Instability and Transition of Fluid Flows
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Module No. # 01
Lecture No. # 37

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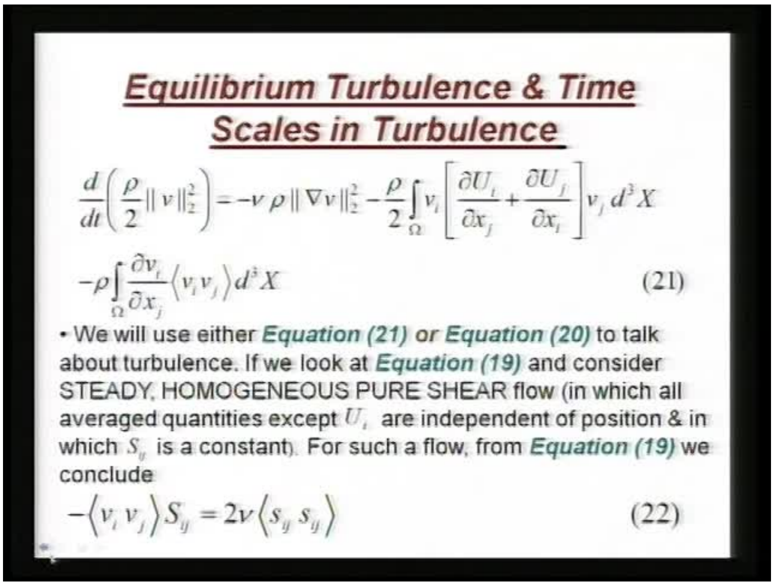

Once again, let us review what we have done in the last class looking at turbulent flows which is in equilibrium and what are the time scales those are involved in turbulent flows, we wanted to estimate by looking at this equation that we called as the turbulence energy budget, which actually looks at the energy associated to the fluctuating component of the velocity.

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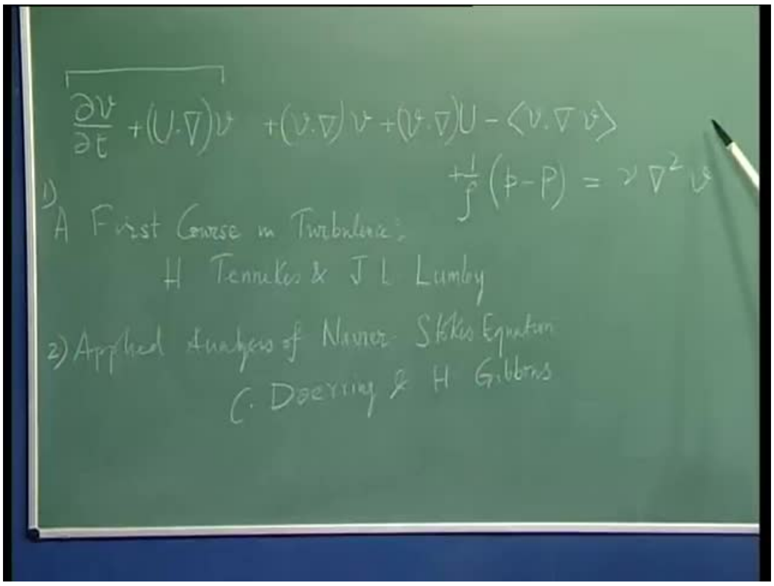

And what you notice is the assembly of this term that originated from this governing equation for fluctuating quantity and as you can see that these two are taken together. When it is multiplied by taking a dot product with v gave us this term; that is what I have identified there. For example, this term comes from the viscous diffusion term here; that is what we get. And this term that comes from here and there is this term that comes from a combination of this and that. This does not give rise to any contribution when you integrate over the whole volume (Refer Slide Time: 1:10 to 1:45).

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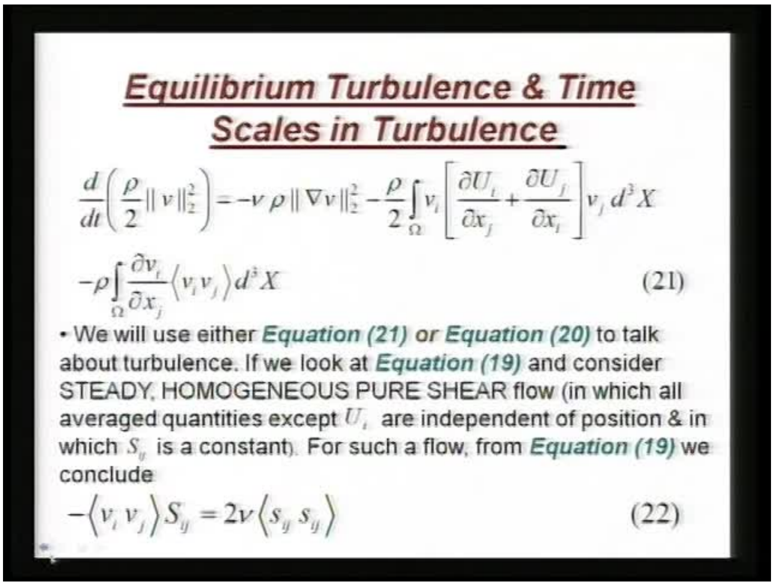

So, then we decided to take a look at a special flow which we called as a steady homogeneous pure shear flow. Steadiness comes from if I put the left hand side in time derivative equal to 0; homogeneous will say that all quantities average quantities are independent of position they are same everywhere except the mean flow that convicts the flow; that gives rise to the time rate also; that is that. And pure shear flow, we will tell you that del u i del x j is pure constant and if you do that then you can see what remains is that, this is the term that comes from the mean shear and this comes from the fluctuating quantity; they are to be balanced.

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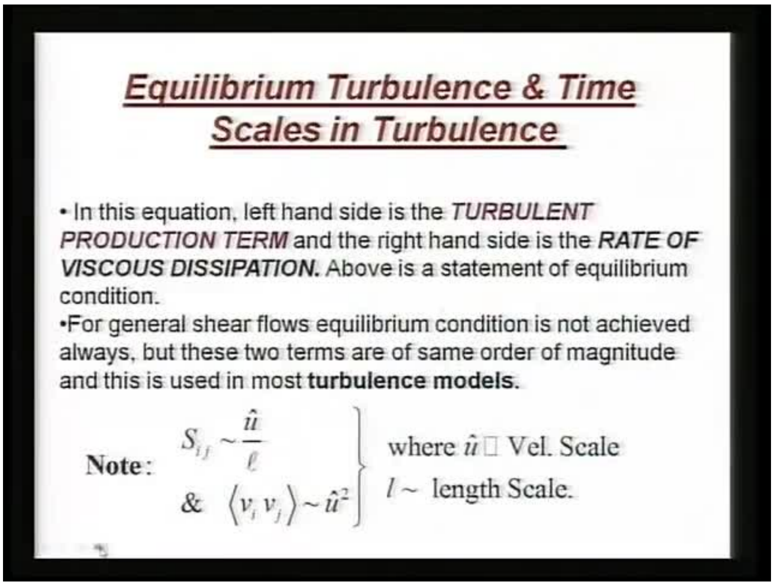

So, that is what we looked at and we further made some order of magnitude estimate and we looked at that. When we wrote that equilibrium condition on the left hand side, we saw what was happening was basically the turbulence production term, because it is in equilibrium, it must equal to the rate of dissipation. So, that is what we are getting. Now, if we circumvent or try to go beyond this specific case of equilibrium flow and look at general shear flow, then also what? We would find that these two quantities are of same order of magnitude. They need not necessarily be exactly identical and this observation is really the basis for developing all turbulence models.

