Instability and Transition of Fluid Flows

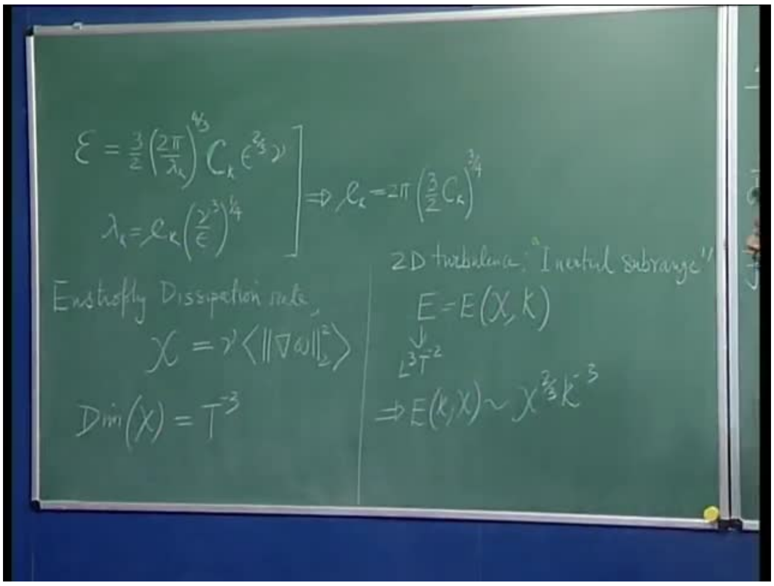
Prof. Tapan K. Sengupta

Department of Aerospace Engineering

Indian Institute of Technology, Kanpur

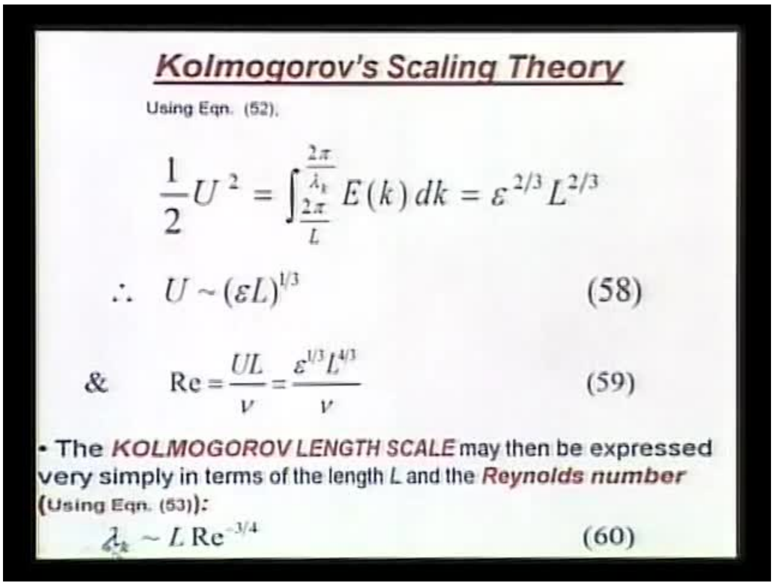
Lecture No. # 39

(Refer Slide Time: 00:25)



Yesterday we were talking about the inertial sub range of 3 D turbulence, and we figure out that there were two constants - one appearing in the definition of the dissipation; another appearing in the micro scale and these two are not independent. This lower case of c k can be obtained in terms of the upper case C k. So, this is one thing that we are noticed.

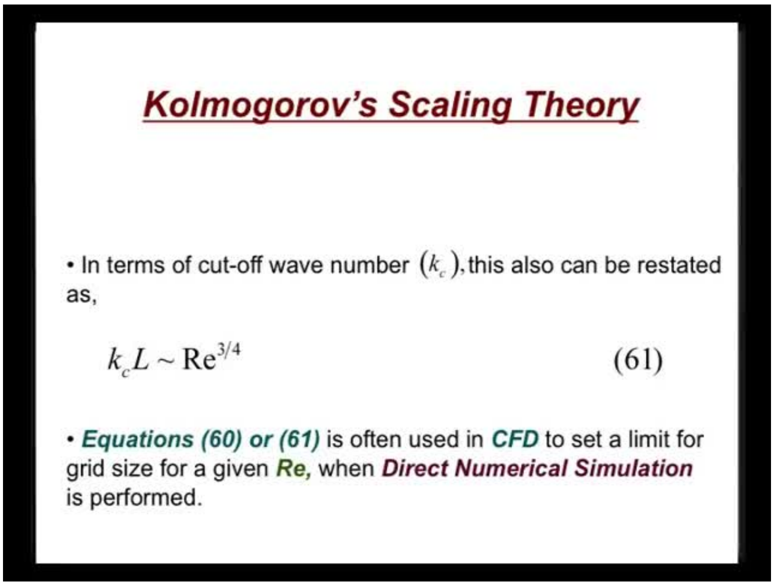
(Refer Slide Time: 00:48)



Now, one more aspect that you would write note is that the kinetic energy, the specific kinetic energy is given in terms of the energy of the energy spectrum here and that you integrate from the largest scale to the Kolmogorov's scale gives you this. If I put E of k as k to the power minus 5 third law, this is what we are going to get. So, that gives an estimate that, U goes as the dissipation rate times the outer scale raise to the power one third. So, if that being the case, then we can see the Reynolds number can be defined like this.

Now, we have already seen the definition of the Kolmogorov's scale like this. So, what I proposed to do is that, instead of epsilon here, I take the epsilon from this expression - nu Re by L to the power four third is this. Plug it in there, then lambda k goes as L times Re to the power minus three fourth so that basically is very important information. What it tells you that if you all looking at the simulation of its let say 3 D turbulence, then in the each direction, the smallest relevant physical length scale as given by the Kolmogorov's scale goes as Re to the power three fourth.

