Instability and Transition of Fluid Flows
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Lecture No. # 07

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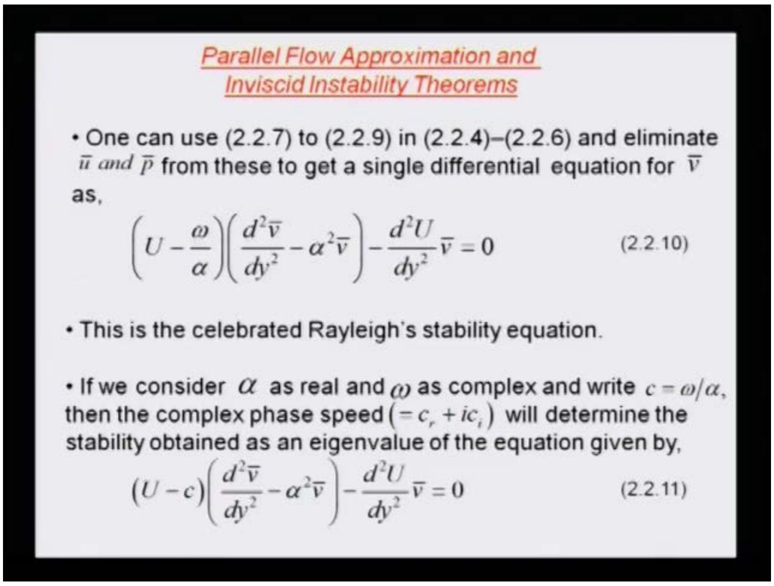

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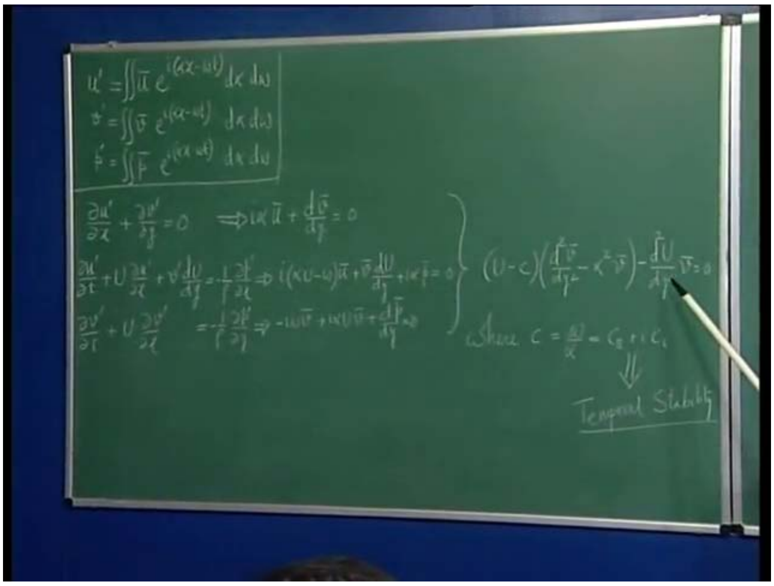

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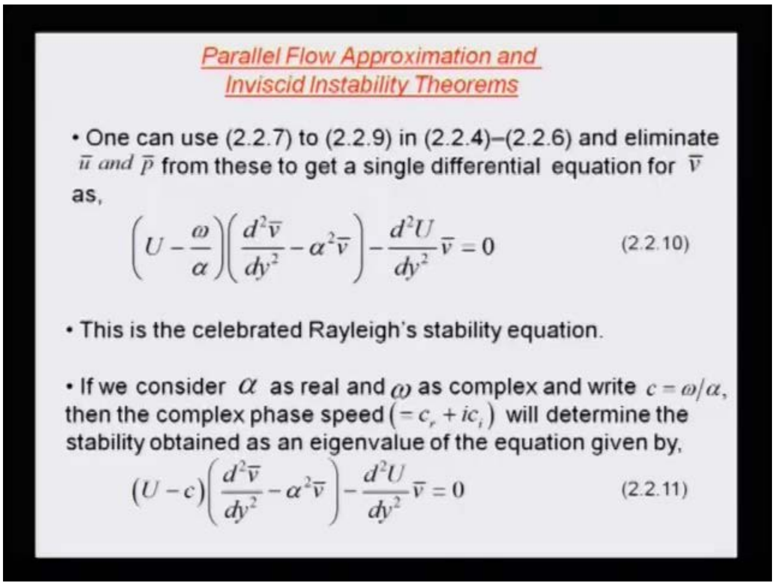

So, in the last class, we discussed about the instability of parallel flows considering the disturbance quantities are covered by inviscid equation and we derived that equation 10 and the board there. This was derived by defining, the velocities, of velocity component and the pressure in terms of Fourier Laplace transform. Plug them into the disturbance equation that was derived and that gave us this equation for the amplitude of all this disturbance quantities u bar, v bar, p bar, and from this equations, we did eliminate u bar and p bar to get a single equation. That is your Rayleigh stability equation. This is quite a famous equation, originally derived by Rayleigh and trying to track the inviscid instability of flows.

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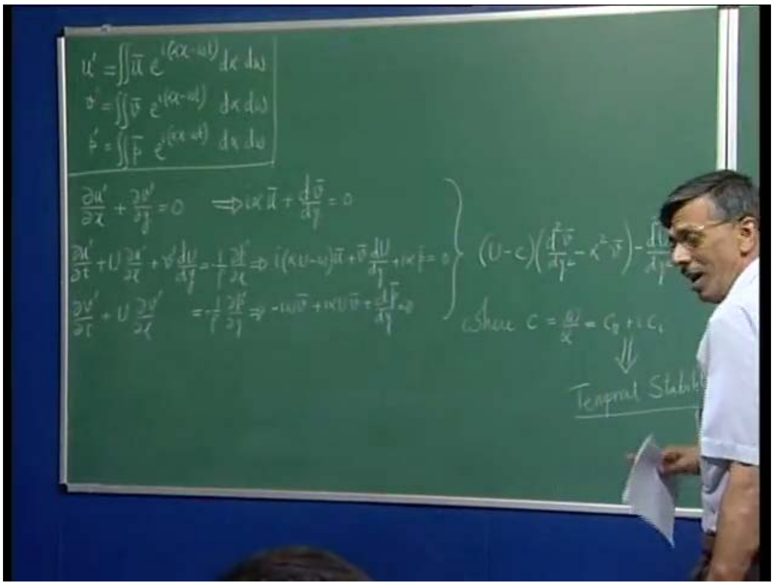

Now, at this stage, we have to make a conscious decision. Which way to go? Now, you can see that, in this representation where we define the disturbance quantity in a spectral plane, we have the wave number alpha; we have the circular frequency omega, and what did we decide to do, where we said that when we are studying stability problem unlike the equilibrium flow problem, we must have the liberty to keep them complex. If we keep them complex, then what happens? The alpha being complex means what? I could think of that alpha the real path will give us a phase. So, what it is right? e to the power i. So, this will be a phase definition.

What happens to the imaginary part of alpha? Imaginary part of the alpha will tell all the disturbance quantity may additionally decay with x or increase with x depending on the sign of the imaginary part. What will be the sign? If the sign is positive, so that will be alpha r plus i alpha. There is i setting outside.

So, that will give you minus alpha i x. So, if alpha i is negative and you are talking about x in the positive direction, then that alpha i would be correspond to the stable solution, that will damp the solution, but please be aware that a priory you do not know. If the flow is going in the positive x direction, the disturbance has to go in the positive x direction. It can go in the opposite direction negative x.

If it does so, that same alpha i which I called as stable for downstream propagating disturbance will become unstable for a upstream propagating disturbance. For that, I mean I will go with the result that is given in most of the books except the one that we have written recently. We would only talk about downstream propagating disturbance. When it comes to upstream propagation, will specifically mention and you will be aware of it; I will make you aware of that. So, alpha r is the phase. Alpha i determines the stability or instability. For a downstream propagating disturbance, alpha i being negative would be the unstable scenario. That is about alpha, what about the time variation? Omega also can be complex quantity. The real part will of course once again define the phase.

The imaginary part now because of this additional minus sign here could be that omega imaginary. If it is positive, it is going to be unstable for downstream propagating disturbance, whereas if it is negative, the downstream propagating disturbance that it correspond to stability. How insert that? You realize that it is quite difficult to talk about disturbances which are simultaneously varying in space in time.

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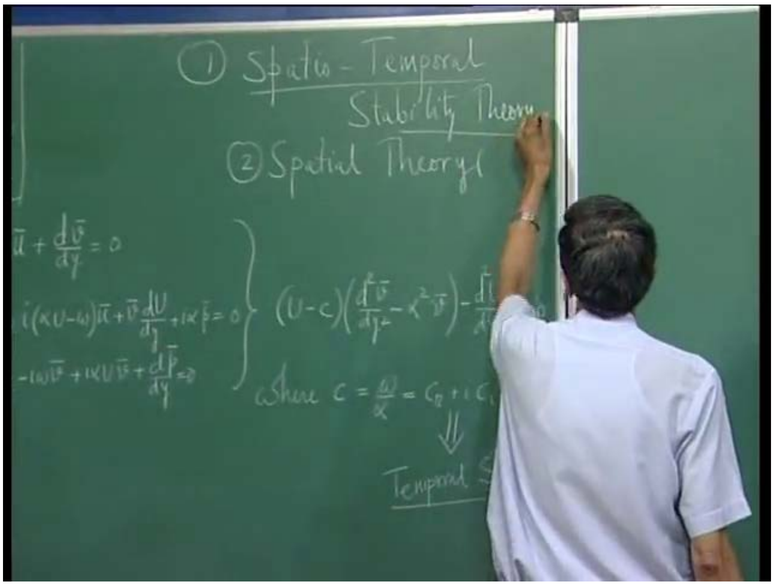

Such problems we will call it as Spatio-temporal problem; that means the space in times would be simultaneously varying for the disturbance quantity and the corresponding study will be called as Spatio-temporal stability or instability theory and somewhat involve it is not so easy to tackle, instead we would take a simpler approach where we would talk about. So, if this is the all-encompassing thing, then we will have sub cases. We can talk about spatial theory. The name itself is suggestive of what? We are going to talk about disturbances that grow or decay in space.

