Instability and Transition of Fluid Flows

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Module No. # 01

Lecture No. # 05

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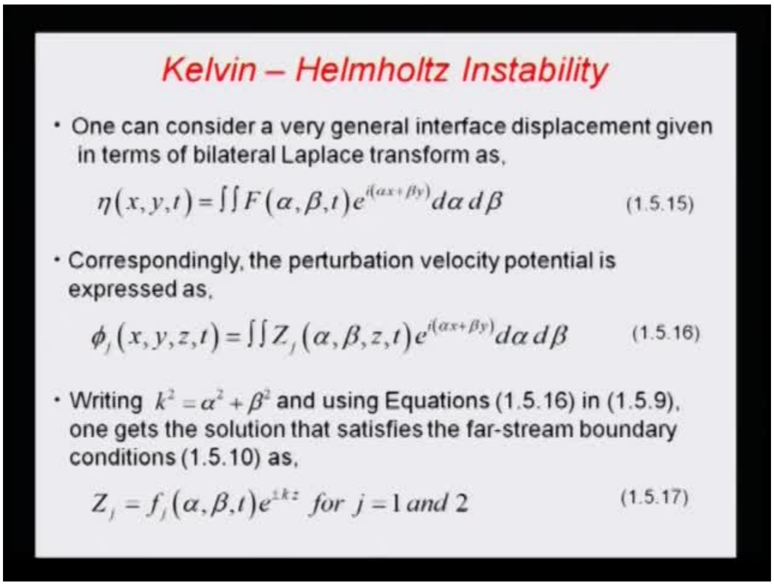

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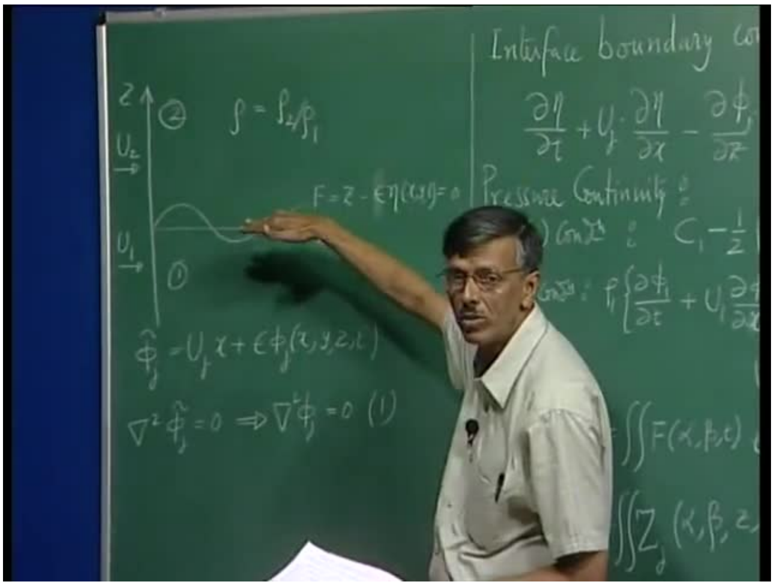

Let us continue our discussion on Kelvin-Helmholtz instability, which basically studies the instability of an interface between two liquids. And the top liquid may have a density rho2 and the flow coming from left to right is U 2, while below the interface you have a fluid of density rho1 and the uniform flow U 1. For such a flow configuration, we want to study, initially let say the interface is given by z equal to 0 and now we part of it by some arbitrary disturbance, which we defined it as z is equal to some epsilon times eta as a function of x y and t.

So, for such a flow we are basically assuming it to be inviscid irrotational flow, but we would be doing than we can use the velocity potential, which I am writing it as a phi tilde and it has a subscript j indicating that we have a fluid 2 above and fluid 1 below. And the uniform flow gives rise to this component U j of x plus the unknown part. This is the disturbance field, so we are basically in search of what this disturbance field is going to be.

So, the governing equation for inviscid irrotational flow is nothing, but the laplacian of the total phi j equal to 0, so if I substitute this, then I will get the corresponding equation for the disturbance velocity potential amplitude in terms of equation 1.

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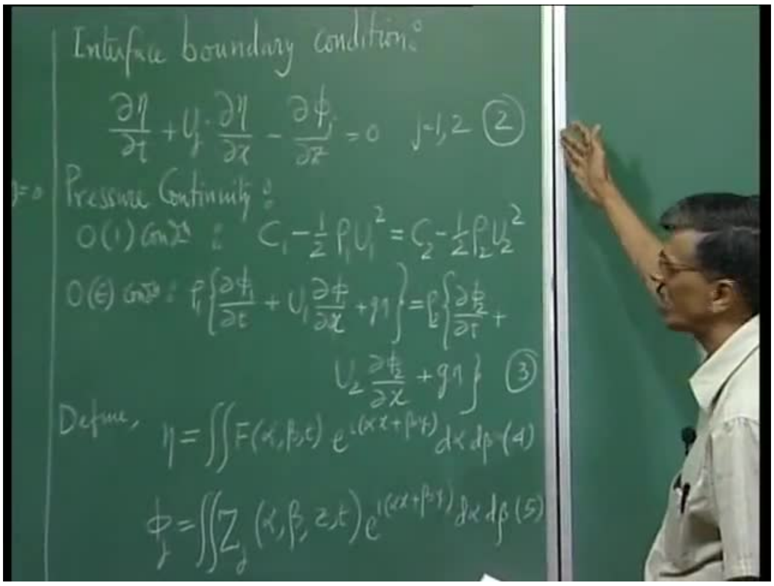

So, this is what we started doing, we also spent quite a bit of time talking about the interface boundary condition, that is the kinematic condition right. So, we satisfy the boundary displacement is equal to the normal velocity, that is what happens in a linear as a sense, we have spent some time and we found out for each phase, each side of the interface j equal to 1 and 2, we get an equation which is given by t. Additionally of course, if we have a interface of two liquids which may have different density or different dynamic pressure, it can also develop a pressure difference. But what we are talking about is, this disturbance is characterized by wave numbers, which are large enough to neglect those kind of discontinuities those arise due to surface tension.

So, what we require here is basically a pressure continuity. So, the pressure continuity condition we can write it down over using the unsteady Bernoulli's equation, in the last class we did that. And we find out that there are two possible conditions that we need worry about. One is the order 1 condition that fixes the constant in the Bernoulli's equation in the dynamic pressure on either side of the interface through this equation. What is more important is, since we are in search of the quantity is the order epsilon condition of this pressure continuity condition and that comes out from the Bernoulli's equation like this.

So, what we are going to do is, we are going to find out that order epsilon condition will be density times del phi by del t. See, please note that this disturbance potential, it is going to be 3 d as well as it is a function of time. So, if we do that, so this is what we get from the pressure continuity equation. Now, those of you may not be completely familiar with Fourier transform. Please do understand that we use Fourier transform in those cases where we want to treat as generally cases possible. So, it is not like Fourier series where you are only talking about periodicity.