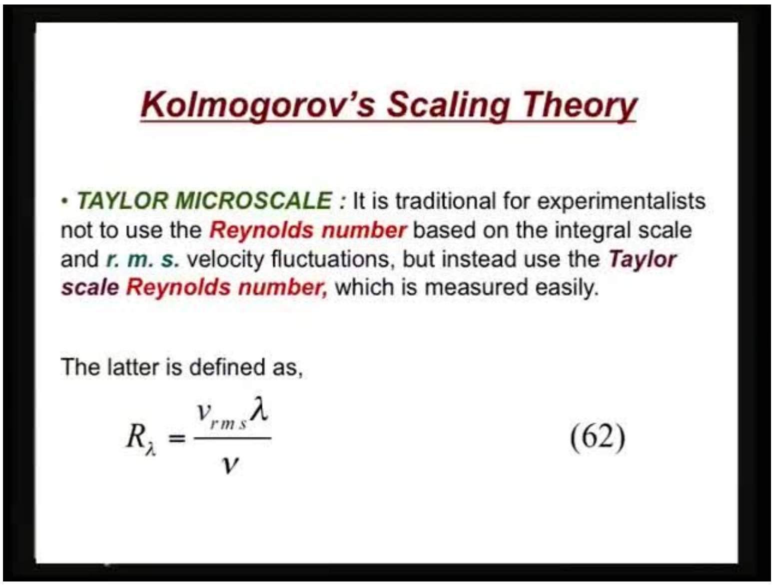
(Refer Slide Time: 02:49)



So, that is lambda. So, if I now converted in terms of the corresponding k, that will go as Re to the power three by fourth inverse of that. So, if I look at it that way then, then what I find that the cut off wave number that we are talking about non-dimensionalize scale size Re to the power 3 by 4. So, if you are doing a 3 D flow simulation, so you will have a k c in the x direction; k c in the y and z direction.

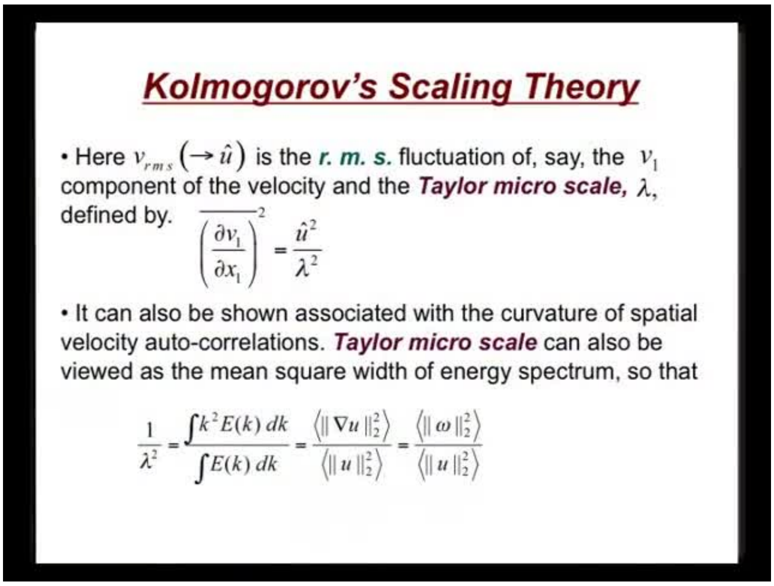
So, the smallest scale - the cube would be given as the product of these three cases, that would go us Re to the power 9 by 4. So, that is what you could hear quite often that, if you are trying to simulate 3 D turbulence, the number of points required in DNS goes as Re square. So, that is Re 9 by 4 is close to Re square. So, it is a kind of a simplification.

(Refer Slide Time: 04:06)



If you have done a simulation for some Reynolds number, you double the Reynolds number; your requirement on the number of points would go Re to power square. So, it will be square. So, this is something that we need to remember. Now, people do also talk about another scale which is called the Taylor micro scale very popular the experimentalist. They basically try to find out some kind of a integral scale and the r m s velocity fluctuation. What you do is with the help of this, you define a Reynolds number based on Taylor micro scale which is that v r m s times that Taylor length scale divide by nu.

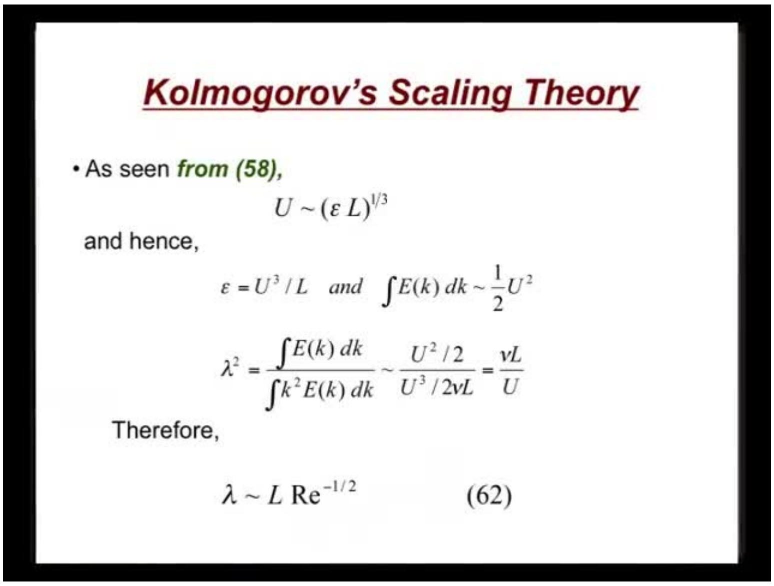
(Refer Slide Time: 04:34)



What happens is then we can go through as the v r m s goes velocity scale itself, and if I am looking at one of the component of the velocity, then what happens is this quantity, the velocity gradient would scale as the velocity scale square by lambda square.

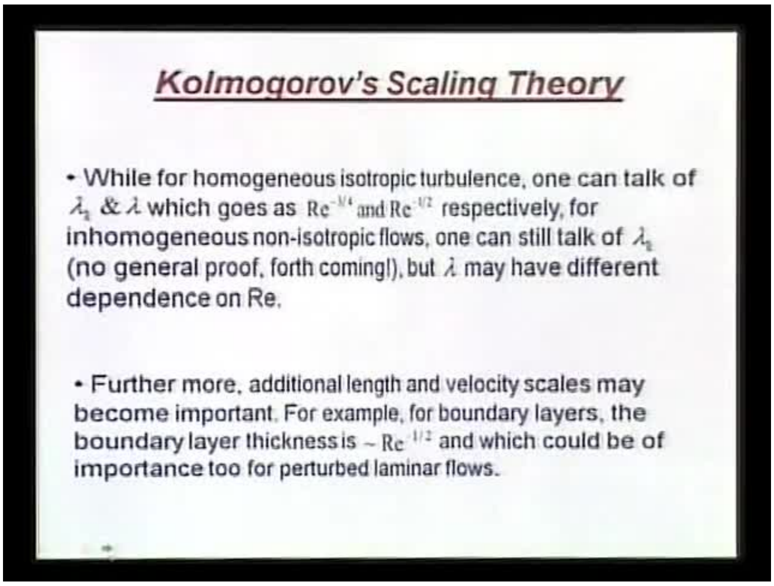
So that there is another way of defining the Taylor micro scale, that gives you a kind of a curvature of the velocity auto-correlation or you could look at it as the width of the energy spectrum. That is what we are doing. So, I am doing k square E k dk divided by this, and what this we have seen already? This is nothing but the velocity gradient scale and this is the velocity scale and this also we have seen is proportional to enstrophy. So, this inverse of the Taylor micro scale squared is the ratio between enstrophy norm by the velocity norm. So, this is something that experimentalist do talk about.

