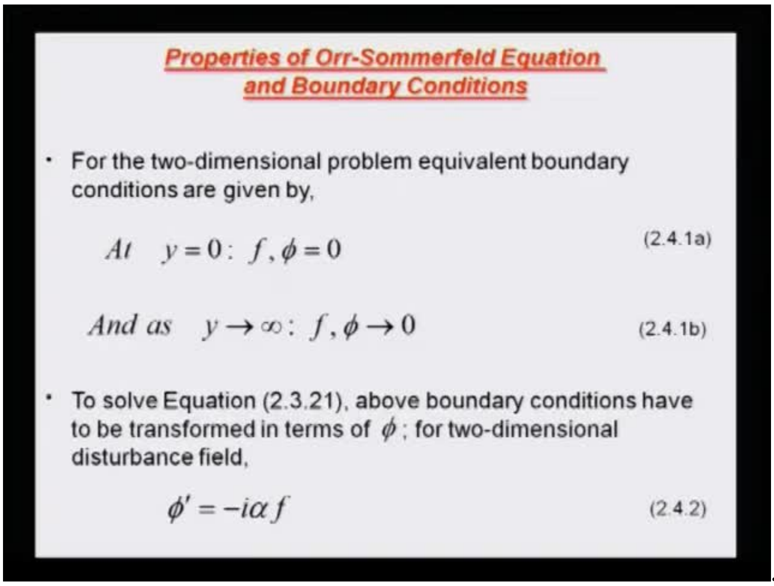

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So these are the four equations, but you see we are we need four conditions and five but we are given conditions on f. So how do you translate that f condition into five condition that comes from a mass conservation. This is nothing but your del v del y term, and this is del u del x term, so that is what we had obtain. So basically, if I give you a condition on f that is equivalent to give you a condition on phi prime, so essentially this four conditions that we have are given for phi and phi prime that is the way we are going to use it . Now we also see that in this equation the source of difficulty is that U and U double prime are going to be functions of height itself from the wall, and however if I am looking at let us say a flow by flat surface with a sharp leading edge boundary reforming like this, I go along this.

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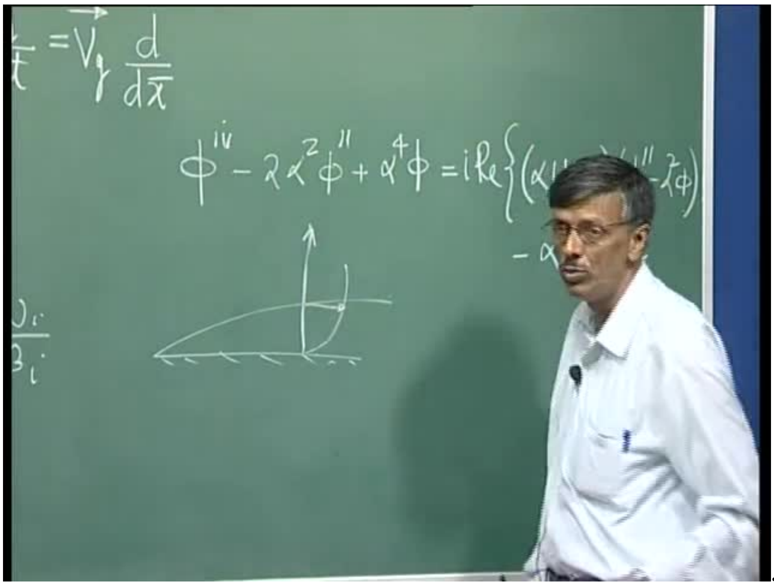

So basically, you understand that Orr-Sommerfeld equation solution basically does the analysis on a particular station x and then solving this equation as a function of y. So what happens here is you have this U of y, so once you come to the edge of the shell you have reached U equal to one. Remember that we have non-dimensionalized all are velocities with respect to the feasting value.

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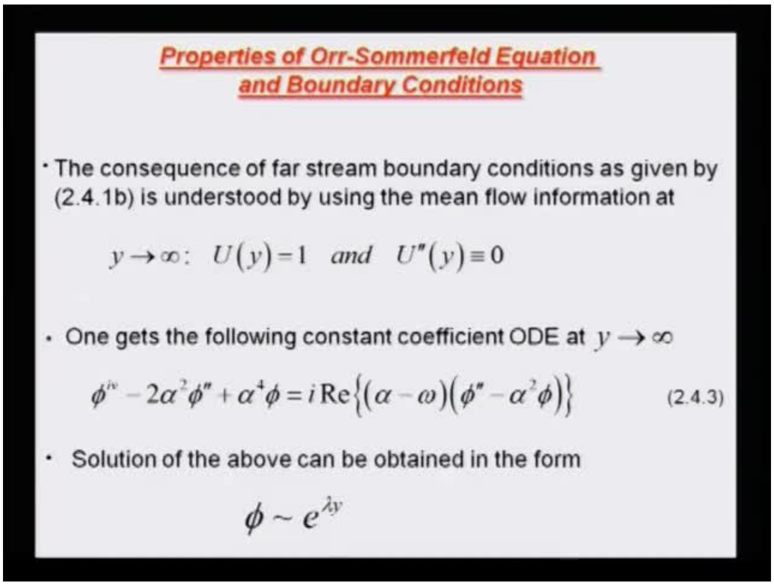

So at the edge of the shell a, that translates into one so that is done, what about the second derivative; second derivative is zero because you are at the edge of the shear layer called derivatives that are going to be zero. and then This equation of course simplifies to this. I have omitted that alpha U double prime term and wherever I had here alpha U that becomes alpha minus omega.

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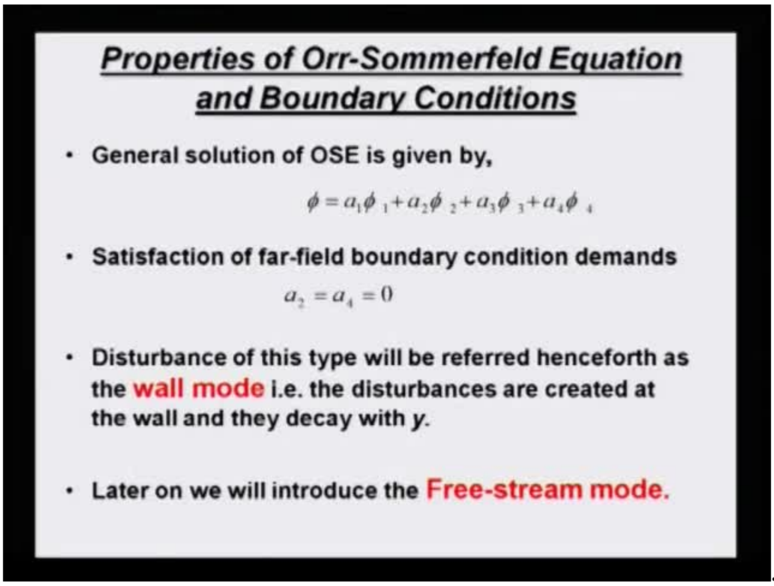

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Now why are you doing this, we are trying to perceive the property of the solution outside the shear layer, so that is what it is and in doing so we note that as y goes to infinity this becomes a constant coefficient ordinary differential equation. and that is I am unable to analytic solution a constant coefficient o d e you can try out a trial solution of the following kind, you write phi equal to some e to the power lambda y and plug in back into this equation.