So, if you have seen any arbitrary periodic disturbance on the interface, then I can define it in terms of its a Fourier transform in x and y direction, which is given by the wave number alpha and beta, alpha in the x direction, beta in the y direction. So, this is the most generic definition of the interface disturbance that we can talk about. So, it will have an amplitude which I am calling it capital F, which will be function of this two wave number components, alpha and beta it is also a function of time. And then of course, this is the phase path and this is what we have to integrate over all possible values of alpha and beta. The same way, we can consider the corresponding velocity potential on each phase to be equal to given by a Fourier transform, what we have here. Please do understand now that this disturbance quantity is going to be function of x y and z and t. So, x y variation we are giving in terms of Fourier transform. So, then what happens to this amplitude? The amplitude will not only depend on those wave numbers, alpha and beta, they will also be function of z and t. So, this is where we stopped in the last class, or I think we went to little further because we used this and plugged it in equation 1.

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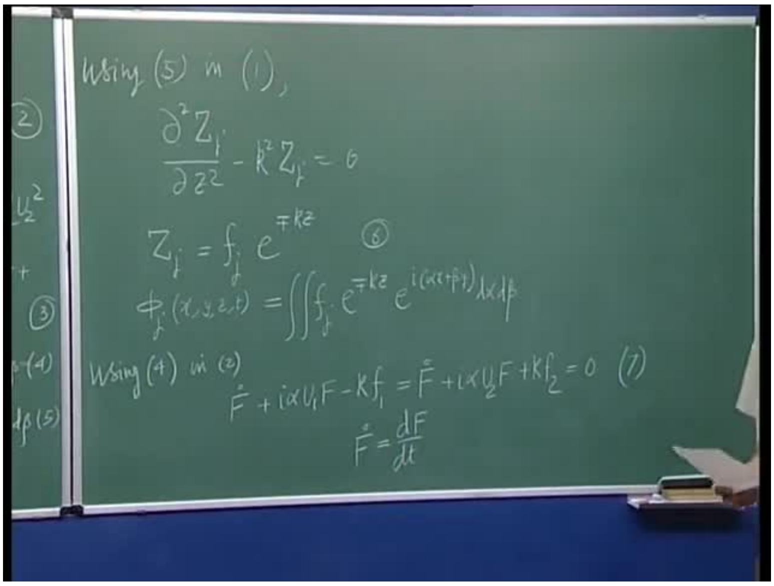

So, what we really found was the following, that if we use this definition of the disturbance potential in the governing equation for the disturbance velocity potential, then we are going to find the governing equation is going to be this. Well, you may object because Z is a function of time also, so,if you do, I would take preventive measure and write it as a partial derivative.

So, let us write this as this, and what happens is when I differentiate it with respect to x twice, I will get i alpha square, so that is minus alpha square, the y derivative will give you minus beta square. So, I can add that alpha square plus beta square and call it a k square and this is what we get. Now of course, this will have two components of the solution. One would go as e to the power plus k z, other will go as e to the power minus k z. And we want to enforce the physical condition, namely that if I perturb this interface, the corresponding disturbance will have a finite amplitude.

So, what happens is, if I go far away from the interface either above or below, the interface disturbance should decay. So, that condition, physical requirement that the disturbance decay away from the interface will basically fix your solution in terms of it, true constant f 1. So, what this f 1 is going to be? f 1 will be function of alpha, beta and time. That is what we are keeping it and same because that is what we are going to get alpha, beta and t because the z variation is going to come out like this. For phase 2, will have minus k z and for the phase 1, I should have plus k z because there z is negative.

So, that is what we are going to. So, please do understand that this is itself a function of alpha, beta and t. Now, so if I do this, I could then write down phi of j which according to our requirement is function of x, y, z and t would be given by this definition. Instead of Z j, I will write this as f j e to the power minus plus k z and then e to the power i alpha x plus beta y and d alpha d beta. So, this is what we are going to get.

So, this I could call this is equation 6, and this is our disturbance potential candidate. So, what does it do, what have we done so far? We have gotten a solution which satisfies the governing equation, which also satisfies one of the boundary condition that it decays if you (( )) this. So, what is required first to do is to ensure the other conditions are also satisfied.

So, what we can do is, we need to basically then go head and explore what is given by condition 2. What we could do is, we could use this definition 4 for interface disturbance in equation 2, then what are we going to get. So, del eta del t will be nothing, but del f del t or I could write d f d t because these are parameters, so this will be like this. So, that d f d t, I will just write out a shorter notation, write like this, F dot, that is this path. And what about del eta del x? That will be i alpha F.

So, that is what we are going to get. And we have U 1 ahead of it, so we are going to get this. And what about del phi j del z? That we can differentiate here. For side 2, I will get minus k and for side 1, I will get plus k times this f of j. So, that is what we are going to get. So, this will be minus K f 1. And for the side 2 that is going to be F dot plus i alpha U 2 f. And now what we are going to get? This is going to give us minus k, so this becomes equal to K f 2. And both of this is going to be equal to 0, that is what the condition 2 tells us.

So, now let us keep this in perspective that whenever I use a dot, I actually mean this kind of a time variation, this is what we mean by F dot. So, what we are going to do is, we are going to use 3. Now, let me use 3, this is the other condition that pressure continuity condition that also we need to satisfy.

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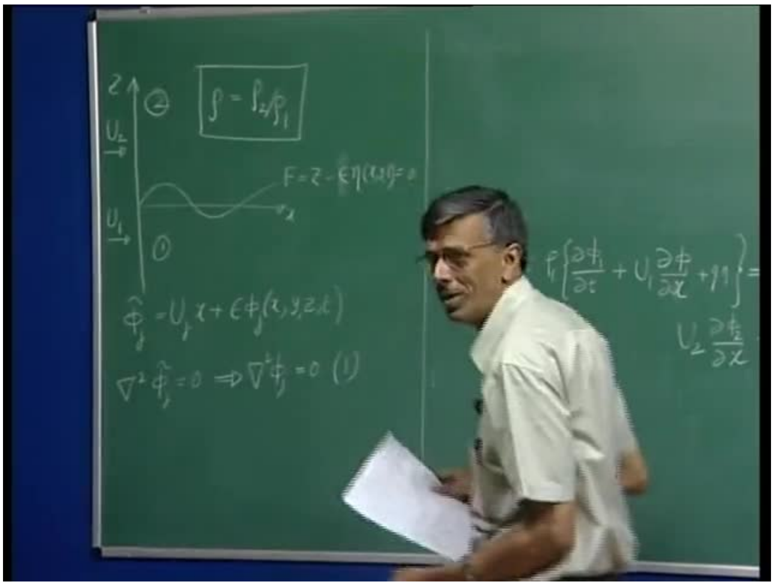

So, if I do that. So, this part is taken care of, interface boundary condition is taken care of that gives a 7. So, this is what it is. So, I will also remove the order 1 condition, which we do not require because we are looking at the perturbation quantities. So, if I really look at this from here, what I could do is I could use this definition, rho as equal to rho 2 by rho 1. So, you divide this both side by rho 1.

