Instability and Transition of Fluid Flows

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Lecture No. # 08

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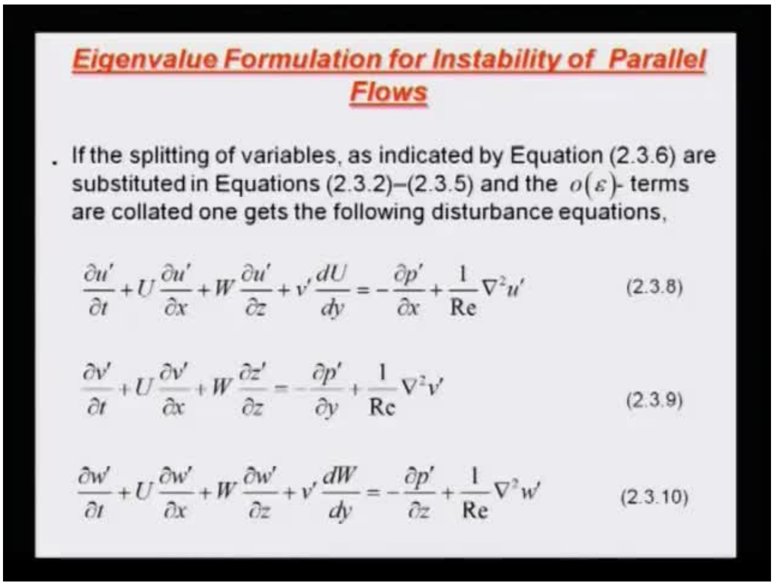

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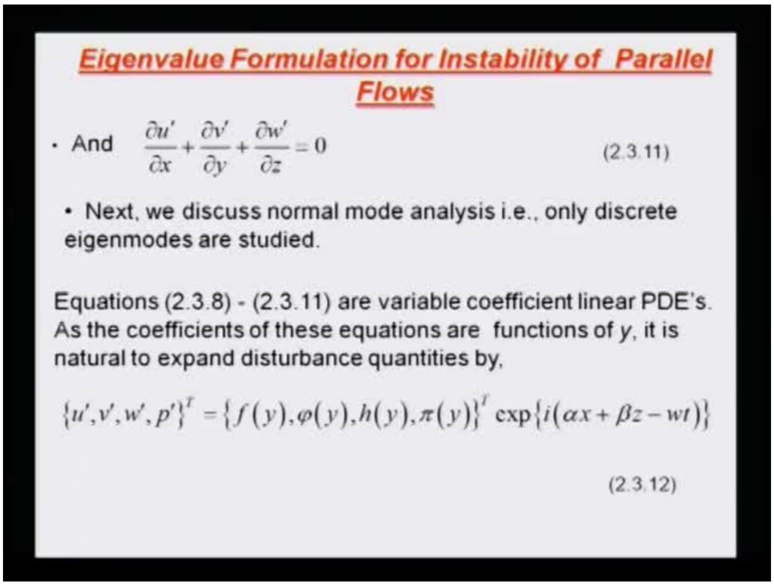

In the last class, we were talking about the viscous disturbance equation for with the parallel flow approximation. We had obtained the disturbance equation indicated by the prime quantities U prime, b prime, W prime and P prime as given by this three linearized momentum equation. And of course, the mass conservation equation there and then we went into that Fourier Laplace transform representation of any arbitrary disturbance. And that would imply that we would have those amplitudes multiplied by a composite phase, amplitude factor, because we talked about that alpha beta omega here, they all could be complex.

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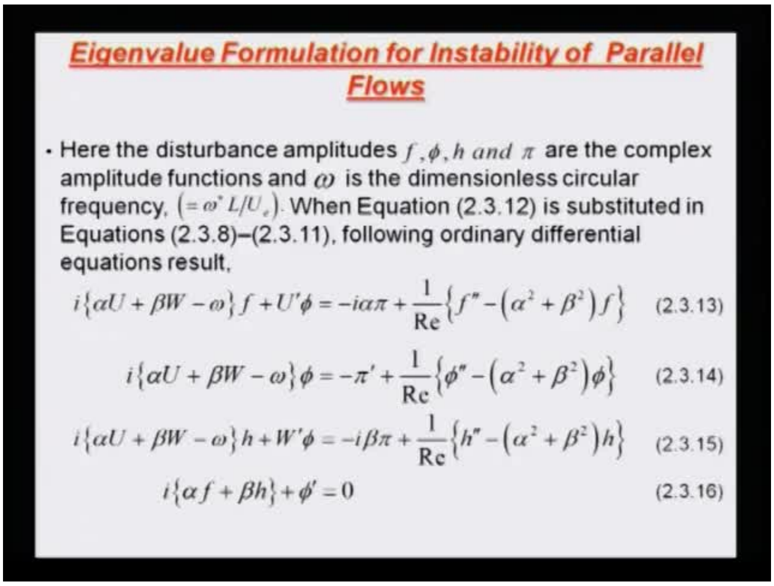

And what we pointed out that, we would be taking a particular point of view, where we will be taking about either the spatial or the temporal problem. So, for the time being, let us go ahead and look at the spatial problem; that means it will take this omega the dimensionless circular frequency to be a real quantity and we will try to find out the complex alpha and beta.

So, those linearized momentum and mass conservation equations respectively give us this four equations. This comes from the u momentum equation; this from the v and this from the w momentum equation and there is a mass conservation equation. So, what I have try to do is tell you that we have a basically four equations for the four unknown amplitudes. So, that is one way of looking at it but there are coupled.

So, if there is a possibility, that we could reduce the numbers. One way of doing it is to write it as a set of first order equations, then of course you can see f is an unknown; f prime is an unknown here. From here, I will get phi and phi prime as unknown, and here, it is from h and h prime is unknown and this relates f and h with phi prime. So, this is basically it is a sixth order system, that is what I am trying to point out to you, but there is an alternative way of looking at it by simplifying it. That is what we have tried to do in the black board.

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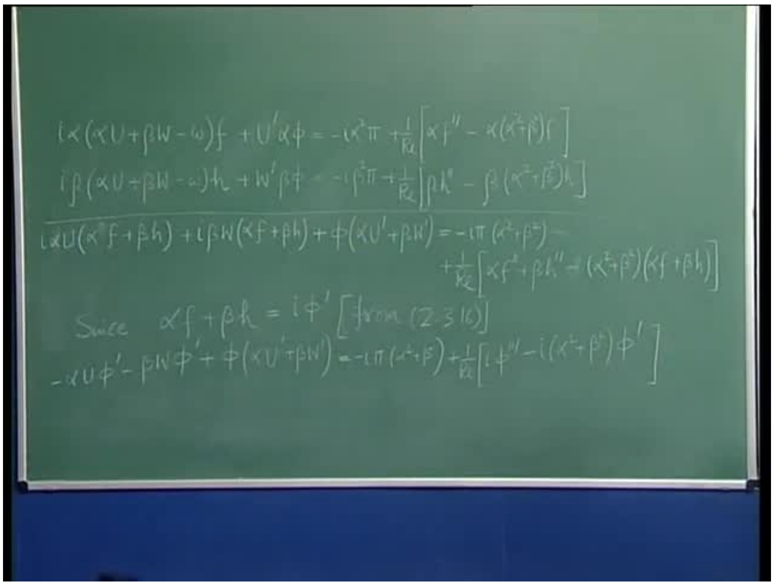

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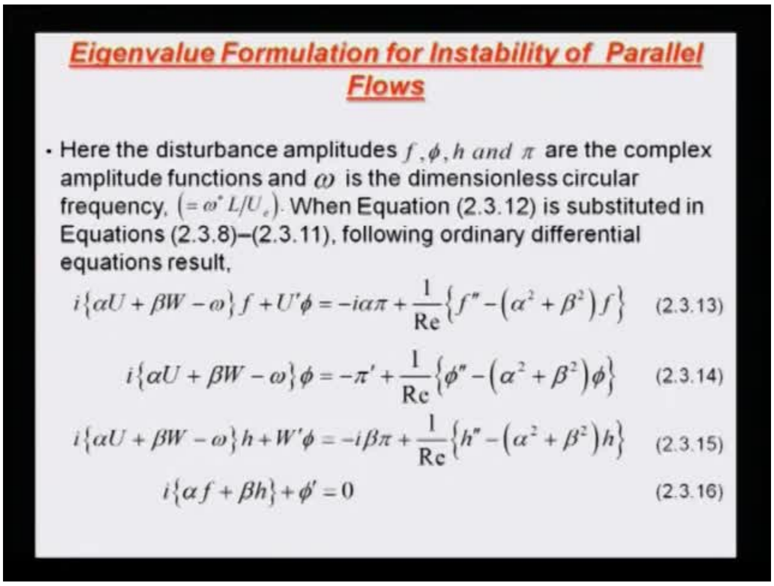

See let us look at equation 13 and multiplied by alpha, that is what you are getting, that is what you are getting. The same way you look at equation 15, the third equation there and multiply this by beta, then we get this equation. If we add this to equation up, then we can see the following combination emergency naturally. There is a combination of alpha f plus beta h in this part.