(Refer Slide Time: 05:50)



Since we have this estimates of the velocity scale going as dissipation rate times outer scale raise to the power of one third, so we can use that and plug it in and this is what we get. S, what happens is the Taylor micro scale actually goes as Re to the power minus one half.

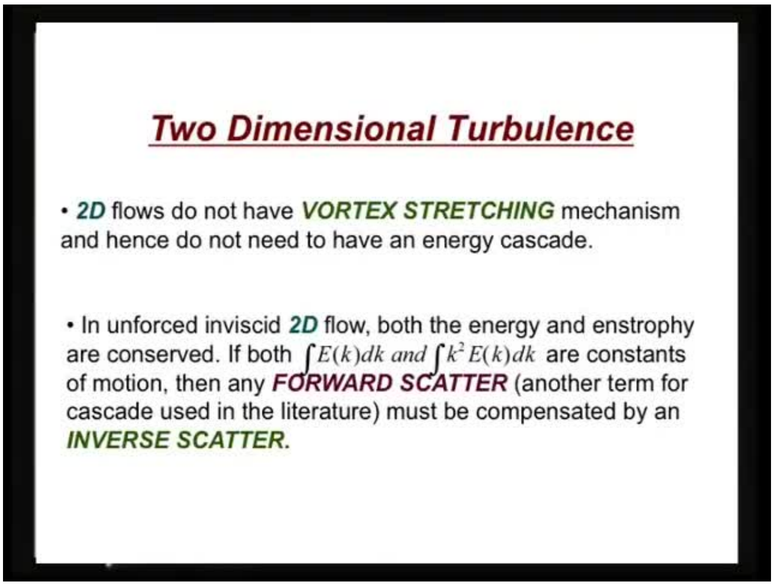
(Refer Slide Time: 06:35)



So, basic idea is that from a different perspective depending on the Reynolds number, you may require to sort of resolve some of those scales. So, if we are let say, for example, looking at homogeneous isotropic turbulence, then we can talk about the Kolmogorov scale Taylor micro scale they all go as a Re to the power of minus three forth and Re to the power minus half respectively.

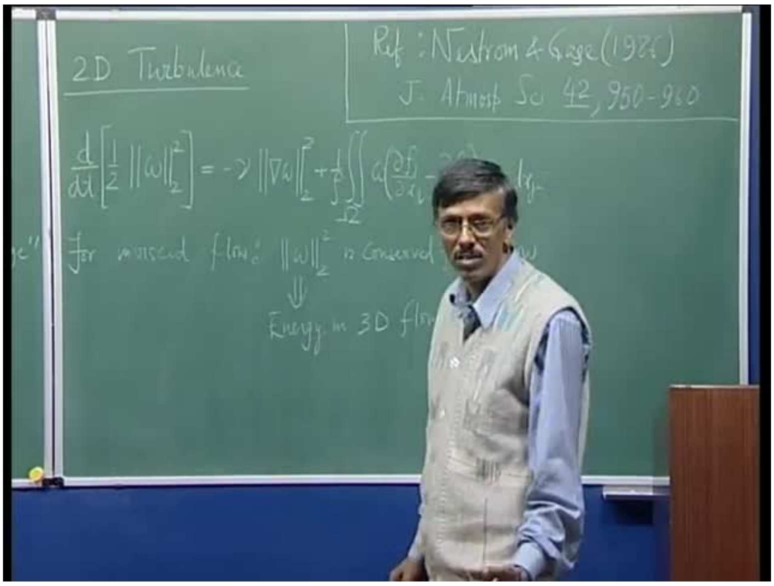
For inhomogeneous non-isotropic flows, we can still talk of the Kolmogorov scale although we have not seen any such definitive proof of it basically. However we can see this lambda, the Taylor micro scale actually depends on the Reynolds number itself. In addition, you may like to resolve various other length and velocity or timescales. For example, if you are looking at boundary layers itself, the boundary layer thickness is proportional to Re to the power minus half. So, you can understand that, that could be also important even though you are not looking at turbulent flow. If you have simply looking perturbed laminar flow, you should be able to resolve those kind of scales.

(Refer Slide Time: 07:45)



So, let me now go and spend little time on 2 D turbulence. 2 D turbulence we can think off does not have vortex stretching mechanism. So, we do not have an energy cascade that non-linear term is missing. If we look at unforced inviscid two dimensional flows then what we can see is that energy and enstrophy are both conserved.

(Refer Slide Time: 08:14)

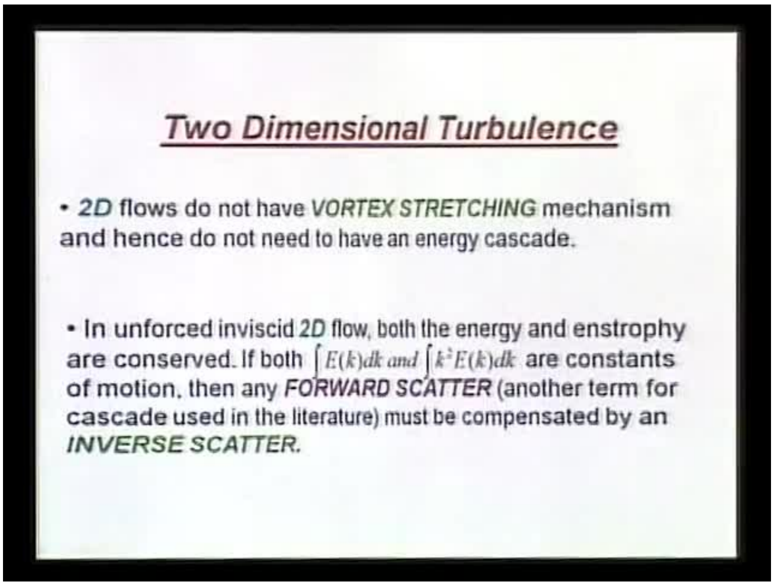


That comes out from let say writing down the equation. The vorticity transport equation for 2 D turbulence in this form. So, we have taken that a vorticity transport equation in 2

