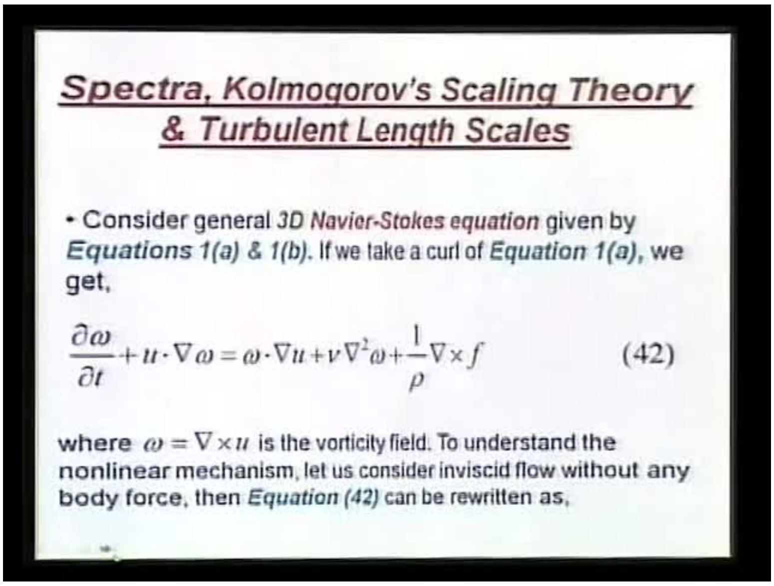
Instability and Transition of Fluid Flows

Prof. Tapan K. Sengupta
Department of Aerospace Engineering
Indian Institute of Technology, Kanpur

Lecture No. # 38

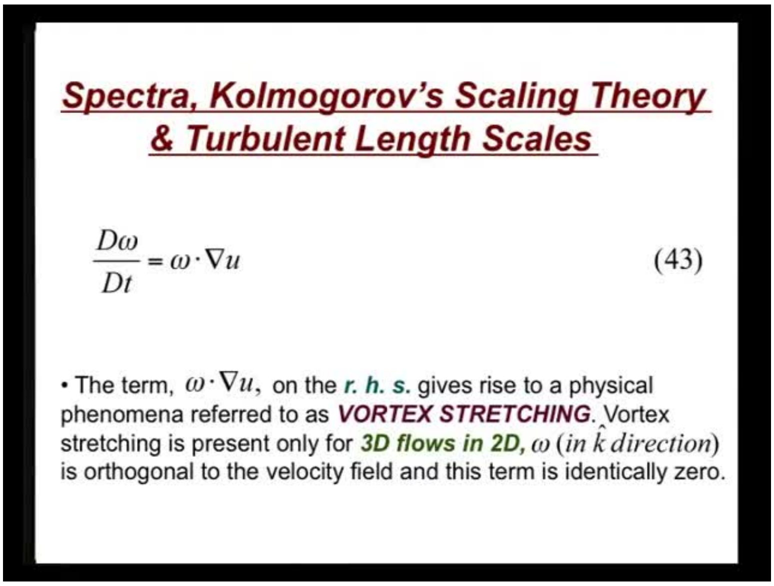
(Refer Slide Time: 00:22)



So, in the last class, we are trying to figure out for turbulent flows, the various scales those are invoke, and to understand that, we wrote down Navier-Stokes equation in terms of the vorticity. So, this is the equation that you would have, I did say it today again I emphasize that in writing that vorticity transport equation, you get better picture of the various processes those are involved. For example, this is your substanship derivative on the side, but this is the additional term which we call as the vortex stretching term is important, because this is only present for three-dimensional flow and not for two-dimensional flows.

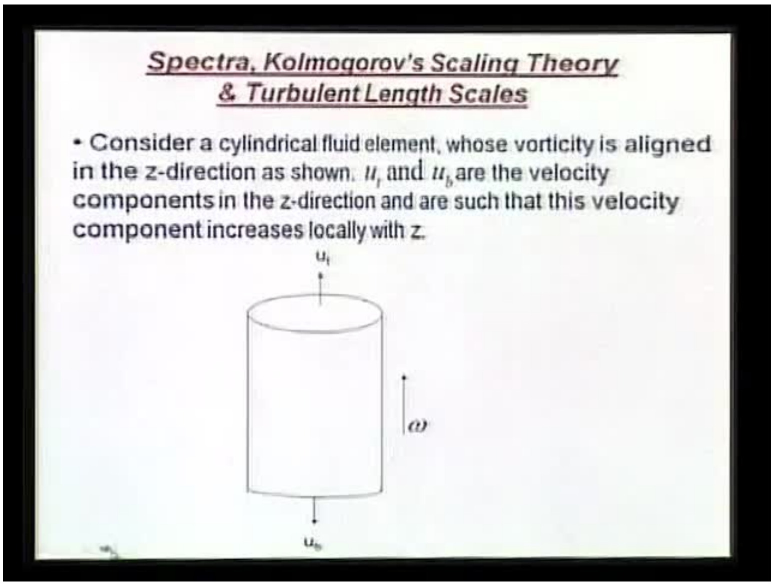
So, this point of view actually helps you in understanding distinguishing between 2D and 3D flow field and also this is a non-linear term. So, we would like to see how this non-linear term affects the flow dynamics.

(Refer Slide Time: 01:55)



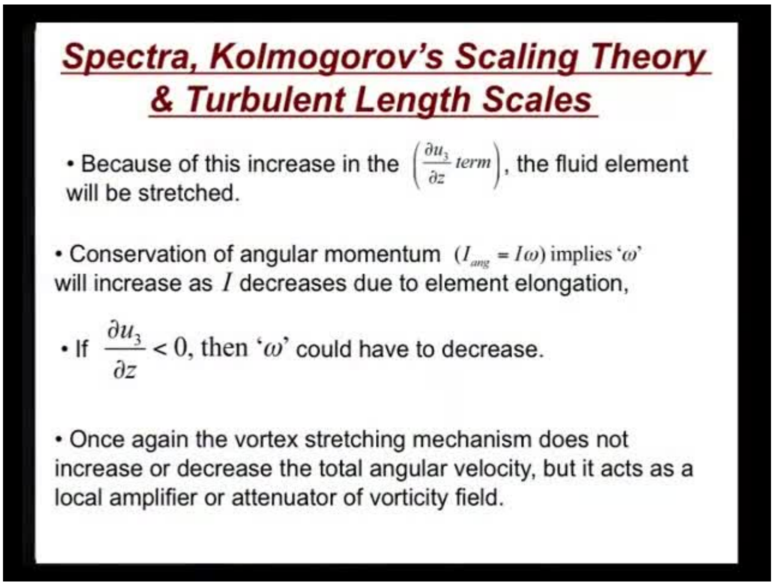
So, if we then consider again for the purpose of analysis, the flow to be inviscid and not have any body force, or even if it is there, let us say this is given as a conservative force, then the body force does not come into the vorticity transport equation. And then, what happens is we get the simplified equation like this. So, the derivative the rate of change of vorticity is solely driven by the vortex stretching.

(Refer Slide Time: 02:24)

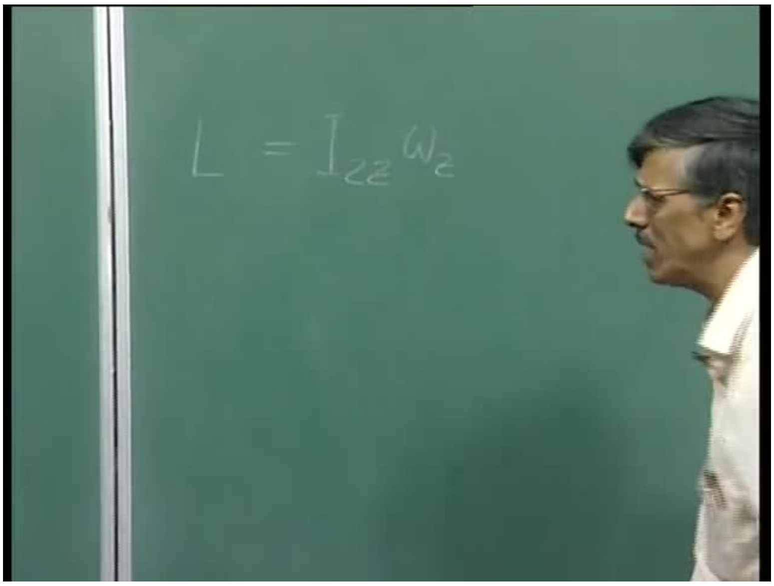


So, this is the term that we have discussed. Let say what happens is this is only present in 3D flows and 2D flows omega and grad operator or in quadrature. So, they would not have any problem, and I was explaining this yesterday by considering a special case, where let us say we identify a fluid element of this cylindrical topology with the vorticity predominantly in the z-direction, and if we look at the velocity of the two ends given by u top and u bottom, so, if it is a kind of stretched, then what is the consequence that we wanted to figure out?

(Refer Slide Time: 03:08)

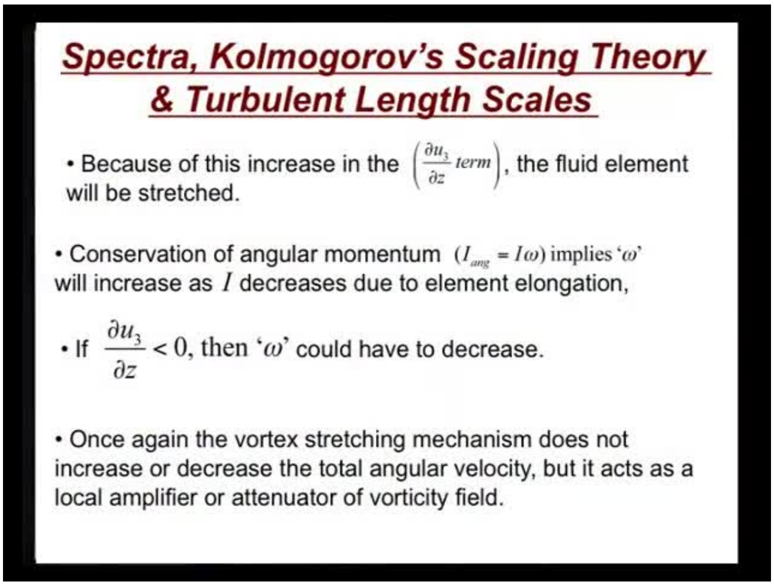


(Refer Slide Time: 03:29)

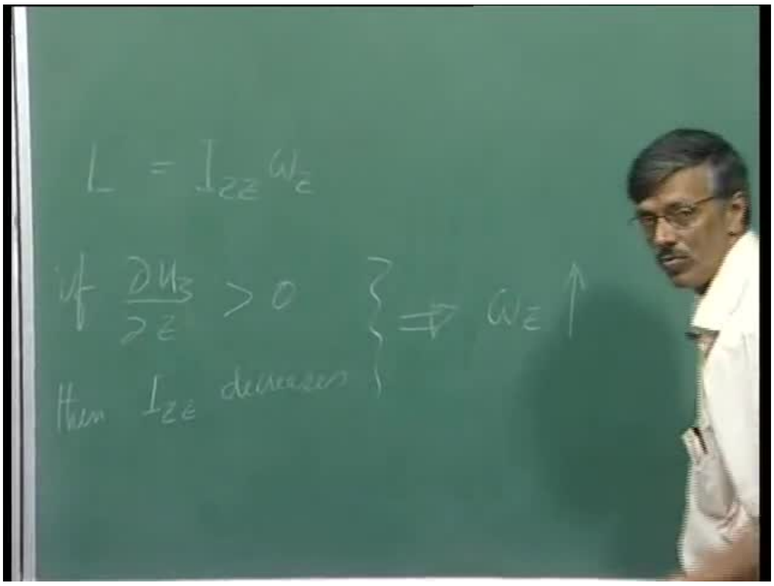


The way we have drawn the velocity vectors here it shows it is kind of a stretching term, and then, once you see that, what happens is the fluid element is going to be stretched and we are talking about inviscid flow. So, our all kinds of momentum are conserved including the angular momentum. Angular momentum is now given by I zz times omega z. So, basically if we are looking at the angular momentum here as simply I z z omega z.

(Refer Slide Time: 03:08)

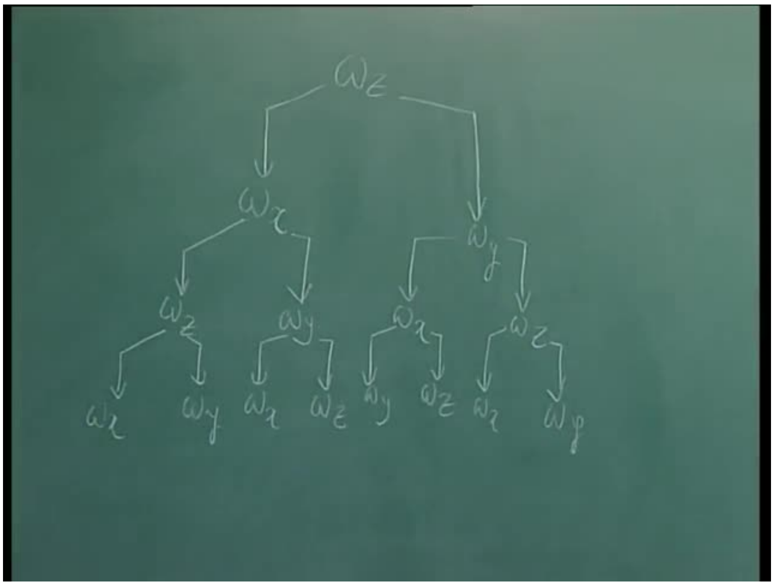


(Refer Slide Time: 03:50)



Now, because of this fluid element stretching due to this rate here del u del z, what will happen is if this is greater, then what will happen? The element stretches and I zz decreases. As a consequence what you are going to get is that omega z is going to increase. So, this is the consequence of that vortex stretching term because to maintain the angular momentum conserve if I z z come down, this must go up. However, if you look at globally over the flow field, the overall angular momentum should be conserve also and the vorticity also would be conserve, and then what happens is if locally there is a variation in omega z, there must a corresponding reduction in other components.