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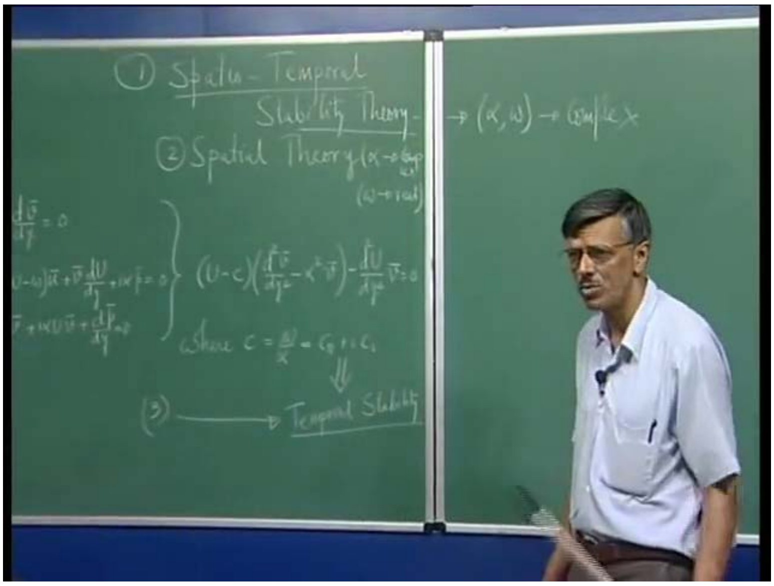

So, that would correspond to what? So, this corresponds to alpha omega complex, both are complex for Spatio-temporal theory, and for this case, what we will find? Alpha as complex while omega will be real. The third possibility is one of temporal theory and this is what we are talking about here. In temporal theory, what will happen? We are going to talk about disturbances which grow in time; so, alpha will be real and omega will be complex.

So, omega by alpha which we have called as the phase speed will have a real part and an imaginary part, and once again whatever applies for omega, we will also apply for C in terms of determining stability or instability, that is, if C i is positive, we have instability for downstream propagating disturbance, and if C i is negative for downstream propagating disturbance, it would indicate stability.

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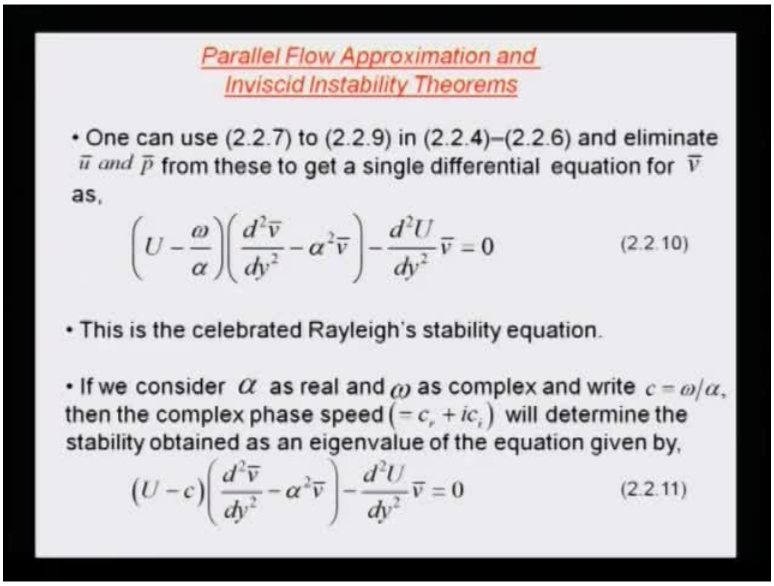

So, this is what Rayleigh did. So, considered alpha as real; omega is complex and written the phase speed in terms of omega by alpha. So, we have a complex phase speed and we will find out whether it is stable or unstable. You see this where the days where you did not have computer. So, you do not just simply go and solve it. First foremost, you do not have analytical expression for U. U is what? U is the equilibrium flow that we assumed it to be parallel; that means it is only a function of y. How do you get it? Imagine this is a Rayleigh’s time triangular has to come out with this boundary layer theory later another 40 years downstream. So, U of y was completely unknown, and one could take something but that kind of assumption would not or approximation would not interest people of Rayleigh's caliber, worth I say let us figure out.

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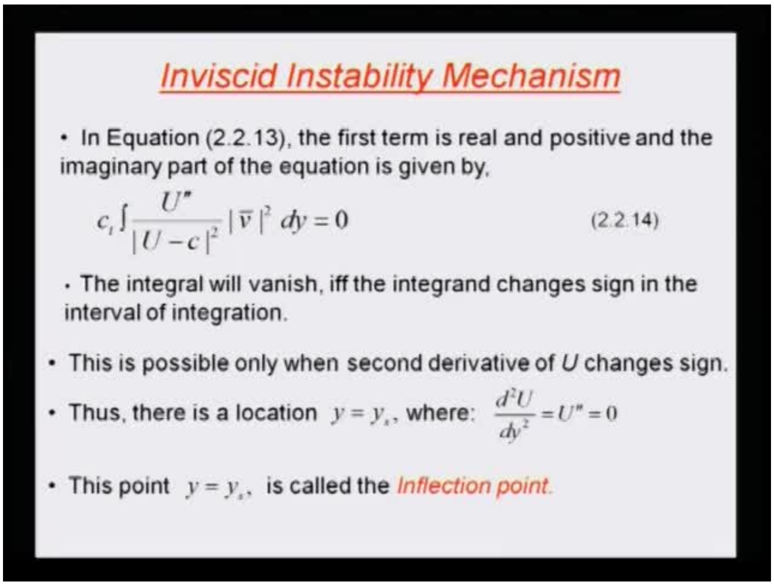

If you can get some information about the stability or instability without you in solving this equation, that is what we are going to do next. So, please pay attention to what we do to equation eleven now. What Rayleigh did was the following that multiplied the equation, the stability equation by the complex conjugate of the amplitude of this.

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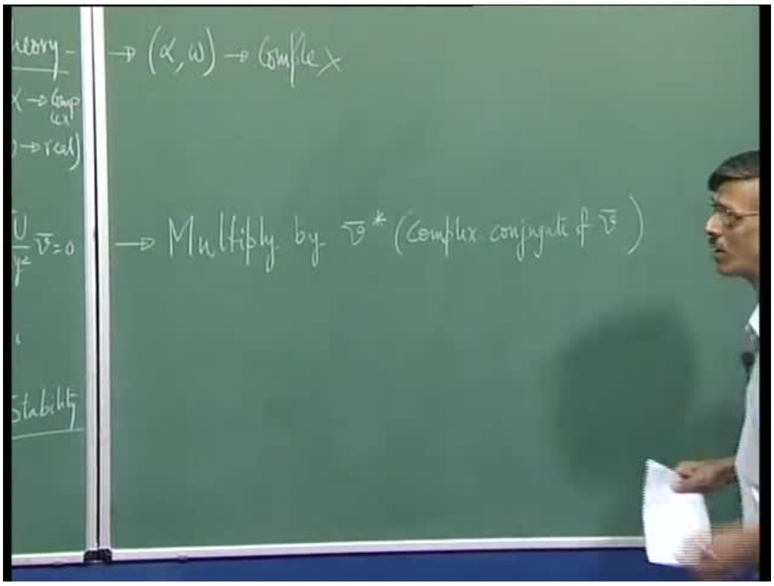

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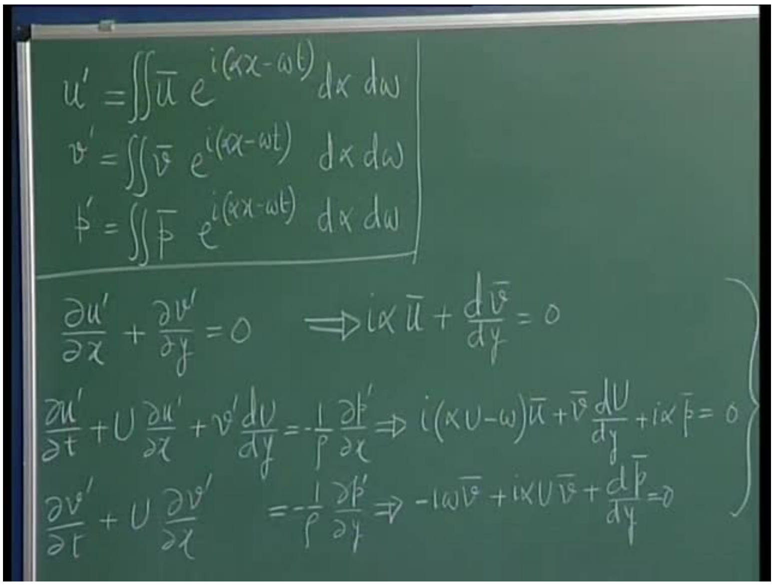

So, basically what we is I have written it down here. So, what we do here? We multiply this equation. What is v star, u r star? This is nothing but if the complex conjugate of, you realize that all this disturbance amplitude would be in general b complex, in general b complex. What does it achieve for you? Well, if it is complex, then you can talk about u, v and p will have their own phase and they need not be all same. That is why you will have a different value of the relationship between the real and imaginary part. So, what we are talking about is done. Take that equation and multiply by its complex conjugate.