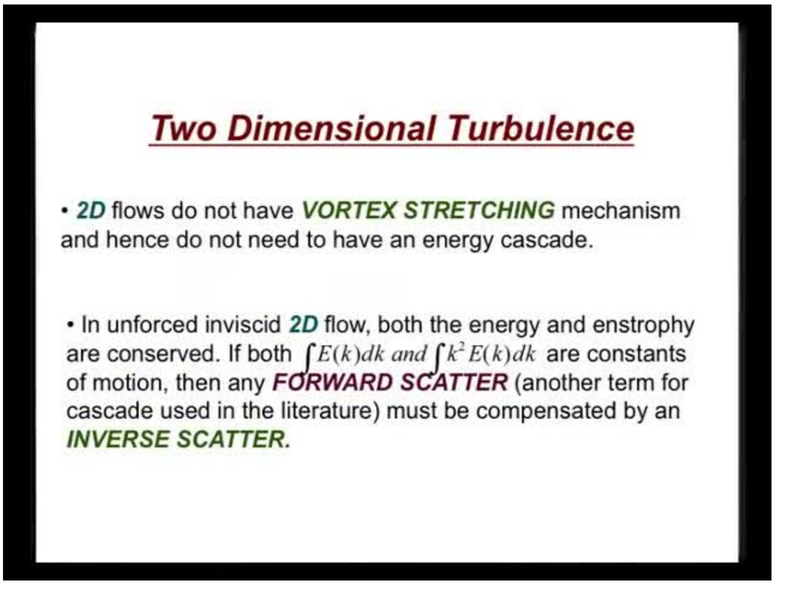
D multiplied by omega and integrated over the whole domain. Then this is the total rate of change of enstrophy that is given in terms of the enstrophy dissipation rate.

So, this is nu times gradient of omega. See in 3 D turbulent, this was omega square itself but it is now gradient of omega square. So, this is we call as the enstrophy dissipation rate symbolically written by chi and you can plug it all the values; omega itself as a dimension of one over time. This is gradient operator is one over length scale; nu as a dimension of L square by T. Plug it in, you will find the dimensionally this enstrophy dissipation rate has a dimension of T to the power minus 3.

(Refer Slide Time: 09:27)



(Refer Slide Time: 09:34)

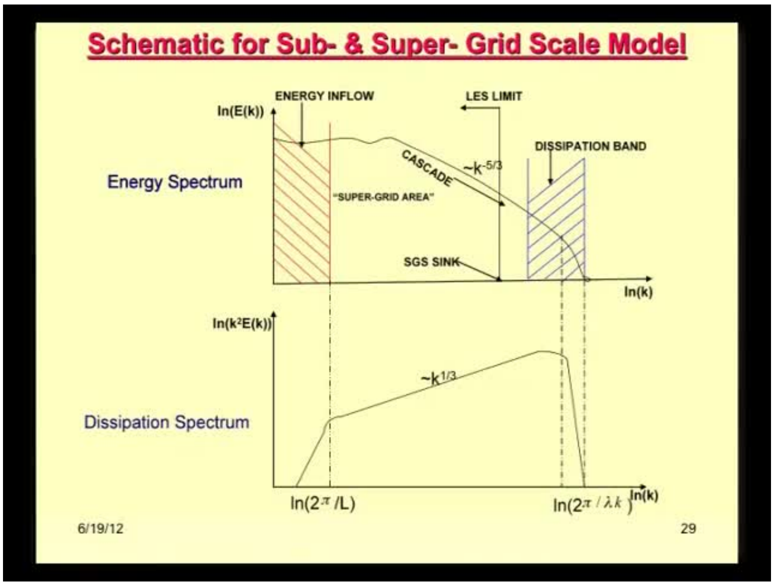


Now, if we are looking at unforced 2 D flow, then what will happen? If it is inviscid, then this term is not there, and if the body force is conservative, this term is also not there. So, if this d d t of the enstrophy is equal to 0, so that is what we are saying. For 2 D flow, you have two requirements both energy and enstrophy are conserved. In 3 D, what we saw? Energy was conserved but enstrophy was increasing because of vortex stretching.

Now, if per save in a flow, we have these quantities as constant. Then by some mechanism, energy is transferred from a larger scale to small scale. That is what we call as the forward scatter or the cascade. So, energy is going from large scale to smaller scale. Then because these two have to be conserved, then we must also have a reverse migration so that these two integrals remains conserved, and this complementary phenomena where energy transports from smaller scale to larger scale is call the inverse scatter or back scatter. In 80s and 90s, there were lots of results related to the back scatter. You can think of it in the following sense.

What was the forward scatter? Forward scatter were larger eddies breaking up into smaller eddies. So, back scatter is just a reverse smaller eddies quailsing and forming bigger eddies, and this is one of the aspect of 2 D turbulent flows. This is something you must keep in mind.

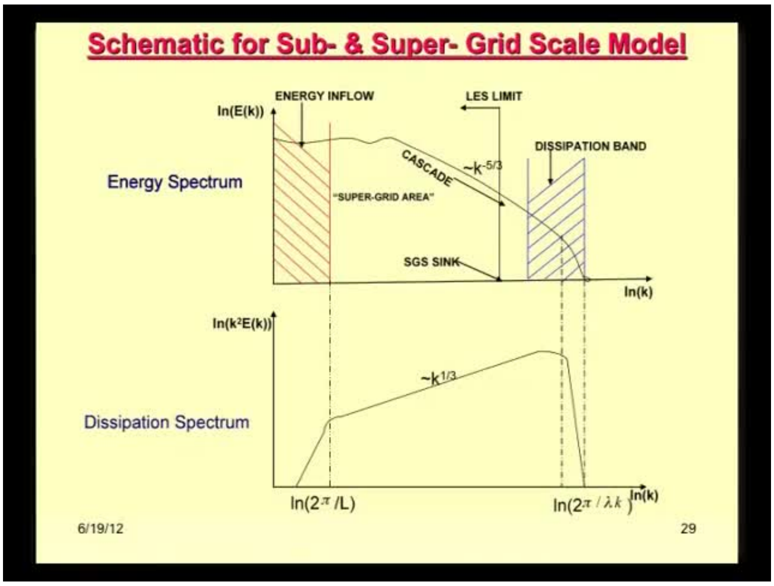
(Refer Slide Time: 11:32)



Now, well, let me now show you something else which I will, let me just show you something else. This is the story for 3 D turbulence. That is the way the energy cascades k to the power minus five third and this is your corresponding dissipation spectrum for 3 D flows. What happens to 2 D flow? We are not talked about it. So, if I have a 2 D turbulent flow, then what am I going to get? What am I going to get is what I have written here. Like if I have 2 D turbulent flow, once again I have some energy coming in and then energy has to be drained out like what you see here.