(Refer Slide Time: 05:05)



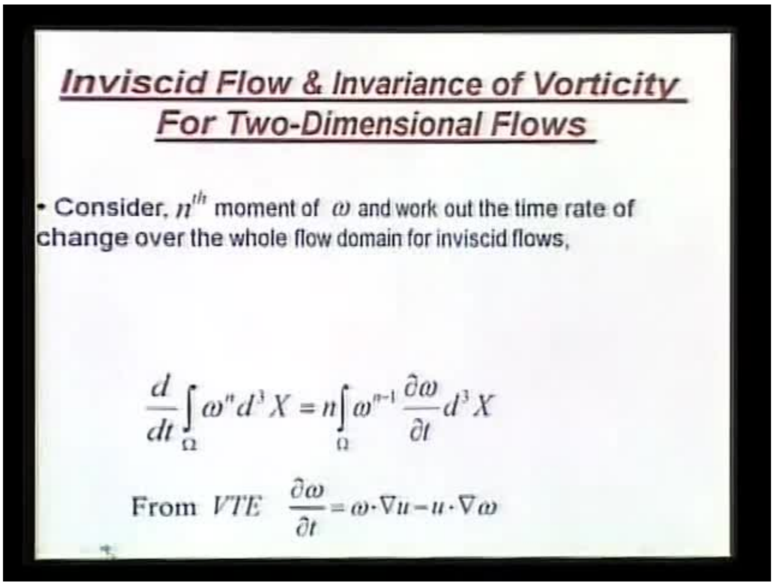
So, that comes out we can show in terms of a tree diagram, like if I have omega z here and then because of this stretching term, that omega z is increasing, but then, that will effect omega x and omega y; so, we get a corresponding variation in omega x and omega y, and again, if the fluid element experiences some kind of a stretching.

Let us talk about stretching compression also will bring in the exactly the same effect but it will be just opposite in sign. Vorticity will be changing sign, but if you look at the corresponding contribution to enstrophy, it will always be additive. So, enstrophy keeps changing. So, if I now have this elements omega x and omega y I change, that can further mode bring change in omega z and omega y and the same way this will bring in change in omega x and omega z.

Then further more if we go one generation down, this omega z will affect omega x and omega y. This will bring in change in omega x and omega z. So, this is a kind of a composite picture that you can keep thinking about, that is going to happen, and if you keep on doing it further down, then what you are going see is that, although you started off with let us say in homogenous change in omega z, subsequent change is going to be sort of all pervade because you are going to change. Change is the happening in omega x in three times omega y, three times omega z, two times, and if you look at the next generation, each one will (( )) another couple of them in Quadrature. So, what happens is this keeps on happening going down them tree. You are going to see these smaller and smaller changes but they are going to be isotropic. So, this isotropic behavior of vortex stretching also is used in turbulence modeling when one does large a d simulation.

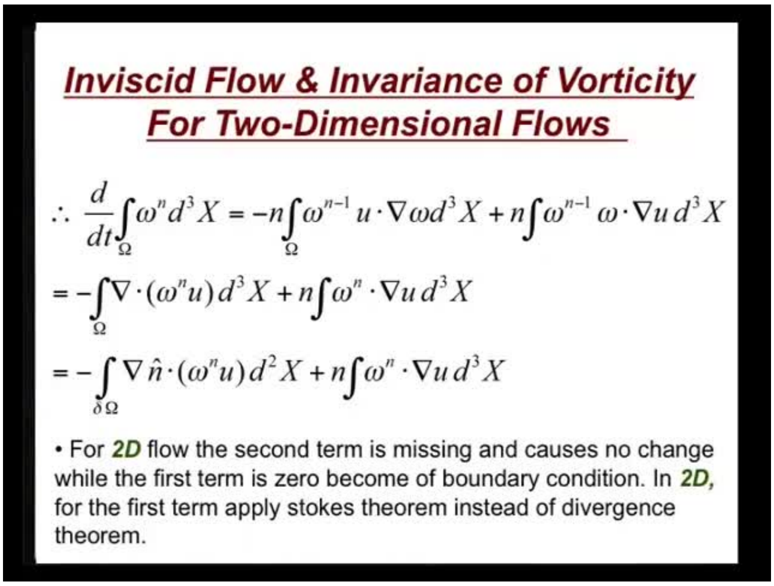
There if you are looking at the vorticity at the smaller scales, they are going to be as isotropic as it is indicated here, and, this, this, if this is not resolve in your computation, then you can model it because now you are armed with that additional information that this is isotropic. See basically modeling is always helped once you know what is happening. For example, previously we saw in case of time average equation, how Reynolds stress was correlated with the mean strain rate and that helped us developed turbulence model. So, in the earliest model also we make use of the fact of that, at the small scale, at the sub ridge scale, we have isotropy. That is a very hopeful sign and good news for one to do that.

(Refer Slide Time: 08:39)



In this following, we just simply said look you do not really need to only talk about enstrophy or anything but you can look at the nth moment of the vorticity and find out it is time rate of change, and then, you are going to get this. This we can put it inside, so, we will get n omega raise to the power n minus 1 times del omega del t.

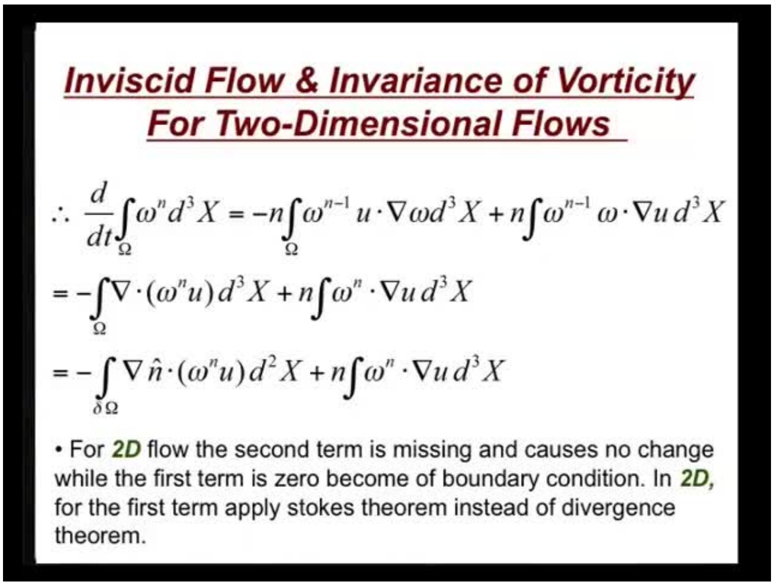
(Refer Slide Time: 09:17)



Now from the vorticity transport equation for inviscid flow, this local change is given by the convecting change as well as this vortex stretching. So, we can put it up there and that is what we did in yesterday's class, and what we really found was that, we get two sets of term one comes out from convecting term another comes out from the stretching term. The convecting term can be written down in terms of this as a volume integral, and this is the stretching term, that is written like this. This could be, because this is written in a conservative form; it is a divergence of omega n u. So, we can converted into a area integral over the controls of s and this is what we get.

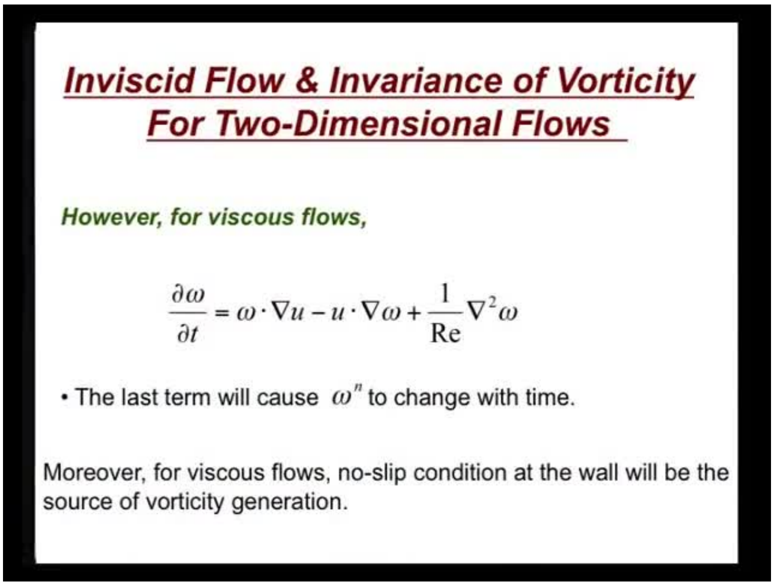
Now, for 2D flow, this does not exist, we know that. The first term is 0 if we consider the boundary condition. What happens is if you are looking at the far field boundary, the vorticity itself is 0. If you are looking at the solid boundary, no slip condition will ensure u is 0. So, this will not be 0; this is already absent. So, for 2D flow, we find that any order moment that you look at for vorticity does not change with term.

(Refer Slide Time: 09:17)



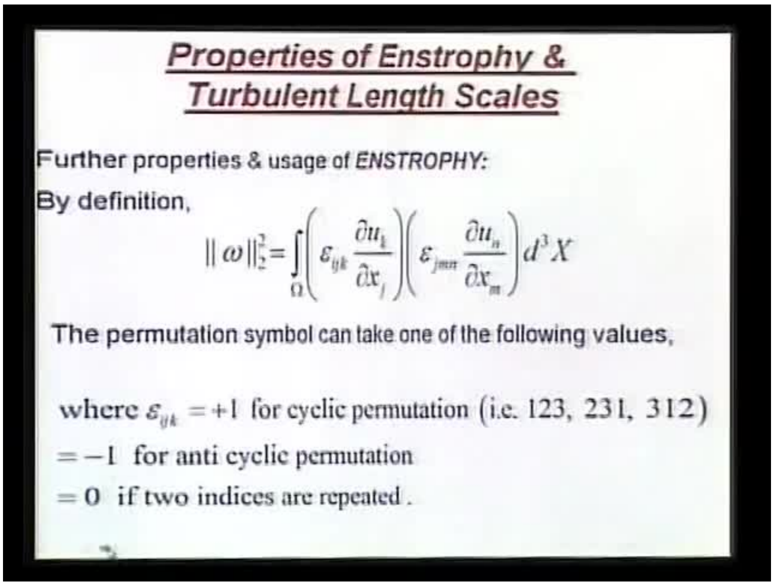
That was what we were talking about. Remember that, if omega z increases, then corresponding omega y omega, that comes about probably if I look at some kind of a knob say enstrophy that will be omega x square plus omega y square plus omega z square. So, if omega z square increases, it better be that omega x and omega y decrease and this whole idea of this tree diagram that we had. So, this is one thing that we can establish for 2D flows. For 3D flows, the time rate of change is given by this the stretching term.

(Refer Slide Time: 11:19)



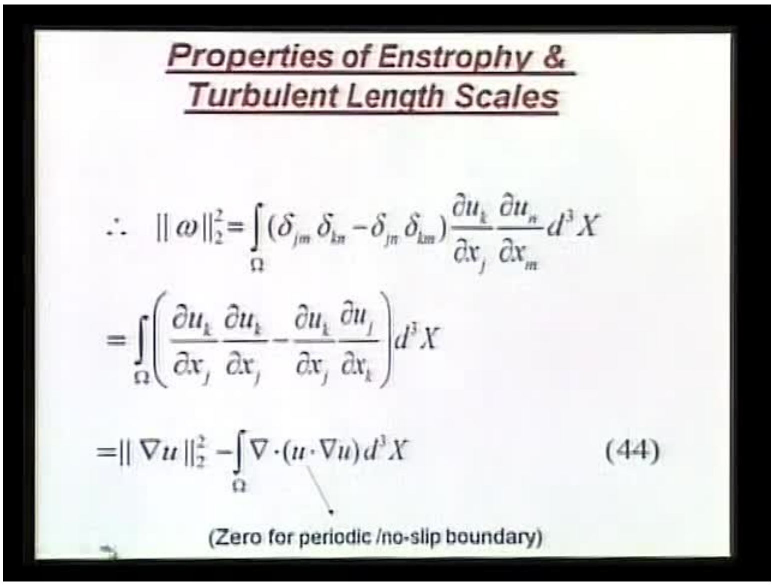
So, stretching is a very important role to play for three-dimensional flow. It can tell you how the various moments of vorticity can change with time. If you add the viscous effects, of course this term will have to come in; this will also affect the moment here with respect to time. However, when we talk about viscous flow, we need to also understand that the solid body itself is a source of vorticity generation, because if the no slip condition, vorticity will have to be generated; so, that adds to the moment.

(Refer Slide Time: 11:59)



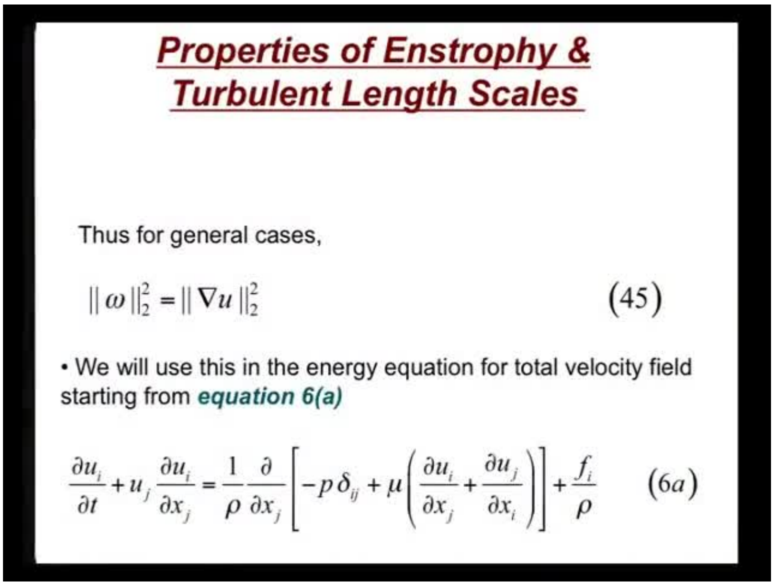
So, whatever we are discussing strictly for viscous flow, they are all together difference. So, we need to keep that in mind. So, let us try to understand what we get by defining the enstrophy here, is a square of the vorticity and this is the generic definition in terms of the permutation symbol e i j k Re j n and this is what we discuss that, if they are in order when cyclic order, then that is positive 1, but if the cyclic order is broken, it become some anti cyclic permutation; then it gets a minus 1, and if any of this two indices are same, they actually do not contribute anything.