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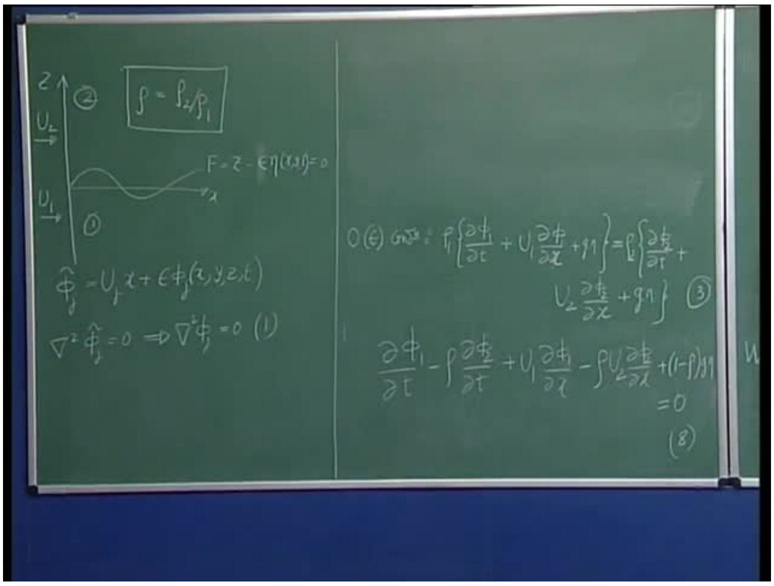

So, what I am going to get? I am going to get this quantity on the left hand side, I will get del phi 1 del t minus rho del phi 2 del t and I have this quantity U 1del phi 1 one del x and this quantity I am transporting on the other side and I have divided by rho 1, so rho 2 by rho 1 will give me rho, that is why we have got this also. So, I will get rho U 2 del phi 2 del x. Plus I have here on this side, I will have g eta and on this side, I will have, if I transport it on the other side, then I will get rho g eta. So, if I write it down, I will get 1 minus rho times g eta that is equal to 0.

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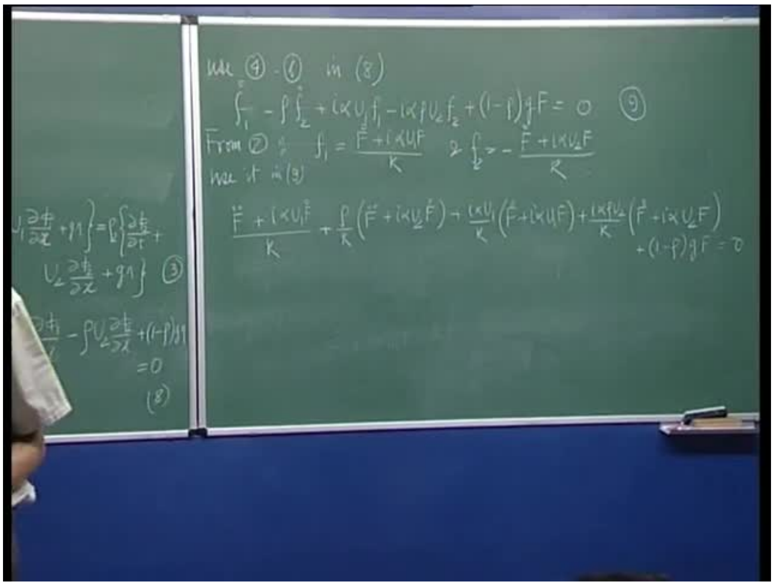

So, that is what we get. So, let me call that as 8. So, now what we could do is, we could use these definitions that we have used and subsequently got the solution. So, basically I am suggesting that you use 4 to 6 and as substitute in it this condition. What I am going to get? See, from here I have this equation, so del phi 1 del t would be F 1 dot. So, I am going to use that. So, basically what I am getting now from there that will be f 1 dot and from here minus rho f 2 dot plus del phi 1 del x I will get i alpha f 1. So, I am going to write that i alpha U 1 f 1. And this one, I will get minus i alpha rho U 2 f 2. And that leaves us with this term, that will be 1 minus rho times g into eta. What is eta in the Fourier space? That was capital F, so this is what we are getting.

So, we are using up satisfying one condition after the other, but now this is the result interrelation. But I can see there are three quantities, f 1, f 2 and capital F, but I have also equation 7. So, I can use equation 7 to relate f 1 with capital F and f 2 with capital F. If I do that, what do I get? So, basically what I am doing that I have obtaining the following relation, f 1 would be nothing, but equal to F dot plus i alpha U 1 F by K, that is the one condition. And the other condition is the f 2, that will be minus F dot plus i alpha U 2 F by K. So, this I can actually plug it in.

So, if I call this as 9, so we get this, so use it in 9, that should simplify our job. So, we could now these are all taken care of. So, what we are going to have is substitute it there and we are going to get F double dot plus i alpha U 1 F dot by K. So, this is your f 1 dot and rho f 2 dot itself has a negative sign, so this will work out as a rho by K. And I will have differential of this that will be F double dot plus i alpha U 2 F dot. So, I have gotten these two term here. Now, here I will get this term i alpha U 1 by K and I have F dot plus i alpha U 1 F. And from here I am going to get again a positive sign here and that would be nothing, but i alpha rho U 2 by K that will be multiplied by F dot plus i alpha U 2 F. And what is left is 1 minus rho into g F. So, basically we have achieved our goal. So, what we have obtained basically a differential equation for a single variable F, where all the derivatives are with respect to time.