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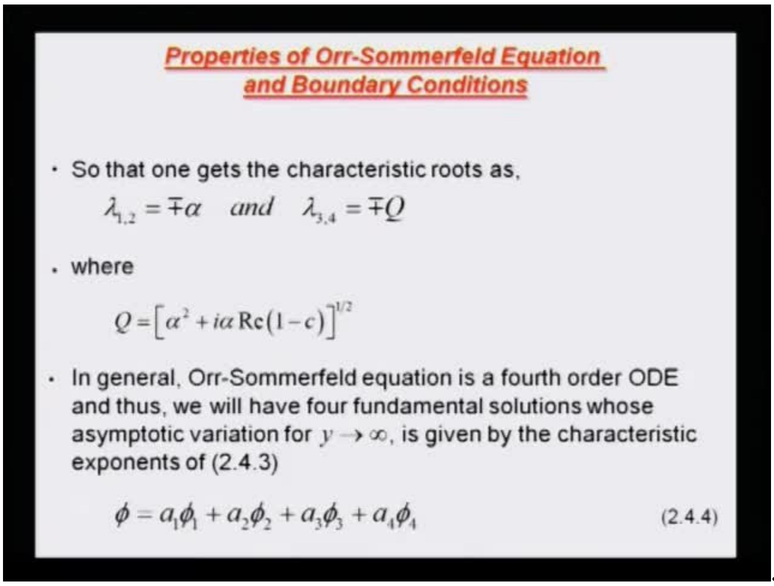

So you are going to get what a quartic in lambda r. So you do that and if you do that I am going to get a quartic and then that would have four characteristic roots, and what you find at that equation that you had just now seen, a pair of roots one and two corresponds to minus alpha and plus alpha and three and four corresponds to minus Q and plus Q. What is Q, Q is this part; remember that let me explain to you what is that we are doing here, a little bit our explanation is not heard.

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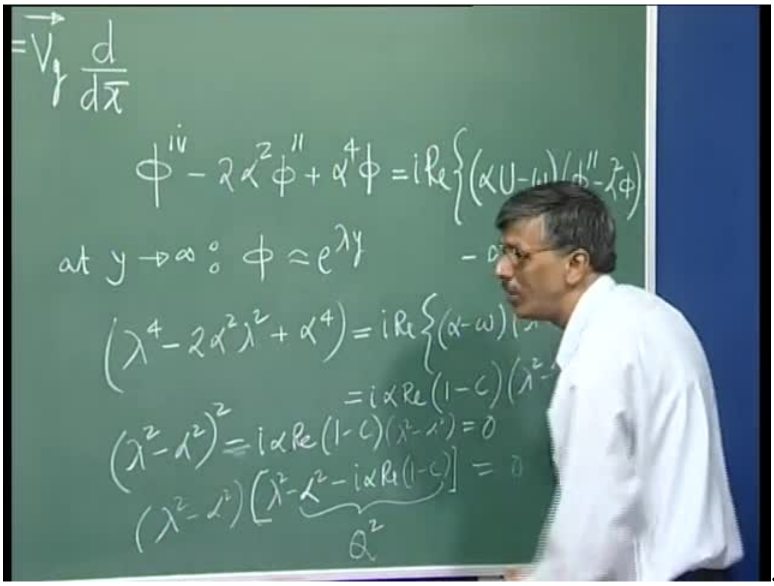

We are talking about y going into infinity, so this equation is constant coefficient o d e. So let us write it down, it is equal to e to the power lambda y, and then from here I will get lambda 4 and there I will get 2 alpha square lambda square, and from here I will get alpha 4. So that is the left hand side, on the right hand side I have i Re and U is one. So this becomes alpha minus omega and this will give me lambda square minus alpha square, this part goes away this double prime is zero.

So what I get here is, I could take alpha out here so I could write here i alpha Re and what I would get this is one, and this omega by alpha is C so that is what we are going to get alpha square. So, you have a quartic now and you can solve it, so what you are seeing that, this is well we can factor it out. This is what is alpha square minus lambda square minus alpha square whole square and that is equal to minus of i alpha Re into one minus C and then I have lambda square minus alpha square. So of course we this C equal to zero, so you take lambda square minus alpha square common and then I will get from here a lambda square minus alpha square and from here I will get this i alpha Re into one minus C square; put it square bracket and this is equal to zero.

So you can see this factor lambda square minus alpha square gives you those lambda 1 2, and this quantity here alpha square plus i alpha Re we are writing it as something like Q square.

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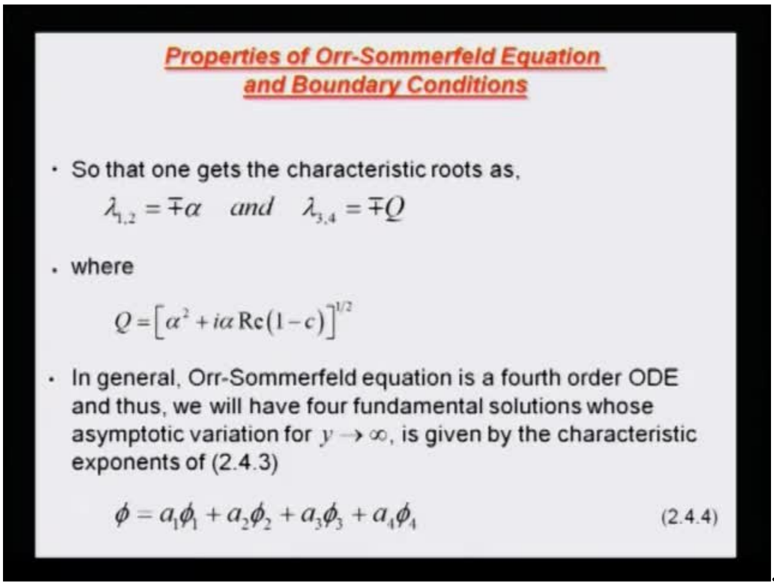

So that is what we have written there, Q square is alpha square plus i alpha Re into one minus e. So you can identify those four words, there is of course very good reason to call this as a inviscid mode. Why we call this as inviscid mode does not depend on (( )) term, while lambda 3 4 are called the viscous mode because a viscous action comes through the presence of re term, so that is what we identify.