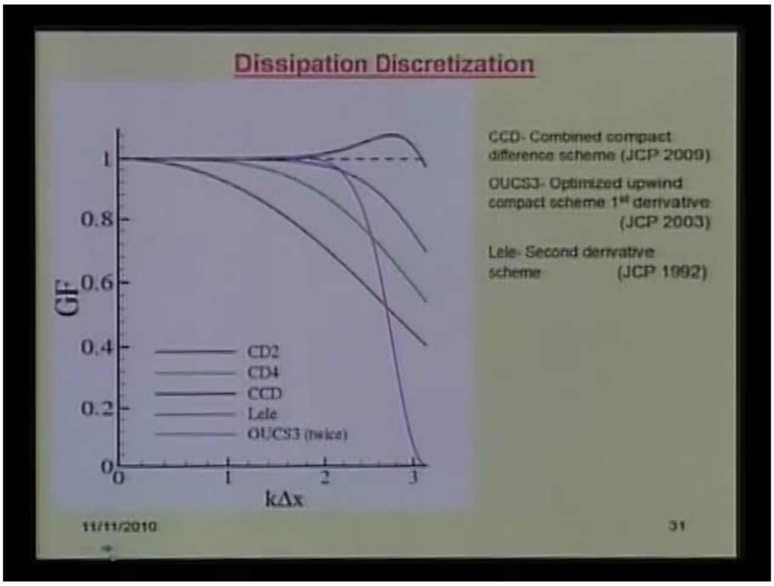
And here, we are talking about instead of energy, we are talking about enstrophy because for 3 D flow, energy was conserved and that was our consideration in getting the inertial sub range, but in this case for 2 D turbulence if we look at inviscid flow, then what happens? These itself is conserved. So, whatever the role that was played by energy in 3 D flow will be played by enstrophy in 2 D flow. There when we talked about an inertial sub range, we wrote the energy spectrum to be dependent upon energy dissipation rate and the wave number. Here, for 2 D turbulence, then I will be talking about energy spectrum and enstrophy dissipation times and the wave number. We have already seen this is the dimension of energy L cube T to the power minus 3; chi we have just now seen here, it is T to the power minus 3; this is 1 over L. Do a dimensional analysis and you find this.

(Refer Slide Time: 11:32)



This is something rather interesting that if you compare it with 3 D flow here, the energy spectrum dependence for 2 D turbulence is given by k to the power minus 3. So, I am not doing it but I am suggesting to you to tell me what happens to the dissipation spectrum then in this range. This is something interesting. This completely different than what we have talked about 3 D flows. 3 D flows we saw in the previous slide that dissipation spectrum was going up like k to the power one third. Here, it will come down as one over k, very interesting thing. So, the requirement of a 2 D DNS versus 3 D DNS are not necessarily the same, we should keep that in mind.

(Refer Slide Time: 14:50)

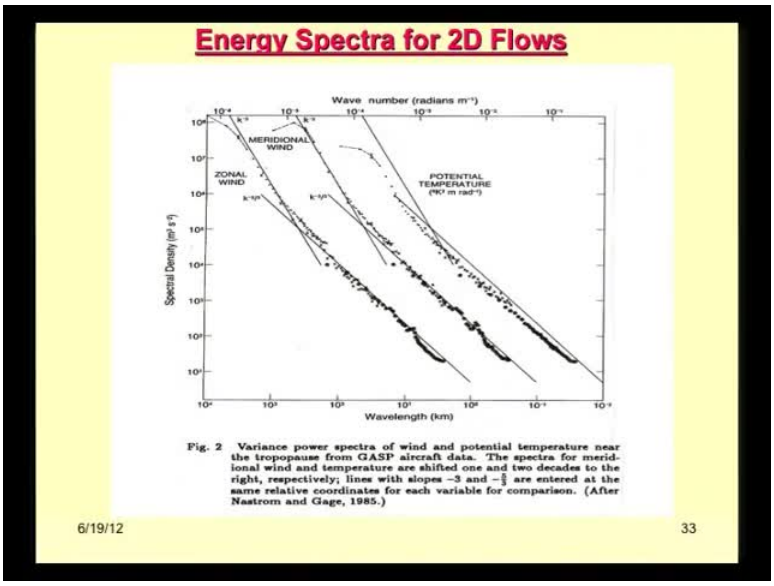


Now, are this somewhat real or not is something that we need to understand, but having seen in the difference between the 2 D and 3 D dissipation spectrum, we need to really understand what we do when we actually numerically solved. This is a picture that tells you about how you discretize the dissipation term by various methods. So, if I take a let say second order central difference scheme at higher k, I actually have significant loss. It is about 0.4 to 60 percent loss of dissipation. This is your c d 4. Then this is the one of that high accuracy scheme compact scheme proposed by Lele.

And this is what we often have used which is called a combined compact difference scheme. We have been doing it for quite some time and last figure is interesting. Last figure is that we use high accuracy scheme for first derivative twice to get it. It is interesting in the sense that when you look at this property that it remains a faithful all the way have to two and more and then it falls off to 0.

Whereas in c c d, you see always that high wave number is over estimated and we have seen those of who have some exposure in computing. You know that aliasing is a major source. So, if you want to knock off, remove the bad adverse effect of aliasing. This kind of over estimation of dissipation is not bad; in fact that might turn out to be very positive.

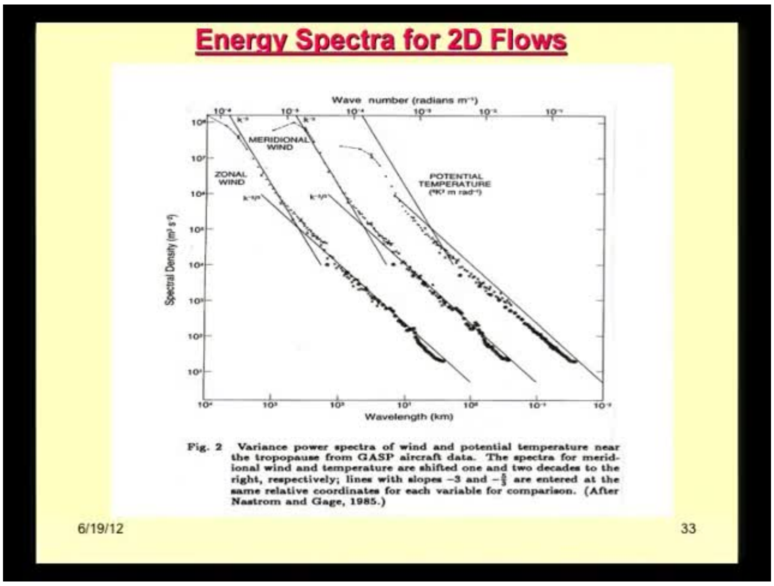
(Refer Slide Time: 16:39)



Now, well, let me just simply close here on a note of what do we actually see in nature. This is a power spectral density for atmosphere. What you are seeing is basically the zonal wave, meridianal wave and the temperature field. This is along the longitude, along the latitude and this is the temperature distribution and it is plotted versus wave length also in the same scale, if you read from the other side, we will also get the wave number. So, this is what you are seeing.