We also note that if we define u cap as a velocity scale l as some kind of a length scale, then the mean shear would be given like this (Refer Slide Time: 04: 03) and the

Reynolds stress that we have is also of the square of the velocity scale itself and we can plug this information in our equilibrium condition to get this following equation.

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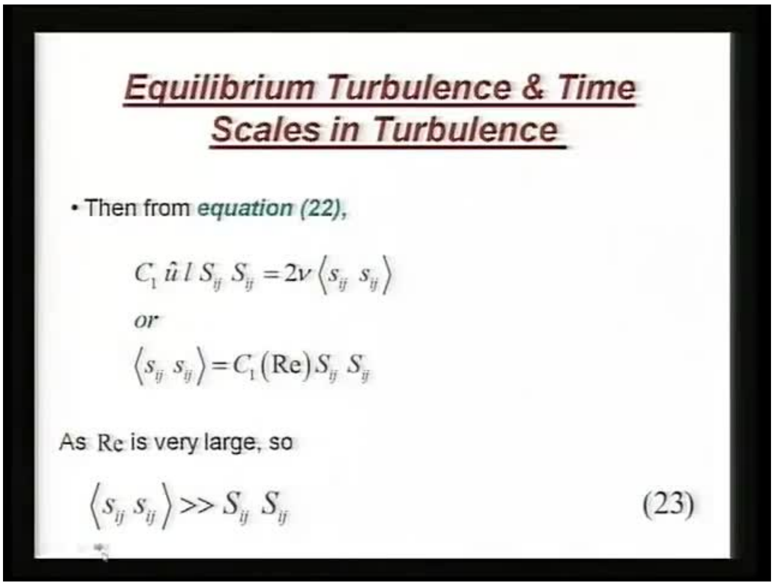

So, what we do get is this that S ij times S ij time averaged over sufficient along interval is equal to the product of the mean shear times the Reynolds number. And we have already noted that Reynolds number is very large for turbulent flows. So, that would show that on a time average sense the fluctuating strain rate is significantly higher than the mean strain rate.

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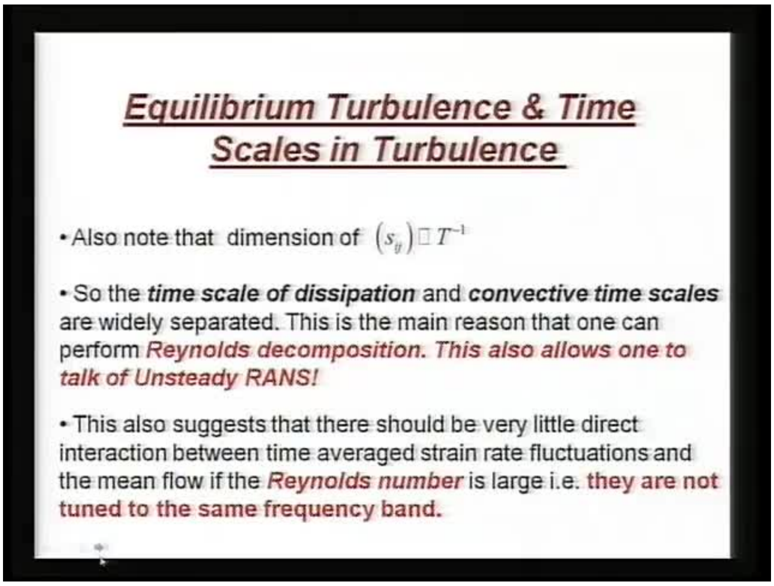

We have a seen the dimension of S ij is of the order of 1 over t. So, the time scale of dissipation and the time scale associated to the convective process, they are widely separated. This is the reason that one can perhaps do Reynolds decomposition. This, for the same reason that we can talk about unsteady RANS because even though the mean may very slowly, but the frequency of variation is so low compared to the frequency of the fluctuating component that we can say that they are, kindly, kind of separated, so that there is no direct interaction between time averaged strain rate of fluctuation and the mean flow provided the Reynolds number is significantly large. So, in a sense, you can just simply say it in one sentence that these two events - the means and fluctuations, they are not tuned to the same frequency band; they belong to a different frequency scales.

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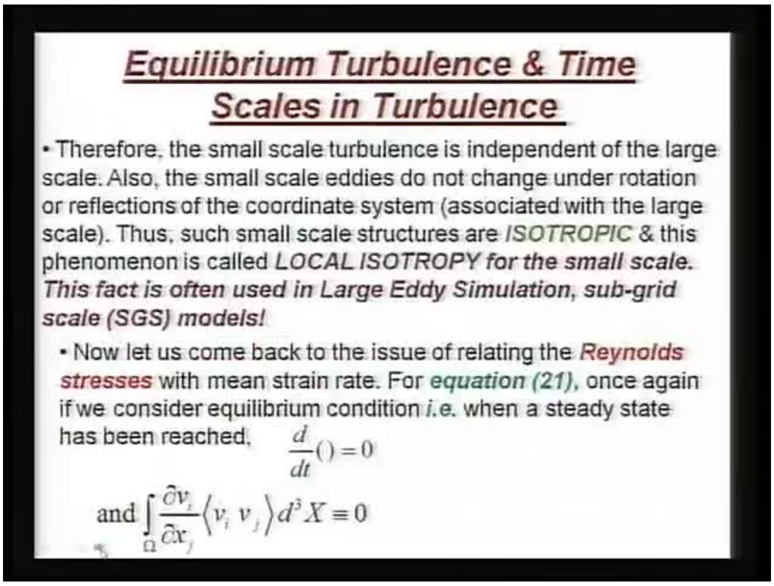

Now, if I look at a general shear flow then we are seeing that the small scale turbulence is independent of the large scale. So, when I am looking at, let us say, turbulent flow over, let us say, aircraft wing, the very fact that I put in a aircraft wing the eddies are created which are of the size of the dimension of the aircraft wing, the card. So, you could basically look at that and then what you can see that this small scale eddies do not change under rotation or reflection of the coordinate system.

So, basically, the small scale is independent of the large scale and what happens is that these small scales are, therefore, considered kind of isotropic because what is happening is here? We are getting this small scale by successive fracture of the large scale. We have large scale; they break down into smaller scale and so on so forth.

So, when you go to the lower scale, they have become kind of all most homogeneous isotropic. This is what is called as a local isotropy assumption - the small scale, and this is used observation is used in large eddy simulation where you try to derive a model for this small scale which are anyway not resolved because of your grid. So, we will have to modulate. So, you make use of the fact that those scales are isotropic. So, basically, then this is one byproduct of what we have studied looking at equilibrium turbulence.