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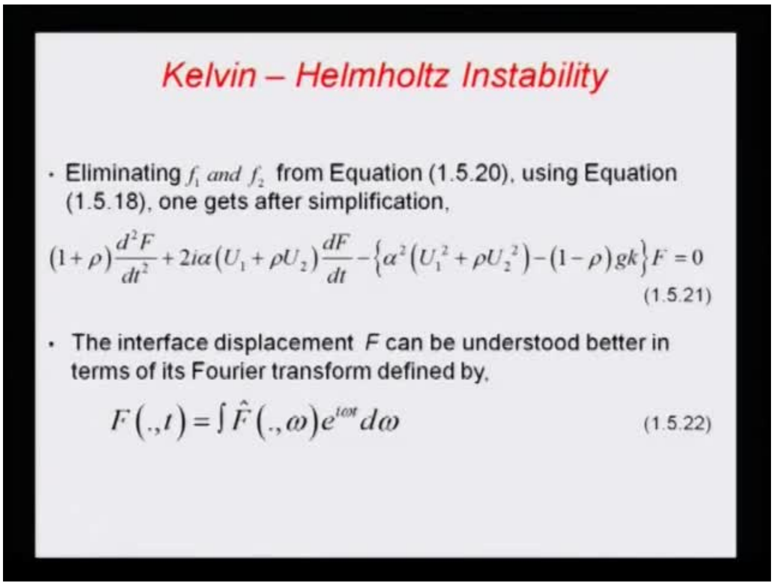

So, we are basically studying the variation of this interface disturbance amplitude capital F as a function of time. So, if I look at the developments that we have obtained, we have gone through all of these and we have actually come to this equation 1 5 2 1 that you can see there. So, this is what we have obtained. So, I wanted to derive all of this step by step. So, this is how it looks like. So, this is how it looks like and you get that equation, it is second order ODE, we can solve it. Now, the time variation, the time variation of the interface displacement also I can write it in terms of a Fourier transform, that is what we have done here, we have done it in terms of a circular frequency omega, that is what it is that. We have written F as basically nothing, but a Fourier transform times amplitude times a phase.

So, F I have written as F hat e to the power i omega t d omega. So, what we could do is, we could substitute this in this. So, this if I call that as equation 10, so you can use this. Now, what happens? Why actually do we resort to Fourier transform? The simple reason is this helps us converting a differential equation into an algebraic equation, that is one of thing that you are seeing.

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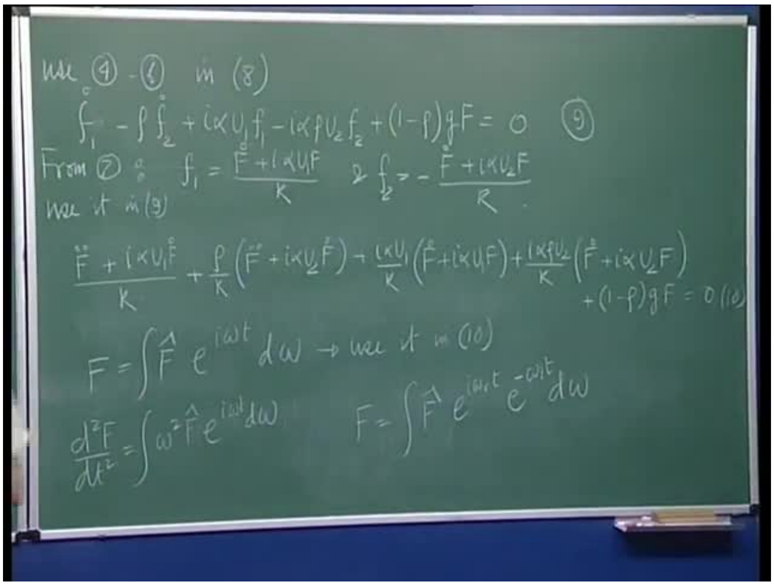

So, if I substitute it there, then what do I get? Second derivative of F will give us i omega whole square, so that will be nothing but this. So, it is going to be omega square F hat e to the power i omega t d t d omega. So, you understand that the whole goal is to basically converting a differential equation into algebraic equation. And when you have partial differential equation, you will also see that it can do the same thing with respect to those particular directions where we take the Fourier transform, it converts the differential equation into corresponding algebraic relation. And if you have to do it in terms of numerics, that is also a great benefit. Taking a differential numerically is all is going to introduce you large error, that can be completely circumvented. And at the same time, you can actually also obtain this in terms of a algebraic relation. So, you can explore it over very large range of omega. So, that gives you additional ability or degree of freedom to explore any range of frequency that you wish to. And mind you, we are not talking about any specific type of disturbance, this is a very arbitrary disturbance.

And what happens? Omega is what? Omega is a real quantity or a complex quantity? See, here you got to understand what is our goal? What is it we started with? We started with to explore if the interface disturbance is going to the disturbance that I have created at t equal to 0 is going to increase, decrease or remain same. So, how do I assess such a thing? That omega must be complex.

If I have omega as complex, then I can see lots things happen. The real path will give you some kind of a time variation. What happens to the imaginary part? So, if I am write that omega r plus i omega i whole square, well let us look at, instead of this, let us look at F itself, that will be more instructural. So, F will be somewhat like F hat and then I will write i omega r t. So, this is the really a phase path. What happens to the imaginary part? That will be minus omega i t d omega.

So, what happens now? The omega I will determine whether it is going to grow or decay with time. If omega is negative, then what happens? This is going to blow up in time. And if omega is positive, that is going to decay in time. So, our goal would be to really explore what happens to this omega and that is precisely what we are doing now.

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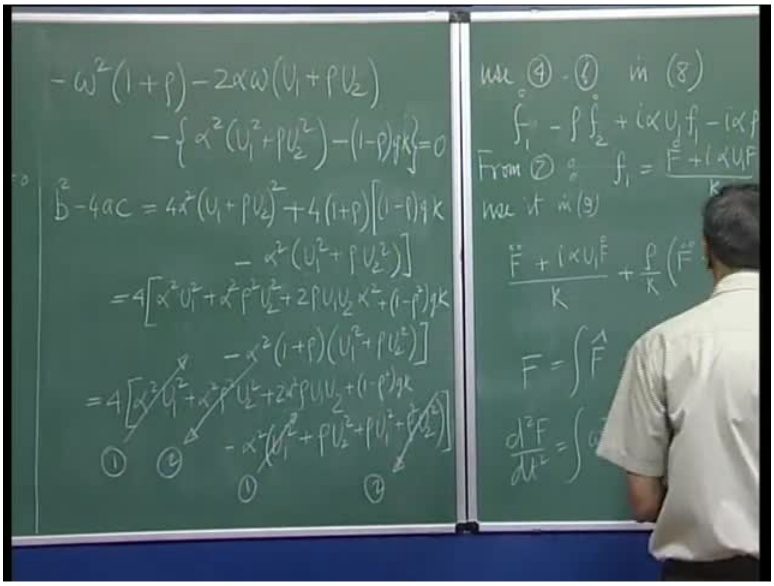

So, we are going to substitute this in equation 10 and see what we get. So, substitution of this Fourier transform in this will give us the following equation. So, from the first time here I will get minus omega square times1 plus rho. So, that is that, omega square times 1 plus rho, that is that term. Then we have the d f d t term which has the coefficient here 2 i alpha and d f d t will also give me i omega. So, this will become minus 2 alpha omega.

So, that is what we are going to get, minus 2 alpha omega times the coefficient that we have there, U 1 plus rho U 2, that is from the first derivative. And the last term of course, remains as it is, so we could keep it whatever we have alpha square times U 1 square plus rho U 2 square minus1 minus rho g K. Now, you see the whole of this is multiplied by, so this is the multiplied by F hat and then you are multiplying it by e to the power i omega t. I am performing that integral over all possible omega and that is equal to 0. So, that is what we have.