Here, you see another part and this of course directly related to phi, you keep it as it is, and the pressure term gives us minus i pi into alpha square plus beta square. The diffusion terms also we can write it in this form alpha a double prime plus beta h double prime, and from here, when we take alpha square and beta square common, inside we get alpha f plus beta h. Now, if you look at the equation 16 here, equation 16 tells you what alpha f plus beta h is going to be. Alpha f plus beta h could be written as i phi prime. So, you can substitute this in this equation and you get this.

So, what have we achieved here? We have achieved the elimination of f and h, we have eliminated f and h. Then what we have? Well we have this equation plus equation 14, 14 remains untouched. So, what you notice that in equation 14, we have the derivative of pi with respect to y while this equation has pi alone.

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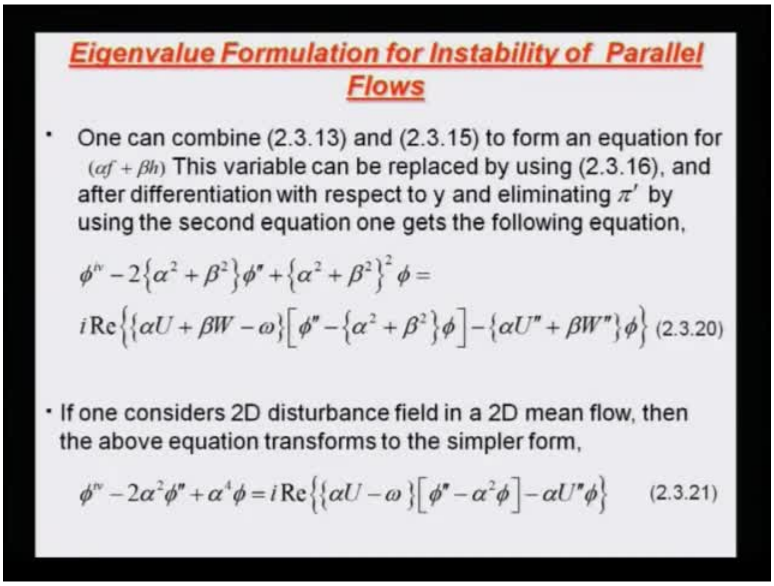

So, what I could do is I could differentiate this 1s. Then from this term, I will get minus i alpha square plus beta square turns pi prime and we can see that pi prime can be exclusively written in terms of phi. So, there by also we can eliminate pi. So, in this process, we get that, we get this following equation. Well, we do get this equation. If we do that, we get this equation.

So, this is a fourth order ordinary differential equation for 5. So, this is a general form that we have written down the corresponding form for 2D mean flow and 2D disturbance field was a obtained originally by Orr and Sommerfeld and that takes this form. So, you can just simply substitute capital W equal to 0, and if you have looking at 2D disturbance, you can put beta also equal to 0 that this equation simplifies to this and this was what was originally derived and presented by Orr and Sommerfeld.

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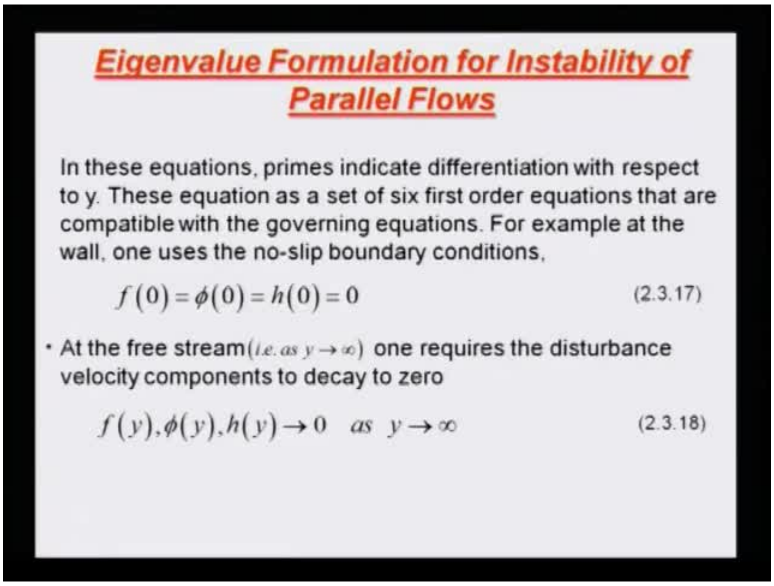

So, this equation 21 is called the Orr-Sommerfeld equation. This is a very famous equation. Now, lots of work gets stands starting from the Orr-Sommerfield equation. Before we do that, let us say that what we have. The equation that we are just now seen, - the Orr-Summerfield equation is a homogeneous equation - there is no right hand side term.

Now, as I told you that those original equations were six first order equations, but elimination of that, we bring it down to a fourth order o d e. So, that is one positive thing of reduction in form. These equations, if we are trying to solve in a stability frame work, then what you should be doing? We should be satisfying subset of boundary conditions.

For example, on the wall, we may say there are no disturbances. This implies that magnitude of u velocity, the amplitude of the v velocity and this amplitude of the w component of velocity, so, those are 0s.

And if we are trying to study how the disturbances propagate, then we will assume that those disturbances are of finite energy. So, if you go far away from the (( )) where the mean equilibrium flow is; that means if you go y 10 into infinity, then all this disturbance amplitude should decay to 0. Please do understand that there is a great deal of difference between saying equal to 0 and approaching 0. So, we are not saying that this is equal to 0, we say that this decays to 0. It is an asymptotic form of the boundary condition. Then what happens is basically, we have what? Suppose we start off from some numerical infinite location, we will not be able to take exactly at infinity that is out of question.

So, in a numerical science, if we take a very large value of y and some of use this condition there and then we keep marching down, then what do we have? We actually have a situation where we have a homogeneous equation with a homogeneous boundary condition, and you know that is a recipe for what? That is a recipe of a Eigenvalue problem.

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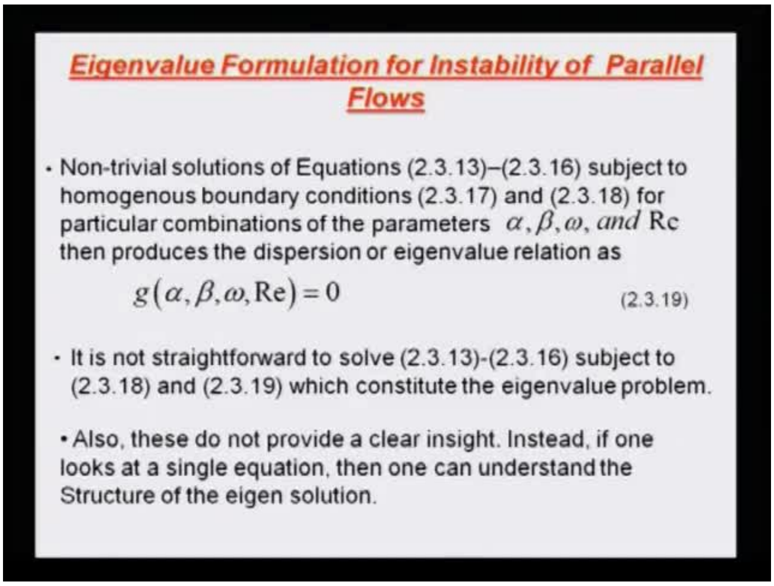

Well, demystify this Eigenvalue concept a shortly, but what we would find that if I have in general, mathematically speaking, if I have homogeneous equation with homogeneous boundary condition, the natural solution is the trivial solution, but in addition, they are all parameter combinations where we would find the despite homogeneous boundary conditions. For the homogeneous differential equation, we will get nonzero solutions and those are the Eigen solutions. That is what you may have been introduce to Eigenvalue problem. For example, the simple problem if I take a column like this and I apply in actual load and then what can happen? It can buckle.

That is a classical instability problem studied by Euler a long ago. So, the same thing that we are looking at homogeneous differential equation with homogeneous boundary condition, but then, we will find out a combination of the parameters of the problem, and what are the parameters in this case? Well, we have the parameters in this case given by those quantities the coefficients in the differential equation, they are nothing but the wave numbers alpha beta, the circular frequency omega and the reynolds number as the parameter of the problem. Then we need to figure out the combination of these parameters such that we get those nontrivial solutions.