Now, let us get back to this equation of relating the mean stress mean strain rate with the Reynolds stress. We find that we are again considering an equilibrium condition and we are talking about a steady state having been reached. And once we have reached that steady state, we get this following equation.

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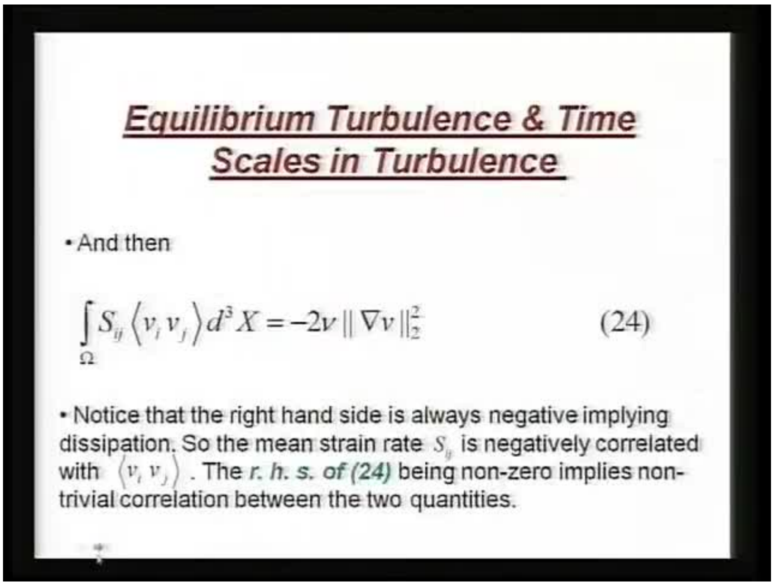

So, this is the observation that we would like to make note of, that the Reynolds stress is related to the mean strain. When you integrate over, the whole domain is determined by the viscous or diffusion term and the negative sign here implies that the overall effect of v i v j integrated over the whole domain has to be of similar effect like what you expect out of dissipation.

So, basically, that is why most of the turbulence model simply as a sort of overall dissipative phenomena. That is what is the sign; it implies that it is dissipation; it also shows that they are not completely uncorrelated. What has happened? What we just now talked about? The fluctuating strain rate is uncorrelated to the mean strain rate, but here we are talking about a different thing. We have talked about the correlation between the mean strain rate and the Reynolds stress.

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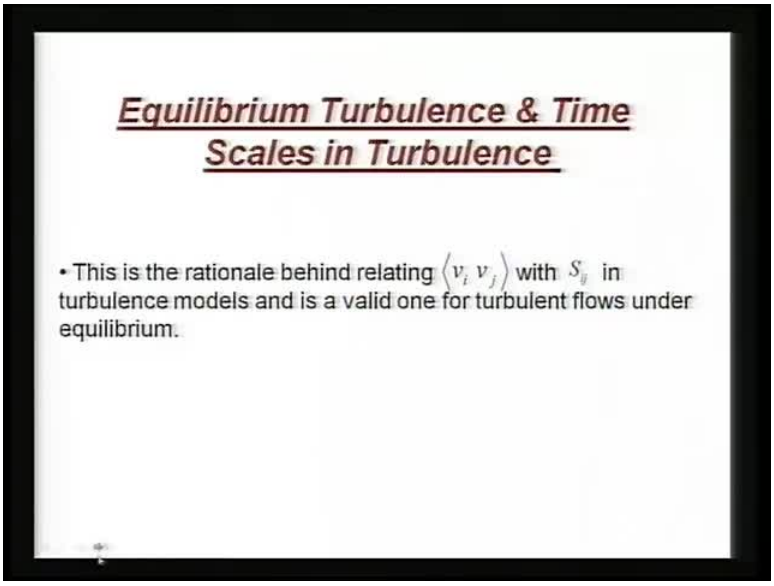

So, this is the velocity second moment, a fluctuating velocity second moment. So, this is also what is done in most of the algebraic are differential equation based turbulence model, where you eventually try to write out some kind of a algebraic relation or a differential equation, which relates the evolution operator of this with the help of the mean strain rate. This is what you do in turbulence models.

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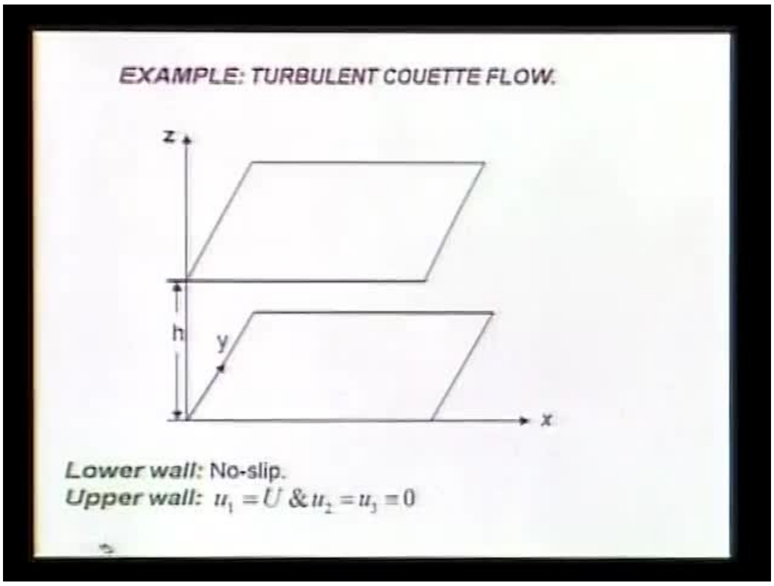

Now, take a simple example for term station purpose flow inside this passage, where the bottom wall is fixed; so, you do not have any slip here; other top wall is a simply slipping by at a constant speed say u or u bar, while the other two components y and z components are 0. So, basically, it is a turbulent couette flow that we are looking at.

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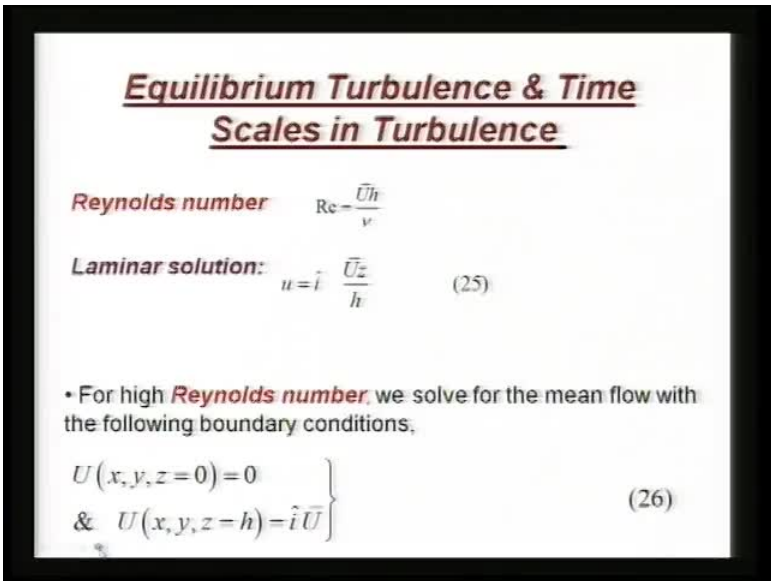

Now, some of the parameters that defines this flow, of course, is very easy for you to see from the Navier-Stokes equation. The Reynolds number is given by the velocity scale that the speed at which the top wall is slipping by, h is the gap between these two walls that promotes you to have an exact solution for the laminar flow. Because it is a parallel flow, you get u is equal to simply u bar z by h.