Now because you have a fourth order o d e as a governing Orr-Sommerfeld equation, so you would have four fundamental solutions which we actually do not know how it is inside the boundary layer, it is only outside the boundary layer. We know their behavior that is what we have worked out, so the asymptotic variation for y going to infinity is known but we at least know it is a fourth order o d e, so it should have four component solutions.

Now if you look at the behavior of this solution, as you go up outside the shear layer that means your y could tend to infinity then what happens to the phi 2 part of the solution. phi 1 and phi 2 part of the solution for large y would be given in terms of this and this will go as e to the power minus alpha y. This will give you e to the power plus alpha y, and this will give you e to the power minus Q y, and this will give you e to the power plus Q y.

Now there is no reason on earth to expect that alpha i is going to be only positive. Alpha is complex but the real part can be positive as well as negative. For the time being, for the ease of understanding let us talk about the real part of alpha being positive. What happens if real part of alpha is negative, what you expect to happen while you can very clearly see that your phase speed is given by omega by alpha; if you are taking about 2D. So if i get that as negative that means the phase will go in the opposite direction well similarly, one can talk about the group velocity and one can truly find out with the disturbances going downstream or upstream.

So we are alive to the situation, we are aware of the situation that we could have disturbances which go downstream or we can have disturbances which can go upstream. At this point in time let us focus our attention only to downstream propagating disturbance and also the case where alpha r is positive, if that is so then what happens to this part a 2 phi 2 as y goes to infinity is going to blow up; it is e to the power alpha y.

So y going to infinity that will blow up, same thing will happen if I do this square root and pick up both the part that are there. Then if I take the real part to be positive, then again the same token a 4 phi 4 will block when you go to the edge and beyond the shear edge of the shear layer. That is what we are talking about here that if we want to satisfy far-field boundary conditions and if we consider the real part of alpha and Q to be positive, then we must have this so that fixes it. Those two constants a 2 and a 4 have to disappear.

This is one of the things, so we are talking about a particular attribute of the disturbance field. What is this disturbance field, it going to be that this is going like this e to the power minus alpha y and e to the power minus Q y. So that disturbance is predominant and was closer to the wall because as we increase the height we are going to decay. So that is why the solution which decomposed of a 1 phi 1 and a 3 phi 3 and will call that as the wall mode; you know the reason now why you call this wall mode. So the disturbances may be created at the wall or near the wall but they decay with y that is why you are calling them as wall mode.

Now we can similarly also conceive of a situation where the shear layer is disturbed from outside; why do we have to really think always the distusrbances are created inside and we need solutions which decay with y. I can have completely the other situation that I do not create any disturbance inside the shear layer but the disturbances are going out. In fact you know one of the very clearest manifestation of this is, in the weather system this rains pattern that we see in the whole belt starting from Iran all the way up to the north western part of India we get this winter rain, and this is called the western disturbance. So this disturbance are nothing but some kind of a vertical disturbances which start of from the Caspian sea region and go in the easterly direction but from Indian perspective, those disturbance come from the west so we call it as a western disturbance but they actually travel in the east and they are outside the boundary layer. This massive vertical structure gets created and as they migrate they create this rain.

So this is quite understood, in fact I will show you some of the lab experiments we did about ten years ago, where we did set up this so called free-stream mode. A good example also comes from engineering side when we look at what happens inside a gas turbine. There you would see beat in the turbine or in the compressor; talk about compressor it is a cleaner flow, you have this stacks of rotor and stator. So from the stator what do you get, you get vertices and those vertices they go down stream because that is the mean convection direction. When it comes out of the stator these vertices are not immediately ingested inside the boundary layer but they remain outside the boundary layer, and those vertices originating from the stator as they convict over the rotor. They create instability that is a scenario what you would be calling as in the free-stream mode.

So the disturbance presides in the free-stream, then you want to find out what does it do inside, we will have lot of discussion on this as we gone out. Unfortunately this is once again going to point out the drawback of mathematical physics stability theory. Stability theory does not distinguish between the two; there just you simply talk about homogenous boundary condition at the wall and at the free-stream. But if you do that you miss this part because here what you are saying that as y goes to infinity the disturbances do not decay, but they do at in a finite volume value; the one that you have prescribing.

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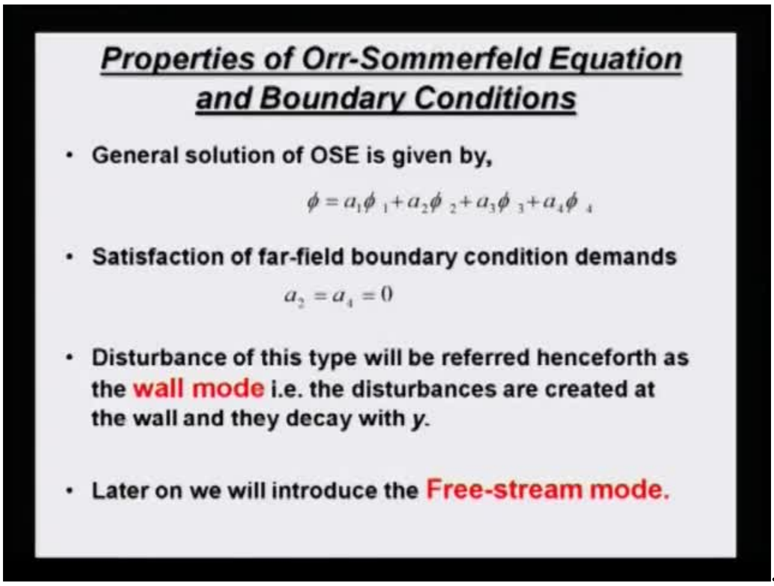

So your disturbance should match up to that condition, it may be zero at the wall, so it will grow with height and it will saturate to the excitation level. But that is what is not very cleanly brought out by the stability theory that all the people have actually missed, it was only late nineties we have made this idea somewhat common to the knowledge of the research community.