So, this is what happens. So, this differential equation turns out to this. So, if the integral over arbitrary omega range has to be equal to 0, then the integral must be equal to 0. So, that is what I had been referring to, that it converts a differential equation into a algebraic equation because this simplifies to what we have written here, that is what we have written as 5.23, that is the equation, that is what you have it within the square bracket.

So, what you have now it is basically a quadratic in omega. A quadratic in omega and we do not have to worry about this, we will just simply write this equal to 0, and that will tell us what is happening. What is happening with respect to what? What are the physical quantities that our disposal that we want to study? We want to see what this density ratio is, rho, we also want to see what happens to U 1 and U 2 or their relative magnitude. So, that is what we are trying to figure out. So, this is your quadratic in omega and you can get the roots. So, to get the roots of a quadratic we need to basically find the discriminant.

So, what is this discriminant? So, this is your b, this is your c and this is you’re a. So, if I do that b square minus 4ac, what am I going to get? I am going to get here 4 alpha square times U 1 plus rho U 2 whole square, that is your b square and minus4ac will give you what? That will give us 4 times a is 1 plus rho and c, I can notice that this is going is to be 1 minus rho g K minus alpha square U 1 square plus rho U 2 square, this is your b square minus 4ac. Simplify it let us get whatever we can, open this up, we will try to get alpha square U 1 square plus alpha square rho square U 2 square plus 2 rho U 1 U 2 alpha square. And this one multiplied by this, I have taken 4 out, so this will be 1 minus rho square. So, I will have 1 minus rho square into g of K, that is the first term and the second term is going to be, there is a alpha square and I will have 1 plus rho and U 1 square plus rho U 2 square. So, this is what we are going to get.

Well, a little more algebra, I am purposely doing it, so that you do not have to fumble cross later. Alpha square U 1 square plus alpha square rho square U 2 square, then we have 2 alpha square rho U 1 U 2 plus 1 minus rho square g K. And I will open this up and that will give me alpha square, within bracket I will have U 1 square plus rho U 2 square plus rho U 1 square plus rho square U 2 square. Now, you can see there is a cancellation, this cancels with this term. So, this is one cancellation and the second cancellation of course, will come from here that will cancel with this term.

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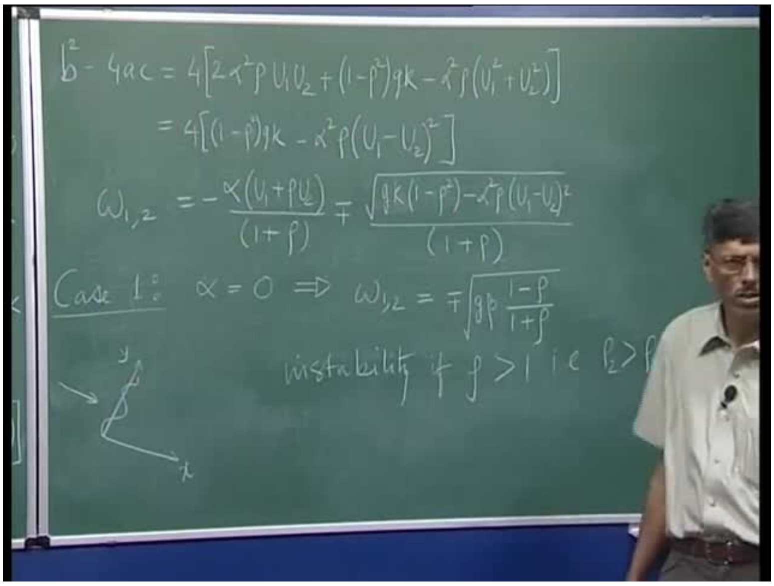

So, we have made significant simplification and we are looking at the discriminant of the quadratic in omega and this is what we have obtained. So, what we find that b square minus 4ac simplifies to 4 times 2 alpha square rho U 1 U 2 plus 1 minus rho square g K and minus alpha square rho, within parenthesis we have U 1 square plus U 2 square. So, this little more simplification is required here. This two term can we clubbed together and what we get basically minus alpha square rho.

If I take common I will get U 1 square plus U 2 square minus 2 U 1 U 2. So, that I could write it as U 1 minus U 2 whole square. I purposely did all this algebra to show to you that what matters in all this depends on not on the absolute magnitude of U 1or U 2 but the difference. Why it is different? Recall this is what Kelvin observed that if you have a relative motion, it is going to tear apart the interface. So, this relative motion is what U 1 minus U 2. So, that is what we are looking at. Basically, what we are exploring? We are exploring the various mechanisms of instability. So, one of the mechanism that we are seeing, that if we have shear the relative motion across interface, that is a very potential candidate for leading to temporal instability.

So, this is what we are going to do. So, we basically get that expression now, I purposely spent a little time to help you with the simplification. So, it has one path is alpha times U 1 plus rho U 2 as given in equation 24 there in the slide. So, you can get this path and there is this other path that comes from that characteristic determinant path. And what we get is g K 1 minus rho square minus alpha square rho U 1 minus U 2 whole square divided by 1 plus rho.

So, one thing is pretty much apparent to you that if this quantity under the radical sign is positive, then omega is real. So, what that would signify? The disturbance that I give will remain the same, it is just periodic variation, it is only the phase path, it will go with omega r. So, that is the possibility, if the quantity under the radical sign remains positive ,then we have a case of neutral stability. So, that is what we understand.

So, we have the expression here and we just noted that if we have the quantity under the radical sign as positive, that ensures neutral stability. We can have instability, when this quantity under the radical sign is less than 0, then that would contribute to the imaginary part of omega. And we are going to have a pair, something will be plus i omega another will be minus i omega. And because of that, this we have seen already that, if omega i is going to be negative, then we are going to have instability. So, come at may because of the quadratic nature of this equation, the quantity under the radical sign, if it is negative, you have instability.

However, let us also look at the other possible cases or let us look at one of the simpler case that we can conceive of this, alpha equal to 0. What does it mean actually? See, basically where this alpha and beta came about? It is the interface disturbance that we have created at t equal to 0, that is what we defined as eta equal to F e to the power i alpha x.

So, we are talking about disturbances for which there is no waviness in the x direction, in the direction of the stream. However, there is waviness in the y direction, so that is the normal to the plate. So, this is going to be a situation of this kind. Let me just show you with a perspective. So, this is your y direction and this is your x direction and z is of course, perpendicular. So, basically let me slightly better for you to appreciate. Let us say this is the what we are doing at x and y direction.