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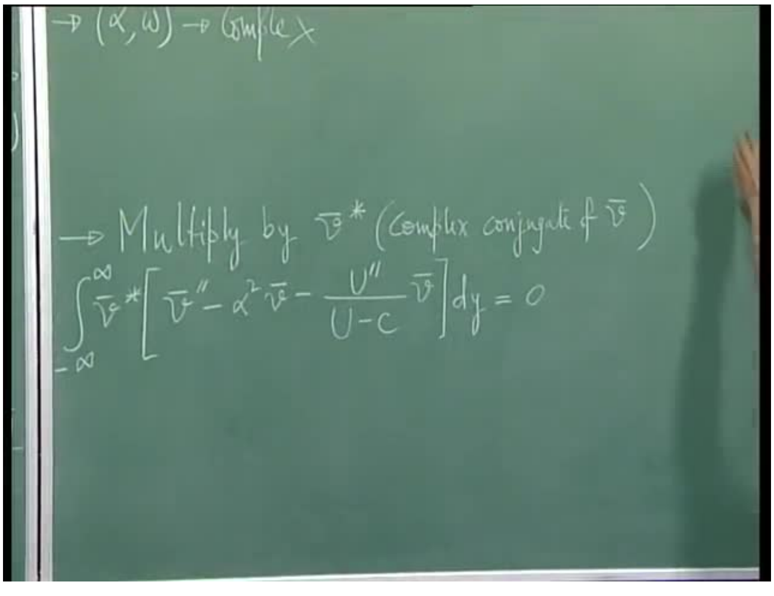

So, basically we will just r investigating v bar star multiplied by I will just simply divide this equation by U minus C, well, and from now onwards just for simplicity, I will introduce prime to indicate derivative. So, instead of writing d 2 v d y square, I will just simply write v bar double prime. So, that is understood. I have this alpha square v bar, and then, what I have? I have this quantity U double prime; so, that is that u double prime divided by U minus C and multiplied by v bar and we are going to integrate over all possible range of y that can occur in the problem. So, without loss of generality, we will say the y varies from minus infinity to plus infinity.

Now, some of you may be already worried why I am trying to divide this, what happens if this becomes zero? Then we will be in all kinds of trouble. However, you realized that U is a quantity jet is real is the velocity equilibrium flow; C is complex. So, the real part can match but still you would be left with the imaginary part. So, the chance of dividing by 0 is the precluded when we are studying a temporal stability problem. Although you would have a situation for neutral stability, neutral stability would correspond to what value of c i would be equal to 0.

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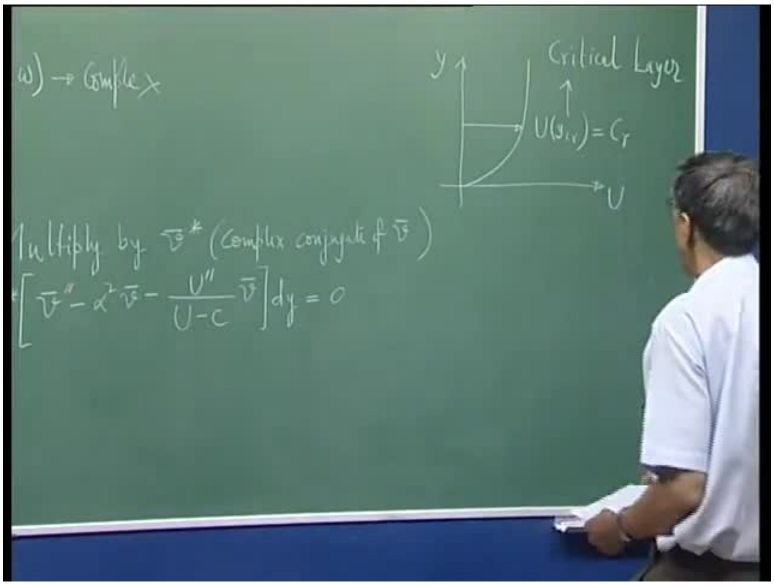

So, for a neutrally stable flow, you will have a height where this can match. What are we talking about? You see basically we are talking about the following that if I plot U versus y, then I could get a profile like this. So, at each height, I have a value of u. At some height, I could have U of y. I will call, that is, I some critical height where that should be equal to C of r.

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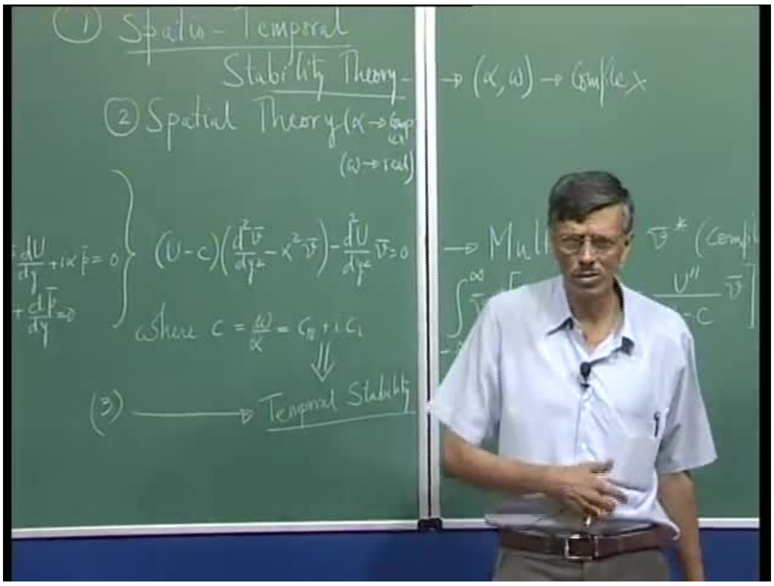

So, what happens is basically I will call that as the critical height. So, why C r is the critical height at which, U minus C r equal to 0, and if you look at this equation, what happens? This equation becomes degenerate. Do not worry about division but if you look at this equation itself.

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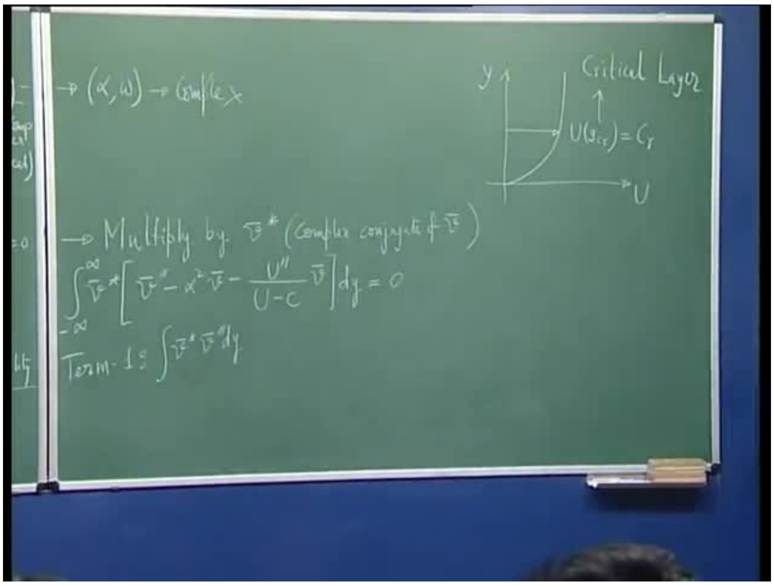

If I have this coefficient equal to 0, so at that height, the order of the equation goes (( )), because that highest derivative is the second derivative, and so, we will have a zero coefficients; so, you cannot really talk about. So, it does create a problem, and also you see, what happens is if U minus C become 0 for the neutrally stable solution, at this height that below this, the coefficient is positive; above it, is going to be negative. (Refer Slide Time: 15:00) So, across the critical layer, the coefficient flips sign.

So, we are basically confronted with a situation where we want to get an inside into a solution of an equation which is an ordinary differential equation, but it is a variable coefficient ordinary differential equation and it does not give you immediately a analytical solution. So, that is a prompted Rayleigh to look at it, but baring that critical layer, specifically for neutral solution everywhere else, there is nothing we are losing by casting this equation in this form. Now, let us look at the terms one by one. So, the first term that I have is this this multiplied by this. So, that is basically integral of…

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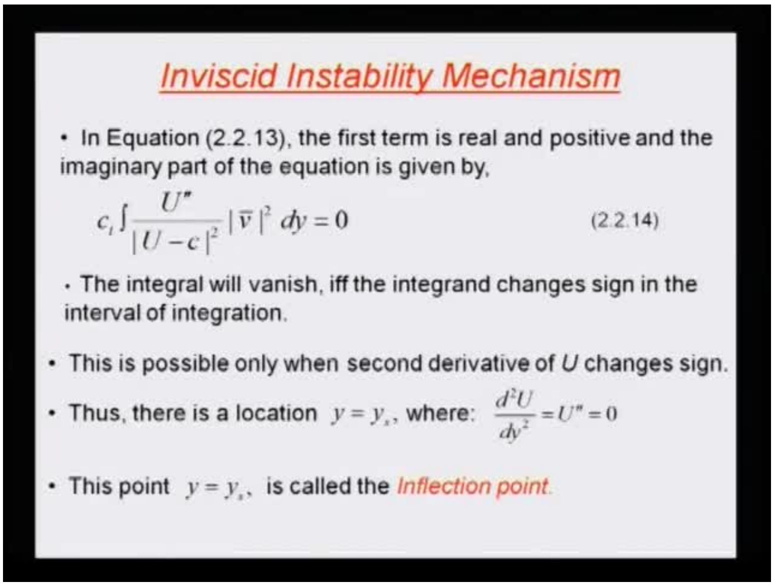