(Refer Slide Time: 12:40)



What happens is this enstrophy can be written down as a combination of two sets of terms which we have written it down here in terms of this Kronecker deltas and you can see that these will only work when your m will become equal to j and k will become equal to n. So, that gives you the first component and this is the other component where j becomes equal to n means this is u j, and when k becomes equal to m, so that will be this, so, that is, that is what we (( )).

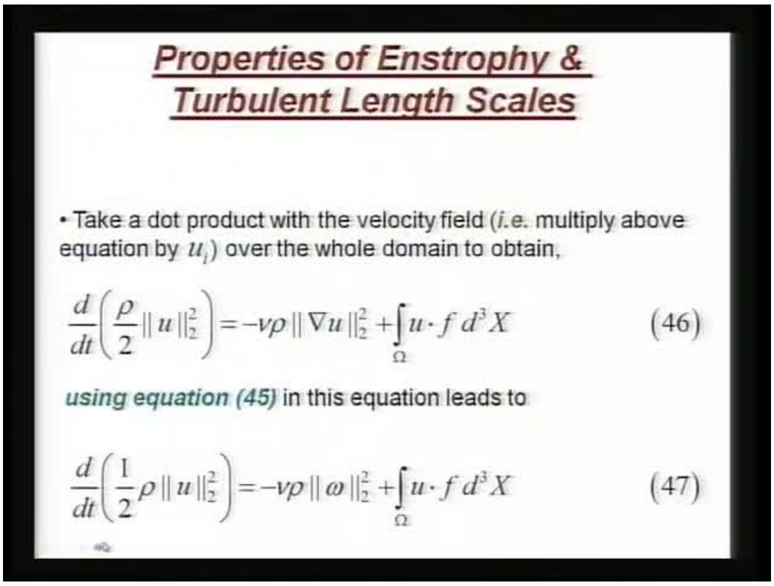
(Refer Slide Time: 13:45)



So, this is basically a two sets a part with this is the gradient u square. So, what you find that enstrophy is related to a gradient of the velocity field that normal. This term that comes about from the second set is going to be 0. If we are talk about a periodic problem or if we are talking about mostly boundary condition will cooperated, there will make that second term equal to 0.

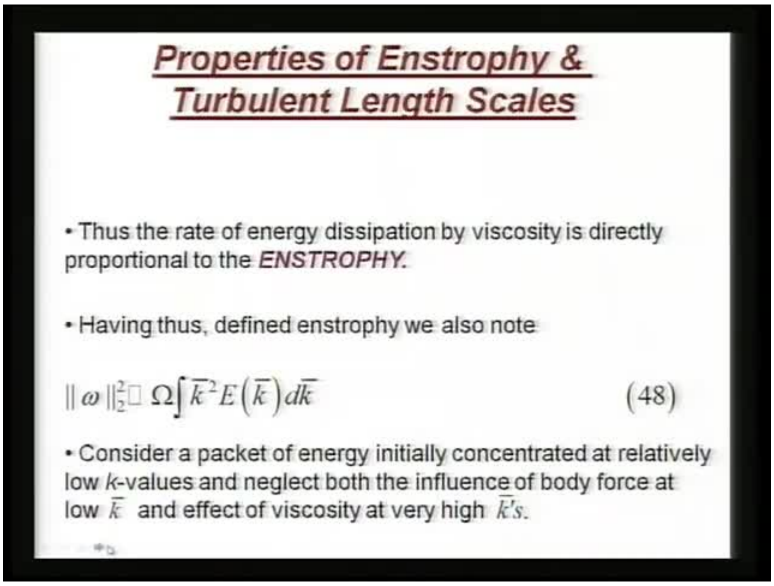
For general case, the enstrophy measure is given by the velocity gradient measure. So, we can make use of this equation 45 in further developing our ideas. If we start writing out, again the energy equation for the velocity field as it is written there. What we can do is we find then this is what we get.

(Refer Slide Time: 14:11)



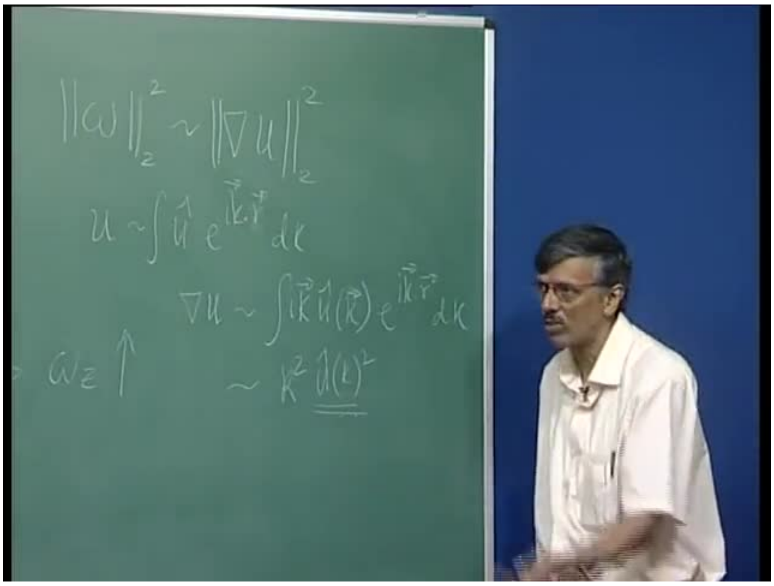
If I take the momentum equation and take a dot product with the velocity field, then we get the time rate of change of kinetic energy that is given in terms of this. This comes in the viscous dissipation term and this from the body force term. Since now we have establish, this quantity itself is equal to enstrophy. So, you can very clearly seen what is the role of enstrophy. Enstrophy actually tells how the energy is grained out because of this viscous diffusion term. So, enstrophy is a very powerful tool.

(Refer Slide Time: 15:02)



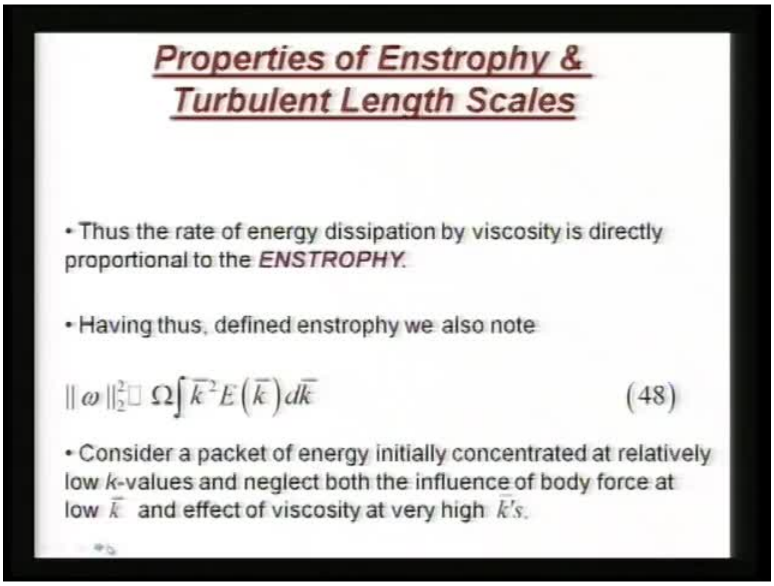
Now, what happens is we could also see that enstrophy could be written in terms of its enstrophy squared is nothing but grad u square.

(Refer Slide Time: 15:20)



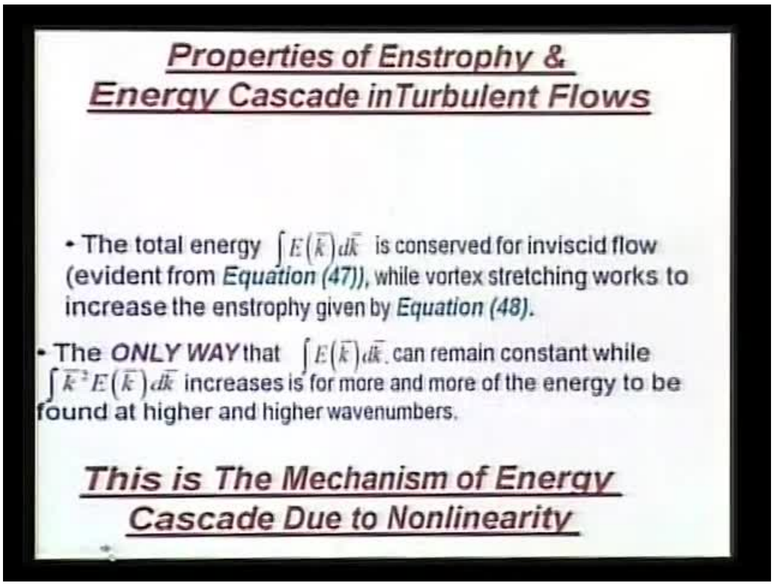
So, what is grad u? Grad u will be, if I am talking about this, so this goes as (Refer Slide Time: 15:24) So, if I write u in terms of u at e to the power i k x d k, then what happens to grad u? So, this is I could write it has k vector dot say r. So, then what happens is I could get this would be equal to nothing but professional to i k vector or u hat which is a function of this k vector times e the power i k that r d k.

(Refer Slide Time: 15:02)



So, now, if I square it up, I can see that this is going to be like k square and this is this. So, this goes like this. However, this is what from the Percival's equality, this also represents e of k. If this is the specific kinetic energy in the k space, that is also the kinetic energy in the… So, this has a role as we have written down here e of k. So, basically the vorticity is something like this.

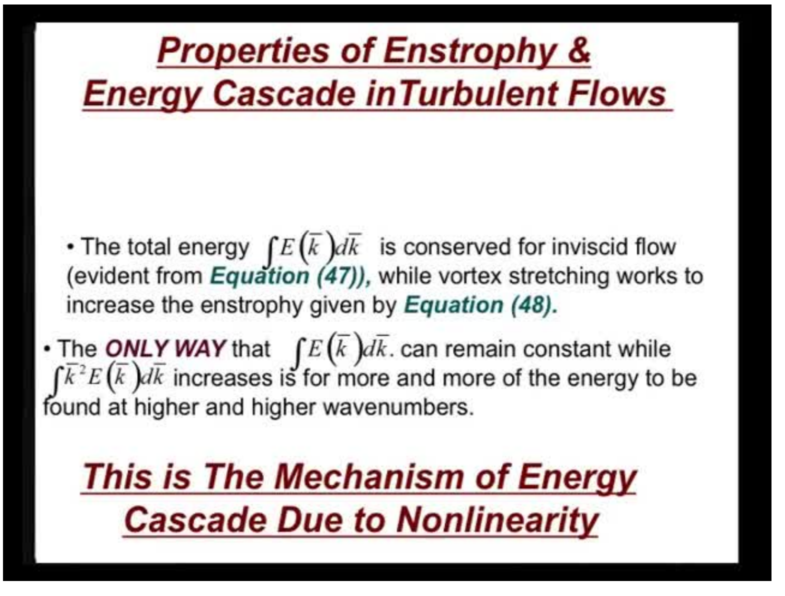
(Refer Slide Time: 17:25)



Now, consider a packet. Let us now consider a packet of energy which is initially concentrated at a low k value. Suppose the body force is not acting upon at the low k and you are also neglecting the effect of viscosity at very high k. Then what happens? If we are looking at inviscid flow, I know the energy is conserved, E k d k is conserve. That is what we have obtain.

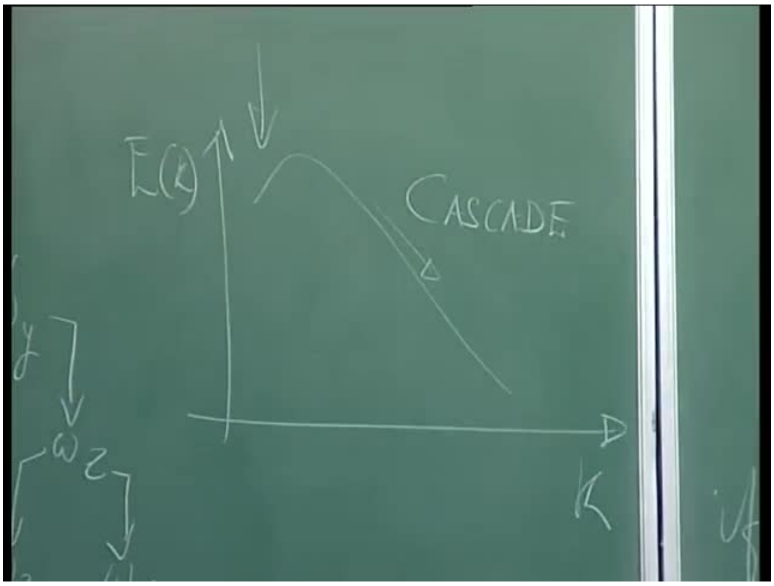
What happens to the vortex stretching work? That we have seen that increases the enstrophy. That is what we have been talking about here that, if I, well, we saw it that if because of the stretching, this will increase, enstrophy increases because the vortex stretching.