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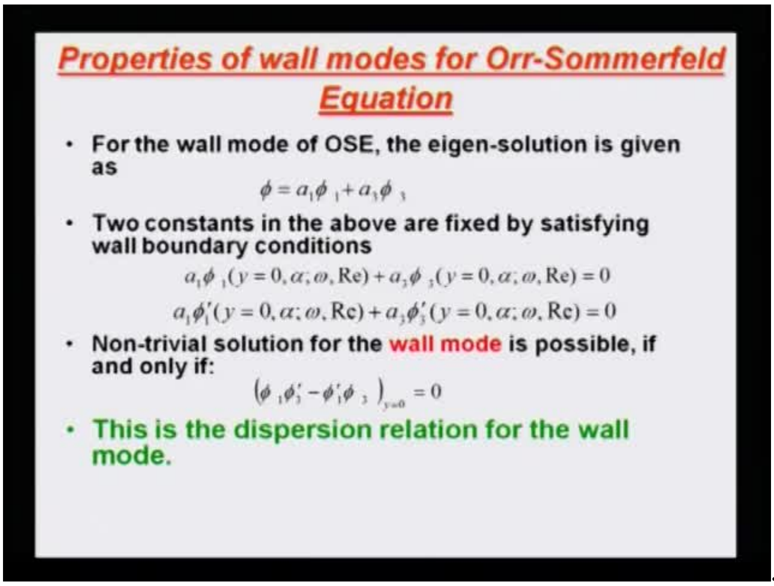

Let us now talk about wall modes and what property is a we have let us lo at the Orr-Sommerfeld equation. Now for the wall mode we will have these two components, as we discussed we will have a 1 phi 1 plus a 3 phi 3. How do we evaluate these two multiplicative constants a 1 and a 3 that is what you want to do. Well of course it is a forth order d, we have chosen the solution in such a way that the two conditions the fast stream have been automatically satisfied. So what is left off for you to do this, to satisfy the other two boundary conditions they happen to be the wall boundary condition at the wall; what we have phi and phi prime are zero.

Stability issue, stability prescription, so I will write that as a 1 phi 1 as evaluated at y equal to zero added to a 3 phi 3 evaluated at y equal to zero. Some of these two has to be equal to be zero, same thing about this. So now what happens is, you know this is where the mathematicians are the phases they do say the correct thing, but they do it in a indirect manner.

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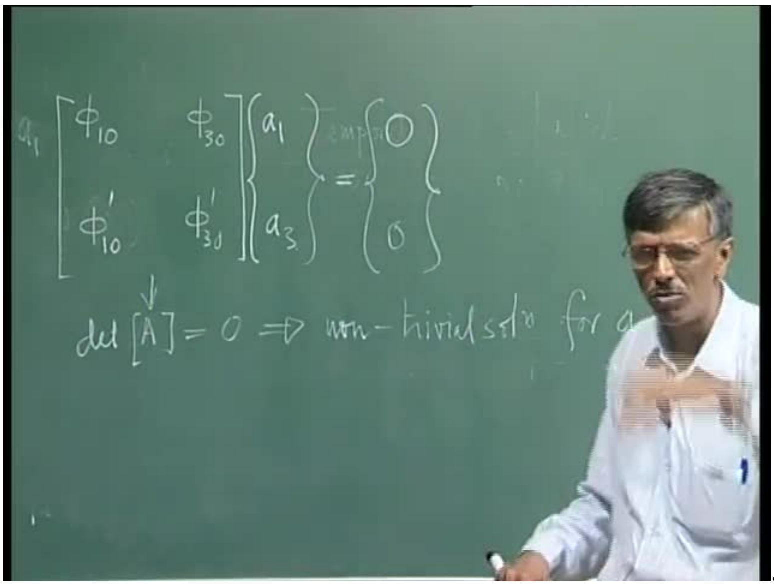

We have two equations for two unknowns, so that we could write it like this. We will write it in the following manner, I have a coefficient a 1 and another coefficient a 3, so let us write it in a matrix form. I have phi 1 evaluated at zero that is what I indicate a, and I have phi 3 evaluated at zero and my unknowns are basically a 1 and a 3. The second equation tells me that phi 1 0 prime and phi 3 0 prime and unfortunately in stability theory the right hand side is given as zero. Then we are trying to look for a nontrivial solution for a 1 and a 3. How can that happen, it can only happen if this matrix is singular. So this matrix is singular, how can it happen, the determinant of that matrix is equal to zero and you see that how they make life complex by just simply saying that this is equal to zero. So that comes from, if I call this is a matrix. So we are going to say that determinant a equal to zero corresponds to nontrivial solutions. You understand what it is basically, if it is singular then you get a 1 a 3 vector as zero by zero form and you know by Lapithos rule you can get a nontrivial solution. Zero by zero does not mean zero that is the whole idea of the stability theory which remains somewhat obscured.

So that is what you are talking about here that, this will be nonzero when the determinant is zero that will have the numerator by denominator both as zero. So that means what physically we are saying, stability relates to that condition that even when the numerator variation is small instead of zero; think of it as the vanishing small quantity that is being divided by zero, so division by zero is a kind of a state of amplification. So, if the determinant go to zero means that you are amplifying the numerator. So that is what instability means that you have vanishingly small disturbance but the property of the system is such that you are amplifying it, that is what it is.

So this is what is required of this two fundamental modes phi 1 and phi 3 evaluated at the wall should be equal to zero, and this will be equal to zero for not all possible combinations of alpha, Re and omega but only for selecting configurations. I think now we realize Now we realize clearly what we meant by stability theory as we have understood basically it comes out that the fundamental solutions should be such that this combination would be equal to zero at y equal to zero and this will not be true for any arbitrary choice of alpha omega and Re, this will only happen for a specific choice.

So I fix Re that means we are talking about a combination of omega and alpha that will help us satisfy this. So there would be a combination of omega and alpha so what is that you call is our dispersion relation. So now you understand that for Orr-Sommerfeld equation the disturbance dispersion relation comes about for the wall mode by satisfaction of this equation. This is what you need to do.