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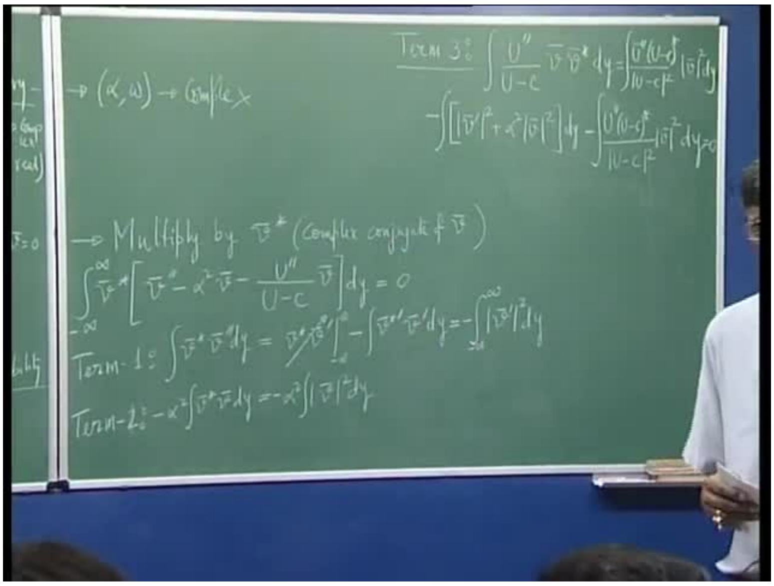

So if I do, what do I get? I can do it integration by parts, I integrate it once, then I will have, sorry, and this is over the limits evaluated, then what do I get? Here, I will get this differential of this that will be v bar star prime. Then, I have already integrated this once. So, that is there, and then, again I have integrating.

Now, you can tell me the fate of this term. If we are going to the end of the region of interest, that is why disturbances go to 0. If the disturbances go to 0, the conjugate also will go to zero. So, this term does not survive, and what about this? It is an interesting quantity. This is a quantity times is complex conjugate. So, what do you get? That will be the more of v prime. So, this then equal to become minus.

So, one interesting thing is whatever may be the value of the disturbance, this is essentially a negative quantity. All of you would agree with me that this indeed is negative. So, first arm gives us this kind of simplification. Term 2, the second term is rather simple, because that is this minus alpha square v bar star times v bar. So, I will get minus alpha square. So, I will just simply write it as the same way that we have seen this into its conjugate. So, this will give me v bar square d y, that is also negative sign.

So, what is a, yet that we are left with now. We need to really worry about that third term. So, the third term that we have of is this U double prime divided by U minus C, and then, we have v bar and v bar star d y. Well, we can bring in some kind of simplification by doing this. Multiply the numerator and denominator by its the conjugate of U minus C. So, down stairs what will we have? We will have U minus C whole square because we have multiplied by the conjugate of U minus C.

What about this two? Now, what you get? You can collect all the terms - the first term, the second and the third term. What do I get? This is negative; this is negative. So, I will take a negative quantity is together, and inside, I have v bar prime squared plus alpha square, that is that and this term is this. So, that is also negative and then we have U double prime and U minus C star divided by U minus C square this equal to 0. That is what we want to get.

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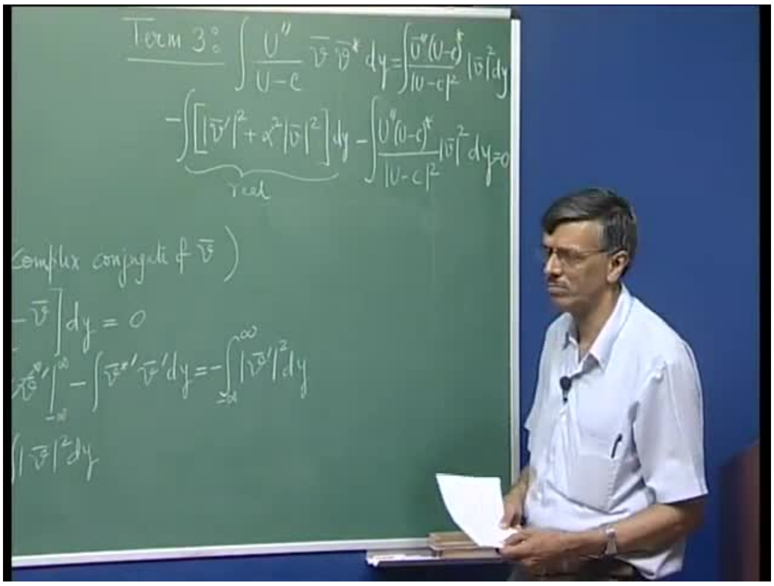

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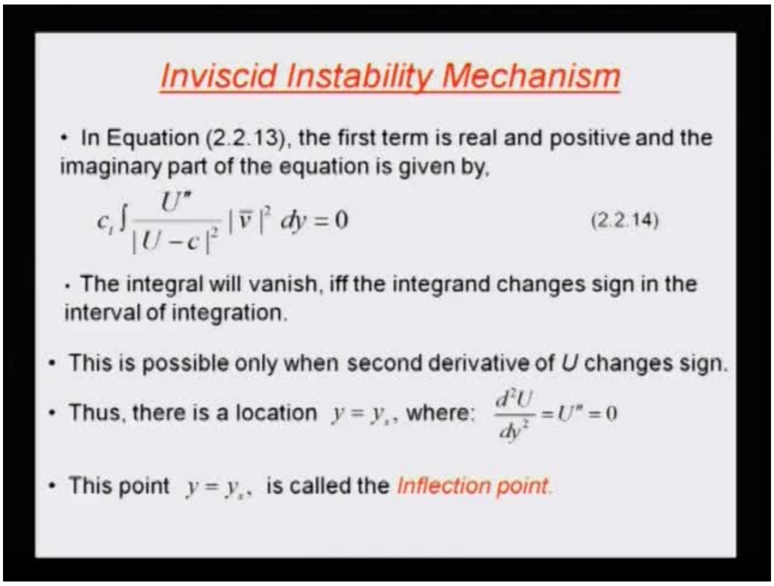

Now, what can you say about the first quantity? Is it real imaginary complex? What is the first quantity? Strictly real. So, that is something that we have a clear view. This is purely real. This of course this part is real; this part is real; this part is real but has a complex nature. So, I can split it into a real part and imaginary part. A real part will go with this. What happens to the imaginary part of this equation? That is what is written on your black. This of transparency here 14. The imaginary part is this.

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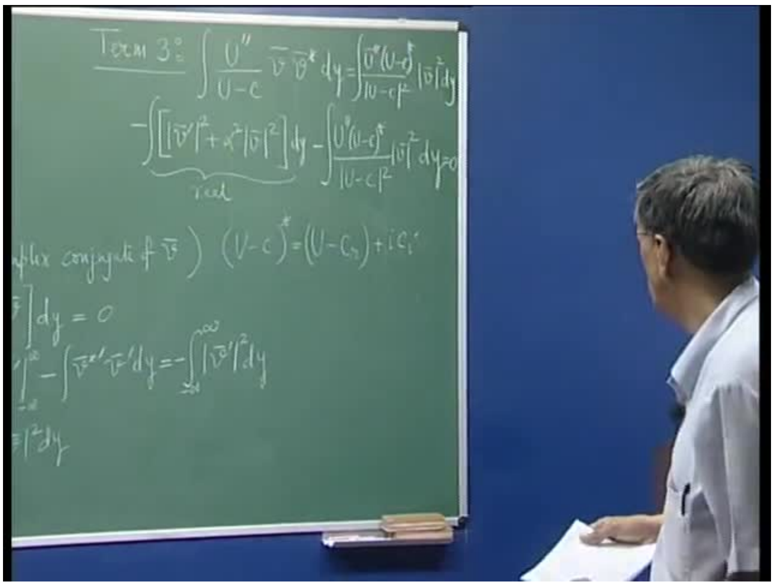

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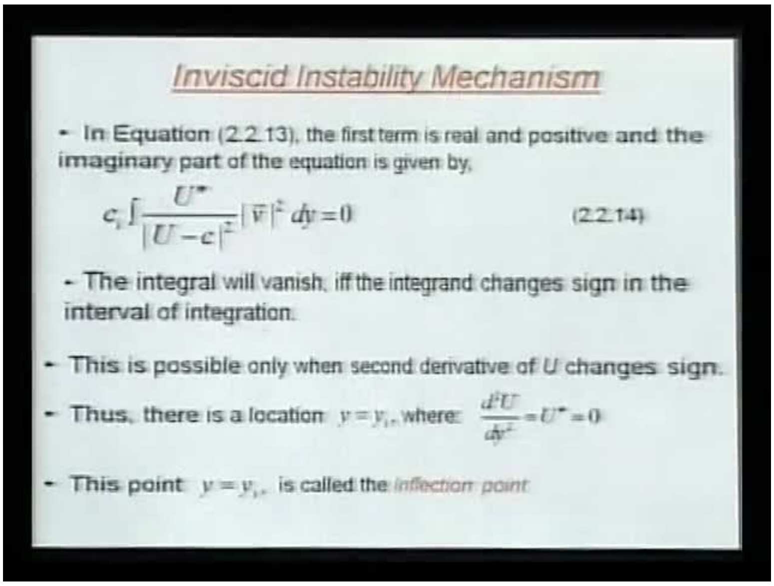

Because U minus C the conjugate is what? U minus C real, that is the real part, and there is a conjugation. So, that will be C r minus i c i. There is a minus i there that makes it plus i c i. So, you have that equation 14 over there. So, this is where the ingenuity of Rayleigh's analysis comes out a glory that you just simply look at this. The imaginary part of that equation, when it is integrator over the whole domain, gives us this following relation.