(Refer Slide Time: 17:25)



So, if enstrophy increases, then what is happening? We have given an estimate of enstrophy as k square E k. So, what is happening is we have a very interesting situation that this is conserved, E k d k is conserved, whereas k square E k d k is increasing. How can that happen? It can only happen if you transfer the energy from low k to higher k because it is multiplied by k square. So, if this contribution has to increase, then I have some low k contribution in E transfer into high k.

(Refer Slide Time: 18:49)



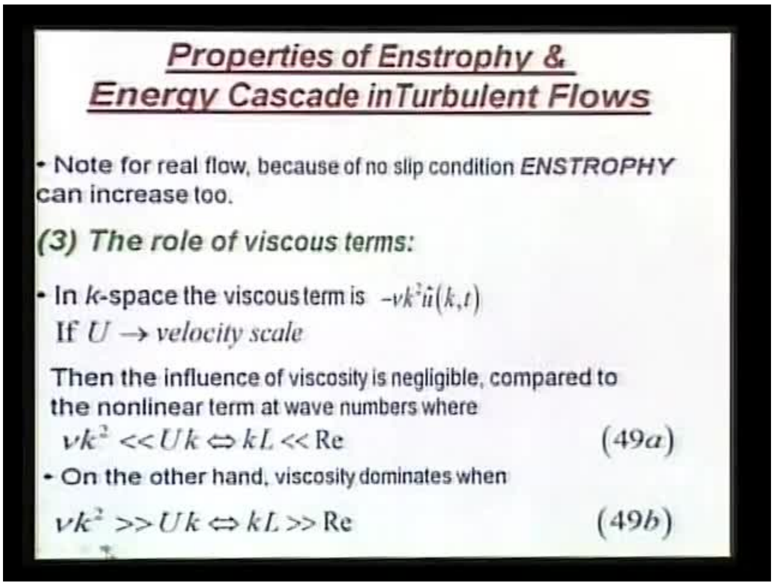
This is the mechanism of energy cascade due to vortex stretching. So, that is we have been talking about. We were talking about those idea for a long time what E k does with k that, if I talk about vortex stretching, this will essentially do that.

That if I campaign energy here at low k, because the vortex stretching term that energy has to shift to this. So, this is what we put, we calling as a cascading effect. The energy cascades. That is what we are talking about. So, that is why I show it with a very bold remark here. This is the mechanism energy cascade and this is a non-linear effect.

In fact, this idea has been so much interest into the thinking of turbulence community that people do not like to acknowledge, there can be any other mechanism of energy transfer. So, whenever you look at source material people only say, turbulence has to be three dimensional. It cannot be two d because there is no stretching. So, how is energy spectrum be explain, but we have lots of counter examples. We have seen like Kelvin-Helmholtz instability is one where we saw what was happening - bigger eddies were breaking down into smaller eddies. So, that means what? And you know, tell me Kelvin-Helmholtz instability is more dominant in 2D flows compare to 3D flows. So, here are counter examples people conveniently forget. Then we have also seen bypass transition effects.

What did we see that, if a single vortex is just simply moving at a constant speed; there is no trace of any unsteadiness, but what happens down below in the boundary layer? We get vortices erupting all over the no slip boundary and that is a very high white band phenomenon. So, there are something, but please do understand that for 3D flows at least vortex stretching helps you explain how energy is transported from low k to high k.

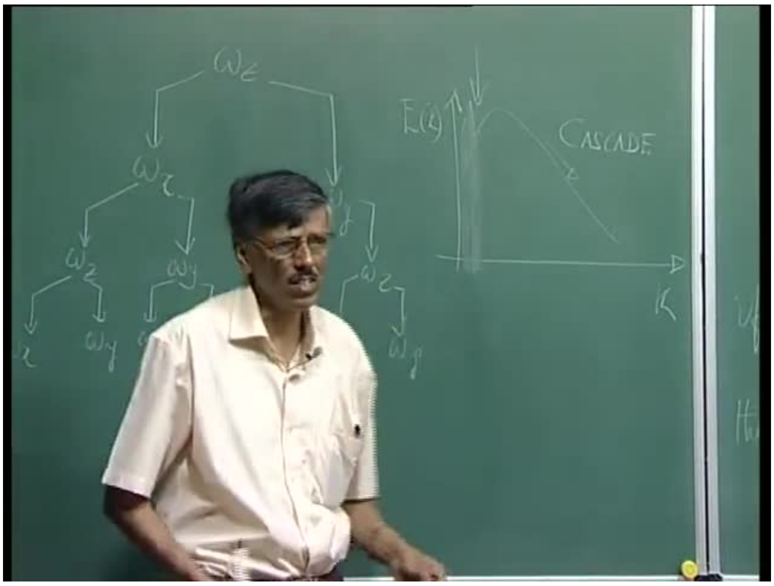
(Refer Slide Time: 21:23)



We have of course noted that if you are looking at real flow, the enstrophy increases. Now, if I look at the role of viscous term in k space, the viscous term is like this. This is a second derivative of you; so, that will be minus nu k square u of t.

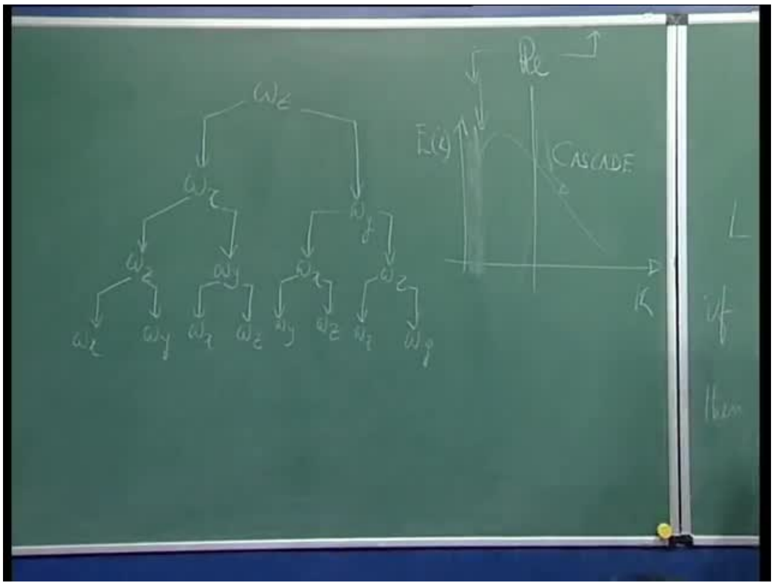
So, if I am talking about u as a velocity scale, then I can say the viscous effects are negligible compare to the non-linear term. So, this is a viscous term nu k square and this is your convection term like del u del x term. So, del u del x will be nothing but i k times u; so, that is this. So, if this is the case a viscous term is less, then what does it correspond to? That this k L is much smaller compare to Re. So, what does it mean? That those events are, because these are the places where viscous effects are going to be negligible for those cases, and which are those cases? The small case. That is what we are saying. Re is large, and to have this condition validated, we must have the corresponding non-dimensional k much smaller than Re.

(Refer Slide Time: 22:49)



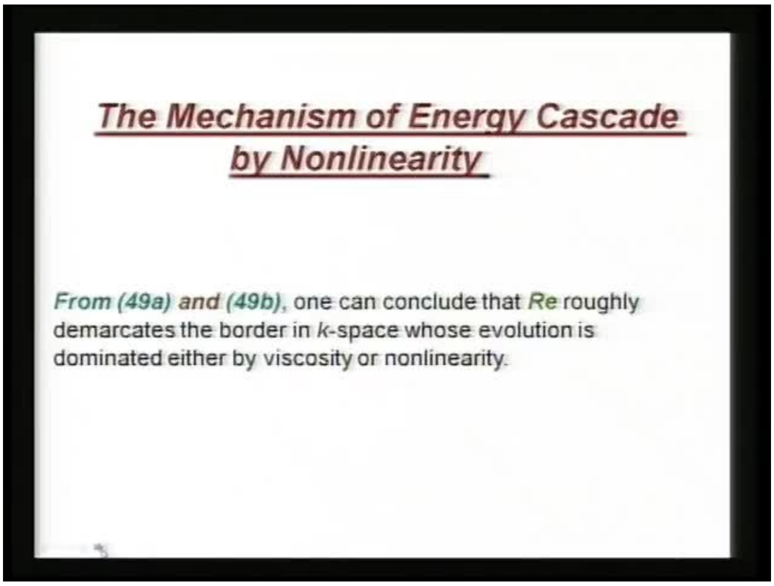
So, if I am talking about it, so this is the region where the viscous effects are negligible compare to the convection terms nonlinearity. On the other hand, if you look at nu k square far greater than this that will be what? This corresponding k L will be much larger compare to Re. So, if you prescribe a flow with a given Re, so it basically gives a kind of a line of demarcation, and one side to the left of it, viscous terms are unimportant; to the right of it, viscous terms are important.

(Refer Slide Time: 23:32)



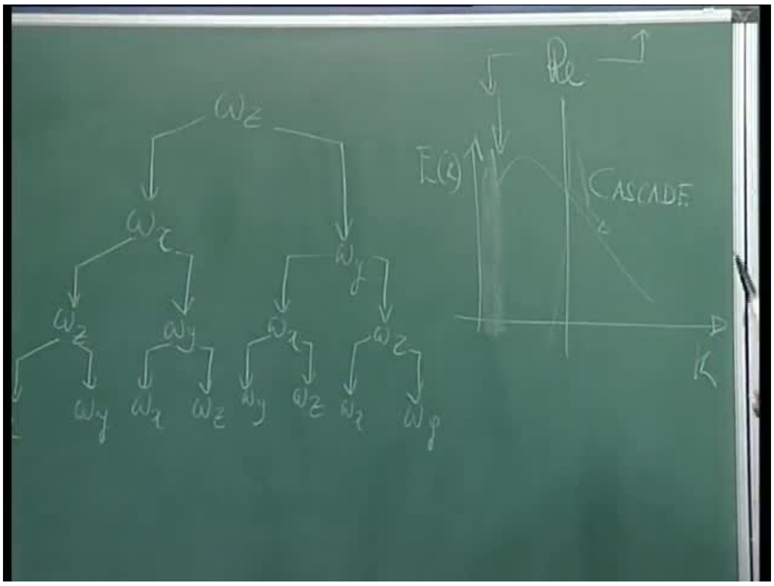
So, you can see that when we are talking about it. So, if I draw a line non-dimensional k along like this, so, this is the boundary that is determine by Re. So, on this side, viscous effects are unimportant; on this side, viscous effects are important. So, that is the one way of interpreting this slide.

(Refer Slide Time: 23:59)



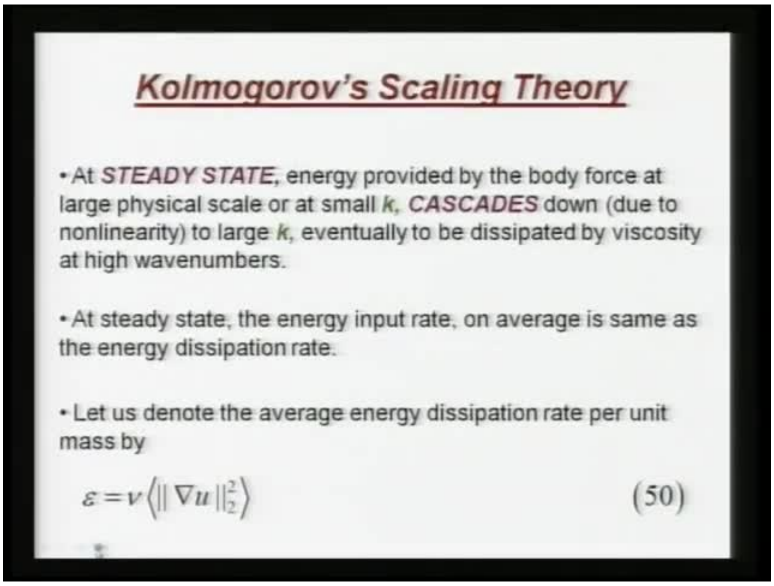
So, that is what O said Reynolds number roughly demarcates the border in k space whose evolution is dominated either by viscosity or nonlinearity. So, we are talking about a truly time dependent flow. Turbulent flows are in time dependent. So, this is not something, you know, it is frozen because what we are talking about?

(Refer Slide Time: 23:32)



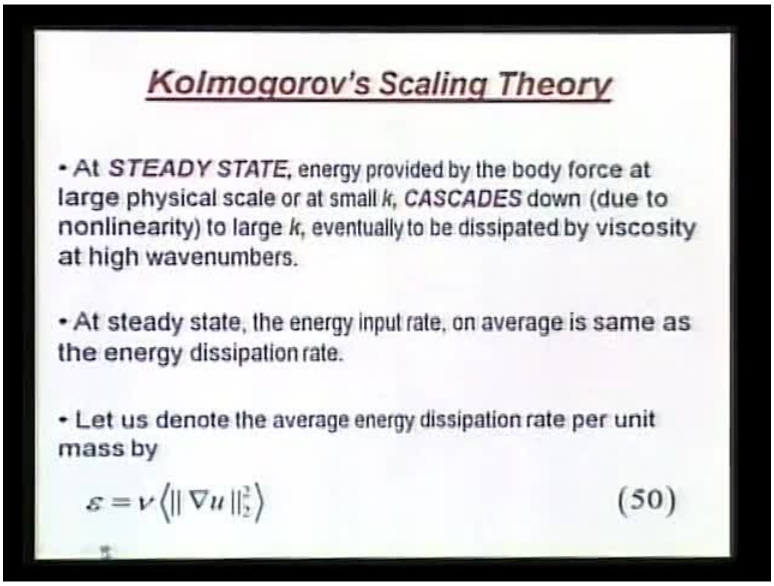
This border itself will keep moving depending on whether the nonlinearity is important or the viscous term is important and both this terms keep changing with time. So, if that is so, this boundary line that we are talking about which is of the order of Re that will keep moving about. This is something we should remember.