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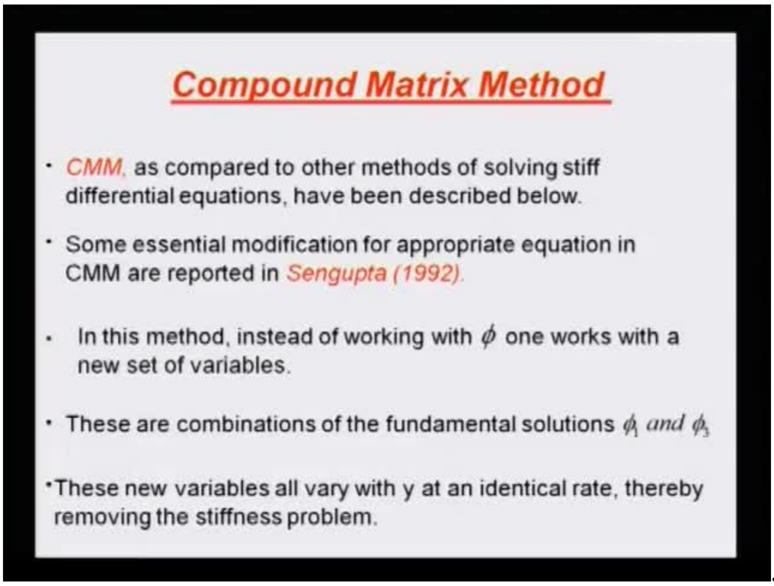

So this is understood, but phi 1 also satisfies the Orr-Sommerfeld equation and phi 3 also satisfies, but each one of them whether I am looking at phi 1 or phi 3 they themselves vary by different rates; do you have any estimate for those rates at which it varies. Yes we do in fact, that is what we are seeing here if we remove all of it this is what we are seeing that we have two quantities.

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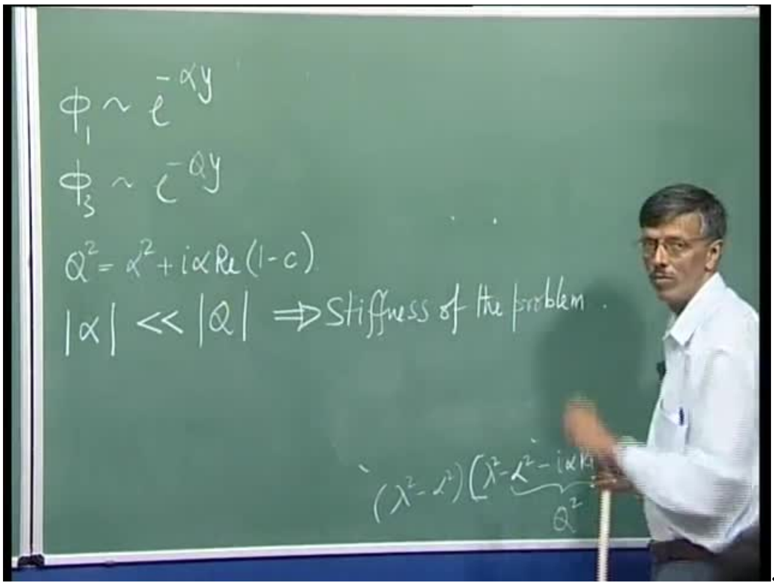

So phi 1 we know is e to the power minus alpha y and phi 3 are varies of e to the power minus y. What did we also find, we have note it that Q square is nothing but equal to alpha square plus i alpha Re into one minus C. So if I look at modulus wise, mode f alpha is complex but still it will have a amplitude that is significantly less than the mode because Re is a large quantity and we are adding to alpha square. So what does it mean, we have two component of solutions, they are decaying with y at a totally dissimilar weight and whenever such a thing happen we call that as the stiffness problem. So this is related to and this is what we have talked about many a time that this particular attribute of the solution makes it difficult to obtain it. I mention very clearly in the last class also that while Navier-Stokes equation was solved long before we could really solved it, this is essentially due to this and this is a topic of mathematics called stiff differential equation.

So we have to worry about how to solve stiff differential equations. What does stiffness do the following thing that if you start off with a solution somewhere; let us say you start off from the free-stream because that is where you have the qualitative nature the solution available to you. Now you come down as you come down, which one is growing more rapidly, this part because now y is decreasing as y decreases this part will be over powering that part. So, although you have two independent solutions, one same to dominate over the other and what happens is any background error, numerical error tends to follow this sopping the components.

So you want to do fundamental components of the solution to be alive at all points, because otherwise you will not be able to satisfy those wall boundary condition in the process of integrating. If my solution is totally dominated by phi 3 I will never be able to solve this equation.

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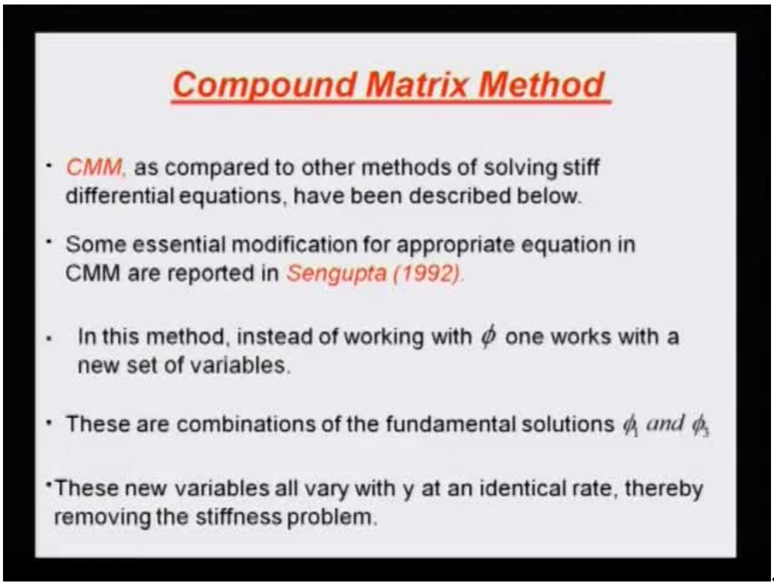

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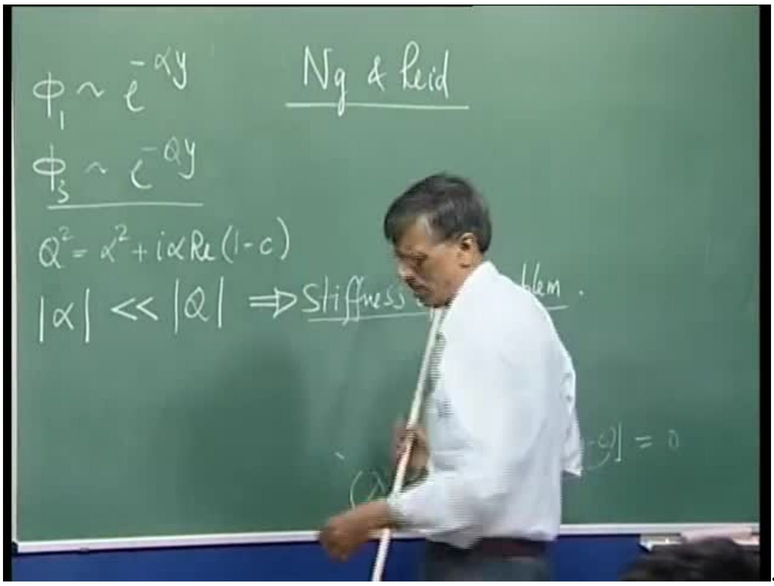