So, that combination as expressed by relation or equation is called the dispersion relation or the Eigenvalue relation. What it actually means is that we are trying to find out the dependence of alpha beta on omega. Let say we keep Reynolds number fix, then this equation 19 gives us a relationship between alpha beta and omega, and what are these? Alpha beta is nothing but the wave number, the space dependence; omega is the time dependence.

So, what we are trying to say is that for different frequency, we will have a different combination of alpha and beta. So, initially, suppose I create a disturbance which is rather coherent at time t equal to 0 which is very localized in time. I have started at t equal to 0, then what will happen? It would have exited a range of frequencies, and different frequencies will correspond to different combinations of alpha and beta and we have already talked about the phase speed in the group velocity.

We have very specifically pointed out the group velocity is the speed at which the energy propagates, and if that is going to with the case, then what we are seeing? We are going to see the different frequency component will travel at different speed; that means if I create some disturbance, it will disperse with respect to each other. Different harmonic component will be found at different place, because they are traveling at different speed.

That is why this is what we call as dispersion relation. So, if I create a disturbance which is localized initially, it will disperse with respect to each other component.

So, this is straight forward said that we those four first order equations, sorry, four disturbance equations and we will have to solve that equations subject to this boundary conditions that we enunciated just now, that leads to this kind of a dispersion relation easily said. Then now, I have mention to you in the beginning of this course that, why Navier-Stokes equation was solved even at the beginning of twentieth century's in best calculator that is non-linear partial differential equation.

And here, what we have? If I prescribe you with in the mean flow capital U and capital W, then these equations are nothing but when they are also even converted into single equation, that equation is a linear equation. The only problem is the coefficients are functions of y. So, this is basically linear variable coefficients, ordinary differential equation and it is not easy to solve. I told you that using a daze calculator; people could solve this stroke's equation in early twentieth century and the first a good quality numerical solution emerged in beyond 1950. So, it is not a very trivial exercise.

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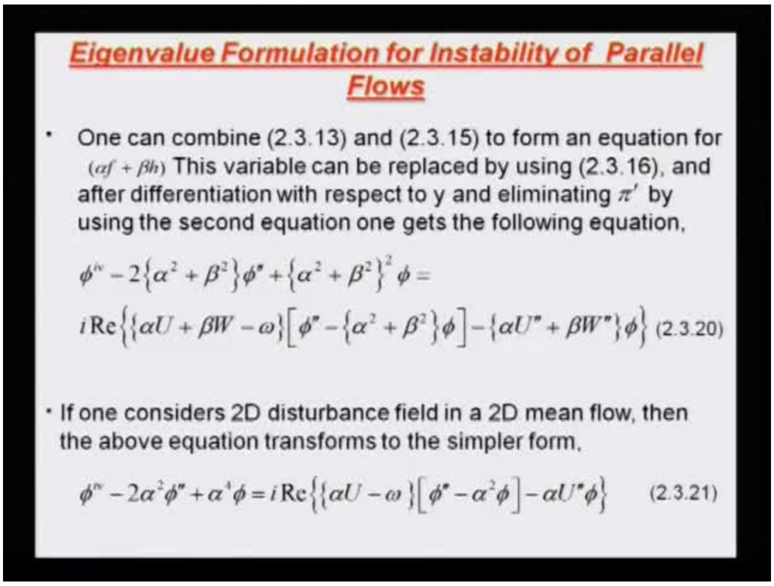

So, this is one of the issue that it is not straight forward. Why it is not straight forward, that is what we intend studying now on. So, let us go on with it, and we will talk about it in details. We have already talked about the elimination of three variables and writing a single equation. The generic form is an equation 20 and it is special form for two-dimensional disturbance field in a two-dimensional equilibrium flow is given by equation 21 and that is what historically has been called the Orr-Summerfield equation.

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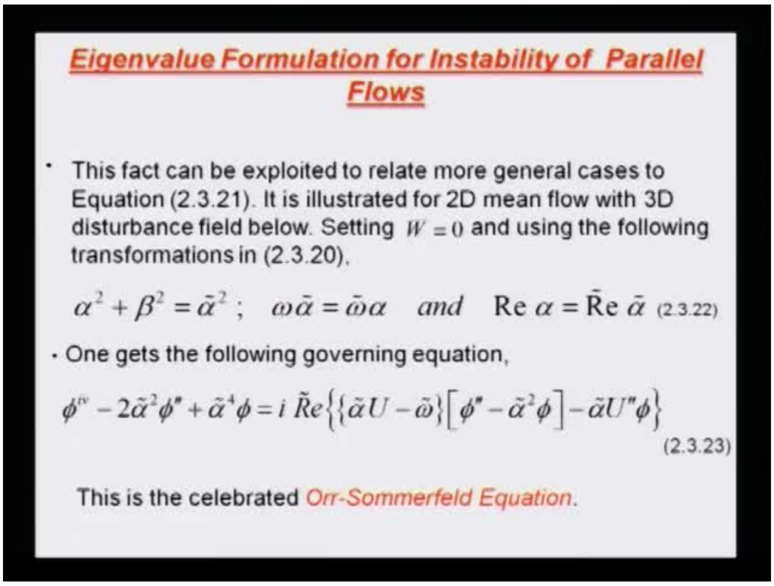

However, we can also see why we would prefer studying one form of the equation over the other, like we have talked about eliminating the variables and bringing it down to a single equation. Then we talked about subclasses of solving it for, let say we focus our attention on two-dimensional equilibrium flow. Now, we have a choice; we have a choice of studying two-dimensional disturbance field v is a v three-dimensional disturbance field.

Which one you should study? It is very legitimate to question to ask. Your initial reaction would be probably let us go for the most generic case a three-dimensional disturbance field, but what we are trying show you here, that is not necessarily this smartest thing to do. Let us a work it out. So, let say we are starting two-dimensional mean flow, then we can set the W, capital W equal to 0. For the mean flow, it is in the stream wise direction given by you, and then, we will defined some kind of a transformation.

Never mind that alpha and beta are complex, but we still can defined it in terms of alpha tilde square as the sum of this two. So, alpha tilde is probably as complex. Then given omega which is let say real for a spatial problem, I could introduce omega tilde through this relation. So, and then the final thing that I could do is Re times alpha I could write it in terms of Re tilde times alpha tilde.

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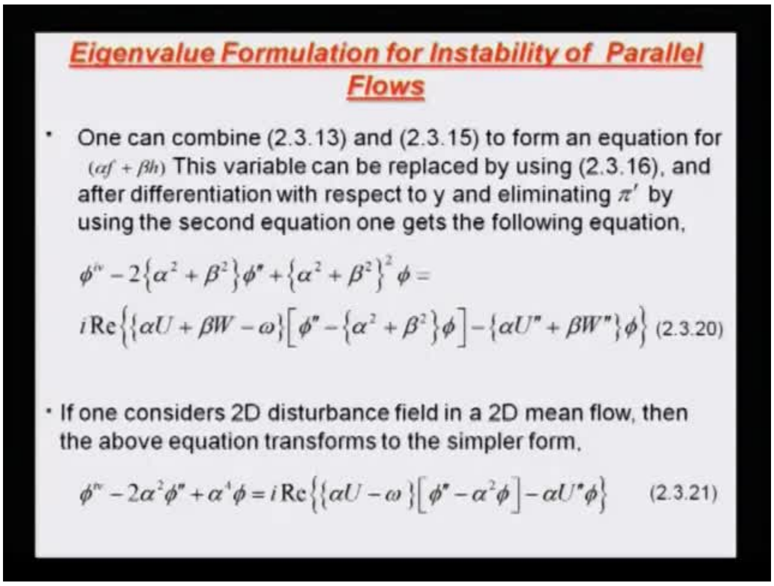

And then you substitute it in that the 3D disturbance equation that we have shown in the previous slide, that is, in 3 2, this 3 2 0, this equation, we omit all this capital W time and then this we are writing in terms of alpha tilde square, and then, you can see a significant simplification results. So, this becomes what? 5 4 minus 2 alpha tilde square and that you can see there is a commonality between that term and this term. All the thing is instead of writing alpha square here, I have replace this alpha square plus beta square by alpha tilde square.

And the same way this will also happen the same, and on the side, this term only survives; this goes away. This is alpha tilde square once again. What about this? This will also be alpha U double prime phi. So, you can see we can introduce, they are all these alpha terms. So, if I can factor out an alpha, then I will get alpha Re and that would be nothing but alpha tilde Re tilde, and if I do that, I will get exactly this equation but everywhere this parameters would become in form of tilde.