(Refer Slide Time: 24:44)



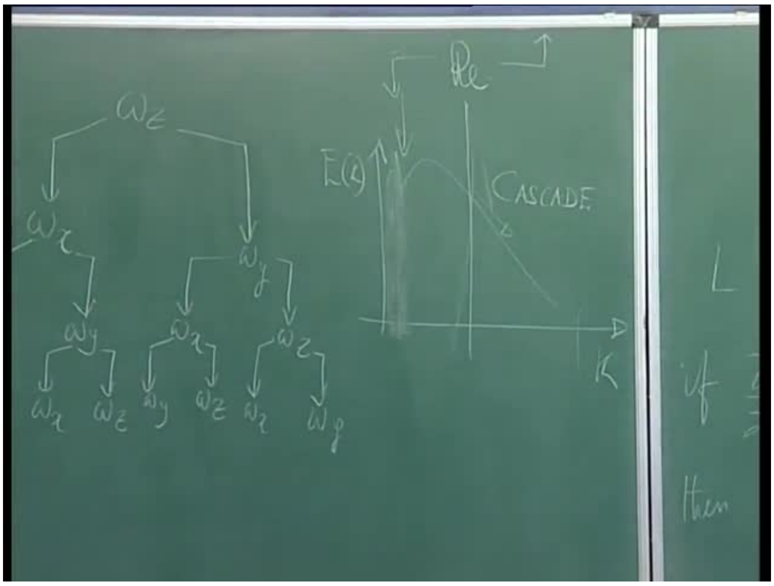
If at all we arrive at a statistical steady state, flow is unsteady but we still can get a statistical steady state. Then what happens? This is the picture that we drew; that we are providing energy at large physical scale or small k that could be due to body force that could be due to many things. If I put in a sort of a cylinder in a flow, that this size of the cylinder determines what is the larger size of a d that will be form. So, there the disturbance energy gets created at small k. Then we have seen nonlinearity takes over and it creates this cascade and it goes over there. It comes there and it cannot go on indefinitely, it goes and stop somewhere, where actually the viscous losses or transported into mechanical energy converted into thermal energy.

(Refer Slide Time: 24:44)



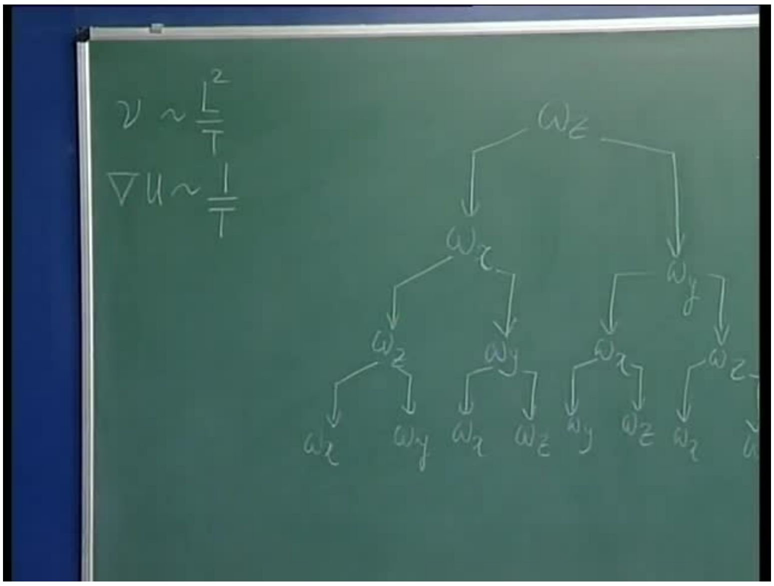
So, heat is getting created. So, that is what we mean by kinetic energy being dissipated by viscous action at high wave number.

(Refer Slide Time: 25:39)



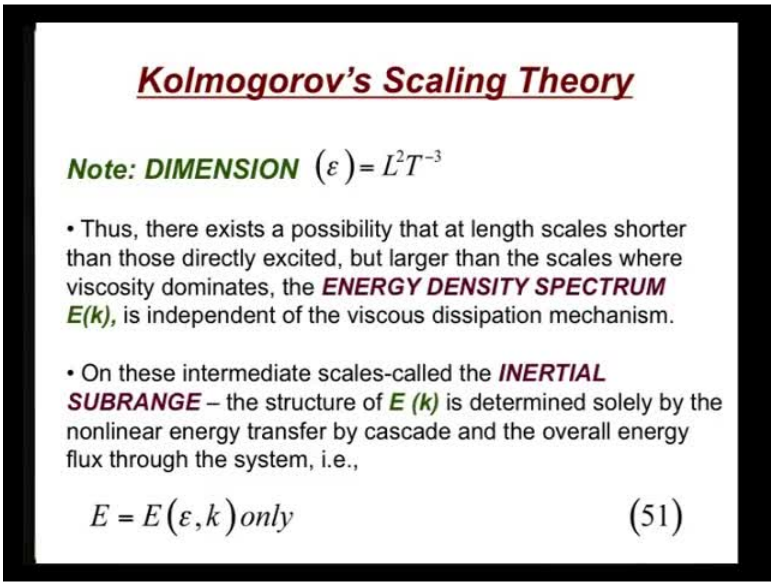
So, if I am now talking about this scenario for a time dependent flow, then what can happen? I am putting in something here; I am taking it out here. So, the steady state has reached. That means what? Whatever is put in there the same amount, goes out there. That is your definition of an equilibrium steady state. That is what Kolmogorov started looking at and he said that at the steady state, the energy input rate on average is the same as the energy dissipation rate.

(Refer Slide Time: 27:08)



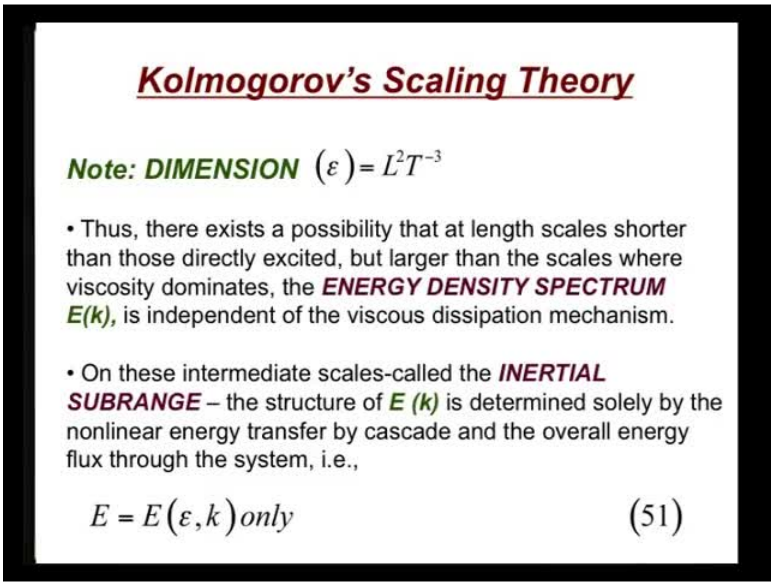
Now, let us look at the average energy dissipation. That we have seen nu times gradient u square. So, what happens? What happens is we can find out it is a time scale. So, what is it going to be nu. Nu is what? nu is a dimension of L square by T.

(Refer Slide Time: 27:50)



Now, what about grad u? What is this dimension? So, what is the dimension of epsilon then? It will go like L square T to the power minus 3, is not it? That is what we are seeing, this into this square. So, that is what we say. The dimension of dissipation is L square T to the power minus 3.

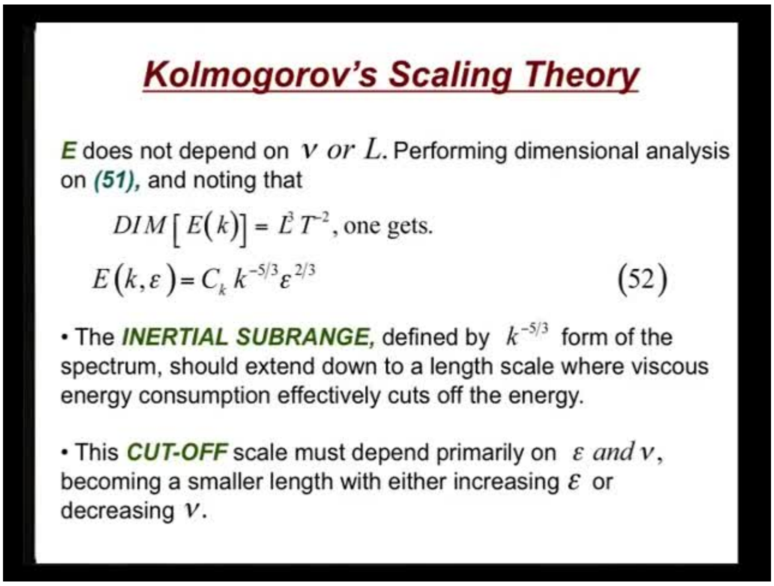
(Refer Slide Time: 27:50)



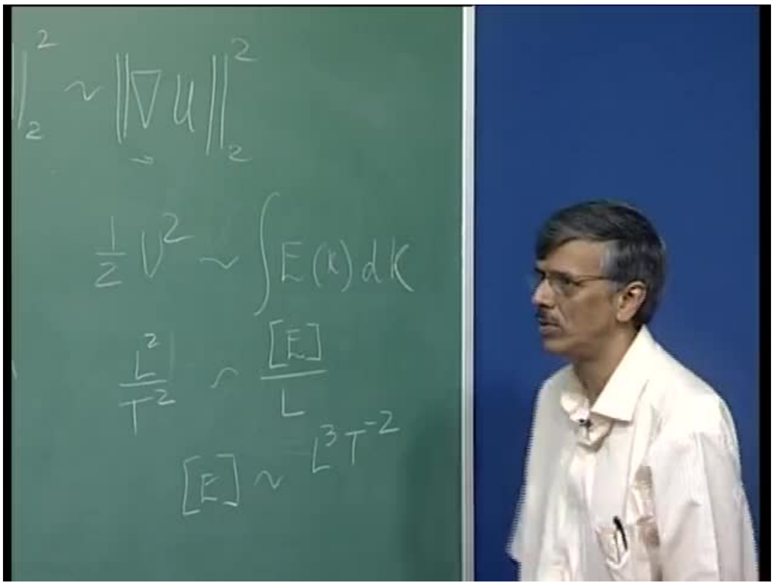
Now, if I have reached a steady state, I am pumping in energy here; I am pumping in energy there. So, what happens? To some range in the middle, it does not matter what you are putting in on the left and what you are taking it out on the right, that is precisely what we are seeing that, we can have some such range, where the energy density spectrum E k is independent of what you are putting in and what you are taking out.

So, what do will happen? If I am looking at there, this intermediate scale which I called as the inertial subrange. The structure of E k is determined solely by the non-linear energy transfer mechanism. How this cascade is going? How far it will go? Whether it comes and stops here or whether goes there? So, it depends on what? Where exactly this energy is taken out? So, the dissipation is important. It will also of course be function of k, because that is what we are looking in the k space. So, what happens to the energy at this k is different that this. So, there is a direct dependence of k. So, basically, in this inertial subrange which is either on to the left or to the right in the middle, we are taking about the middle ground which we called as the inertial subrange, where this epsilon would be a function of epsilon and k. Why do not we talk about the energy supplied, because that is itself equal to epsilon, because this is we are talking about the steady state; so, they are same. So, that is why I did not put three terms two terms are adequate because energy input is equal to energy out go. So, that is why we can write this E as this.

(Refer Slide Time: 30:02)



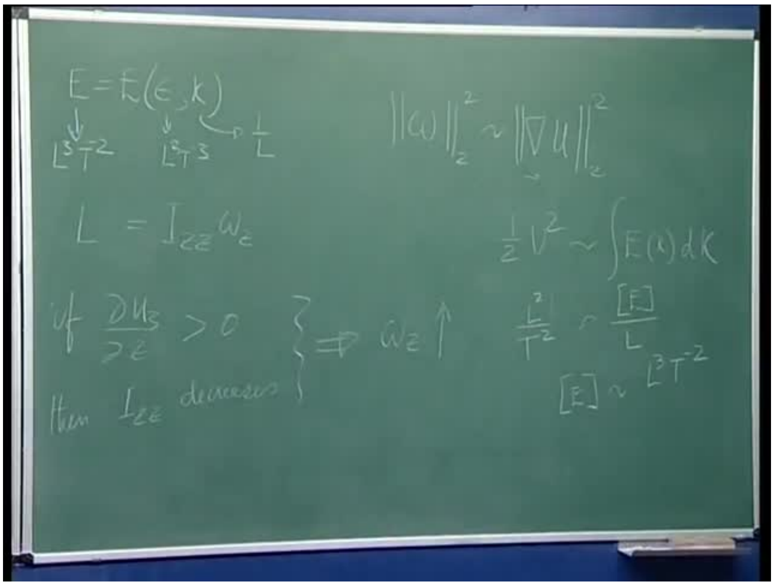
(Refer Slide Time: 30:19)



Now, what about dimension of E? How do we go about doing that? See, dimension of E if you recall, it is something like this. The energy is something like this and what was it that we saw? That was like E k d k, fine? Do you agree with me? So, what about this then? This is the right hand side dimensional E is dimension of energy, and what about dimension of k L? On this side, what will have? L T to the power of minus 1; so, L square by T square.

So, E. dimension of E then is going to be L cube T to the power minus 2. That is what we have written - dimension of E k, it is L cube T to the power minus 2.