So that is the problem of stiffness. People have tried to do many things; we will not talk about many things, we will just talk about one thing in which we our self participated and that is method which was develop to solve state differential equation is called a compound matrix method which was originally introduced by 2 mathematicians; (( )) and Reid. Also they are other mathematicians like Persadavis in UK, they have worked on it but (( )) and Reid wrote some very definitive nice paper available with general of computational physics. You can take a look at this compound matrix method which was developed basically to solve stiff differential equation; we have done some of this. I will tell you what this is, what we do actually we see that phi 1 and phi 3 grows or decay at dissimilar rates. So instead of working with the phi's we work with a combination of phi 1 and phi 3 such that particular variable set how of the component varies with the same rate so that we do not have this stiffness. This is the basic philosopher very simple, so what we are really looking for some combinations of the fundamental solutions of phi 1 and phi 3 and as a result we desire that the new variables should vary with y with an identical rate and help us remove the stiffness part.

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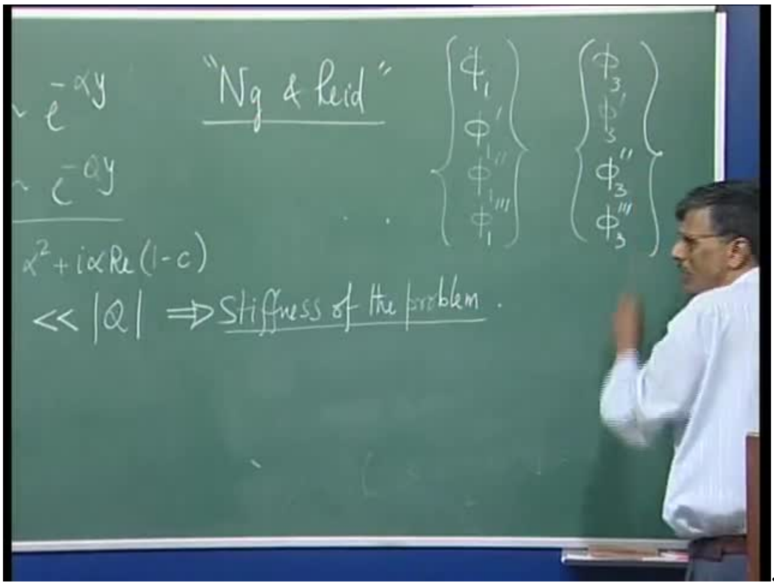

Now, I have talked about in the context of solving the Orr-Sommerfeld equation that let us say fourth order equation. So if I say I have a solution for phi 1, what am I talking about I know phi 1, I also know phi 1 prime. I think those of you have been following how p d e are classified that is what we define the solution. So this is one solution vector and we have another solution vector that is phi 3 phi 3 prime phi 3 double prime.

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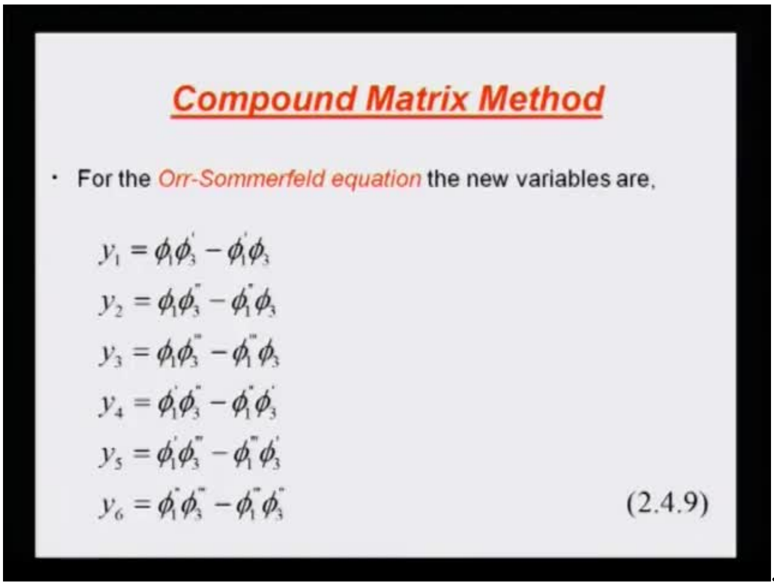

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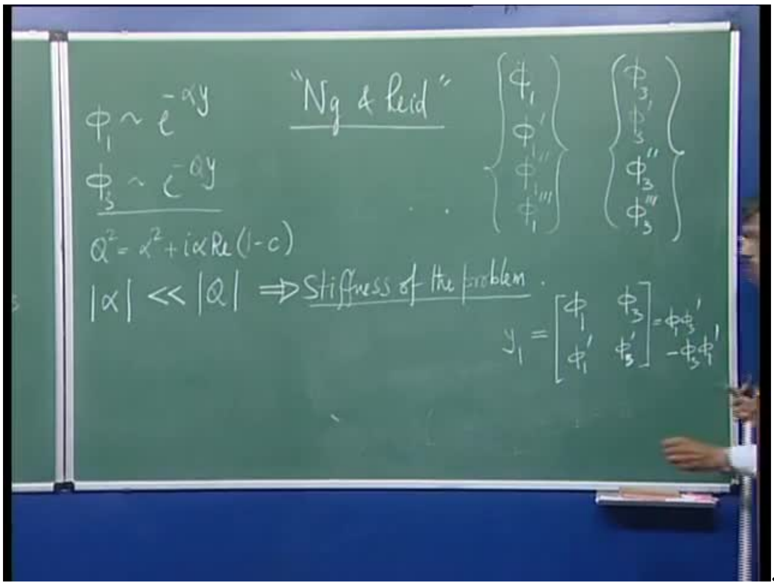

Now what we try to do is, try to generate a set of new variables and what we want to do, because we have two fundamental solutions so we try to form combinations of these two solutions in a two by two mode. So for example, what I could do is I could take this, I could form one of the solution y 1, I could write it like this phi 1 phi 3 and phi 1 prime phi 3 prime.