So, the integral has to vanish. This is a positive quantity; this is a positive quantity and C i is something we are looking for. So, C i cannot be equal to 0. So, we have to make this part equal to 0. How can that happen that U double prime have to flip sign somewhere, but the positive component will negate the negative component. That will yield you a non-trivial solution. So, what it essentially says that only when this integral is equal to 0, then your C i did not be 0. Wherever this integral does not yield the value of 0, then C i has to be equal to 0.

So, that is what we are saying that this integral will vanish when they integrand change a sign in the interval of integration and this is possible only when the second derivative of U change a sign. Thus there is a location y equal to y s, I will call that where the second derivative equal to 0. This point is called the inflection point. You are familiar with the concept of inflection. If I am defining a curve surface y as a function of x, then the first derivative provides us with the slope. A second derivative divided by something gives us a un-estimate of the curvature.

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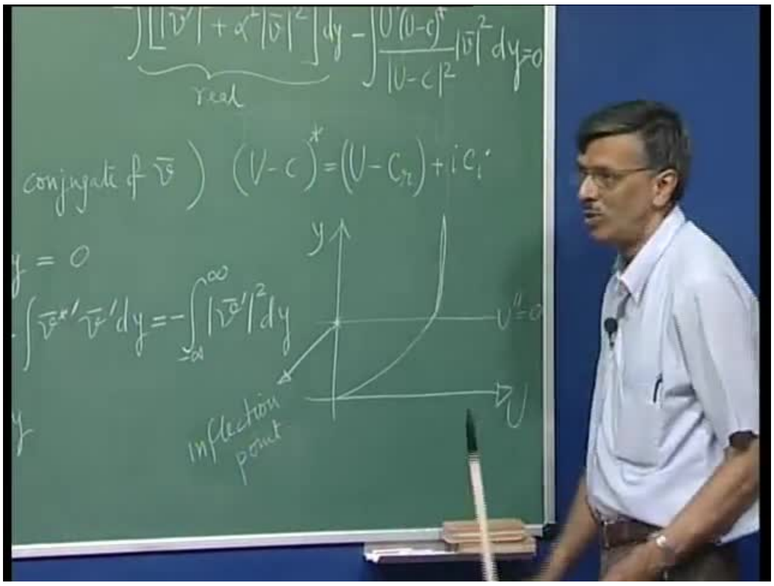

So, when the second derivative becomes 0, the curvature is 0. So, if I am talking about a velocity profile which we sketched before, if I am looking at this, I would have a velocity profile where it will have, let say the positive value, then it will have a negative value, I mean zero value somewhere. So, there would be a place where I will have U double prime equal to 0.

So, this point is called the inflection point. So, what happens is that allow Rayleigh to draw a very (( )) conclusion which enunciated as a theorem which is called Rayleigh 's inflection point theorem, which states a very simple observation that we have discuss so far that, if I want to get a unstable solution, that means C i non-zero stable or unstable solution, then what I need? I need that integral to vanish and I must have the second derivative to vanish somewhere in the interior of the domain. Please mark my word. I am using the word in the interior of the domain, it is not necessarily at one of the end the of the domain.

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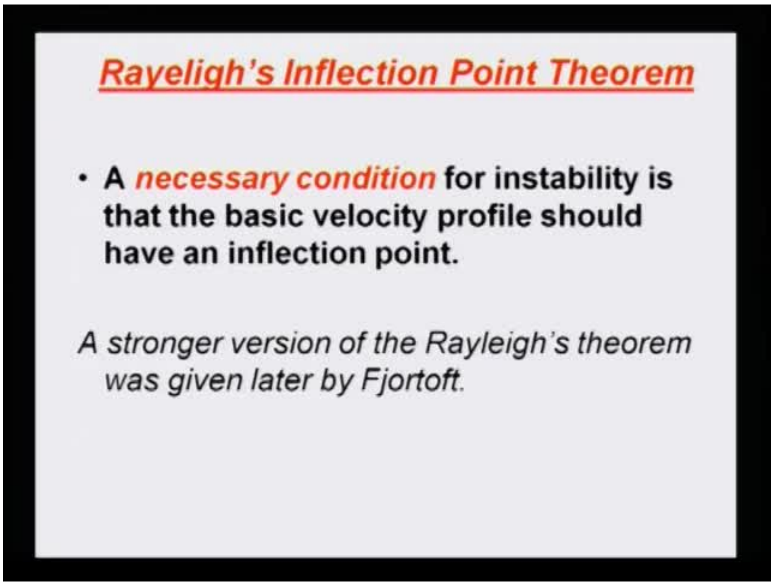

If I try to recall, remind you of boundary layer profiles, you have all heard of (( )) profile. The zero pressure gradient profile. What happens to its second derivative? There the second derivative is 0 right at the wall. So, this Rayleigh's inflection point theorem which gives us a necessary condition that the second derivative must vanish, are there must exist an inflection point. This inflection point has to be in the interior of the domain, it will not be necessarily on one of the end of the domain. Then of course this theorem does not work and we gave you an example. A zero pressure gradient boundary layer is boundary layer for which the second derivative other inflection point of the velocity profile is right at the wall. I would ask you to show it yourself.

Just show it how do we do it, how do we do it? Now, look at the boundary layer equation. Boundary layer equation is a reduced x boundary in term equation and there you see the viscous term comes with the second derivative, and on the no slip wall, what happens? The second derivative is related to the pressure gradient, stream wise pressure gradient. Now, you see, when you have a zero pressure gradient flow, the second derivative is automatically 0. So, we do not have to do anything, we can just enunciate it and you can realize also that this is a necessary condition.

So, if you have velocity profile and you do not have a inflection point, do not worry about in viscid instability. The good news is - whenever you have a velocity profile with inflection point, you would find that there would be some frequencies for which the flow is going to unstable. Basically in viscid instability gives you a much more of a critical scenario unlike what we would expect from viscous analysis.

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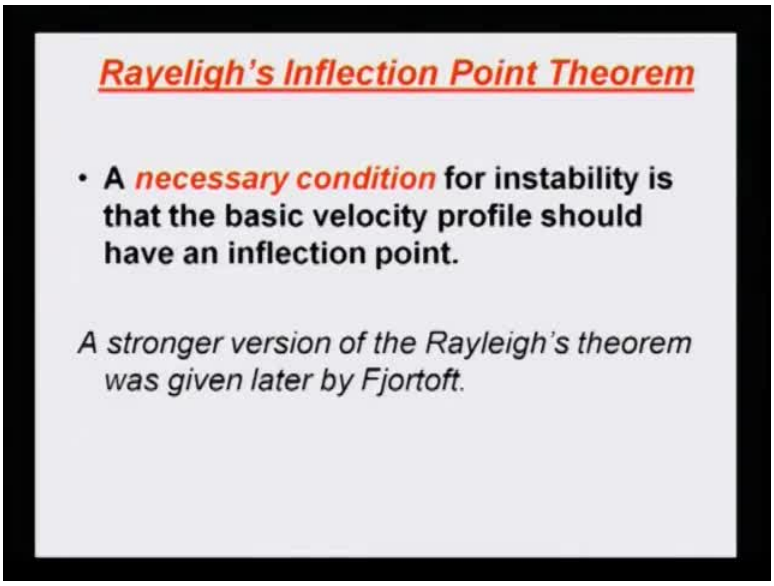

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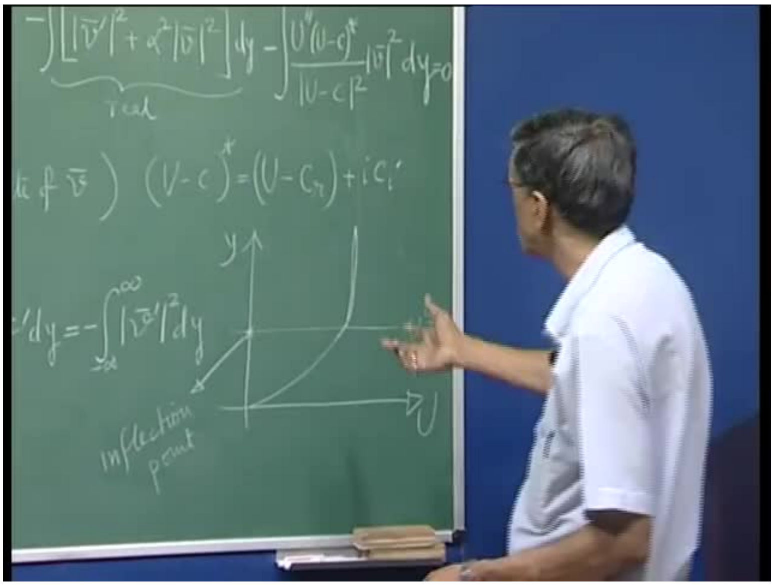

So, this is what was done by Rayleigh, and as I told you that he did it without the equation. All he did say that if I can measure the velocity profile or I can get an analytic expression by looking at the velocity profile, the equilibrium condition itself I can say whether that scenario depicts a stable or a unstable condition. However little later, some time later Fjortoft came into picture and he said look in enunciating that inflection point theorem what we have done? We just simply looked at the imaginary part of that reduced equation.