(Refer Slide Time: 31:26)

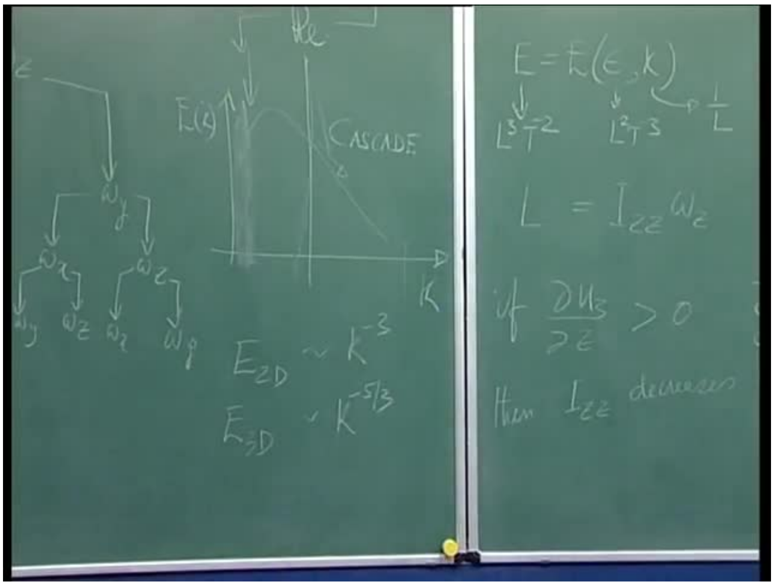


Now, you have the dimension of E. We said that E is a function of epsilon; E is function of k. So, we have already seen what is the dimension of epsilon, dimensional epsilon is, and this we have seen the dimension as we have given is L cube T to the power minus 2.

Now, k is dimension of course we know is 1 over L. Now, all you will have to do is dual element of dimensional analysis. Those (( )), this is it that this in the inertial subrange, energy spectrum is given by this. This is a fantastic result; very simple and approach but very far reaching in consequence. What you find that in the inertial subrange, the energy varies as k to the power minus 5 third.

So, this is the result by which the whole community swards that we have a inertial subrange in turbulent flow which does not depend on how you put in the energy; how you take out the energy. The energy spectrum has a universal future. That future is given by its dependence of k by this exponent minus 5 third. So, this is the story that we are talking about, but please do understand that this is a story for a 3D flow, because here, we are talking about nonlinearity the vortex stretching coming into picture. So, this will not be the case for two-dimensional flow.

(Refer Slide Time: 33:29)

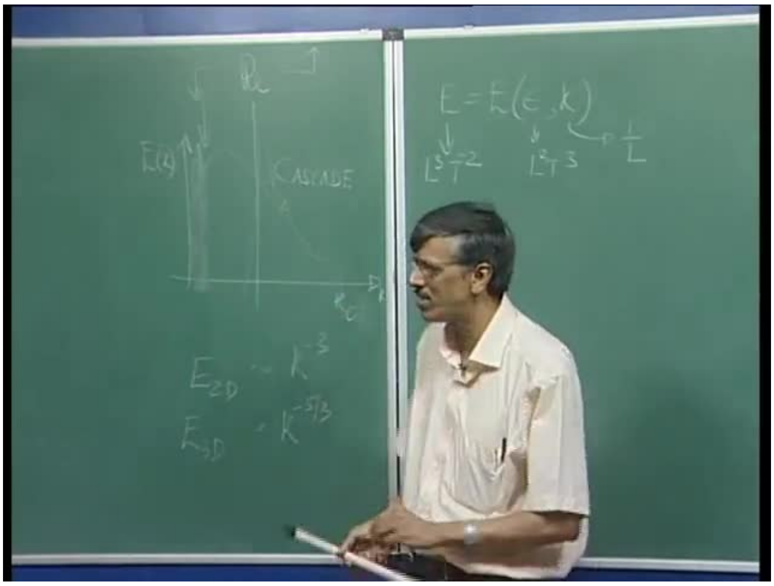


Time permitting we will see that for two-dimensional flow that E of 2 D, that goes as k to the power minus 3, and here, what we are seeing is E of 3 D goes as k to the power minus 5 by 3. So, the, do you understand that if you have a way of measuring the energy spectrum and we look at different k range and you see, what kind of slope it has? If it has a slope of minus 5 third, you can see three dimensionality is important, and if you see the slope has minus 3, then two-dimensional aspect of the flow is important. And interesting part is what you know?

You look at our weather system, you will find out. First, you get this - k to the power minus 3. So, whether prediction actually survived by treating this flow problem layer by layer. So, in that each layer, it is a kind of 2 D problem. Three dimensionality is stack one over the other. So, it is like your onion peal; you can peal out one layer over the other but mostly the dynamics is dominated by this two-dimensional motion. Anyway, that is what is also the reason you know why water wave equation play such a central role in weather prediction shallow because that defines in two-dimensional motion in that plane.

So, what we have now? We have learn something very interesting about turbulence that we have a inertial subrange in all turbulent flows and that is where we get k to the power minus 5 third. However, it gets cut-off at a very high k. So, there is a cut-off scale. What it will determined upon by? The rate at which the energy is supplied, that will also the rate at which it is dissipated and also it should depend on the property of the flow, and what better quantity of that is then a kinematic viscosity itself.

(Refer Slide Time: 36:20)



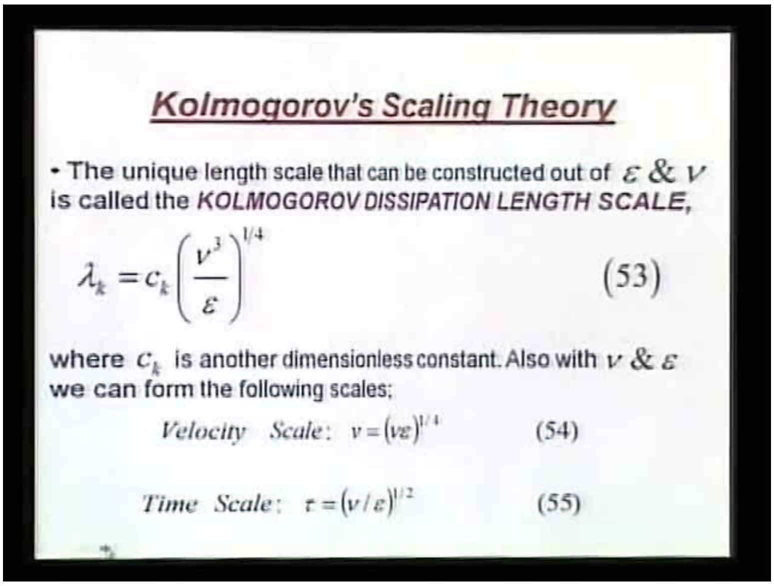
So, this cut-off scale must depend on epsilon, and so, now I think we have a fairly a decent idea. What happens in a turbulent flow which we get by very little investment? We have mostly dependent upon dimensional analysis and we have say something which are very true that, if we have a statistical equilibrium steady state, then the energy that comes out here cannot recite there because the importance of the role of nonlinearity versus the viscous term. In this small k range, nonlinearity is important; that pushes the energy to higher k and it keeps on going, and somewhere down the line, it must dissipated itself into thermal energy and that is what we are talking about. At there is something if I am plotting this k and there is some k cut-off, I will call it as K c.

This cut-off scale will depend upon the rate at which the energy is supplied or taken out and also the material property in the fluid property.

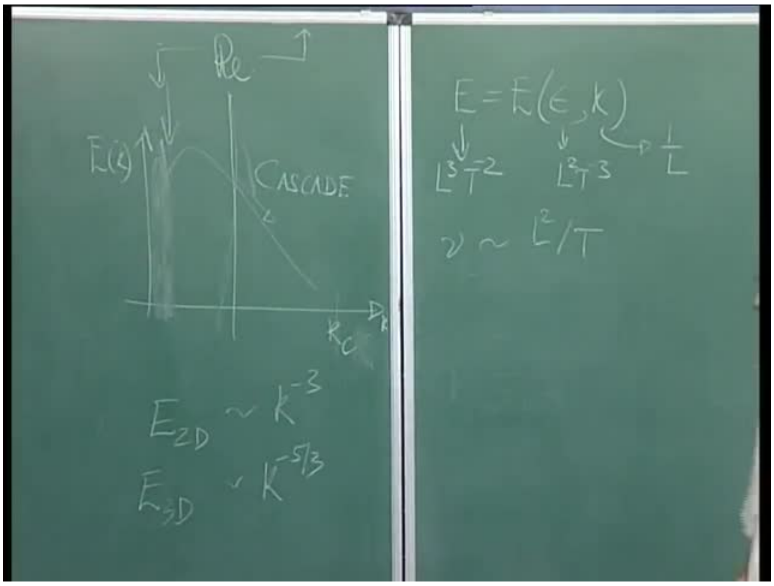
And what is this fluid property that we have already seen, that is Reynolds number in the problem. Reynolds number directly incorporates is the kinematic viscosity. So, what happens is this cut-off scale can then change. What happens? If the value of reynolds number is large, we can actually go to much higher k. The smaller at the reynolds number; that means larger of the value of nu. It has to end earlier because that is where viscous dissipation takes over.

So, this is something that we must keep in mind that this cut-off scale will become smaller if we either increase epsilon or decrease nu. Increase epsilon means what? I am putting in more energies. So, I am allowing it to go for larger range of k. Increasing nu means what? I am talking about higher and higher Reynolds number problem. So, that allows me to sustain larger rates of strains. So, that also allows into go there. So, that is precisely what we have stated out there.

(Refer Slide Time: 38:43)

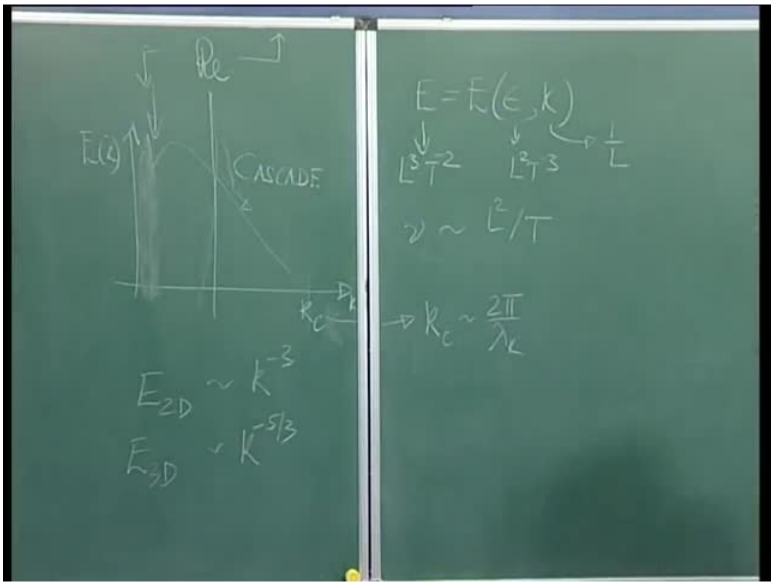


(Refer Slide Time: 38:53)



Now, what you do is we have epsilon; we have nu; we have seen the dimension. We have the epsilon here; nu dimension is given by L square by T. So, what I could do is I could use it to define a length scale or which I call as lambda k that I am going to divide purely for epsilon and nu whose dimensions are given, and if we look at this, this is the only possible combination that you can take. So, basically that is what we are going to see that this lambda k is c k times nu cube by this thing.

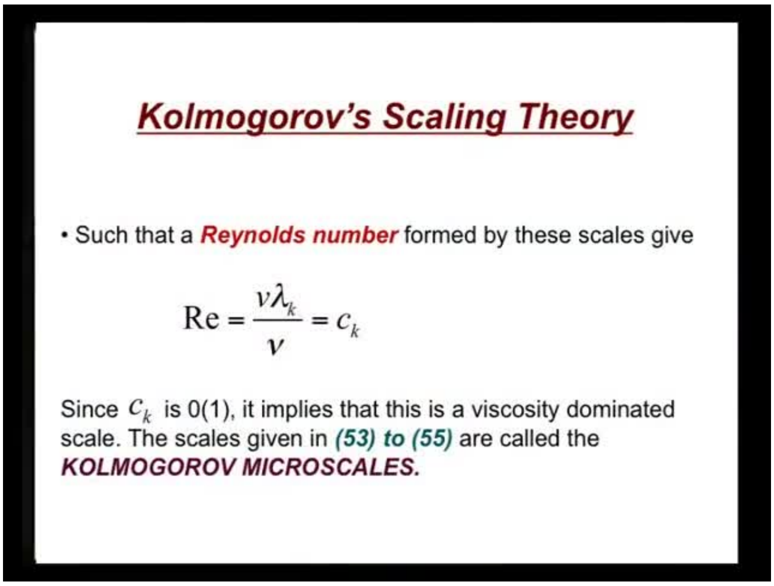
(Refer Slide Time: 39:49)



So, that is rather interesting. This is what exactly we are saying that how far we go this k c. Kc is what? If I look at this k c, k c is the wave number. So, that will be 2 pi by lambda k, the corresponding wave length. So, that lambda k the length scale in terms of nu and epsilon would be written like this.

So, what we are talking about? We are trying to estimate how far we can go and that is what Kolmogorov's said that at that scale, at the cut-off range, everything would be determine by epsilon and nu, and then, the corresponding length scale is this. You can obtain a corresponding velocity scale exclusively in terms of nu and epsilon. You can construct a time scale exclusive again in terms of nu and epsilon. So, we get this through scales that more or less defines everything. Is not it the way we have been non-dimensionalizing Navier-Stokes equation? What we needed there a velocity scale and time scale and a length scale. So, we have all through of them.