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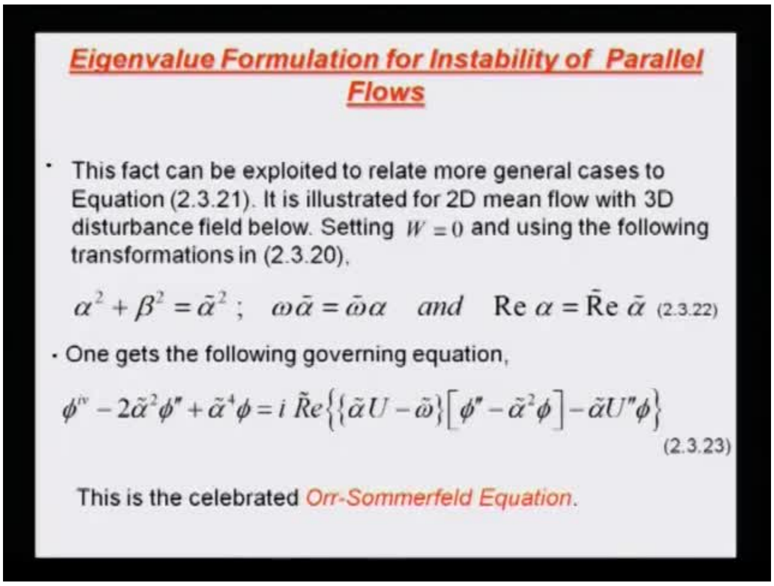

So, that is what was done by square. So, this transformation is what is called as a square transformation, and what it does basically? It reduces the 3D disturbance equation in an equivalent 2D disturbance form. So, this is also that Orr-Sommerfield equation that we talked about. So, whether we are looking at 2D disturbance field or 3D disturbance field, for a 2D mean flow, we can always bring it down to this.

Now, let me basically talk about this equation. Now, if I compare the results of 2D disturbance field and 3D disturbance field, how do I compare? What about the magnitude of alpha tilde? It is greater than magnitude of alpha. What about Re tilde? Re tilde is nothing but Re into alpha by alpha tilde. Since alpha tilde in magnitude is greater than alpha, so, Re tilde is going to be less than Re tilde.

So, if I now find out a critical Re tilde, let says it is 500 but the corresponding Re for the 2D flow will be worked most in that. So, what you are seeing basically that the 2D disturbance field is going to give you more conservative estimate. It will indicate instabilities faster than the corresponding three-dimensional disturbance field. This is what is called as a square's theorem.

So, square worked it out and he said that suppose if we all looking at in terms of magnitude wise, then it 2D disturbance field will become unstable earlier compare to with 3D disturbance field. That is the reason I suggested to you that it may not be a good idea to look at the 3D disturbance field. So, this square's theorem very clearly identifies that it would be better that you studied two-dimensional field. It will give you the more conservative estimate. This also brings to your mind what we talked about in terms of Kelvin-Helmholtz instability also. There we found the disturbance where essentially 2D. The group velocity in the other direction was 0.

So, what it basically tells you that when it comes to stability studies etcetera, it is not necessary that you make the problem more complex and try to study it. Even the 2D flow will give you an early indication of whether the flow is stable or unstable, and if there are simultaneous presents of 2D and 3D disturbance, 2D disturbance are most catastrophic then 3D.

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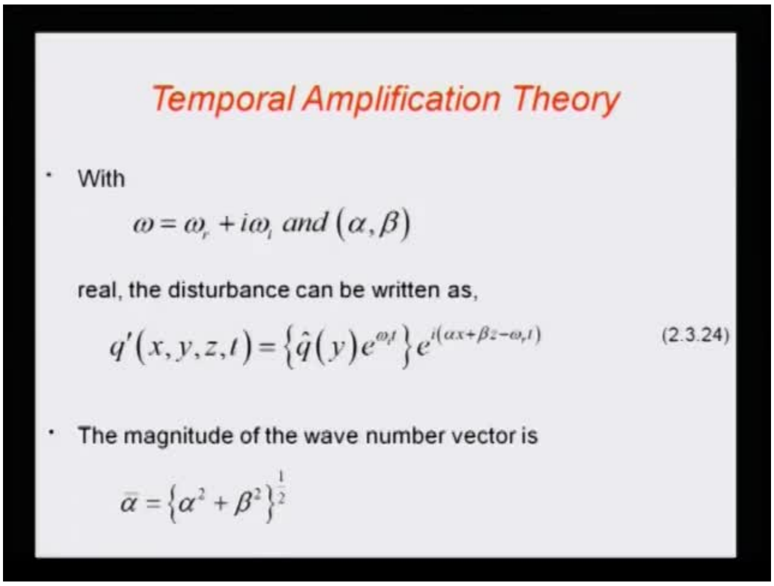

So, that is why you will find that many a times, you will focus our attention on 2D flows more than on 3D flows. This probably goes against the common brain of thought among practitioners people always think that anything bigger and more complex or to be more interesting from a physical point of view. Here, we are saying just the opposite. Sometimes the simpler thing gives you more inside than the more complex things. So, it is a philosophical point but it happens to be the case in many real life scenario also conduct simpler studies, but go deep and draw better conclusions. Never try to hide behind complexities, it does not yield too much of results.

Now, I would like to bring to attention on attributes of instability studies as I told you that ideally it would be nice to talk about disturbances which grow both in space and time. Despite that, we have classes of flows where we would see the disturbances grow in time and there would be other class of flows where it will flow in space. Give you simple example flow plus positive black body like flow positive cylinder.

If you are positioning yourself in the week of the cylinder, you would always notice that the disturbance is actually grow in time, of course, those disturbances migrate. You have seen those shade vertices, they do move, but if you look at the near wake point, the disturbance actually at a given point grows in time and after a certain formations of this convection starts. So, if you are in the near wake, things are happening in time not in space. There a bubbles there and grow, and once it achieve some critical relative strength, then one of it initiate the other one a subdominant and the scenario reverses itself in each of cycle.

So, basically the near wake of a black body is a very good example where things to happen to grow in time and those kinds of disturbances could be studying using what we called as a temporal amplification theory. So, that is a euphemism for basically considering omega as complex.

So, here, omega will be complex and the real part will determine the phase and the imaginary part depending on sign, it will tell you whether the disturbances are growing with time or decaying with time. That is what we want to do. That is easily perceived if I substitute omega as omega r plus i omega i. So, omega r part remains here in the phase part and omega i part I could put it in terms of the along with the amplitude of the disturbance quantity. So, this quantity in parenthesis here is basically the net amplitude, while this part grows with varies into y; this part tells you how it is going to grow or decay with time. Please do understand though I did not write it explicitly apart from y, it is also function of alpha beta and Re if the implicit dependence is very much there.

And if I am looking at temporal amplification theory, what I could do is basically I am treating alpha and beta is real. If alpha and beta is a real, then I can define a magnitude of the wave number. What is alpha and beta? Then the, for the disturbance field, the wave number in the x and in the span wise direction, the z direction. So, basically then, what is alpha bar alpha bar is the resulted wave number vector magnitude.

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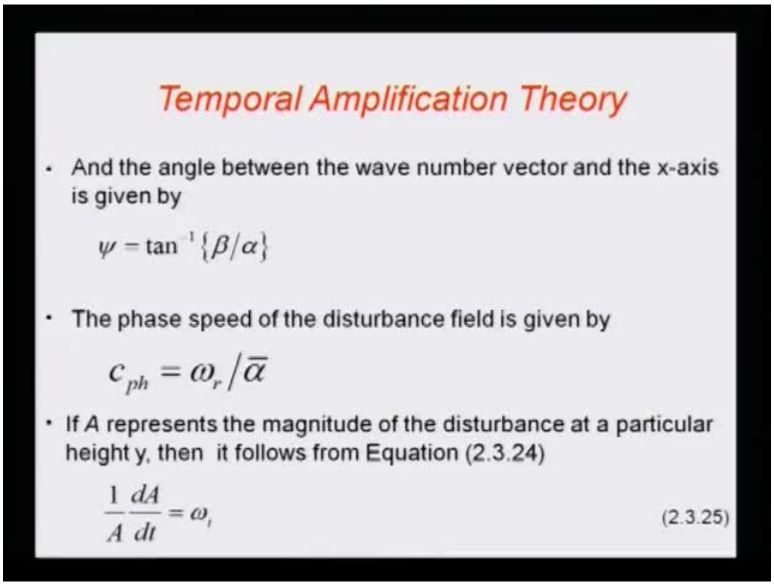