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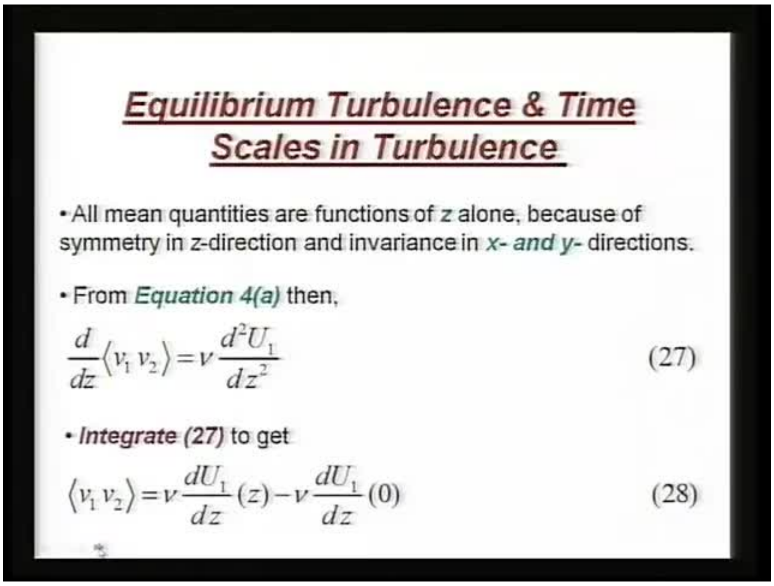

Now, if I want to solve for the turbulent flow, we need to apply these two boundary conditions. Those two boundary conditions are considered be time independent, even though the flow is turbulent. And as a consequence, what happens is, all the mean quantities are going to be functions of z alone because couette as parallel flow, we have a perfect symmetry in the z-direction and there is no variance in x and y direction.

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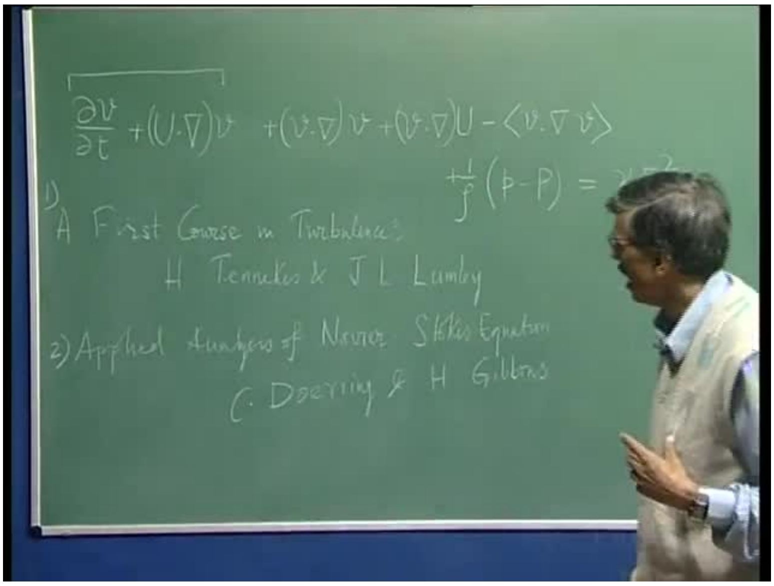

Then, if I write down this equation, I will get this term. You can verify that this term is not there; this term is gone and this term will not be there because of your invariance with respect to x and y direction.

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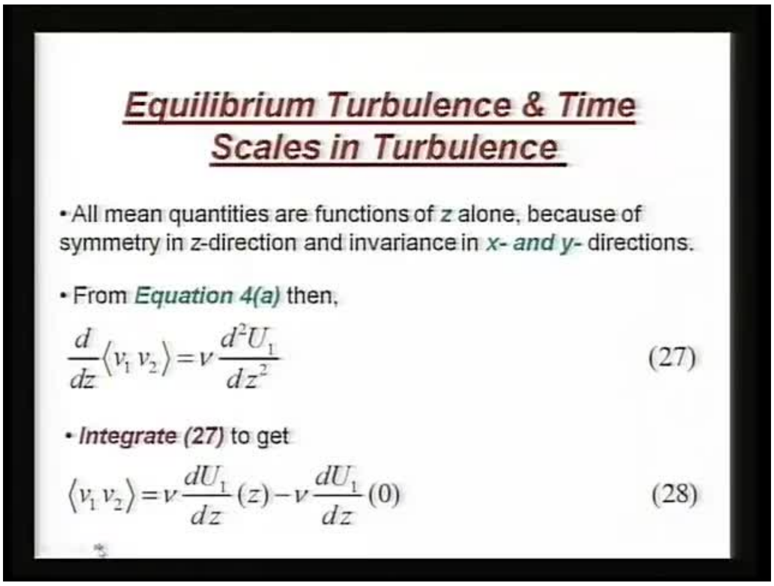

So, what you end up getting from the RANS equation is this shear gradient of the Reynolds stress should be balanced by equal to the stream wise diffusion term. So, it is fairly simple. This equation is simple. All you need to do is integrate this equation once. So, on the left hand side, I will get v 1 times v 2 is equal to nu times d u by d z, d 1 by d z or 1 is associated with the x direction at height z minus nu of the same quantity evaluated at the wall. So, we integrate it from 0 to a running height z.

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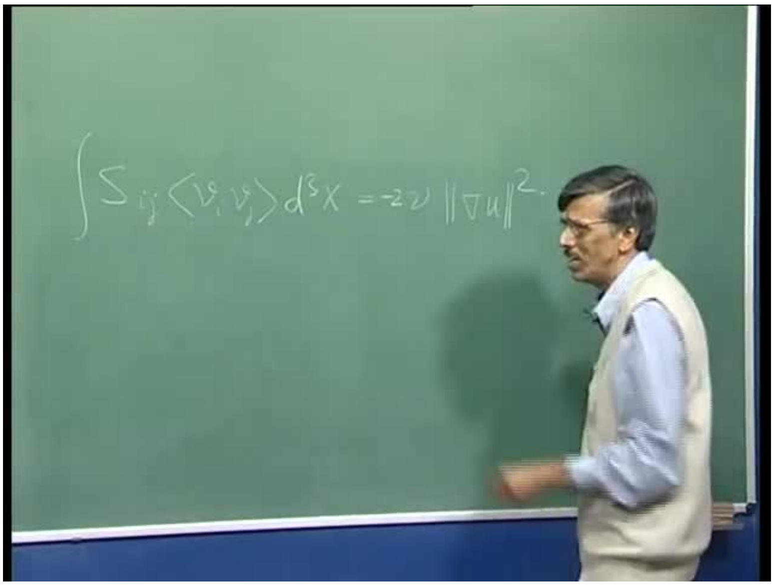

So, what happens is we have seen that equation 24 was what? If you recall, equation 24 was S ij, we had written this. This volume integral is equal to minus nu times this. So, that is the equation that we have. There is a factor of 2 missing here.