So, what you are noticing a very interesting thing that this is at the order of the planetary scale, you knows, the sort of the periphery, the 2 pi d of earth in that range tens of thousands of kilometer, and here, these are in kilometer, this is 10 to the power minus 2 minus 3, etcetera. What you are seeing is the first part, the dependence is like k to the power minus 3. So, in the large scale in atmosphere, you do see attributes of 2 D turbulence. So, this is not something that is just simply academic; we do see that. In fact what you see that this this 2 D turbulent feature is followed by 3 D turbulent feature. This is your minus five third slope. They have been shifted laterally for viewing. So, otherwise there is it would look clattered if it were all put one over the other. So, basically on this mode, I would stop saying that do not keep your mind closed because you can see as the greatest fluid dynamical lab could be your atmosphere shows the presence of 2 D turbulence.

(Refer Slide Time: 16:39)



So, people tend to over emphasize on vortex stretching 3 D turbulence but 2 D turbulence is equally relevant, and this is a direct evidence. This was given in this reference by Nastrom and Gage published in a journal of atmospheric science in 1986. They actually employed a fully instrumented aircraft and made it fly around the globe. That is how you could sort of get the whole scale. So, there you have it.

So, having come this node, I will just stop it and let us have a discussion of whatever we have gone through the whole semester. So, this is your time to really have this meaningful discussion. Let us go add ahead do it. Let us start it. So, you all completely comfortable in (( )) questions. You get started if you wish to. If you want, you can take the…

Sir, I would like to ask you about the difference between the excitation with acoustic (( )) trigger that t s instability. So, let what is it basic difference?

Well, the acoustics can create. It has the receptivity but the mechanism is little more settle length. Let us say the vibrating ribbon experiment of schuhbauer and skramstad. They originally started with the acoustic excitation to generate t s wave. They were not successful simply for the reason was calculations were for 2 D Tollmien-Schlichting waves.

So, you have to create a disturbance field that is two-dimensional. Can you create a 2 D acoustic wave? So, you see this is also we touched up on when we talk to about klebanoff mode. Why Taylor could not detect t s wave, because he was using a mechanical vibration like the vibrating ribbon but his frequency did not match. So, it is very important. If you are trying to compare your theoretical result with experimental result, you must compare apples with apples, not apples with oranges. But if you do have acoustic excitation like we did some calculations few years ago for supersonic flow, you can create disturbance wave but it may not work out like t s wave. There we saw it is more like a bypass event.

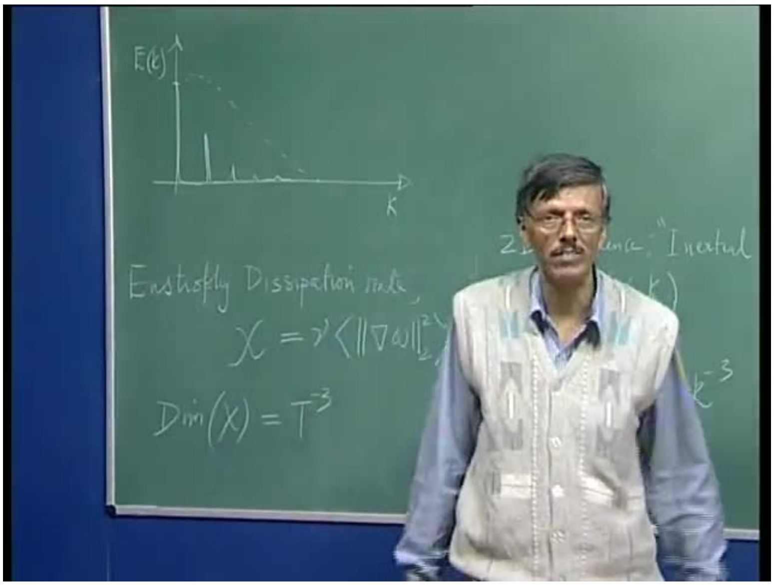
Sir further based convection problem, how do we exactly arrive at flow assisting and flow resisting forming because (( )) of k?

Well, with the help of your direction of the gravity, see there the body force was due to the buoyancy term that is your (( )) approximation. So, if the gravity assists the convection, how the convection would go from a hot to a, it will rise up if we are looking at on the top side of a plate. If I look at the natural tendency, lighter fluid will rise up.

So, that is your the direction. So, if your buoyancy is also in this direction than of course it is resisting, but if you look at the bottom side of the plate and if the plate is still hot, then what will happen? Then it will be opposing flow. So, that is the way to interpret it.

Sir, in the case of laminar flows, we are saying that energy spectrum contains peaks of a particular points. Do they correspond to the vortex if the largest vortex sizes?

(Refer Slide Time: 23:28)



So, if I look at the wake of a cylinder as a very economical example of what we see. If I plot E of k is k, you may just simply see sufficient energy at the, means zero k means the largest possible scale that you have and then it may just simply have this, and then, corresponding to a strouhal number, you will get this and it is a non-linear dynamical system. So, if I excite the system, the system is susceptible to that frequency; it will also be susceptible to its super harmonics, sub harmonics. So, you do get that.

We did not talk about let us say take a cylinder and start doing this rotary oscillation. Then you can really see the presence of various sub and super harmonics. So, you will see that kind of a picture like this, whereas if you look at a turbulent flow, but now, we have seen that it is wide band phenomenon. So, l and g resides very large range of… You have any question?

Sathyanarayan, at the blasius point of inflation, stagnation (( )) blasius point of inflation,

No, let us try to get the question clear. You are saying if you have a stagnation point flow, that is not called the blasius flow. That is called the (( )) flow. So, what is the question if the stagnation point is a point of inflation or not?

See, the thing is if you also look at the blasius profile, what is the value of the second derivative of the velocity at the wall? It is zero pressure gradient. So, second derivative is related to the pressure gradient. So, if I take a zero pressure gradient flow, the second derivative is zero on the surface.