So, the disturbances that we have given have variation in the y direction. So, that will be like corrugation in the span wise direction. So, flow is coming in this way and you have the surface corrugated in this. So, it is like a corrugated surface and flow is going along the group, so that is what it means. So, what happens to this? If alpha is 0, so this part goes to 0, that is nothing there. What happens to K? What is K? It is beta. K square was alpha square plus beta square. So, if alpha is 0, so this becomes this. So, then what you have is omega 1 2 will we equal to minus plus, I will get this, this part also goes away because alpha is 0, so I get square root of g times beta and there I will get 1 minus rho divided by 1 plus rho.

So, these are very interesting case. See, even if you may have shear, a span wise disturbance does not depend on what this value of shear is because alpha is 0. Shear was larking here. So, if I put alpha equal to 0, nothing happens. So, the span wise disturbances do not matter. So, here is the very simple case that tells you the three dimensionality is not going to be very important because you are only going to see this. And what about this? Now, you have to think about what is rho, rho was rho 2 by rho 1.

So, you are going to have instability when this rho is greater than one, that means what? A heavier liquid is resting over a lighter liquid. And common sense tells us that is going to be unstable. If I try to do that anywhere, even with rigid body, if I have to balance heavier object on top of a lighter object that is a tendency (( )), fluid also shows the same thing. This creates instability if rho is greater than 1, that is rho 2 is greater than rho 1. In fact, this is what was that, this is the scenario that was studied in this mechanism called Rayleigh-Taylor instability. You know Rayleigh-Taylor instability was studied when there was no U 1 and U 2, but what we have come through this, that is why I did not even spend time discussing about Rayleigh-Taylor instability, that falls as a special case of Kelvin-Helmholtz instability, you put U 1 equal to 0. There also you would see the same thing, even if you look at the most general case, if I put, look at this U 1 and U 2 equal to 0 this part of course, will not be there, this part will not be there and will get this.

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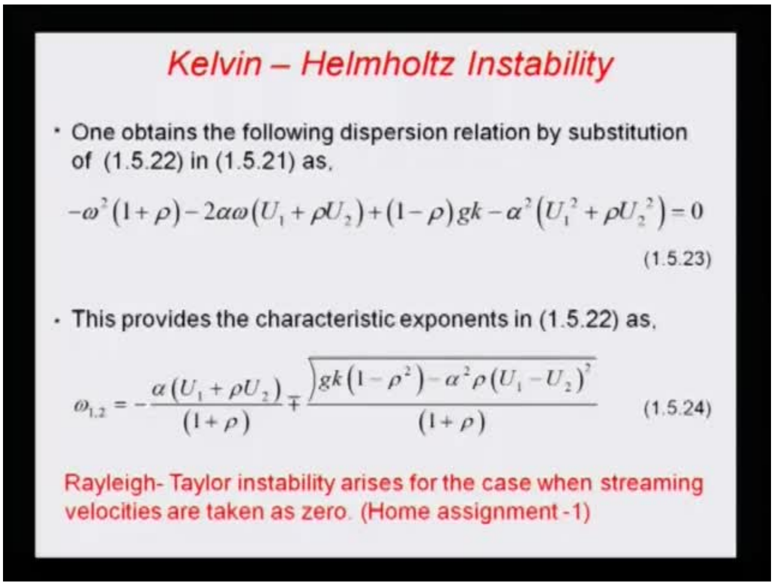

And the instability is totally is going to be determine by g K 1 minus rho divided by 1 plus rho. So, the disturbance could go in any direction k direction, their condition of instability is given by this. So, I want you to really go ahead and do it as a home assignment and submit it. That is going to be your first home assignment, you study Rayleigh-Taylor instability from the first principle, the way we are derived all this thing, you can write down the same set of logic and argument, except that your U 1 and U 2 equal to 0. It is a much more simpler case. So, we will get to practice what we are been talking about.

So, this is what we would call as the case 1, where we find that if we have span wise perturbation, not the steam wise perturbation, then that is going to be unstable only if we have a very unstable configuration, even when this is not moving by the Rayleigh-Taylor instability case.

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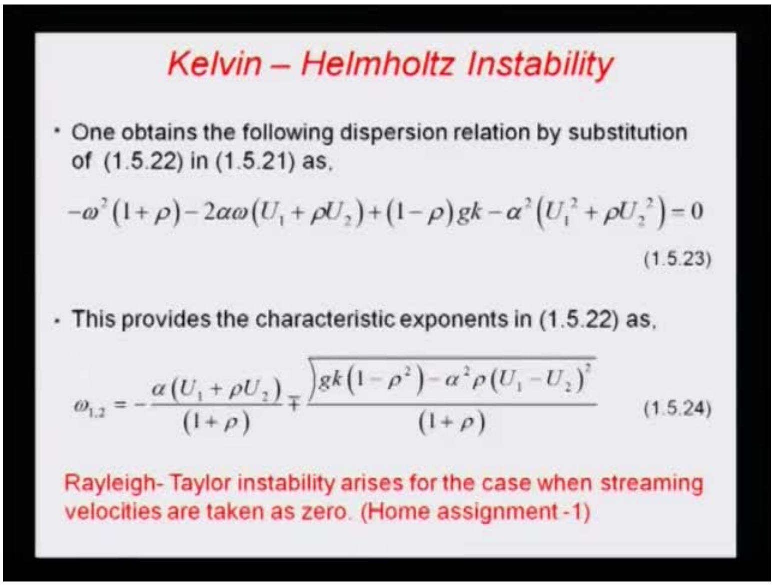

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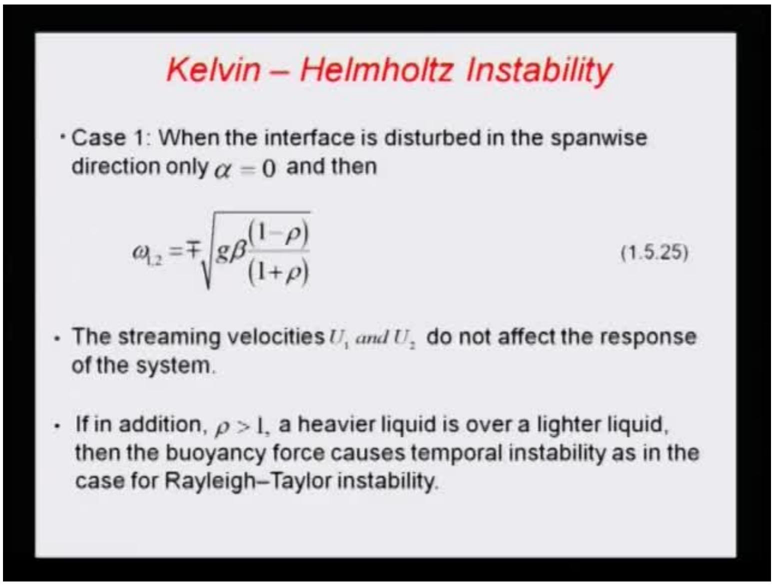