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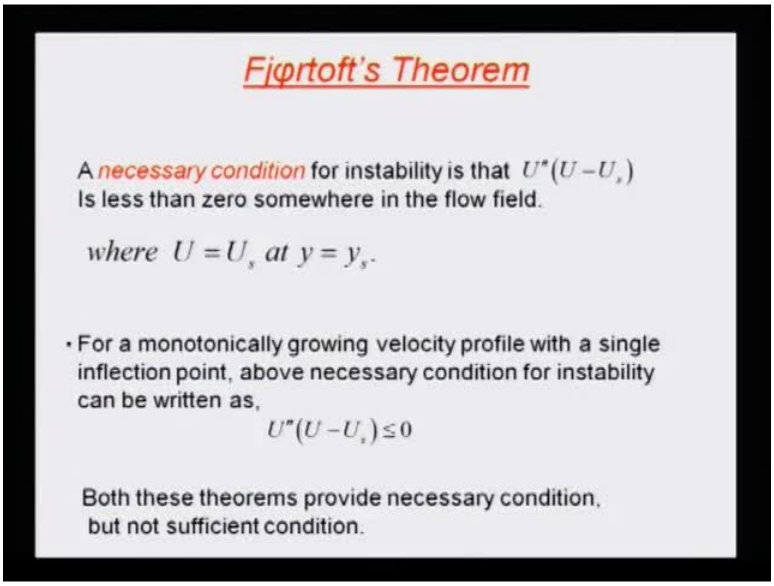

What happens to the real part, that is what Fjortoft investigated and he came out with an improved version of Rayleigh 's theorem which now goes us by the name of Fjortoft's theorem, and which says that a necessary condition for instability is that U double prime times U minus U s is less than 0 somewhere in the flow field. What is U s? U s is the velocity value at which the second derivative is 0. So, basically it is the velocity value at the inflection point. At the inflection point, the second derivative is 0 but velocity is non-zero that is U s. How is this condition? Well, obtained we can do allow investigation and find out how few a top conditions comes about.

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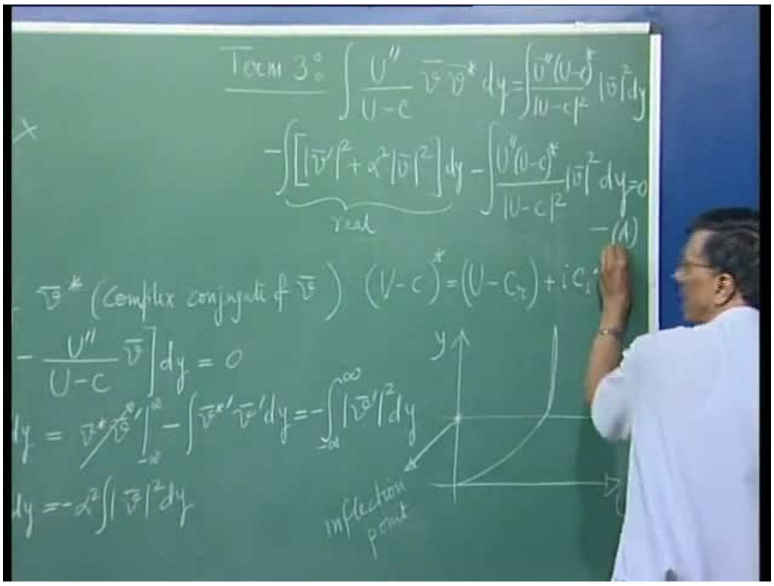

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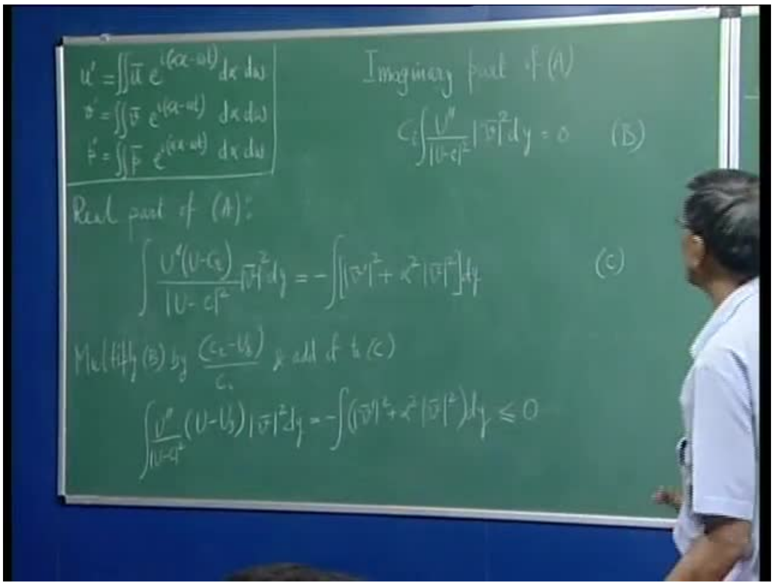

Now, if I call this equation as A, let say the Rayleigh's theorem investigated the imaginary part of A. Let us look at a real part of A. What do we get? Well, we get the following that the equilibrium flow information that comes in from this. This is related to the disturbance quantity by putting all the disturbance quantity on the right hand side and that we have seen already what this is going to be. So, this was a and the corresponding imaginary part we keep it handy, because we require that also, that is of the following C i times U double prime divided by U minus c whole square, then we add a square dy. Let us call this equation as B; let call this equation as c, capital C. So, we have this.

Now, how will do a little bit of manipulation? Nothing great, we will multiply B by C r minus U s divided by C i and the resulting expression will add it to equation C. Hwat we get is the following. See, whenever I multiply this quantity, so, whatever will happen to see will happen only on the left hand side, nothing will happen on the right hand side. So, the right hand side remains the same. On the left hand side what we get is the common fact that will be U double prime divided by U minus C square and then what happens? You see, I have multiplied this equation by this. So, what will happen? The C i C i will cancel and then I will have C r minus U s. So, I add that, so, what do I get here? C r C r will cancel out and I am going to get this equation U minus U s and times v bar square dy equal to minus of where we have this expression is this and of course, this is less than equal to 0. Now, you can clearly see what Fjortoft's theorem is giving us.

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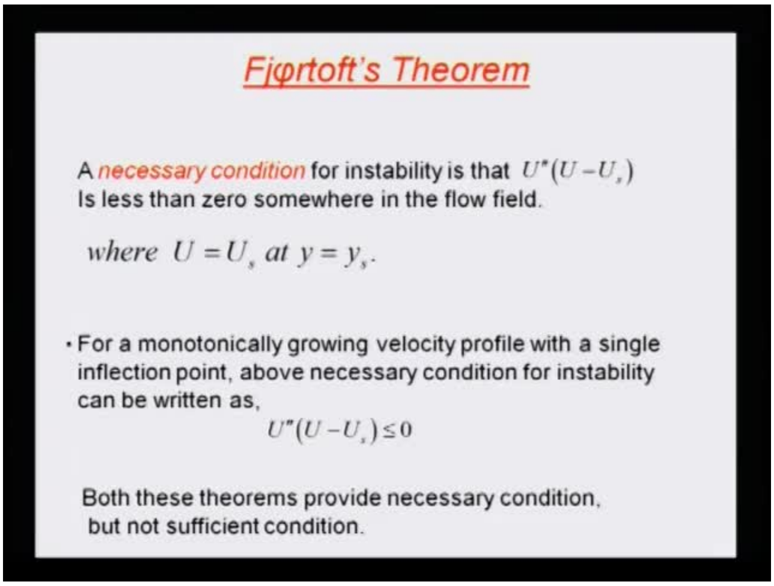

Fjortoft's theorem saying that this quantity has to be negative. The imaginary part was strictly equal to 0 but here that this quantity has to be negative. If this quantity has to be negative, then what happens? This part, the integrand has to be some on negative. Otherwise, it will not give as the thing, because this is a positive quantity; this is a positive quantity and that is exactly what he is saying here. Fjortoft is telling us that we must have U double prime into U minus U s should be less than 0 somewhere in the flow field.

So, basically, since this condition given by Fjortoft includes both the real and imaginary part of the original Rayleigh's equation, it is considered to be more general stronger condition. So, basically you should remember both of this theorem together and you can see that condition of instability. If you write like this, this is also includes Rayleigh's inflection point theorem also.

Rayleigh's inflection point theorem is also a subset of it, because that equation has been used in arriving at this. So, that is what is known as the inflection point theorem of Raleigh and it is improvement by Fjortoft. However we must keep this thing in mind that both these theorems provide necessary condition but they are not sufficient, because who guarantee is that after doing that integral, you will indeed get a zero contribution. It is not understood immediately. That is why please do understand this is a necessary condition but not a ((poor audio quality)) condition.