(Refer Slide Time: 41:25)

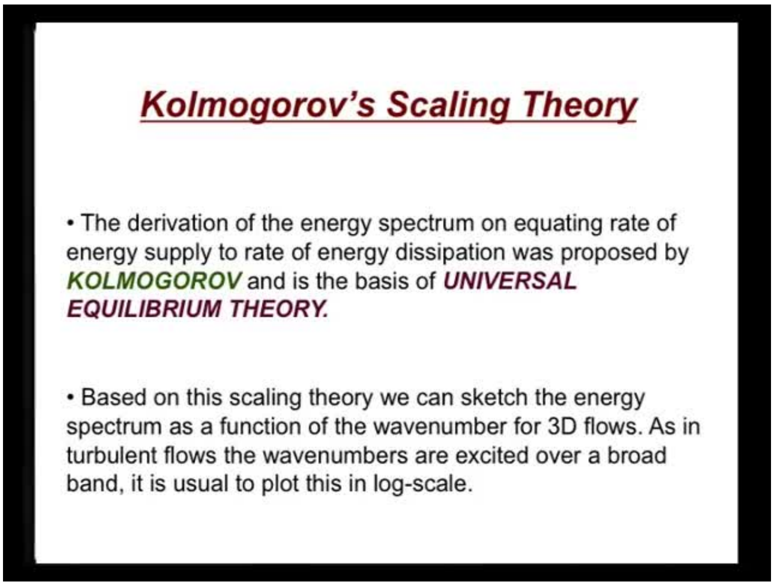


What is interesting is sometimes of course what we have done? We have converted the velocity scale and length scale into a time scale. We can do that, but here, we are talk about, let say these three things are independent. If it is indeed so, I could construct also a Reynolds number. This Reynolds number is not the flow Reynolds number. This Reynolds number is the one where I have taken a velocity scale which I am calling as v; the length scale as a lambda k and this is nu, and the velocity scale, we have seen in the previous slide. We have seen what is a lambda k scale substituted there and then we get this.

So, lambda k was in terms of a undetermined constant C k which is of the order 1, which has to be order one the way we have reasoned out. So, what we are finding that Reynolds number is order one. What does mean? We roughly speaking we always estimate Reynolds number as a kind of a relative weightage between the convection term; it is a v, a viscous term. So, if Reynolds number is 1, so, this is the scale at which the viscous terms are as important as a non-linear term.

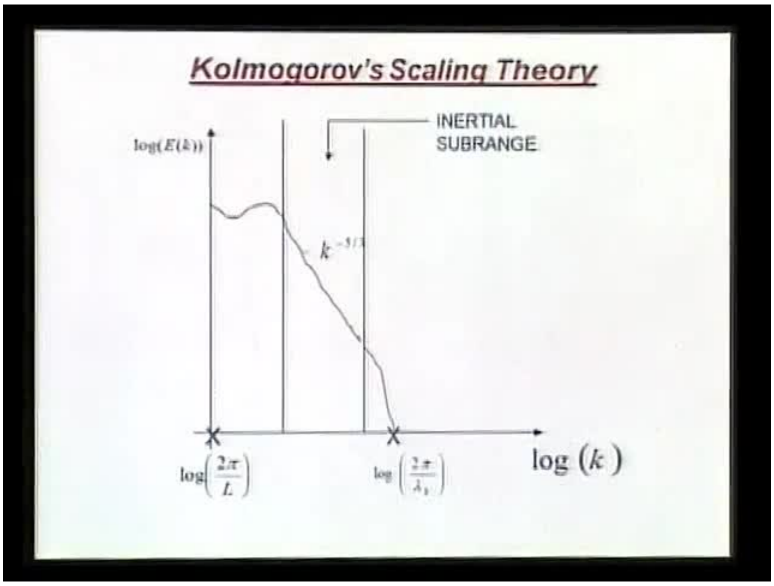
So, these scales where the viscous effects are as dominant as this is what are called as the Kolmogorov Microscales. This is where we can go at the most given a flow field, given the flow property, you can go up to maximum Kolmogorov microscales, and one of the thing properties of turbulent flows that we discussed remember, we said that is microscale is compared to molecular dimensions it is significantly larger. So, Kolmogorov microscales are still much larger compare to your molecular dimensions, and that is why we can actually view turbulent flow in terms of Navier-Stokes equation itself, because the smallest length scale those are exited. They are significantly larger than the mean free part and that is one of the reason that we do use the Navier-Stokes equation as the governing equation for.

(Refer Slide Time: 43:38)



So, now, we have a very decent view of the energy spectrum that we can obtain this energy spectrum by equating rate of energy supply to rate of energy dissipation and this was a kind of a equilibrium condition proposed by Kolmogorov and this is goes by the name universal equilibrium theory of Kolmogorov. It is not necessarily the true that as simply as we have done in terms of dimensional analysis. Kolmogorov (( )) with statistical physics; so, that is the much beyond the scope of this course. We will not talk about it but we have the just of it in front of (( )) we can see. Based on this scaling theory, we can sketch the energy spectrum as a function of wave number for 3 D flows. In turbulent flows, the waves numbers are excited over a broadband, and this is what you can see. See what happens when we talk about laminar flow, we then get to see something like this. We may get one peak here, another peak their and another peak there. That is what you see. You have very discrete peaks not wide band spectrum, like what we are seeing here, it is not a broadband (( )).

(Refer Slide Time: 45:18)

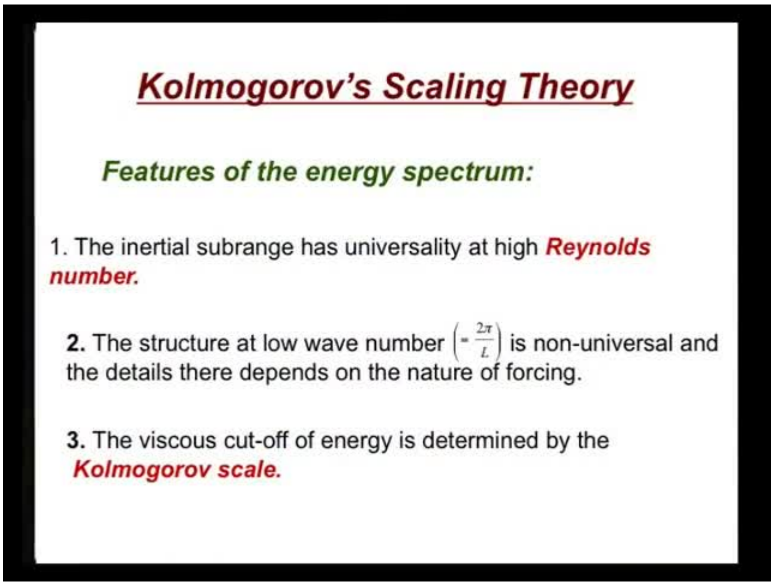


So, basically this broadband is also quite important because what happens is we are going to plotted like this in a log scale. That is the way we are going to plot the energy spectrum will log log plot. What is happening energy also spans over many decades, and so, is the wave number. Think of nothing better than, say our weather system. The largest length scale that you can think of is the peripheral. If you circumvent, we get the whole world. So, it is a, to buy in to 6,000 something. So, it does work out above 24,000 kilometers something of that kind.

And at the small scale, you can see even when a leaf is squealing in the wind that is the smallest scale that you can see think of. So, you can think of this huge range of scales that you can think off coming from a few centimeters or even millimeters all the way up to 1,000 of kilometers. That is why we plotted in the log scale, and this is how it is. If you are looking at a three-dimensional flow, this is how it should be. This is your inertial sub range where the dependency upon on like this k to the power minus 5 third and this is where you are pumped in all the energy. This is how the turbulence is created.

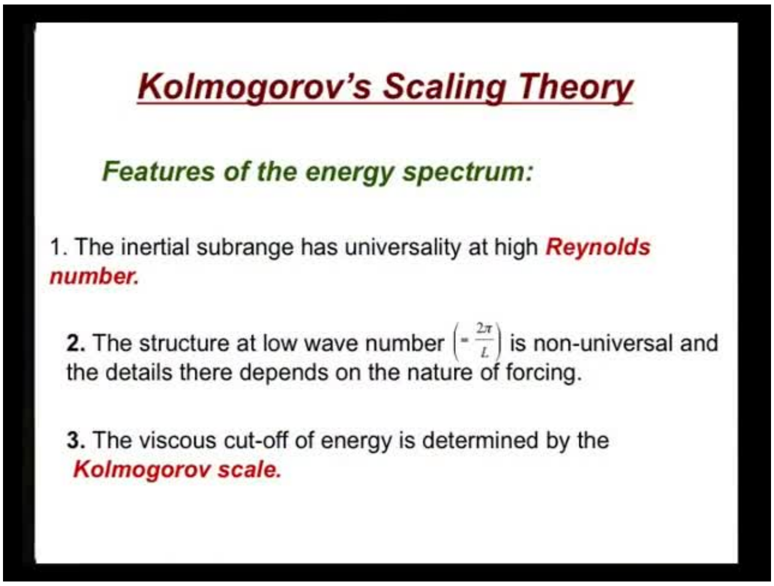
I put in body that is where I put in the energy. Energy migrated along this and got dissipated in Kolmogorov micro scale. So, that is what you see that this cross belongs to log two pi by lambda k. This is your log of two pi by L characteristic dimension of the problem. Talk about flow positive chimney, it is a diameter of the chimney. Flow positive aircraft wing, it is a cord of the wing. So, it is that kind of thing that you talk about the outer scale or characteristic dimensional larger length scale, and the smallest length scale is to determine the Kolmogorov dimensional scale.

(Refer Slide Time: 47:32)



Now, what we could do is we could talk about little bit about the future of this energy spectrum. The inertial sub range has universality only when you are talking about really high reynolds number because flow has to suffer those instabilities to go from laminar to turbulence flow, and you do need to have a significance stretch where this is independent of what is happening on this side and what is happening on that side. What happens if I look at a low Reynolds number turbulence flow, it could be something like this. So, this dynamic range probably comes down by relative factor 100 of factor of 1000.

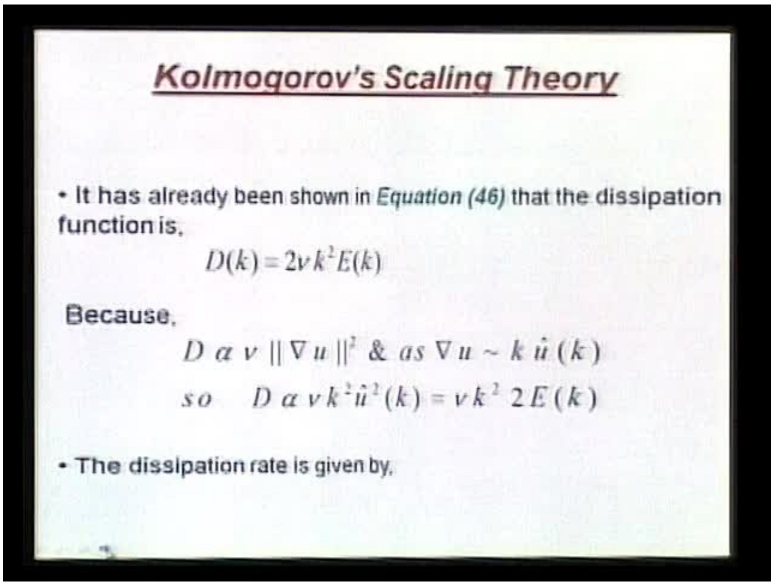
(Refer Slide Time: 48:27)



So, low Reynolds number flows, you may not actually even get this part that k to the power of minus 5 third, whereas the structure at the low wave number is non-universal, the details depend on high usage of flow. That is what we are talking about. However, if I had looking at the viscous cut-off that actually cuts-off the energy, that is determined by Kolmogorov's theorem.

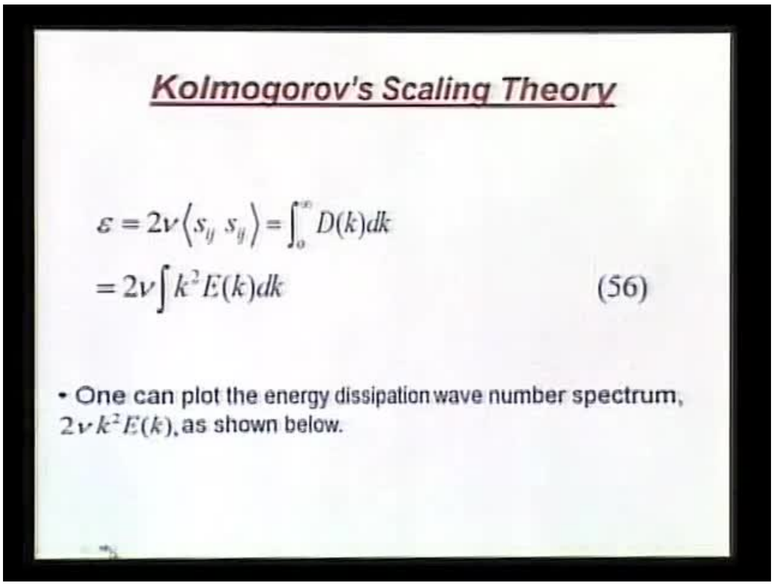
So, at least three things we have and two are kind of determine if reynolds number is a high. One is of course the micro scale and the second is inertial sub range. What happens in the low scale range? We have no idea because it is flow dependent. We are going to have different flows; we will have different.