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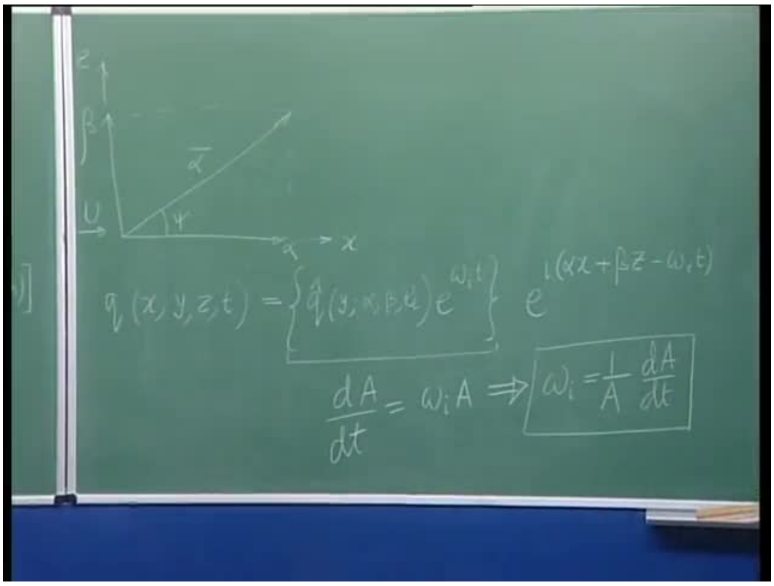

So, I have a component in the x direction alpha; I have a component beta in the z direction; this alpha tilde is the net resulted. So, that is how you interpreted that alpha bar is essentially the magnitude in the wave number vector propagation direction. So, what we are talking about then? We are talking about a component alpha and beta, and then, what we are getting is alpha bar vector that is this, that is the alpha bar.

And this angle subtended alpha bar with alpha axis that is what I will call it as psi. So, psi basically gives me the propagation angle with respect to the x axis, because this is also the x axis and this is your z axis. So, this is what you are getting. So, what happens is we have the real frequency given by omega r, and then, we have found out the result into wave number vector alpha bar. So, omega r divided by alpha bar would be the phase speed of propagating disturbance in this direction.

Now, what we also see it in the previous slide that, if I write down the disturbance quantity as a function of space and time, then we had this factor q hat which we called as a function of y, and then, I mention implicitly. They are also functions of alpha beta and Re, and what else we had? We had also e to the power omega i t.

And the phase part is given by alpha x beta z minus omega r t. So, this quantity that I have written here every in braces is nothing but a. So, this quantity is what I call as a. So, a depends on of course y alpha beta means what? x and z. So, it is like this; it is a function of phase this part times a function of time. So, what I could do is if that is a, I could write it as dA dt. If I do, what do I get? I will get omega i times this, and what is this? This is a. So, this is going to be omega i times A.

And this helps us in defining omega i as nothing but equal to 1 over A dA. That is what you have in equation 25 here. You are saying that by some means, say suppose you go the lab and do the experiment, measure the amplitude. So, you will be getting A, and if I measure it as a function of time, I should be able to evaluate this derivative and I divide by the amplitude instantaneous there. I am going to get an estimate for omega the temporal growth entry.

So, this is something that one does in temporal theory, things are pretty much simple, because what you have? You have a flow may be coming in which direction? We are talking about two-dimensional disturbance. So, the flow is coming in this direction but the disturbances are not aligned with the flow, they are actually going at an angle psi.

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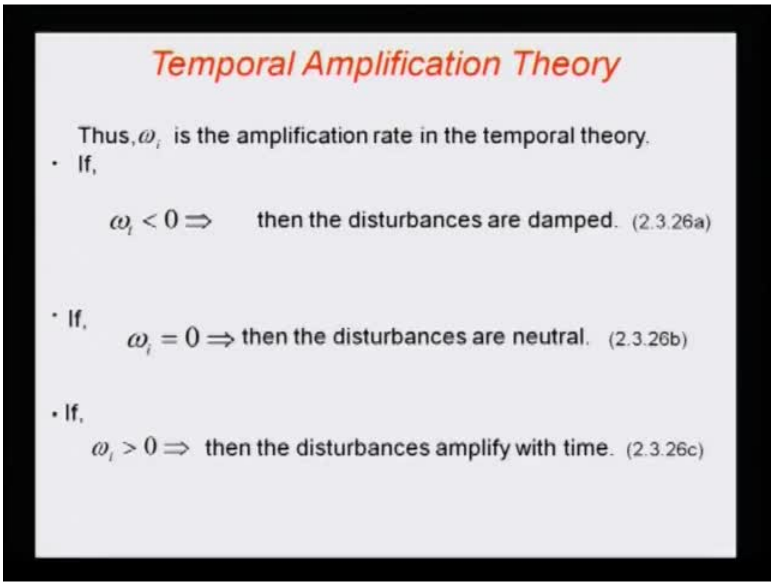

So, that psi is an ambiguously obtain from there and then we can also calculate the growth rate by following the amplitude along that. Next, what we could do is we could look at the complementary picture, but anyway, we have done this, but let us nonetheless summarize what we have omega if basically is going to tell us whether the disturbances growing or decay.

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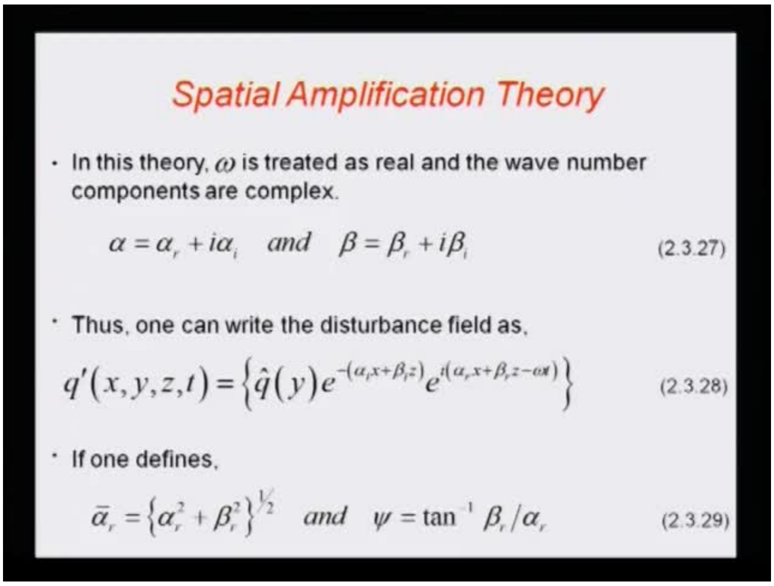

So, for example, if omega i is negative, of course disturbances are going to damp with time. If omega i happens to be equal to 0, it will insensitive with respect to time; it will not change in time, and if omega is positive, then those disturbances amplify with time, this kind of a pretty much obvious. Now, we could also have flow scenarios where the disturbances are seen to convict to space. As they convict in space, they grow.

So, what is the example? Very simple flow where a flat plate there you will see. The disturbances do not grow with time; they convict and grow in space. So then, what we should have there as a recipe, we cannot talk about disturbances with real wave numbers, a wave components. So, what we would be talking about? We will be talking about disturbances for which alpha and beta are going to be complex quantity, but that still does not say that omega should be real or complex. What happens is the complete spatio temporal problem where all of them are complex is later on intractable.

So, what happened is when this stability theory is based on Orr-Sommerfield equation, we develop Prandtl and (( )), they did see that this disturbances actually can be handle where they grow in space and treat omega as real. So, we are talking about a pure spatial theory, where omega is real; alpha and beta complex. This is what we actually would be doing for the sake of simplification on the theory, and as of an I joked about it in earlier times, we have a situation like we have a key. We do not know where the lock is. So, we first generate some solutions which we can effort to get the solutions and then you look around in the nature and try to find out where that depiction seems good.

So, it was one such effort, because of the difficulty, we have force to adopt spatial theory, and then, as let would have it, it works fine for flow over flat plate or boundary layers. This is something that we have seen all are I must say based on what we are doing currently. You in only last week we will looked at some results. We do see that if we excite a system with a fixed frequency omega, what we find that the response of the system is not necessarily be also with omega, there is some amount detuning.

So, if I give omega, it could be omega plus minus d omega; there would be some detuning. So, in reality, in most of the flows that we really come across, we can predominantly talk about temporal theory or spatial theory, but strictly speaking, we should really talk about spatial temporal growth, which you will be talking about when we come to lot of any of the simplified module that we have looking at. Suppose we are able to solve the full Navier-Stroke's equation, then we can work in frame work we do not have to make simplifying assumption.