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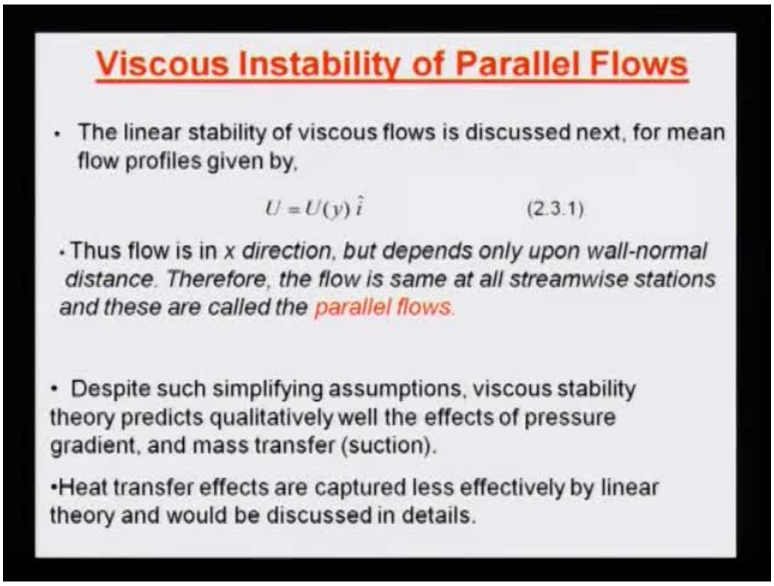

So, having exhausted the discussion on inviscid instability, let us focus our attention on viscous instability a parallel flows. Tell you what that idea given by Rayleigh supported by kelvin and other luminaries of the time always doubted this that whether there was at all in a need to study viscous instability. We have talked about it in the introductory lecture.

We said that was a major turning point that was a debate, a paradigm shift. Whether we need to stay with inviscid instability or we need to do viscous instability studies. Viscous instability studies equations of first written down by or in Ireland and some of in Germany. The corresponding equation is called the Orr-sommerfeld equation which is different from your Rayleigh's equation. Rayleigh's equation talks about inviscid instability corresponding viscous instability. Governing equation is called the Orr-sommerfeld field equation. That is what we are going to discuss now.

What we are doing once again? We are looking at linear stability; that means the perturbation fields are smaller so that in deriving the disturbance equation, we would make small perturbation assumption, omit the non-linear terms. However we retain the same equilibrium flow description. The mean flow or equilibrium flow is once again given by a parallel profile, that is, U is a function of y only and it is in the stream wise direction.

Since we have already talked about it you know, this is the parallel flow. You are saying that all appoints are different x's at the same y has the same value of U; so, that means the stream lines are parallel. That is why it is called parallel flow approximation.

However we know that the spite have a making such simplifying assumption. Viscous stability theory is able to predict many things in detail like the effect of pressure gradient, effect of mass transfer in terms of blowing and suction. They are predicted quite well by viscous stability theory. Now, we know in (( )) that this is an indeed value of a edition but it had its moment on uncertainty. In those days when it was propounded, for quite some time till it was verified theoretically, experimentally.

We would also talk about heat transfer effects. I would depart from most of the books, they say that it captures effectively. I disagree with that the based on some of the work that we have been doing. In recent times, one of our student just different of the thesis where we see that the moment we add heat transfer, all the linear stability theory in surmountable difficulties depending on the flow configuration.

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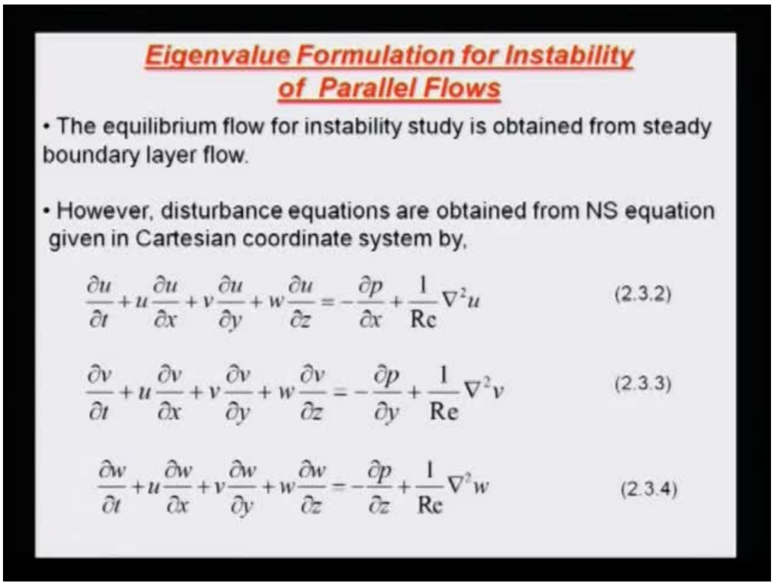

For some cases, it does show good affects. You can use linear stability theory, but in another major subclass, you cannot (( )) that. So, we will come to that at a later stage but let us now go ahead and re derive those equations depicting us a stability or instability of the parallel flow. Now, we have noted by now that instability analysis, we start off with identifying an equilibrium flow. So, in this case, let us say that we have a mean flow which is quite generic of what Prandtl date. Prandtl came out with the boundary layer analysis or boundary flows, and if we consider such study boundary layer flows, we want to study it instability.

So, the mean flow is obtain from boundary layer flow. Boundary layer flow is a kind of a simplified version of the Navies-Stroke equation, but when it comes to obtaining the governing equation for the disturbance quantities, please mark it, it is important that we do not make any assumption about the disturbance quantities, they are still given Navies-Stroke equation, and what you are seeing here written down given in a Cartesian frame, and this is what I have written down the Navies-Stroke equation for the total quantity. So, u is the total component; the v is the total component; w is the total component of the velocity profile.

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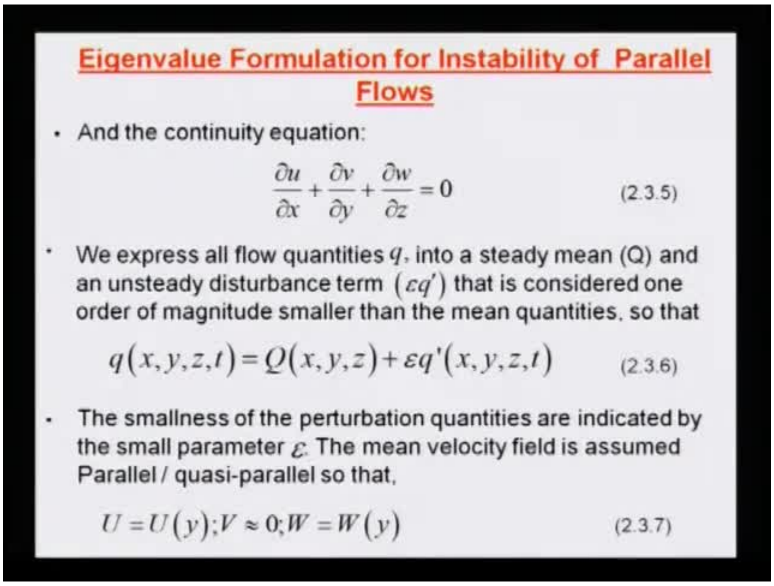

Now, we will split it up into an equilibrium flow and a disturbance quantity. So, those three momentum equations are to be supplemented by mask conservation equation written like this, and next, we define any flow quantities which I have used it as lower case q would have a mean flow or the equilibrium flow which is steady. So, this capital Q which defines the equilibrium flow is a function of space only not time, whereas all time dependence come into the disturbance quantities, and since we are doing a linear theory, we put in a small parameter epsilon trusses stress in a collating terms.

Now, in addition, we say that we go by the same parallel flow approximation. So, now, we have focused our attention on three-dimensional flows. So, the mean flow will have also three component capital U capital V at capital W, and the moment I say we are looking at parallel flow, we immediately realize capital V as to be equal to 0. Capital V non-zero does not allow you to make parallel flow approximation, whereas U and W component once again have to be a function of y to keep the stream sheet.

Now, please understand stream line is a concept of two dimensional force but that again depends on our point of view. If I go along the flow, along the direction of the flow, I can once again define the flow as two dimensions. So, again, I can consider the stream line along the flow direction and view it as a vector and that is as the vector potential. Most of you have done computing course, you know that vector potential is a generalization of stream function in three dimension.

Do not worry about it. We will not get it in theirs until unless we come to genuine three-dimensional flows. At this point in time, let us just simply appreciate that for parallel flow approximation to hold, we will make U as a function of y; capital W as a function of y such an assumption is called the parallel flow approximation or quasi–parallel flow approximation. Below what happens? When we had a boundary layer, it is quite sacrilegious to talk about no growth.

We all know boundary layer grows, but if the growth is negligible, I can still study the flow locally, and say look, if I am studying at this station, I take the velocity profile they are dependence on x is weak. So, let us omit that path. So, basically in the argument place, we should have a y, x but the x dependents is much weaker, it is a stronger function on y. That is what is called as a quasi-parallel approximation.

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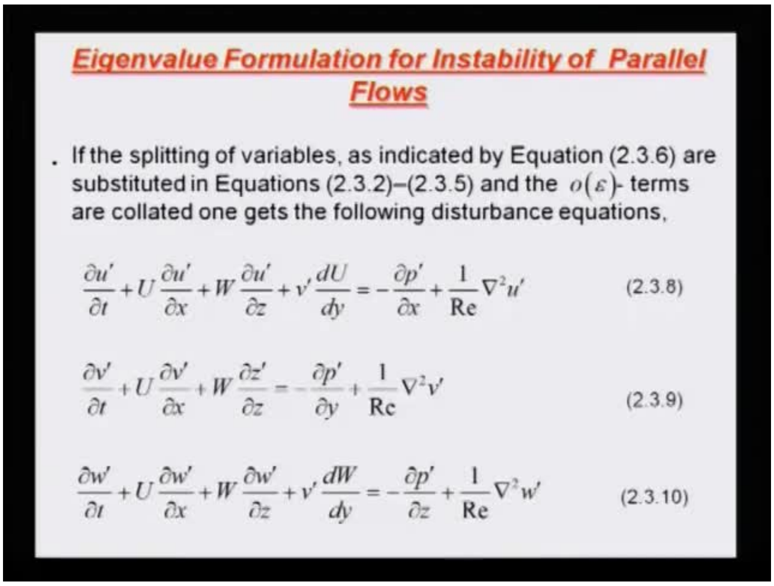

What do we get? Well, we do get those disturbance equations and we have done it for in inviscid analysis and I would let you to look at these equations and decide for yourself if they are correct or not. This term directly comes from del u del t because the mean flow is not time dependents, all the time dependents would come from the disturbance quantity. What about this term? This term comes from u del u del x. The u as capital U plus u prime. So, I will get this capital U times this. This is order epsilon term, this is order epsilon term.