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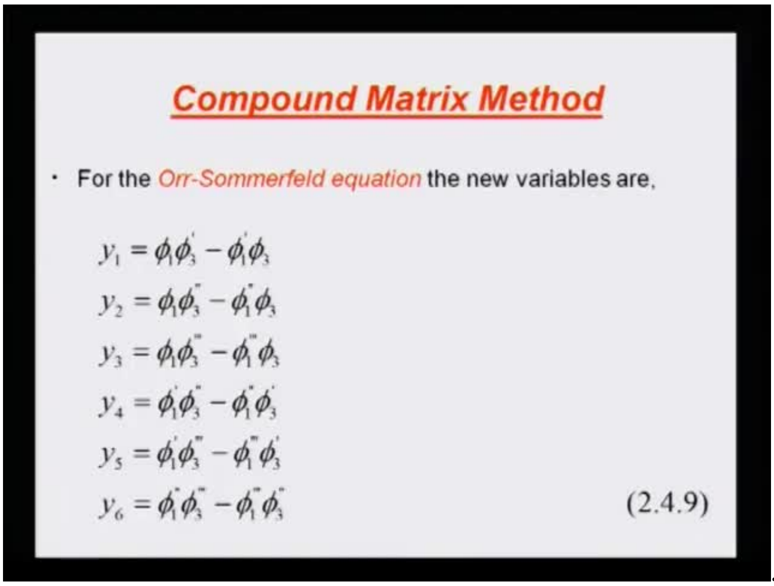

So this becomes phi 1 phi 3 prime minus phi 3 phi 1 prime. So there you have it, phi 1 is this now you can see that if you go outside the shear layer where phi 1 and phi 3 goes like this what happens to this phi 1 prime phi 3 prime; the exponential part, it will be e to the power minus within bracket alpha plus Q. What about the second term, it is also the same thing. So the growth rate has been like e to the power minus alpha plus Q into y. Now if I take this one and this one then I will get y 2 you see phi 1 phi 3 double prime minus phi 1 double prime phi 3.

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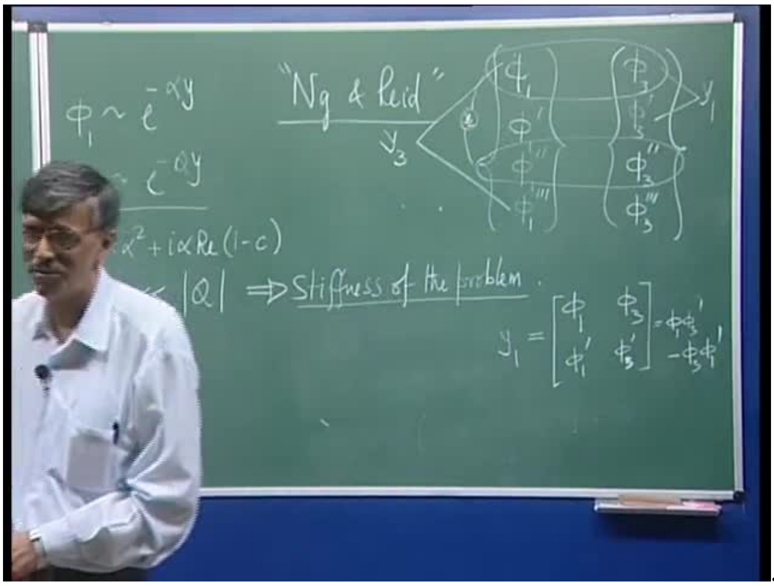

Now, once again I write this phi 1 and phi 3 like this and substitute it there then what do I find, the exponential part will still be same thing. So at least we have ensured that y 1 and y 2 grow or decay at the same rate, so what we could do is this to give us y 2 and this to give us y 1. So if I take this one and this one, I will get let say y 3 that is what we have. Then what we could do is we can see all possible combinations and how many of those combinations would there be, it would be 6. How do you get that four 4 C 2, that is what it is. So you are going to get that, so you see there will be this. So what happens, well we have been a better price because he started off with a fourth order ode and here we are introducing 6 variables what about their governing equations.

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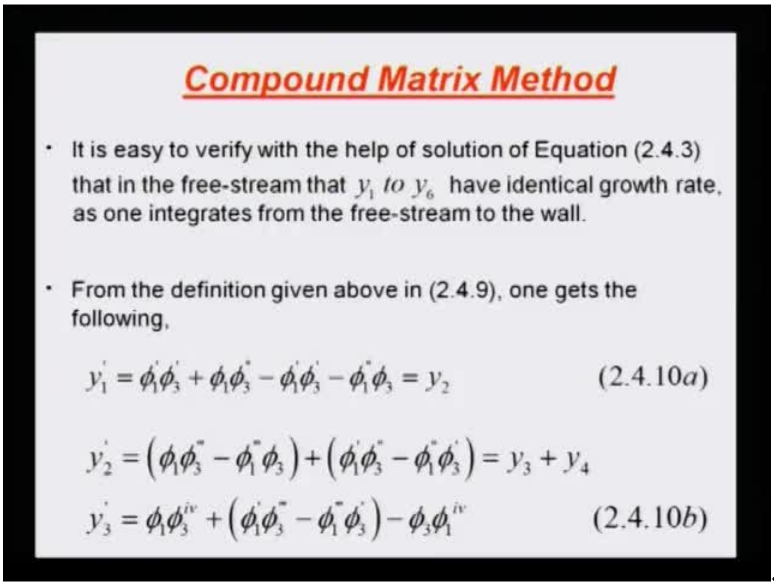
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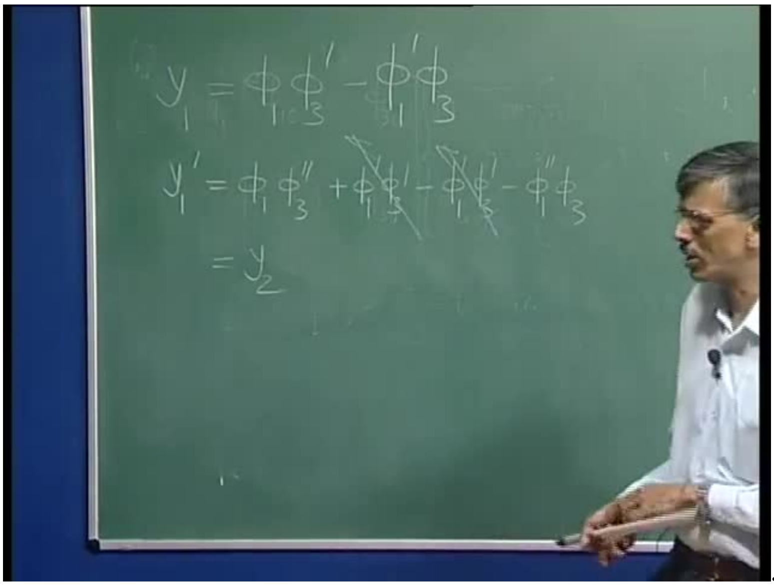

The governing equations can be also worked out from the definition itself, see why wall was like this if y 1 is phi 1 phi 3 prime, I will just show you a couple of them and then leave the rest to you to work it out. So what happens is we defined y 1 as phi 1 phi 3 prime minus phi 1 prime 3, so if I do or differentiate it with respect to y so that will be y one prime, and then and you can see you get phi 1 phi 3 double prime plus phi 1 prime phi 3 prime, and from here I could get a component phi 1 prime phi 3 prime minus phi 1 double prime phi 3.