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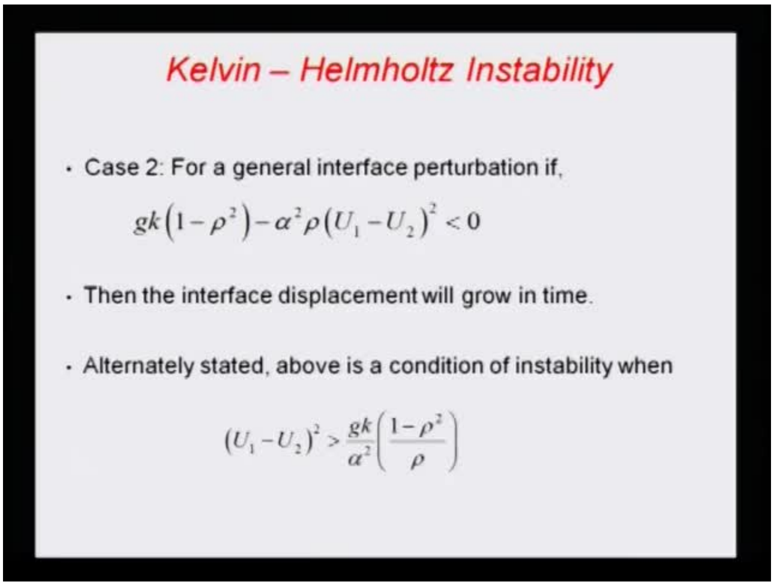

So, we can talk about the next case that we can talk about. The case 1 is just now we talk to about, so we will go to this next case. This is the case where the general interface perturbation is such that the characteristic determinant is negative. We have already said that is going to be unstable and the interface will grow in time. I can also write it like this, U 1 minus U 2 whole square should be greater than g k by alpha square into 1 minus rho square by rho. That is the condition of instability. Now, what you need to do is explore it little further, so this basically tells you that what kind of shear is going to be causing instability. The relative motion that U 1 minus U 2, if you square it, that has to be greater than this, that is determined by this rho.

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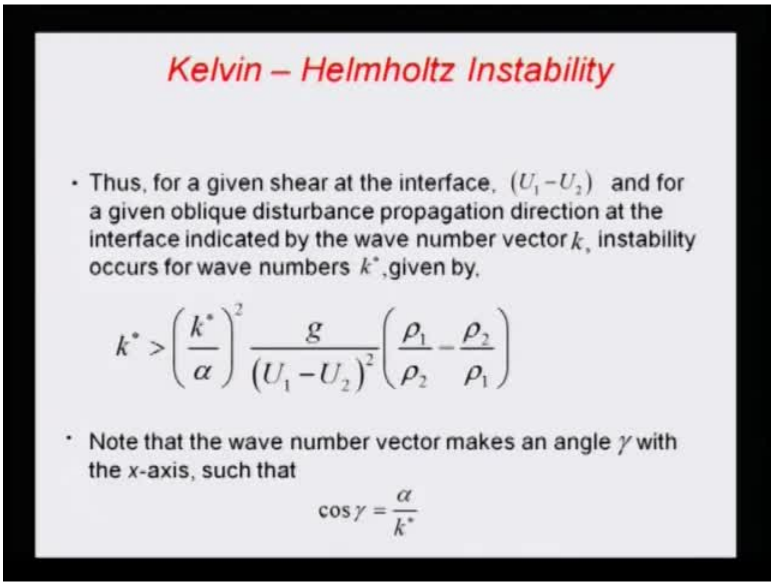

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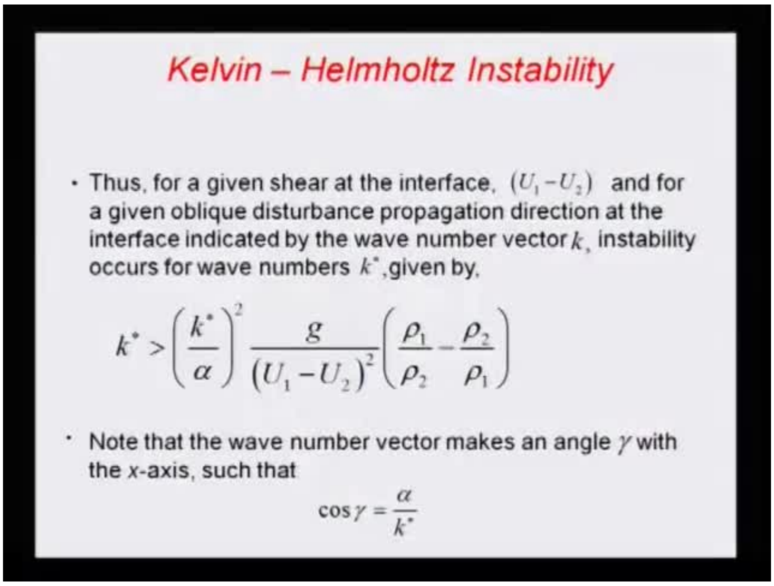

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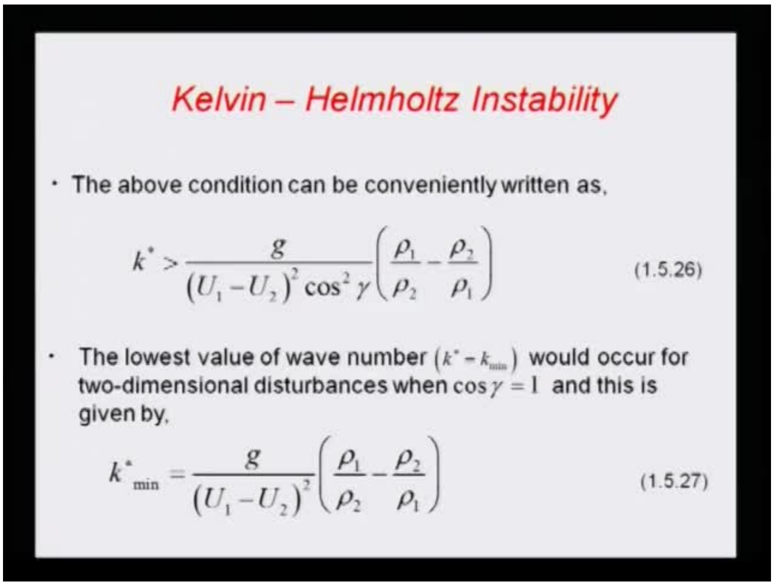

So, what we do is, we simplify it. How do we simplify it? We are talking about k. what does k mean? k means it a oblique disturbance, it is neither in the x direction nor in the y direction. You can take the wave number vector and you will get two components, the components are alpha and beta. So, if I talk about that, then I could write down the same condition that I have written, I will get the instability and the corresponding wave number k star is given by this.