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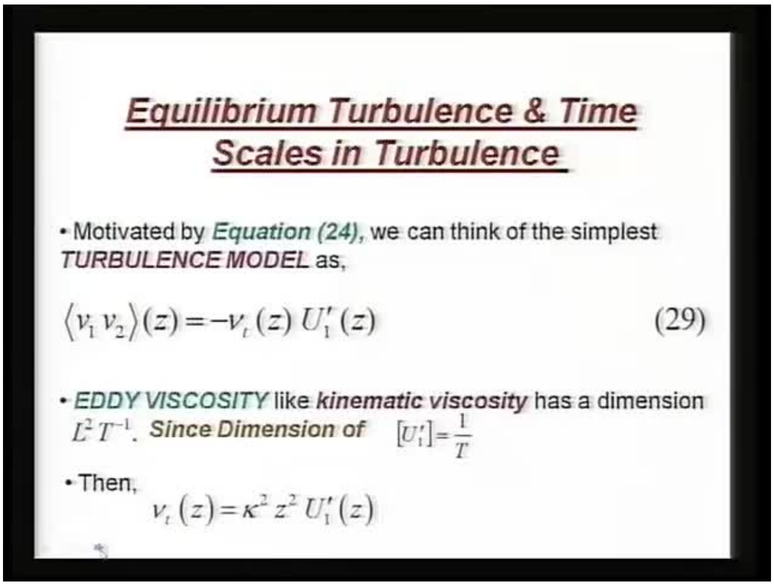

So, what will be the simplest possible turbulence model that should be given by this because that is what our previous equation has shown. We just simply looked at it in this form that 28 tells you that v 1 v 2 is going to be proportional to d 1 d z. (Refer Slide Time: 14:40)

So, if we try to do that, we can write it down in this particular fashion. Then, this will be equal to proportional to this. So, proportionality constant is what we would like to call it as the eddy viscosity and we have talked about enough to justify why we call it a viscosity because we saw that v i v j negatively correlates and it is proportional to the last term. So, this also should make a last term.

So, that can come about if we position it as a sort of a viscous term. Now, what is the dimension of eddy viscosity? It is area per unit time. So, that is l square t 2 minus 1. What about the dimension of this? Dimension of this is 1 upon t this d u d z; so, it is one up on t.

So, then, what happens is this must have this kind of a dependence (Refer Slide Time: 15:59) because if I want to have this dimension is l square t to the power minus 1. Then, that l square part comes from here. z square and t to the power minus 1 comes from this and they are proportional. So, this proportionality constant, I am calling it as a kappa square and purposely wrote it as a square term, So that the new t will be positive whereas, the Reynolds stress will be strictly negative. So, that is the motivation for writing down this expression for nu t the kinematic sorry the eddy viscosity.

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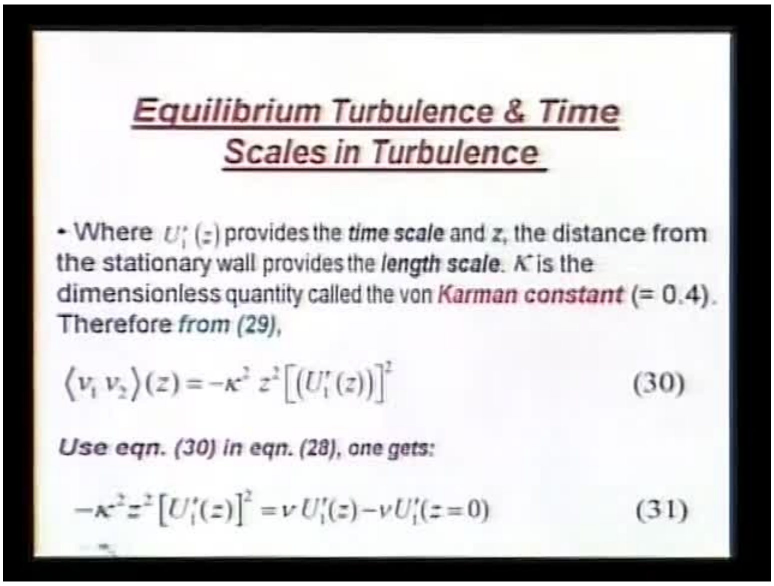

So, we have now seen that u prime provides the time scale, then z - the distance from the bottom static wall to any particular height provides as the length scale, kappa is the dimensional quantity and it has been shown, time and again, in various other studies, as it is a constant due to von Karman taking a value about; point 4 1 can show that.

So, basically, then what happens is we have a solution for the Reynolds stress. Reynolds stress is simply given by this. So, what I do is then I substitute this in that governing equation after integration we have gotten. So, basically, v 1 v 2 z is this quantity; that is written here and this is equal to this (Refer Slide time: 17:36). So, this is the equation that would basically tell you how the shear is varying with z; is not it? that is what it is d u 1 d z at different…

So it is a kind of a quadratic equation so you can solve it. If you solve it, you will be able to obtain the expression for u 1 z.

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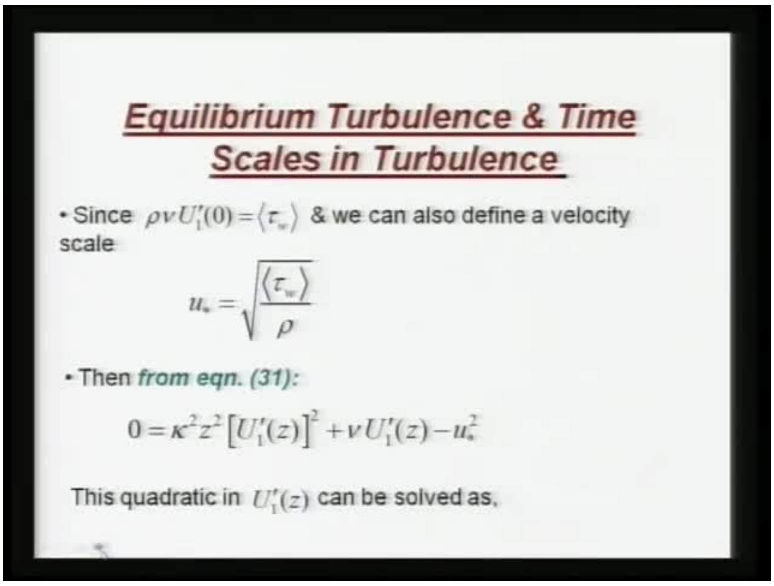

However, we can also define a velocity scale in a somewhat different way where, we talk about a wall shear that is dimensionally equal to rho times nu times d u d z. So, that is that is the definition of this. So, since we are talking about a mean. So, that is why this angular bracket has come. And on the left hand side, we have written everything in terms of mean quantity.