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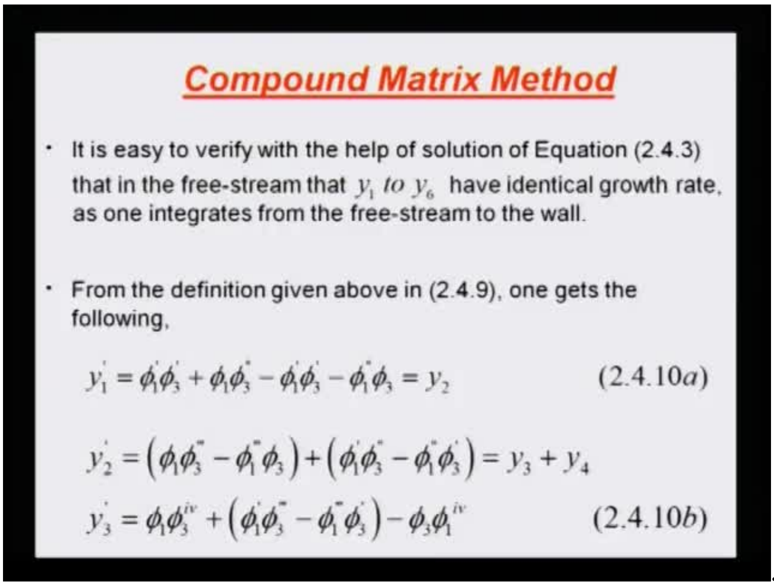

So of course, you can see this cancel from this and what is left with, so that is what we have done. So y 1 prime is y 2, you can do the same thing with y 2 prime. You will have these 2 components if you see phi 1 phi 3 triple prime minus phi 1 triple prime phi 3 plus phi 3 prime phi 2 phi 3 double prime minus phi 3 double prime phi 3 prime. What is this, this is by definition y 3, and this is how we defined as y 4. So we are generating equations for this compound matrix variable they are also called the second compound. So y 1 prime equal to y 2, y 2 prime equal to y 3 plus y 4.

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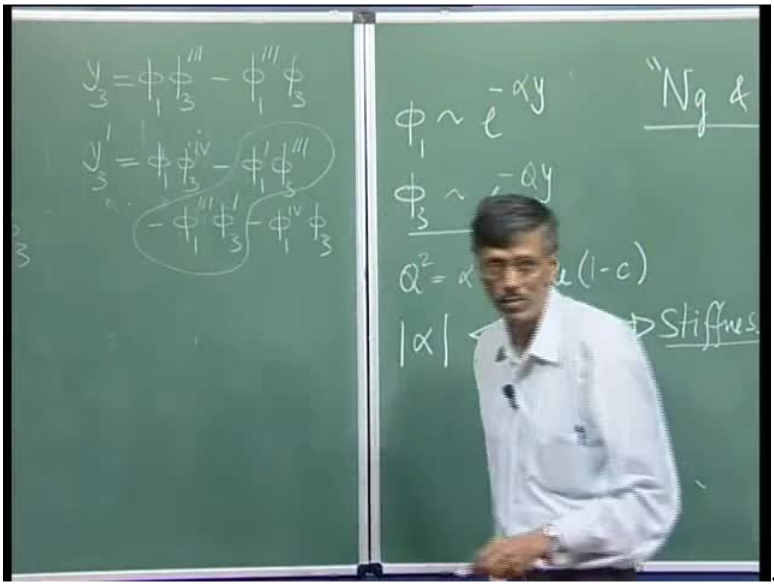

What happens to y 3, well y 3 as you can see y 3 is nothing but phi 1 phi 3 triple prime minus phi 1 triple prime phi 3. So if I do y 3 prime, I will get one part of the phi 1 phi 3 fourth prime minus phi 1 prime phi 3 prime triple prime minus phi 1 triple prime phi 3 and then I will also have phi 1.

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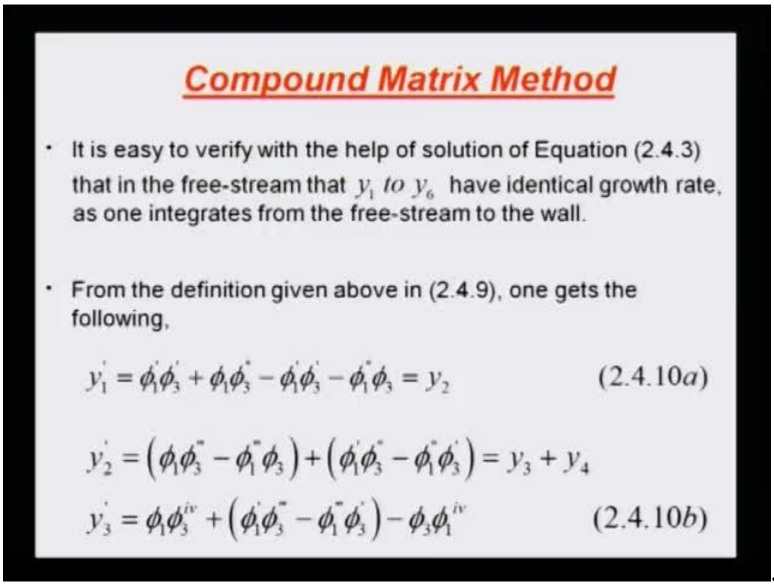

This part is one of the compound right that we have defined. What about this part and that part, that is what we have identified that this part is that one of the compound variable; I do not know I think it is y 5 or so thing. Take a look, it should be y 5 but what happens to this phi 3 fourth derivative and phi 1 fourth derivative. Because phi 1 and phi 3 are the solutions of Orr-Sommerfeld equation they themselves satisfy the Orr-Sommerfeld equations so, he could write it down and I think that is what we will be doing in the next class. So when we meet next time we will obtain all this solutions, I will stop here.