So, some of the results in the later part of the course, I will bring those to your attention and we will see what happens. For the timing, let see historically how things are develop; people still continue to use. Please do understand that these are not trivial exercises. In designing an aircraft wing, we actually use instability theories in designing the wings. Now, talking about spatial theory, let us consider alpha as this; beta as this. Then the disturbance field as a function of space and time, we could write it like this.

Now, you can see what happens is the real part here of wave number component, and this, they actually get together to defined the phase of the problem. The imaginary part of alpha and beta brings out this factor, and please note that here, it is with a minus sign unlike what we had there in the temporal theory, there it is a plus sign. So, what happens is we can obtain the alpha i's and beta i's, and if they are such that they are negative, there net some effect is negative, then of course this will grow in space.

So, we will talk about it slowly. Now, spatial theory is somewhat reliable complicated than a temporal theory. Temporal theory we saw that we have an alpha and beta which are real that signified that we were going along a particular direction, but here, the scenario is a little more complex, because the real part here alpha r and beta r will help us in defining the phase speed, the group velocities.

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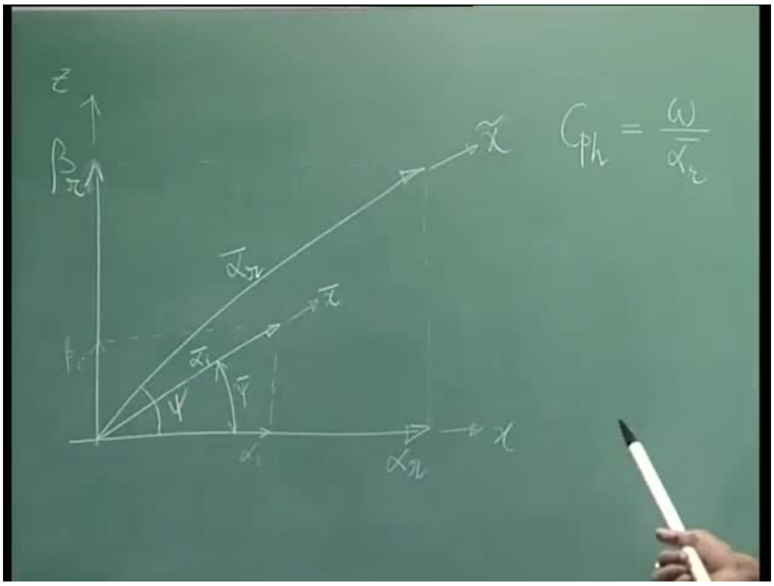

So, that would indicate the direction in which the disturbance propagating. However, alpha i and beta i did not necessarily be in the same direction; that means what? The wave mid go in one direction but it can grow in some other direction. So, this is something that we must appreciate. To appreciate that, let us once again talk about the flow and once again along the x axis let say I define alpha r. So, flow is in this direction which we are talking about U. That we could talk about beta r. So, that is in the z direction, and then, we can talk about a quantity called alpha r bar so that well, it, for beta where we could keep it.

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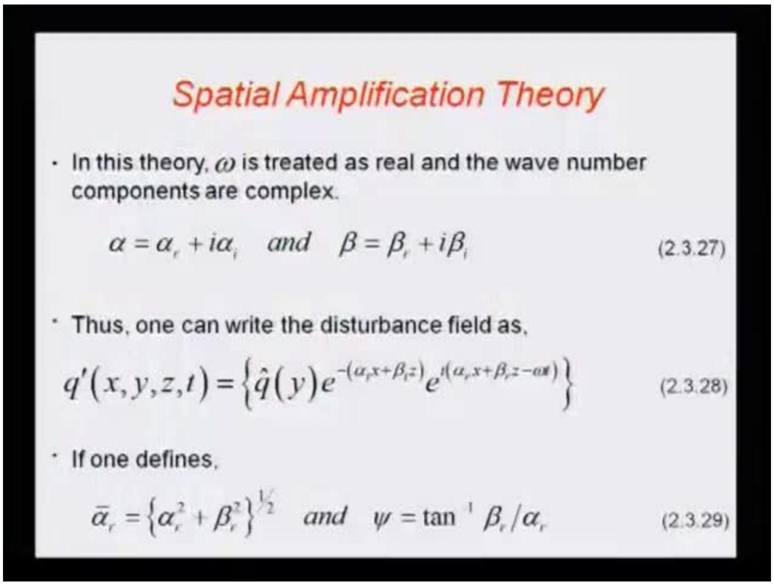

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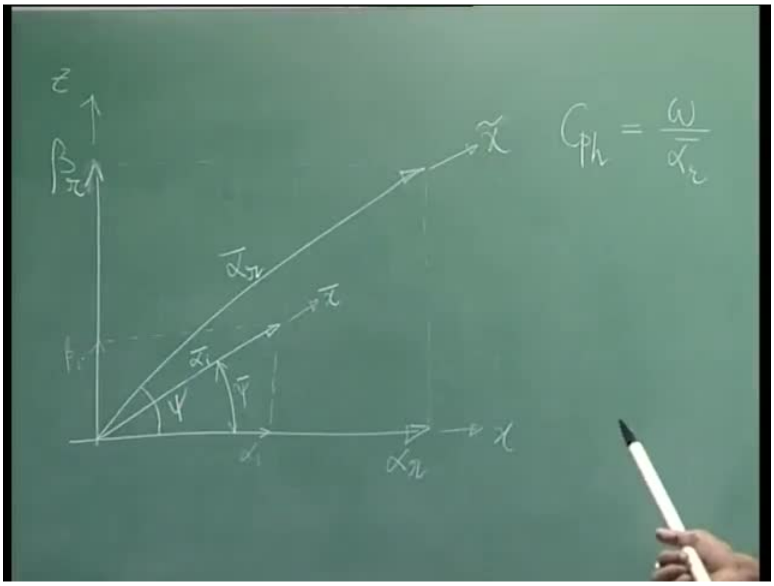

So, what we are going to have a basically alpha r vector in this direction. So, this is the wave propagation direction and tan inverse of beta r by alpha r is going to give me psi. That is what your last equation here is. So, that is what we would be looking at. It is almost like what we seen in the previous case, and what happens is since omega is real from the spatial theory, we can very clearly work out this expression for the phase speed that is nothing but alpha r.

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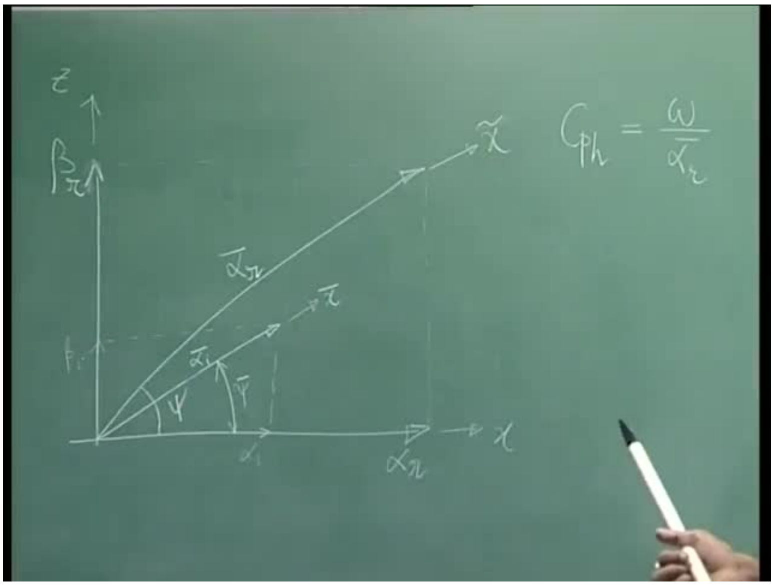

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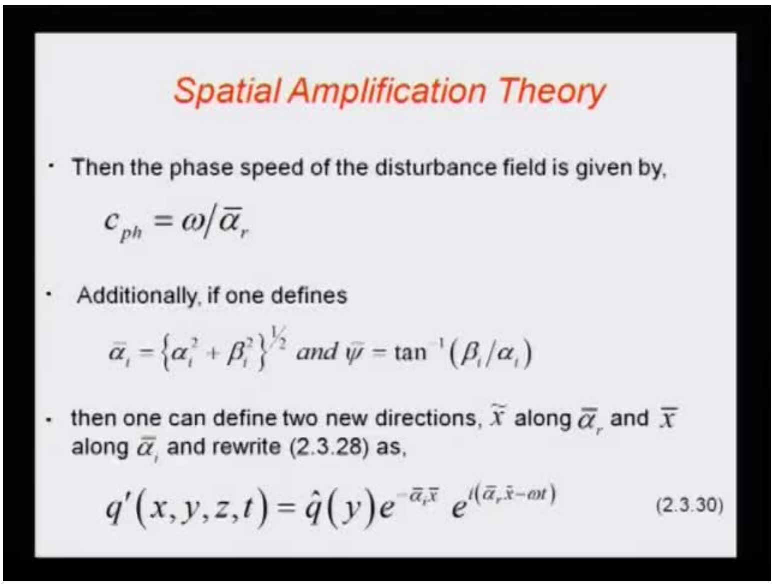

Now, you have also seen the disturbance will grow or decay will depends on alpha i and beta i. So, I could perhaps defined beta i along this axis and let say alpha i along axis. So, what I can do here is I can identity a resultant quantity which I will call as alpha i bar. So, this is my alpha i bar. Alpha i bar as you can see is this square root of alpha i square plus beta i square.