So, what happens is, if I now divide, bring rho on this side, tau w by rho gives you a velocity square time scale. So, this is what is called as the frictional velocity; u star is the frictional velocity which is given in terms of square root of tau wall by rho. So, substitute that expression and you get this equation that is a quadratic.

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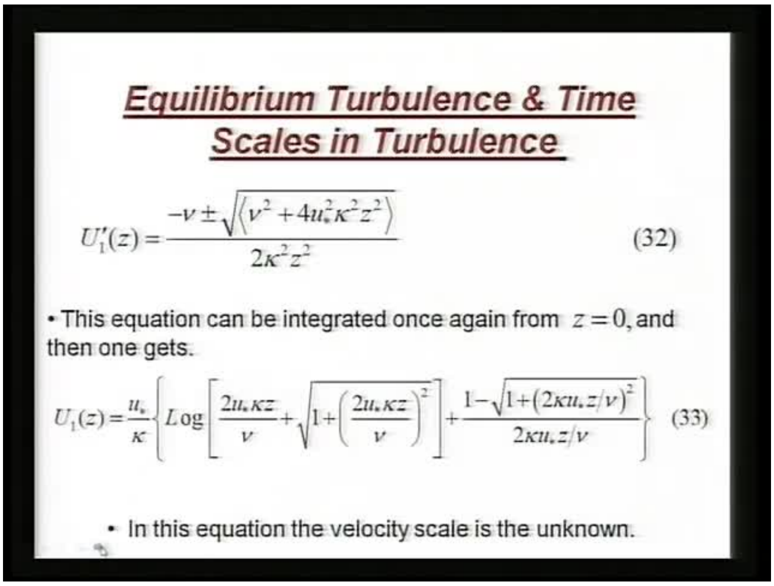

So, we have just introduced the friction velocity and we have obtained a quadratic for U 1 prime z and that is the solution straight forward, and so this is an expression for d U 1 d z. So, we can integrate it once again to obtain the velocity as a solution; Whereas the So, how is it that this is different from a laminar flow. Where has the thing come about? Well, it has come about because of the presence of the v 1 v 2; v 1 v 2, those Reynolds stress would be negligible for laminar flow.

So, where does this thing come in in this equation? In this equation, that information comes in through the way we have defined the velocity scale u star because a tau wall has a built in effect of the turbulence in it; the wall shear itself has information of turbulence being built in there.

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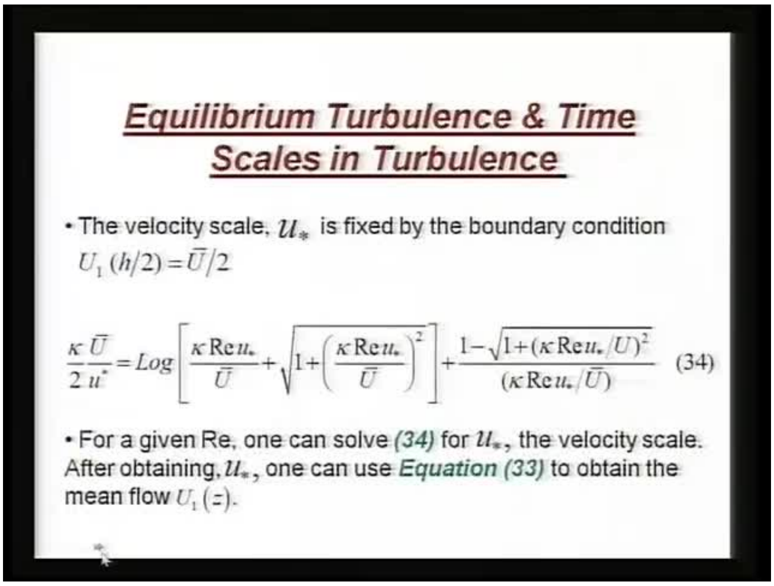

So, once we can get this velocity scale, then we can get the solution for the velocity profile. How do you get this velocity scale is very easy. You have that equation, the solution and apply the boundary conditions. The boundary condition is that if I go at the midpoint, it is linearly growing; so, I get this. So, I substitute it there and I get this equation. So, what happens is if I choose a Reynolds number, I can solve this equation for u star because everything else is known. Kappa is a constant and I is known; u bar is known; only thing that remains unknown here is this u star. So, I can use this equation for u star, the velocity scale. Once I have it, I can go back and obtain the velocity profile. So, that is a very simple application of the ideas of relating the mean strain rate with the Reynolds stress; that is what it is.

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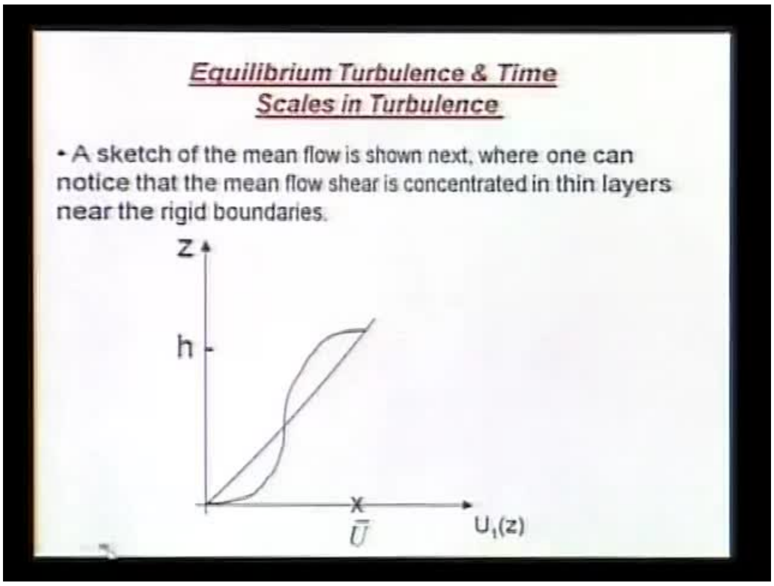

So, if I now plot the U versus U 1 versus z, then this is the way this velocity profile is going to be. So, it is perfectly symmetric about the midpoint. On this side, you have one sign of shear; on this side, you have the other sign. So, the plate is dragged along like this. So, that is what you are getting. So, you get the maximum velocity u bar at z equal to h.