What is omitted? The term that is omitted is basically u prime into del u prime del x, that is epsilon square. So, that term has been thrown away because we are studying linear stability theory. There is also a mean flow term. The mean flow term is what? Capital U times del u del x but del u del x is 0.

Capital U does not depend on x, so, we just have this term. Same thing we have W into del u del z. So, W will have a order one quantity and this is order epsilon quantity; so, this comes in order epsilon equation. What about other term, what about other term? We will have v del U del y and that contribution only would come from the mean flow part which I am writing here by an ordinary derivative. So, that is what you get. The pressure gradient term I have written it like this, and you realize that this equations is written in non-dimensional form. That is why I have gotten the Reynolds number coming here.

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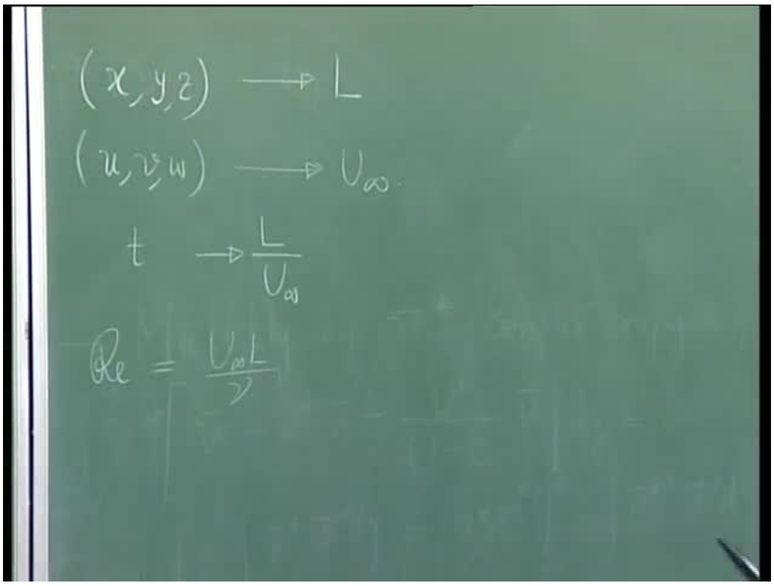

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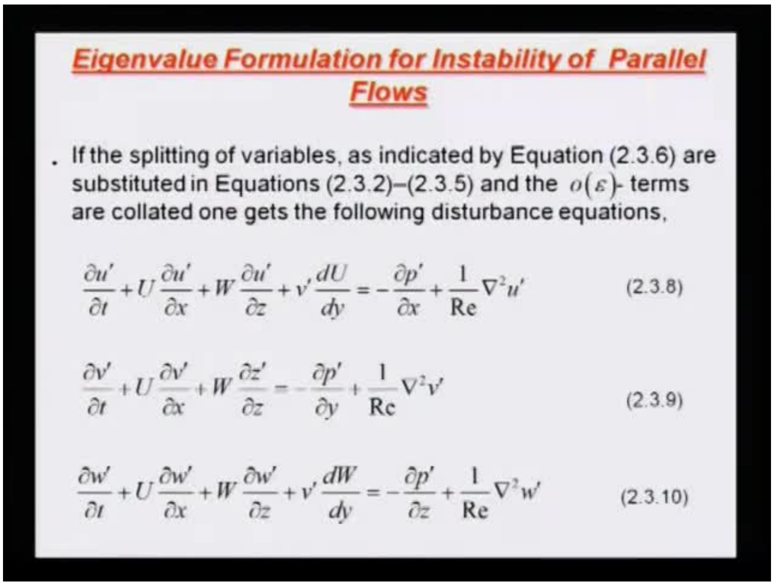

So, what you do is basically you assume some kind of a length scale and some kind of velocity scale and write down the Navier-Stokes equation in those things. So, let us say for length scale, if you have x y z, you refer it to some length scale L and the velocity components whatever we have a total quantity, we will talk about this. What about time then? Time will refer it to time scale, time scale will be brought out by L by U infinity. Use to non-dimensionalized equation, and then, of course Re that we have written there, we will get it as U infinity L by nu the kinematic viscosity, and that is why you see that from p will be non-dimensionalize with respect to what? rho times U infinity square.

We are focusing our attention on incompressible flow; so, rho is constant. So, I will non-dimensionalize the pressure by rho infinity square. That is what we are getting here that del p del x; rho has disappeared and you get in front of the diffusion term, follow over hurry. The same way y momentum equation can be simplified to yield this differential equation for v prime and the corresponding equation is written for the z momentum equation written in the bottom.

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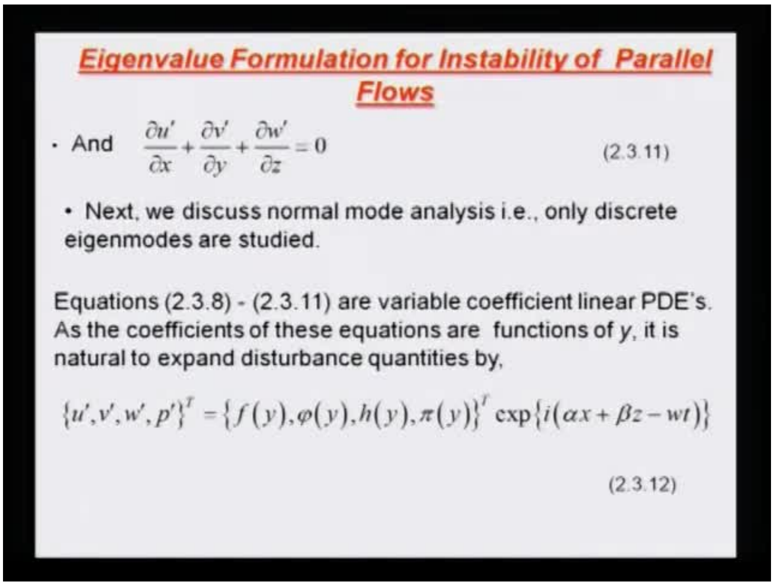

So, these are three equations, this has to be supplemented with the perturbation equation obtain from equation of continuity, that is, (( )) instead forward, you can directly write it down. Next, we discuss what is called as a normal mode analysis. This is something that people have been doing now for nearly 100 years and it basically does is a tries to study one mode at a time.

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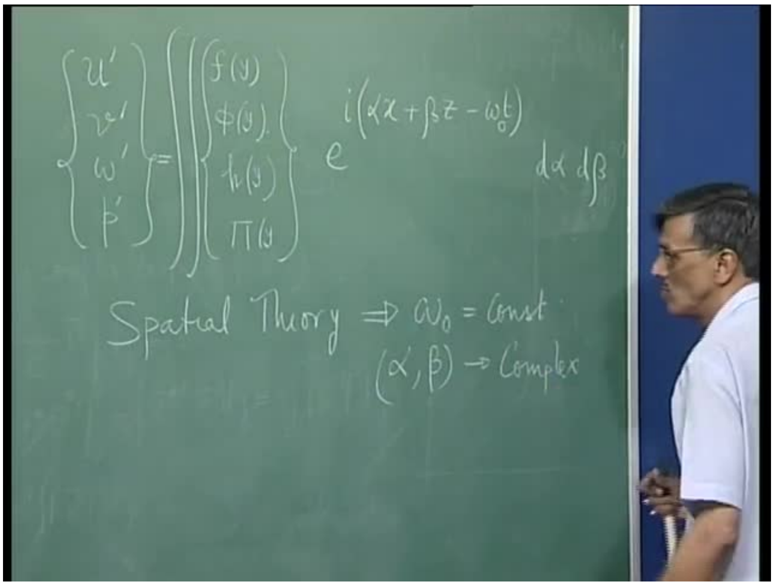

So, what happens is the disturbance quantities you are going to write it like this. So, what we are writing? The disturbance quantity is that we are writing here is in terms of its all possible harmonic components written in terms of a Fourier Laplace transform. So, these are those disturbance quantities, we are writing them down in terms of for their amplitude. So, u prime has amplitude I am calling it as f of y; v prime I am just writing it as phi of y amplitude; w prime let us say we have used the expression h of y and p prime I will call it as pi of y.

So, this is the pure y dependence of the equation. In addition, we will have to write all its phase. Now, phase term once again we will first write it down and then we will justify what we have written. We are looking at three dimensional disturbance field; so, y dependence is here. So, I have x dependents written here. I will also have to write down the z dependence and the time dependence comes out like this.

Now, what you will have to do? Ideally speaking if we are looking at generic case, then we should have written it like this that this equals to all possible values of alpha and beta, and what about omega? If I also would have done written down an integral over omega, that would correspond to Spatio-temporal analysis and I mention to you that is quite tough like in the Rayleigh's equation. We just simply looked at the temporal analysis.

Here, we will do something different. We will instead look at; let us say this spatial analysis by fixing a real frequency omega naught. We will do that, and then, what we are going to do here? We are going to talk about alpha and beta as complex that will be yours spatial theory. So, we are looking at spatial theory which corresponds to omega naught equal to constant and alpha beta as complex. So, I think we will stop here. We will begin from here the next class and we will see where it leads us to.