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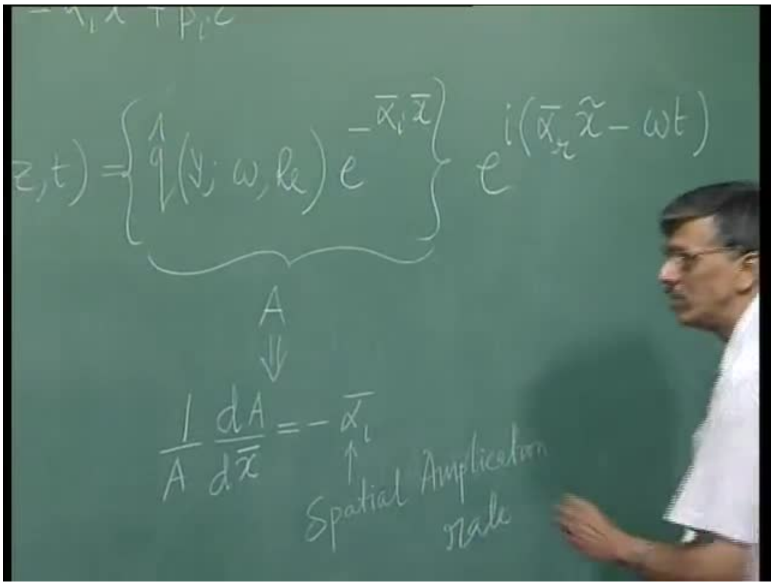

And then, I can talk about this angle as phi bar. So, what we could do is we could define axis system. This I will call it let say x bar and this I will call it well, I think here I have use this notation; this is x tilde, and along this, I will talk about that in x bar. Then what happens is the disturbance field that we are talking about as a function of space and time. Then I will write it as once again the amplitude function of y, then I will have alpha beta, well, they are going to be now the parameters which I will write it somewhat little more carefully. I will write it as omega and Re, and what happens is I have that growth top which was nothing but minus of alpha i x plus beta i z that I could write it as simply nothing but alpha i bar times x bar.

So, I could do that. So, basically what we are essentially writing is essentially this alpha i bar x bar is equal to alpha i x plus beta i z. That is what it is, you can see. So, alpha i x plus beta i z resultant is alpha i bar x bar. So, that is what we have a written there. So, this is this part once again put it in a side the brace, and phase part it is going to be this, that is nothing but alpha r bar x tilde minus omega t. Once again we can call this as the amplitude, this is the amplitude part. So, we have got it an amplitude which gives us this kind of dependents. So, from here, what I could do? I could differentiate this amplitude with respect to x bar. Then what do I get? Minus alpha i times a. So, if I divided by A, I will get this is equal to minus alpha i bar. So, that is the spatial amplification rate where amplification would decay will depend on the following condition that we have it in the slide.

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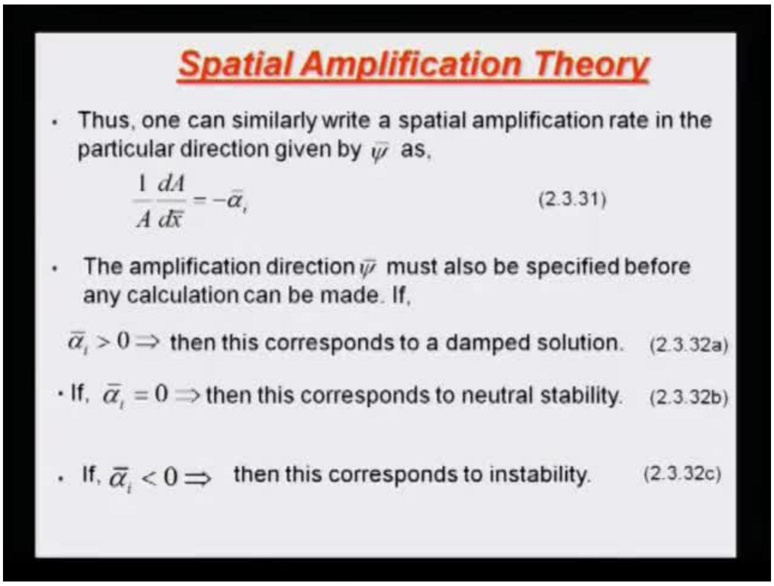

So, if I somehow contrives to prescribe psi bar and then obtain this quantity numerically or experimentally, then I will get the whole quantity as minus alpha i bar. So, if alpha i bar happens to be positive, then that would be a decade solution while alpha i bar equal to 0, then we have neutral stability, and of course if alpha i bar is less than 0, then we have instability.

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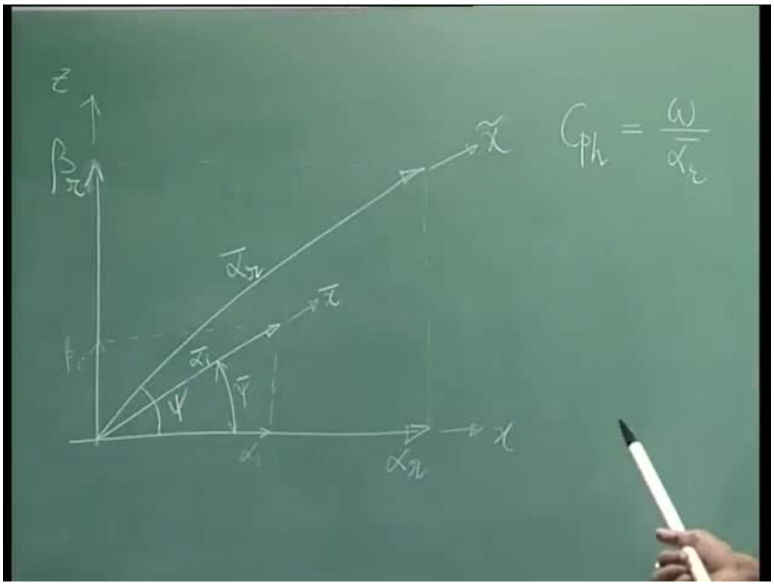

Now, this of course presupposes something that we know this direction x bar unfortunately in the spatial theory. We are going to find out both alpha r alpha i and beta r and beta i. So, we do not apiary know what is our prefer direction x tilde x bar none of these are known. So, what we could do is we could actually really try to find out the different alpha r beta r combination and corresponding alpha i and beta i we find out and we find that alpha i and beta i also would be different in different direction, I could have that.

So, what would happen that this is a very funny scenario. The flow is coming like this; the wave may go in this direction, but the growth may be in the Re another direction. So, this is far too many unknowns and this makes life very complex. How can we get around that is not an easy answer.

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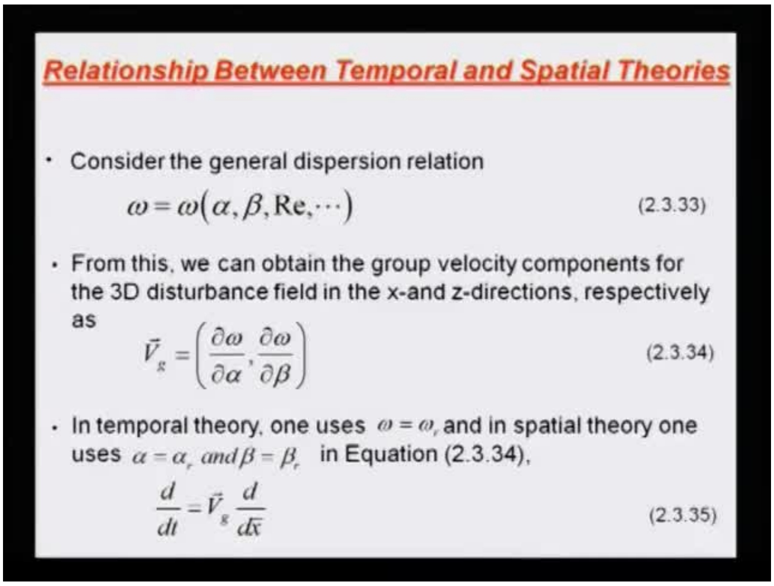

However, what we know is that we have a general dispersion relation of this form that is how the circular frequency is related to the wave number. If we have some such relation, then we can differentiate those omegas with respect to alpha and beta, and then, we get the corresponding group velocities. When we are differentiate with respect to alpha and beta, we are here talking about differentiating with respect to alpha r and beta r.