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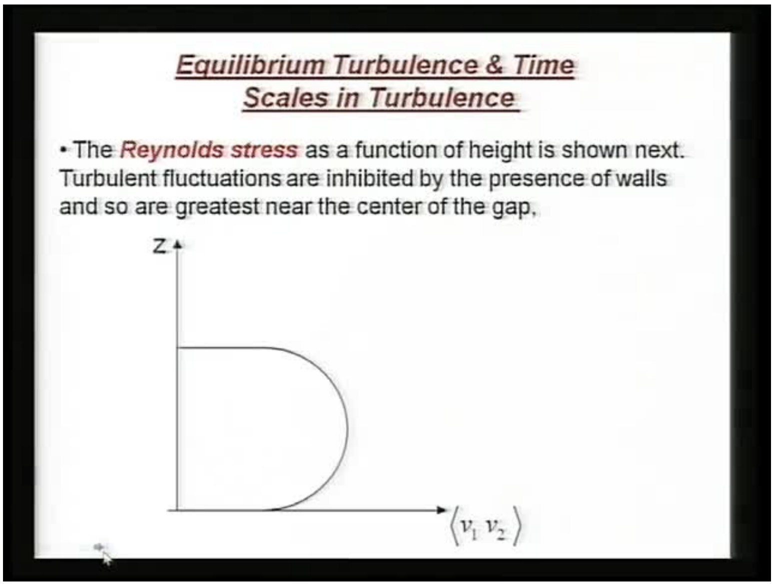

So, this is your mean flow solution what we called as u 1 as a function of z and having obtained that, we can also calculate the Reynolds stress v 1 v 2 as a function of z, which will b equal to 0 at the wall; is not it? Because at the wall, we are going to apply no slip conditions or no fluctuation; so, its time average also has to be 0. Where will it be maximum? It will be maximum somewhere in the center of the passage. So, that is what, we are going to see at the center of the gap. We are going to get the maximum Reynolds stress.

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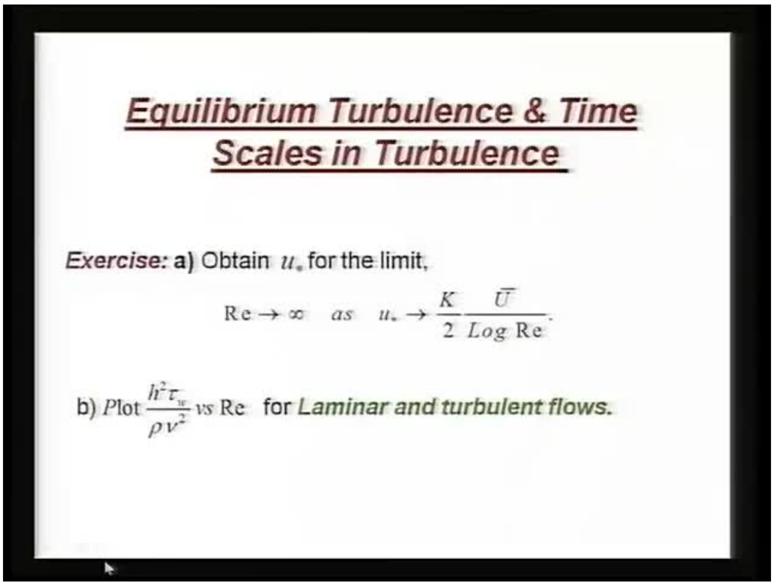

Well, this is just, kind of, an exercise you can spend some time looking at it that what happens to this value of the frictional velocity in the limit Re going to infinity; that is truly a very high Reynolds number solution. You should be able to show that the frictional velocity has this kind of a log dependence on Re; this is that kappa.

This is that kappa. What you could do is you have a laminar flow solution; you have also turbulent flow solution; so, from those velocity profile you can calculate tau w and then you try to plot this quantity versus Re, and then you see what you are going to get. So, do take a look at this and you will be able to see very clearly the distinction between laminar and turbulent flows. Let us now move on to a different approach. See, so far what we have done?

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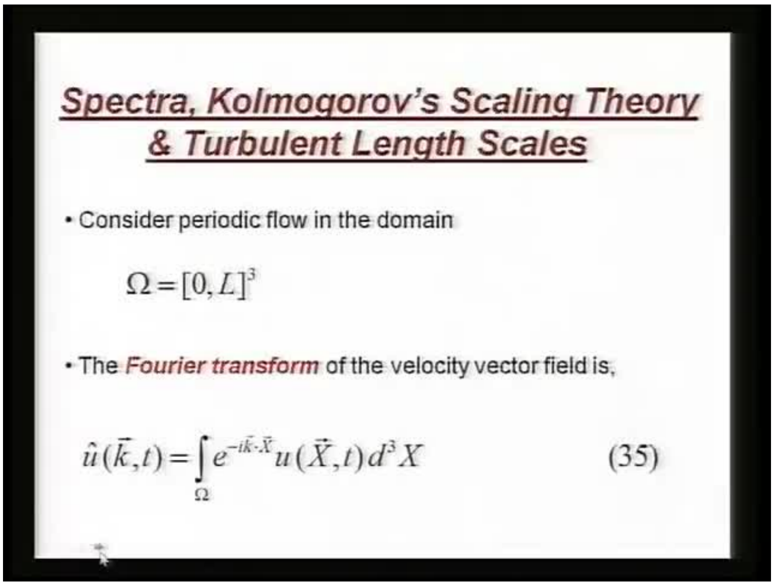

We have looked at turbulence as a kind of time varying quantity and we looked at it in terms of its time averaged equation - the RANS equation. Now, we have also talked a little bit about large eddy simulation, but not to a great extent. And talking about time average quantities, we have made some of the observations like the role of Reynolds stress is going to be dissipative in a time average sense, but is it true that it happens at all length scales?

We are talking about a time averaged value. So, what we would rather like to know is - how the turbulence quantities vary as a function of wave number, and this was what was the studied in great detail by Kalmogorov's by looking at a periodic flow in a box of size l.

So, I have a cube of size l and I am also going to look at a velocity vector which I am writing as u as a function of X and t; I can take it is Fourier transform; I can take its Fourier transform to get the spectrum. So, u hat as a function of wave number k, and k is vector; so, you could have all three components. Now, I could get the spectrum or if I had the spectrum, I could perform an inverse transform to get the velocity field in the physical plain.

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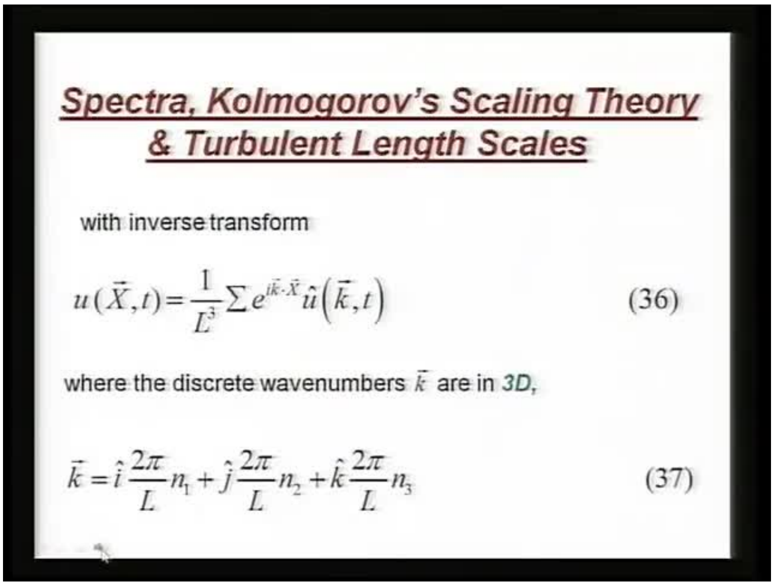

This is what we are doing with the idea of we are looking at a box of size l and what happens? We are also making an assumption of the flow being perfectly periodic. So, what happens is you are going to get succession of wave numbers in this three directions which are super harmonic of the fundamental and the fundamental is given by 2 pi by l.