(Refer Slide Time: 49:10)



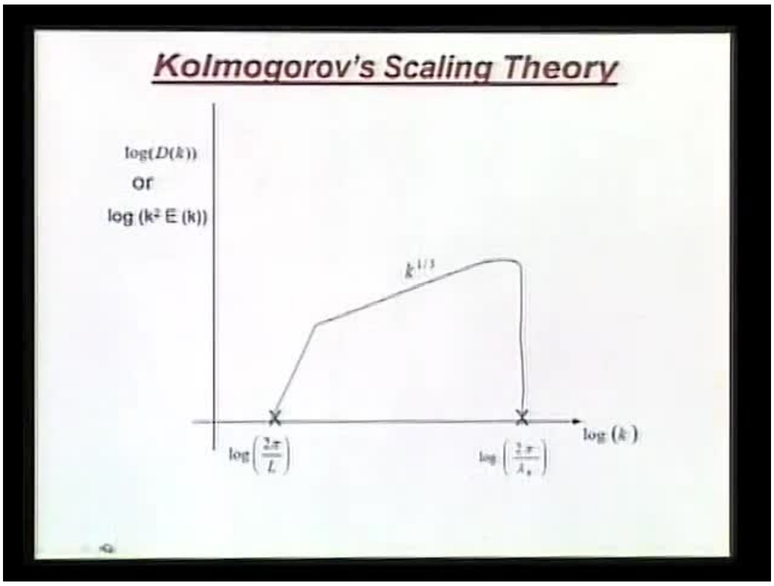
Now, this is one thing that I wanted to talked to you about - it is about the necessity to view the dissipation spectrum. There is also you can look at in the k space, and this is quite straight forward dissipation is if you recall nu times the grad u square. So, that was nu times k square E k. If you look at this, you can very clearly see. That is what have written here that D is proportional to nu times grad u square; grad u itself is k u hat of k. So, D is then propositional to nu k square this. This itself is like your E of k u hat square.

(Refer Slide Time: 50:23)

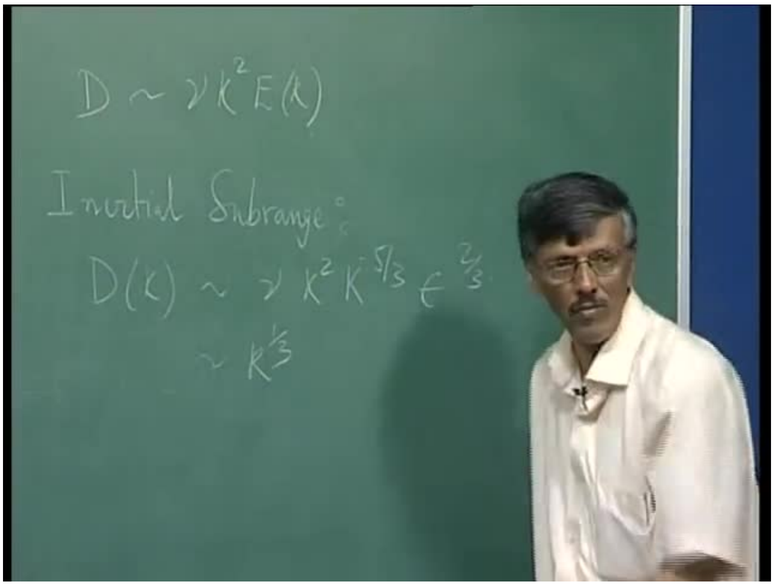


So, that is what we are saying that D k goes like this. So, what happens is if I have the energy spectrum sketch from there, I can also work out the dissipation spectrum. That is the relation given there and this is what we can also see that, we can plot the energy dissipation which is given by nu times the fluctuating strain rate. So, that is your D k E k, that is the definition of D k. So, basically done I can see what D k is? D k is nu times k square E k. That is what we are going plot.

(Refer Slide Time: 50:52)



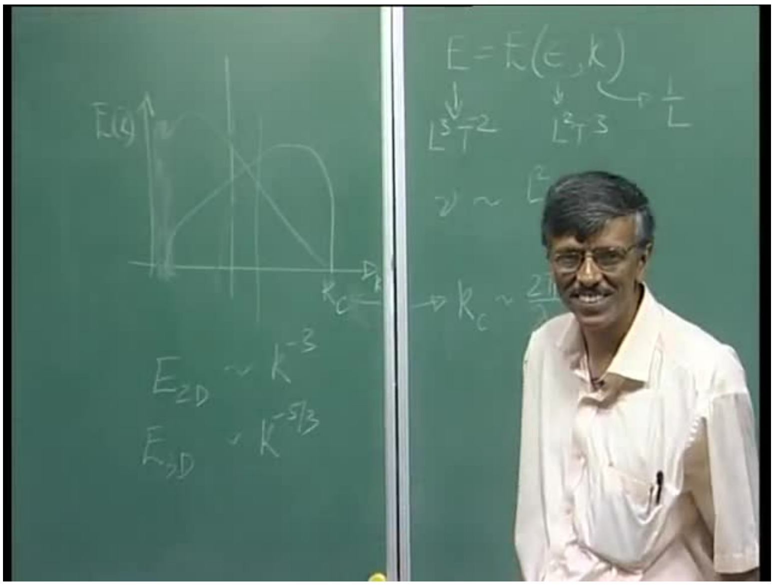
(Refer Slide Time: 51:04)



This is how it is going to look like. It is a kind of a sketch; so, do not take too figuratively. Now, how do you get? What did we write there? D k goes as nu k square E k. Now, what is E k? E k we have just now seen goes like this in the inertial range. So, in the inertial subrange, what do we get? D k goes as nu k square and E k is k to the power minus 5 third and of course those epsilon terms is there. So, that is of the issue but you can see D k then goes as k to the power plus 1 third. So, that is what you are seeing here.

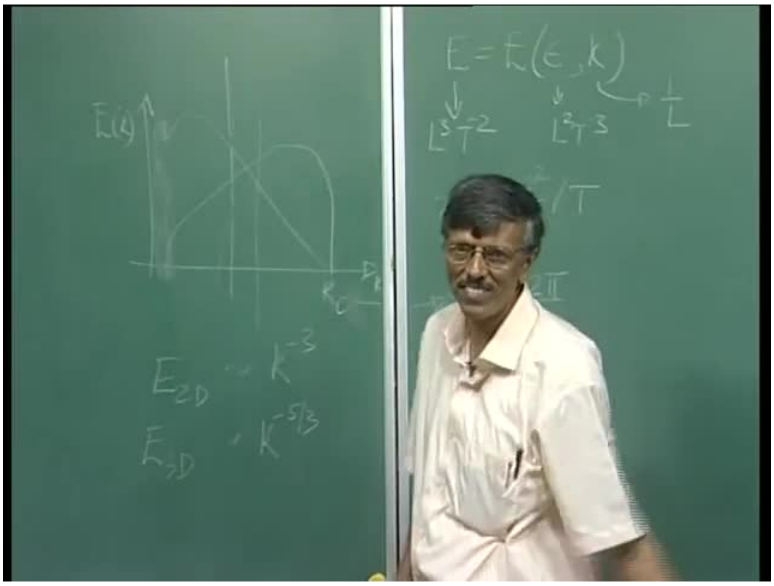
In the inertial subrange, it goes as k to the power 1 third, and at the low k range, that we already seen, we have said that is the place where viscosity is not important; so, behalf it, it is negligible. So, it kind of starts off from where the flows begins, it picks it up, but in an inertial subrange, it goes up as k to the power 1 third, and then, again it is all lost to due to dissipation. So, that is at the Kolmogorov scale.

(Refer Slide Time: 52:44)



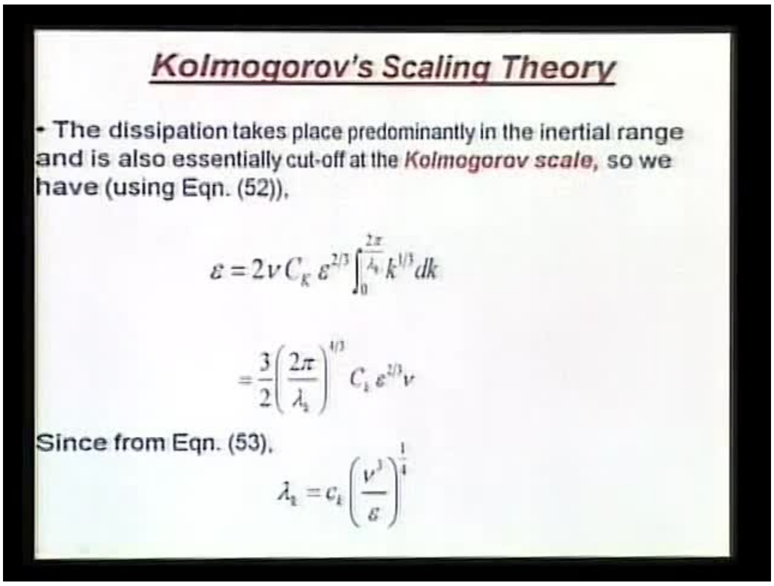
So, what you find is this. This is a very instructive picture which many people do not keep track off is, if I plug this as E of k, so, if this is my E of k which goes like this; my D of k goes like this. So, you see, when you are, let us say trying to compute, this is important in computation that people tend to think that I will do earliest, and all I need to do is worry about E of k. I will short circuit this E of k somewhere and I say how I am doing it, but people do not talk about what they have to do correspondingly to the D of k but the D of k as this structure.

(Refer Slide Time: 52:44)



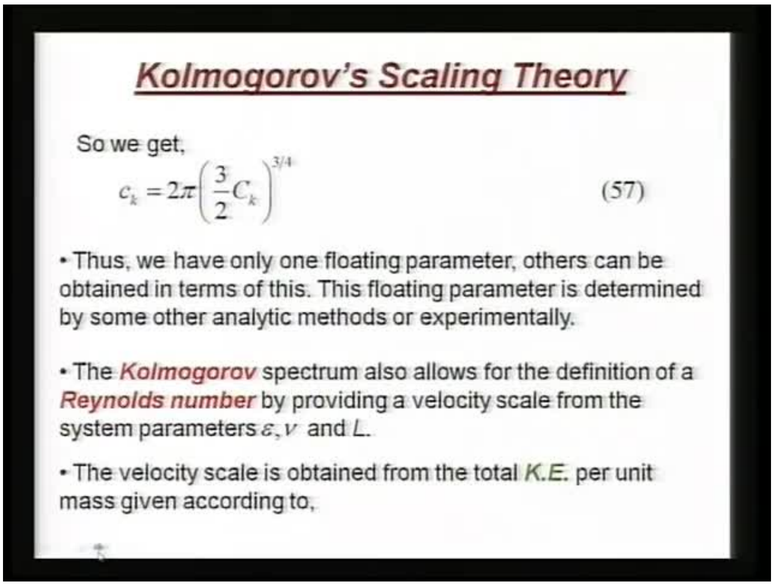
D of k is more important may be in the scale which is beyond this. So, that is set your submit scale. So, think of the submit scale modeling, it is not a trivial exercise. You will have to model it appropriately so that it mixes that D of k property. So, this is something one must do when you generate a submit scale model for l e s. Make sure that at least in the inertial range, it should have this kind of attribute. If it is not, then you have a very poor model.

(Refer Slide Time: 54:11)



So, this is your dissipation spectrum and we can do all kinds of things. We can play around with all this numbers that we have seen, and epsilon we can use this previous equation that we can now talk in terms of the range where it is becoming important. So, D of k we have obtain. So, I substitute D of k expression here and integrate over all possible k, and k now goes from 0 to 2 pi by lambda k, that is a Kolmogorov's length scale and I get this.

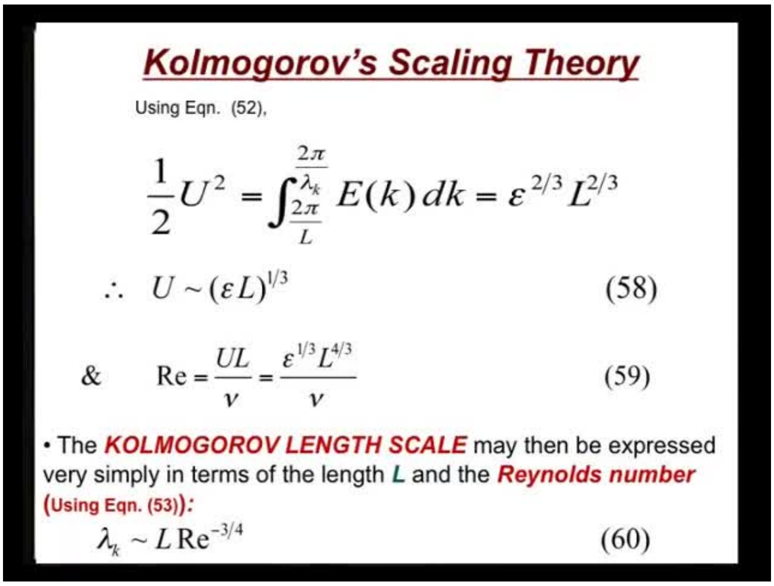
(Refer Slide Time: 55:11)



Already we have seen that lambda of k goes like this. This is your Kolmogorov length scale that we have obtain from dimensional argument. So, we can plug this last expression in the previous one, and what do we get this C of k, they are lots of floating constants that we are talking about. So, C of K, this lower case c of k is related to capital C of k is like this. So, what do we mean is there are not too many c of ks that you will have to be looking around for. Now, there is a unique relation between the two. If you obtain one of them, the other one gets frozen. That is that we are saying this floating parameters determine by some experiment or by some statistical analysis, tools, etcetera.

Now, we have talked about that Kolmogorov's spectrum allows ask to define a Reynolds number, because it provides a velocity scale by this following parameters – epsilon, nu and the length scale L. The velocity scale is obtained from the total kinetic energy per unit mass. That is quite easily understood.

(Refer Slide Time: 56:15)



So, that is given according to this, and then, we find that this is your specific kinetic energy that is by definition given by this E of k d k and this E of k I have already gotten that expression I substitute it there. So, I find U goes like this. So, if I look at Reynolds number, that is, UL by nu, it goes like this. So, we have obtain the value of epsilon in terms of Re here and I can now plug it in back into the expression for lambda k, that goes as this. I think I will stop at this point. Tomorrow I will take first few minutes to expand up on this, and then, will have our general discussion session.