But that is not a inflection point, because inflection point is across which the slope changes side but you do not have anything across, you stop there. So, that condition that we have talked about you probably are hinting at Rayleigh condition, flow tops condition that you have a inflection point to be invisibly unstable, but if you have it right on the surface, it does not cause a problem, but if you have it of the surface, if you have a inflection point, then you would get a violate instability. In fact, for any velocity profile which displays inflection point, they are going to be very violently unstable. For example if you look at (( )) or mixing layer, a critical reynolds number are less than 100 unit something like 3, 4 in that range you can find those flows to…

Well, the question is at those reynolds number whether it is right to consider the flow to be parallel or not it is a different issue altogether. So, any other question? Anybody having any particular question to ask?

Sir, we discussed about Kolmogorov scale. So, when we do computing, these streams are

(( )).

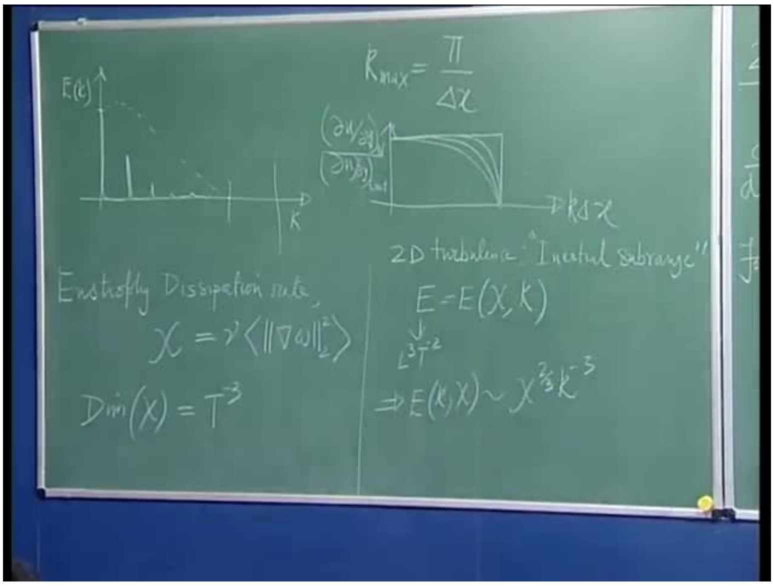
The scales is not affected by computing but your computing should be able to resolve those scales. That is what we made the point that Kolmogorov scale is a smallest scale, that is exited in a flow. Beyond that, everything is dissipated into heat. So, you cannot have eddies smaller than that size. So, you cannot have fluid flow entities which will be retaining its identity and having a kinetic energy beyond that scale. So, that is what you want to do. In direct numerical stimulation when you are trying to solve the equation, you are trying resolve all the way up to Kolmogorov scale.

(( ))

Up to

but then there is the (( )) also the (( )) that also…

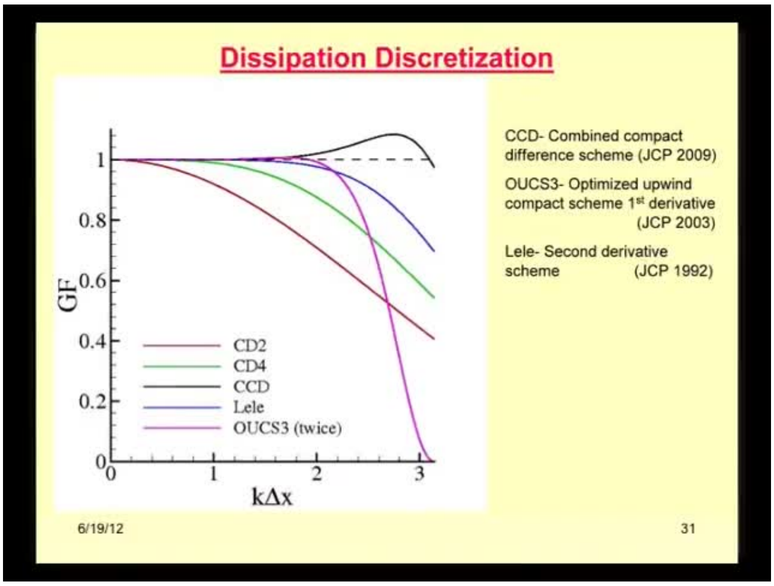
(Refer Slide Time: 29:02)



Well, we would we what sharatho is raising a point is something more that what we talk about, what do we exactly mean by resolution. Suppose I solve numerically and I have a resolution given by this grid spacing. So, what we know that this is your maximum result wave number. Theoretically speaking this is the maximum that you can resolve. That is what you do if you let us say do Fourier spectral method.

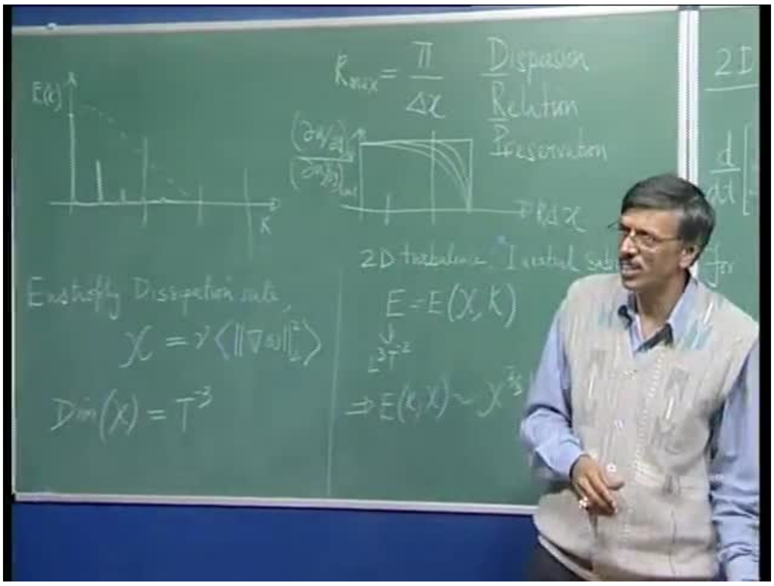
You take any other method, what will happen? You will not be able to exploit this whole thing. That is why he is saying that if the Kolmogorov scale is here, your method should be further on this side because you do not, if you are looking at transfer function of the method, say if I am trying to show the derivative, if I have to calculate this, one of the shear derivative let me say that this I am trying to do numerically divided by the derivative which I have for exact condition. Then Fourier's spectral method gives you this kind of a picture.

(Refer Slide Time: 30:42)



Any other method you get a part of the range better the method, you will get more and more. See this was something that we saw. We saw here as far as the second derivative representation was concerned, ideally I would have liked it to be equal to 1 all the way, but different methods filter it in a different amount. So, he is saying that do not draw your conclusion based on this maximum limit because your method starts deviating. For example, if you look at this purple curve, I get only two thirds of it result.