The real part, the real part defines that will helps us define the group velocity. So, that is how we have to do. Now, in temporal theory, we used omega equal to omega r, and in spatial theory, we use bar; this is not strictly a correct thing, but what it basically says that in calculating the group velocity, we will have to use alpha r and beta r; omega is already real, and so, in temporal theory, what you do? You take the real part of omega and differentiate it with respect to given alpha and beta which are real so that sign and we goes. Now, in the spatial theory, omega is real but you do have combinations of alpha r's and beta r's, and if you differentiate them, and then, you get the corresponding group velocities.

So, different alpha different beta r's will play in different directions find in a group velocity. However, we can see that some of the time growth can be related to the spatial growth if we know the speed at which the disturbances troubling with.

So, that is what is written here heuristically. We will prove it, we will prove it rigorously somewhat later but this is what Schlichting did while Tollmien was the first to calculate the critical value of Reynolds number above which flow becomes unstable. Schlichting actually worked out the various growth and decay rate for using spatial amplification theory and please do understand that this kind of complications that we are getting it is, because we are talking about three-dimensional disturbance field.

So, alpha and beta making all this things, but if suppose I too look at 2D disturbance field, then what happens? Then everything goes in the x direction. So, all this complications are with respect to three-dimensional disturbance field; it is not somewhat imaginary or illusory to consider that.

Because let say if we are designing in a craft wing, we do out very often see that a very good exercise for one to undertake would be to drop away an aircraft a dark night and harm yourself with an infinite camera and just take the picture of the wing alone and you will see lots and lots of interesting thing happening, because who know the heat transfer rates are going to be different depending on whether the flow is laminar or turbulent.

So, you could actually pick up the foot print or the portrait of which part of the wing is laminar, which part of it is turbulent, and then, as you go alone, we will see which way this laminar and turbulent flows are there and they are not going to be necessarily studied. Depending on flight condition, you will see all kinds of instants and the better thing is to do is sum up an experiment.

You put an some kind of a exciter on the wing, a point exciter on the way, and let say we create a mechanical vibrator on in the way, then vibrate it, and from that single point, disturbances will come out in three dimension and there we will see all this nice things we should be able to pick up which way the disturbances are propagated. Well one has to do additional work; you may have to probably heat the wing from inside to amplify those pictures better, but you see that spatial theory is little more complex than temporal theory.

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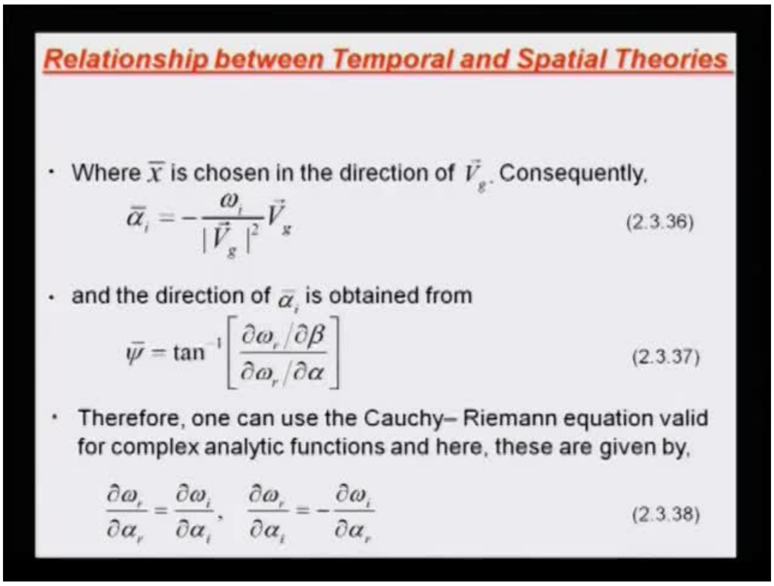

So, that is what we need to look at. Now, that is what we are talked about that we can actually relate the spatial growth rate with the temporal growth rate provided. We know the group velocity and the group velocity components we are just now talked about. This is the group velocity in the z direction; this is the group velocity in the x direction. The ratio of the two defines that angle psi bar that we have shown here. So, that is what you do. How do we get this? This is something is a heuristically arrived at by Schlichting. Schlichting did all that, because you see, temporal calculations are easier to make compare to spatial and he was doing temporal calculation.

So, he was basically analytically and numerically obtaining omega i, but then, if you know the group velocity, then you can convert that temporal growth rate into spatial growth rate. This is what the implication of this equation is. Now, why does it happen that way? Why does that happen is given out by properties of complex functions. So, what we have? Think of the real part of the frequency omega r.

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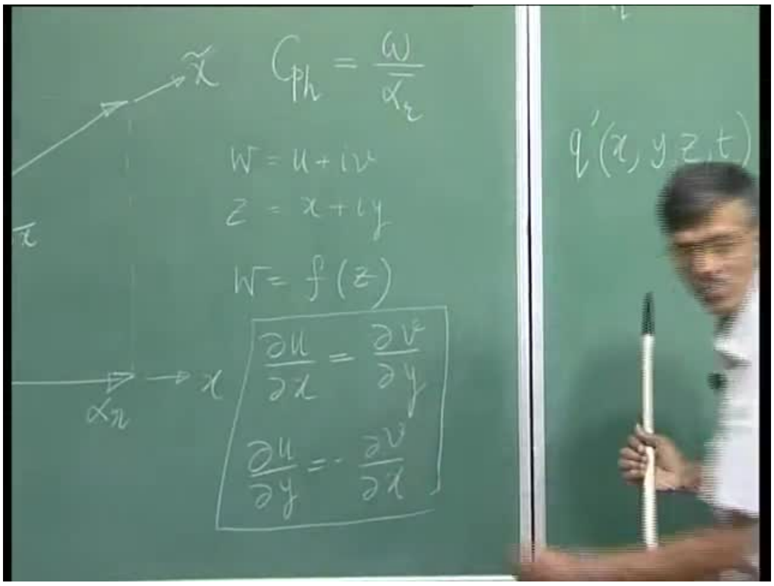

So, what I could do is - I could write this complex omega is a function of complex alpha. If I have that, I could write down the Cauchy-Riemann relation. So, this is basically a Cauchy-Riemann relation; omega written as a function of, I do not know if you recall that is what we do. Suppose we write W equal to u plus iv, and then, we write Z equal to x plus iy. Then we talk about omega being a function of z, and for the derivatives to be analytic, we get the Cauchy-Riemann relation.

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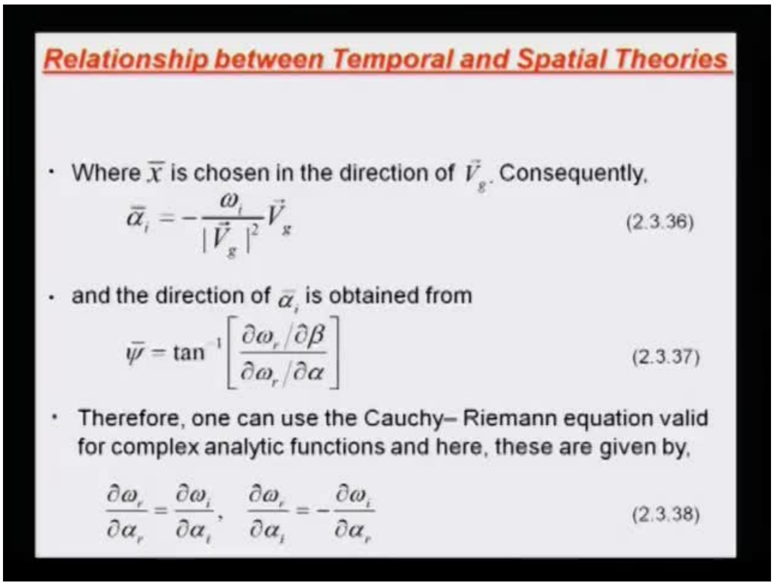

How do you get, how do you get? Tell me how do you get. What is that? And this is how we get Cauchy-Riemann relation and that is precisely what we have written. So, you see that; so, omega as a function of this, we have derive this where it will take little more time. So, we will continue with this discussion in our next meeting tomorrow, well, amplify this issue, how we get this equation 36. I will stop here.