I have just read it in the same thing. So, what happens here? Look at this quantity, what is the angle between alpha and k star? If that is gamma, then cos gamma is alpha by k star. So, that is the reason I have put k star also on the right hand side because this is essentially a geometric parameter that 1 over cos square gamma. So, I find that k star or those unstable waves, which are greater than this for a given angle.

So, for every angle gamma, I am going to get a different value of k star ,which are going to be unstable. For every value of propagation direction, oblique direction, we are going to find that out. However, for any arbitrary gamma, since cos square gamma is in the denominator I can get a minimum value of k. Minimum value of k star will be when gamma is equal to. No, it is in the denominator. So, what is the maximum value of cos square gamma? It is 1. So, cos square gamma equal to 1, means what? That it is going in the stream wise direction, k star and alpha in the same direction. So, we have already seen here in the span wise is direction, if you do not have that potentially unstable positioning, then we are going to get stable condition. If I do not put heavier on top of lower, the span wise is disturbance is not going to do. And here we had seen that I am going to get k as minimum, what does it mean? k minimum corresponds to longest wave length, longest wave length disturbances, those are going to be unstable and that is given by this condition that we have written.

So, that is what we are talking about. So, this value of wave number, k star is equal to k mean would occur for two dimensional disturbance. The disturbance is two dimensional, it is going in the stream wise direction, there is no span wise components and that is this. Now, you can really understand that when it comes to study of stability, many a times we find two dimensionally instabilities are more potent than more harmful and so called more general three dimensional cases. Here is a very clear example and here of course, one can note that we have taken a very simplified model. If you now summarize what we have seen so far. What we have seen so far is basically we have a considered a uniform flow below and above that interface and that is also we said a very simplify assumption it is irrotational. And then we see that the potentially unstable temporal instabilities occur only in the direction on the flow directions.

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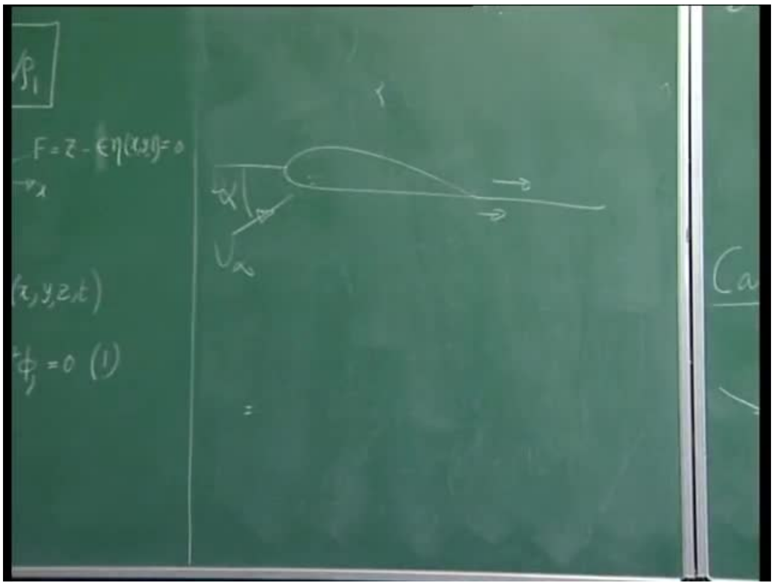

So, 2D disturbances are the most harmful one and 3D disturbances are less harmful because you can see that amplitude is given in terms of that. And what is the value of the shear is given by that expression that we just now seen. Now, what you can see that we can also consider a case, where this two are of same species, where would you see this case? Does it have any relevance to physics? I have an interface somewhere, I have a different velocity above and a different velocity below, where do we get it? People from aerospace engineering should not have any difficult in visualizing that, where would you see that?

No, I say that aerospace engineering, aerospace engineering do not worry about atmosphere. They do, but not to that extent. Where would you see that? Yogesh. Jet. Well, then I am going to create a jet, two different jets, then they interface is where we will get, and what will we call that? That interface is giving to be called the mixing layer. But you also can see it. If I take the, say a aircraft wing at an angle of adopt, what happens to the flow here? On top I have one type of velocity and bottom I will have another type of velocity, that is why you generate lift, but it is the same fluid.

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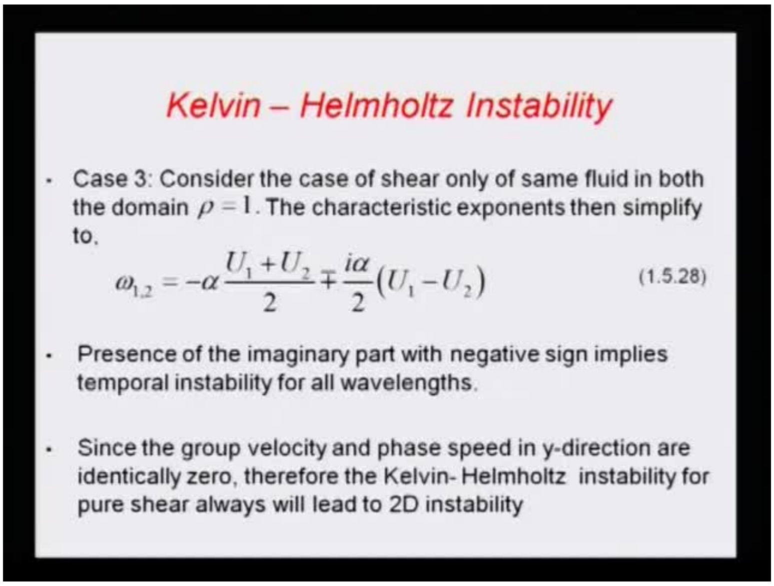

So, we are studying case 3 as that, that we are talking about rho equal to 1 and we still have dissimilar velocities across the interface. And that is what you see as a trailing edge of a wing and we can simplify that, whatever equation that I have written, so I will put rho equal to 1, then I will get those characteristic exponent would be given by this. And it has got a real path that is a given by the average velocity. What does the real part tell you?

Yes Morashi, what does real path tell you? The expression that I have written for interface disturbance, what does the real part gives us? The frequency of variation. So, the frequency of isolation of that interface. What we are saying that if I create any small disturbance, the disturbance frequency is given by this, the average velocity. And whether it is going to grow or decay is determined by the shear. And you see now we have no choice, it is unstable, you have a plus quantity and you have a minus quantity.

So, what you are seeing if the flow passed a aero fall at the trailing edge, you are going to get a potential instability. So, we have temporal instability and that is for any alpha, you see that instability is for all alpha. That is a very dangerous scenario, that is why you see you can hardly think of the flow behind any stream line body to remain laminar because this is a very potentially unstable configuration, you are going to see the disturbance are going to grow.

I will stop here. Before I will do that, I just simply will ask you, since you have an expression for omega 1 2, you can calculate the group velocity and phase speed and tell me what do you see. So, I will stop here.