(Refer Slide Time: 31:53)



So, his question is a very valid question that I may not be able to resolve the derivative itself properly. Now, add it to that is the main thing about this course is are we faithfully reproducing the physical dispersion relation computationally or not? That comes about from the group velocity we have seen, and the what we find that the physical group velocity at the numerical group velocity usually as mismatch over a large range of k delta x. So, you know, this is all about resolving a derivative that still does not give you the full picture.

A better picture would be where you resolve this, you also satisfy the physical dispersion relation. This is what is called as topic which is common. When people quote it very common in misunderstanding it, there are lots of people who can claim would do claim that they have method which preserves. So, they call it DRP schemes. So, they say that we have developed numerical methods which preserve dispersion relation.

That what one should do actually is look at what your numerical method is doing. What is that methods dispersion relation is the numerical dispersion relation compare it with physical, you have a very small intersection. So, talking about this, if I am happy with this method, my dispersion relation may actually bring it down further. So, what we are saying is DNS as it is practiced by many people across the world. They have serious limitations; they do not even understand what is really a DNS are to be.

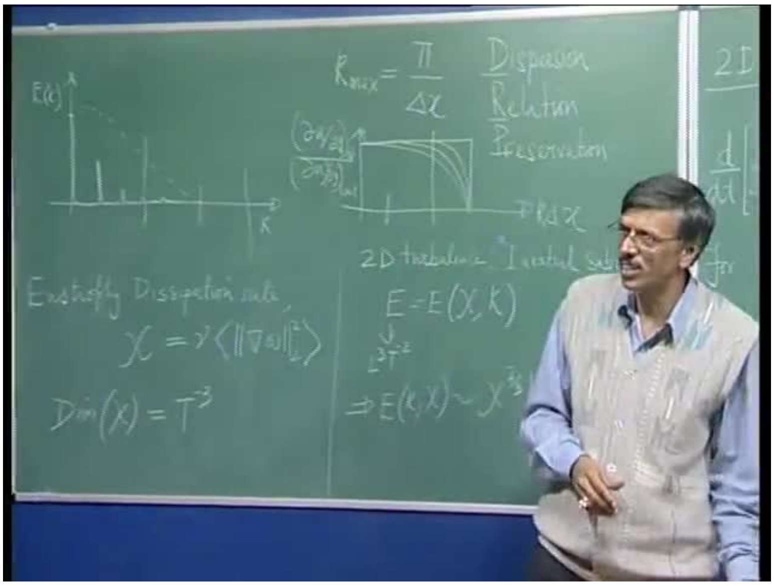
If you do not understand your numerical dispersion relation, you have no way of knowing what you are doing. Unfortunately that is the state, but it is good that sharatho pointed this issue. What I was doing? I was parroting what everybody says that the lowest denominator is, at least you should be able to resolve this. If your numerical resolution itself is here, then you are not even going up to the Kolmogorov scale. That is a last case anyway.

So, there are various other issues, for example, we have not talked about another source of error that comes about because of aliasing. That is also a non-linear product terms that is where evaluating this product terms you associate aliasing. So, that also limits your resolving ability significantly. Those of you now, my point of view in this course is that we learnt quite a bit about theoretical results related to transition and instabilities. If we want to compute them per say have been issue without making any assumption, then this sort of issues come into picture.

We have to really take out and the good news is the ability of high accuracy computing is increasing. So, dramatically that we are now in a situation where we can talk about non-linear flow instabilities being computed and we have done one such thing you have seen. We talked about proper orthogonality composition in this course.

We did also relate those p o d modes with the instability modes. This is a new subject in the arisen I mean it is it is being done here. Currently we have explode a whole range of (( )) flows starting from flow past a stream line body, flow past a bluff bodies. We are seeing that if we could relate this p o d modes with the instability modes and we have come out with point of view where we see some universal features appearing, like if you compare flow past a cylinders, it is external, flow but this flow inside a driven cavity, they may look completely different, but when you look at their p o d modes, we will see those building blocks are identical.

(Refer Slide Time: 31:53)



So, there are lots of exciting things happening or possible. Now, we are now in the threshold of talking about non-linear instabilities in a full-fledged way. I think the future looks absolutely bright. There are lots of things those are happening here in our lab. People are paying attention and this is very hopeful sign.

Let me tell you do not think of computing in isolation, you can be at the most glorified clerk. You will be able to get some code and you can done it but you have to understand the physics of the flow and I hope this subject actually exposed due to some of the nonsense of instabilities, turbulent flows. Without that, you would not even know what you are computing.

For example, we just talking about dissipation discretization, this may look good. If I understand, what the dissipation spectrum is. We know that is where it peaks up at the highest wave number. So, I cannot just simply say I am doing perfectly alright up to here this side it should be… Lots of people talk about discussion on resolving scales only in terms of energy spectrum.

But what you need to do is basically energy spectrum information should be supplemented with the dissipation spectrum because you saw the whole idea of Kolmogorov spectrum and all relates to the balance. That is what you were putting in that is exactly going out, and if the dissipation is not model properly, you will go ever; you will settle down to a wrong equilibrium condition. So, any other thing?

If you have some questions, you can ask; otherwise we will just say good luck to you. Hope you will be continuing some of you solving some of these problems and come out with some new interesting finding. Some of the topics which I could not do, which I could have touched upon but lack of time prevented is the instabilities of three dimensional flow specially from the perspective of aerospace engineering. For example, if you look at flow over a aircraft wing, then you can have the flow going in one direction disturbances may go in other direction. We talked about it when we are talked about spatial and temporal instabilities and that is what you see that, the phase goes in one direction; growth is in another direction; mean flow is in the third direction.

The currently the state of art is of course one way is to be able to do a complete DNS. It is not so trivially done, we are making some preliminary effort in that direction, but people do theoretically? They isolate the problem like if the disturbance were growing in the stream wise direction, then I get some instability criteria. If instability was going in the cross flow direction, then I get another criteria.

So, we are just simply talking in terms of various criteria depending on if the disturbances were going preferentially in one particular direction but actual flow, it may not be so. So, there are lots of such things which I would have wanted to do but lack of time prevented, but nonetheless the basic mechanisms have been covered and one of the distinctive aspect of this course would be emphasis on receptivity. We do believe, continue to believe; we have not seen anything contrary to our believe is, that all flows are excited. You cannot just simply talk about a flow past is fear which is perfectly spherical in shape, flow is completely uniform and then you get turbulence, it is not so. We got to understand what the background disturbances doing to the dynamics.

Like the dynamical system point of view is the main thing. We should look at the dynamical system. Instability study relates to the transfer function but we also say the input is important to understand what we see in the output level; otherwise the picture remains incomplete. So, I think we will close here. Thank you.