So, I have it in the i direction; I have it in the j direction; I have it in the k direction; x y z directions and subsequent harmonics will take different values of n 1 n 2 and n 3. So, it is a periodic flow; so, n 1 could go from minus infinity plus infinity. So, can be said about n 2 and n 3 also; that is what we are looking at.

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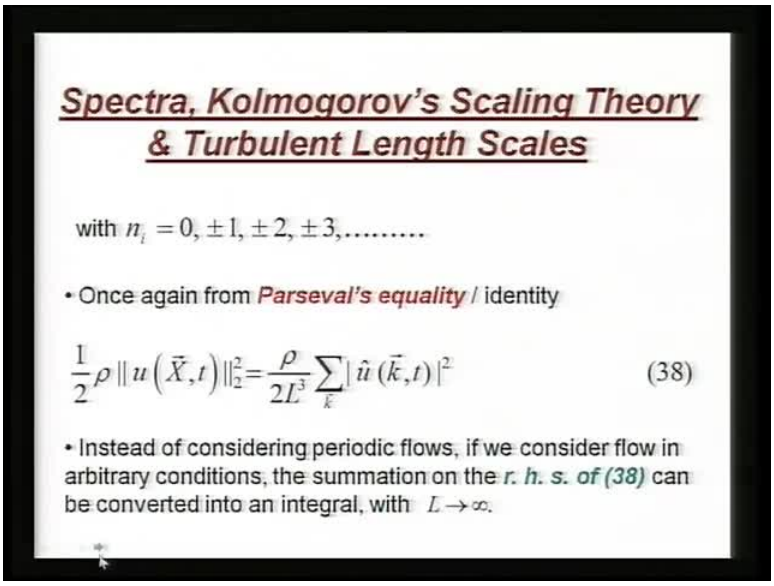

So, once we have defined the velocity field either in the physical plain or in the transform plain, we can invoke what is called as Parseval's equality or identity which is the very simple observation that if I look at the kinetic energy per unit volume, that is what I am going to see it in the physical plane, that is half rho times modules of u as a function of x and t square. That is exactly the same thing, if I would have written on the velocity in the spectral plain. So, that is what we are doing.

So, what happens is whether I look at it in the physical plane or in the spectral plane, kinetic energy represented would be the same. Now, if I consider a non-periodic flow with arbitrary conditions, then this summation here would be replaced by an integral. I will have to replace Fourier series by Fourier transforms.

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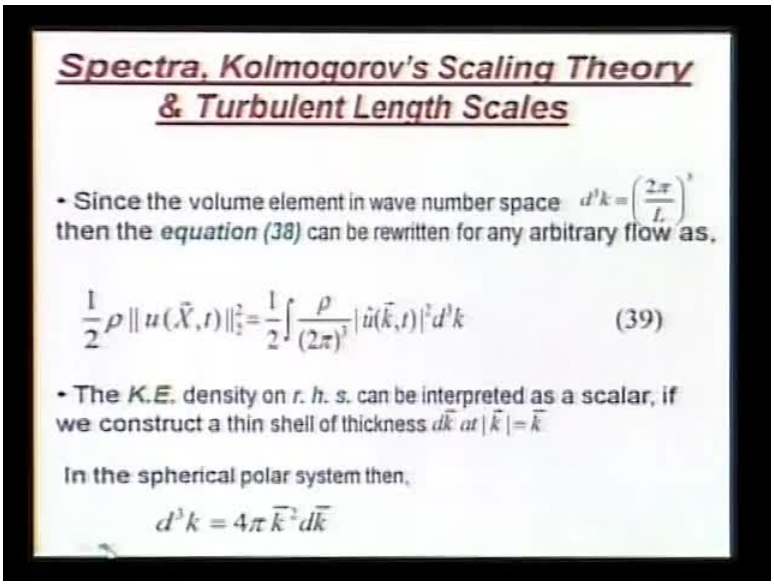

So, that is what we do and what happens to the box size? The box size is becoming infinite to the extent that it will take L going to infinity; replace this summation by appropriate integral. Now, what you do is this, the appropriate integral would imply that I will have to be talking about an elementary volume in case space; that I am writing it as d cube k. What were the individual fundamental frequencies? Those were 2 pi by L.

So, I have a in each direction, I had 2 pi by L as the interval. So, d cube k is nothing but 2 pi by L. So, then, for arbitrary flow, I can write the Parseval's equality in this form. Now, you can see, we have replaced that summation by integral and we have written it; that L cube, we have written it as 2 pi whole cube by k; so, that is what we have got. So, this quantity, the kinetic energy per unit volume is called the kinetic energy density. This is a scalar quantity. I am going to do integral over all possible case.

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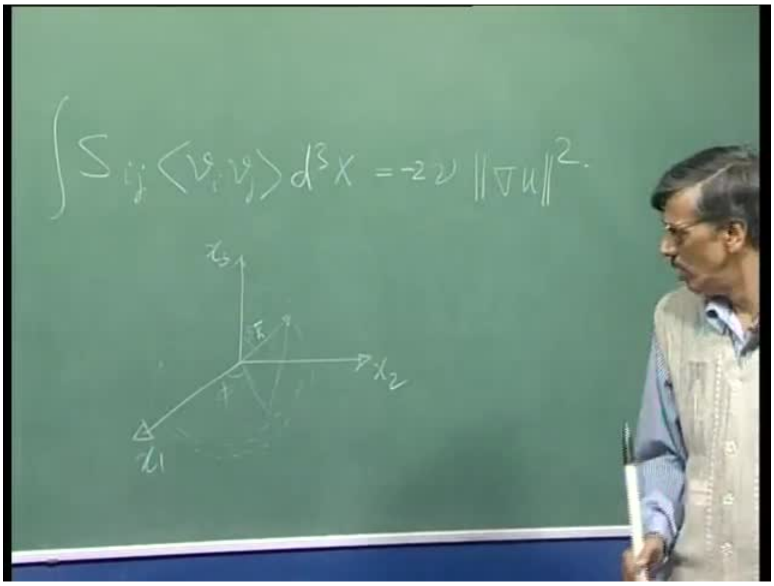

So, it is a 3 dimensional case space. So, what I could do is I could do it in a slightly different way. So, for example, if I have these three directions, I could write it in, let us say, Cartesian frame. I could also write it in a spherical polar system.

So, if I do it in a spherical polar system, then what happens? I am going to talk about the radius vector of the… so that I could call it, let us say, as k bar and elementary volume in spherical polar system would be what? Would be a sort of a spherical shell of width; I will call it as d k. So, what happens to the elementary volume? In this case, what we are going to do is, see there, we are talking about 3 variables in the x, y and z directions. So, what we are doing? Here also, we are doing the same thing k bar and then we will talk about this angle theta and this angle phi; that is what we do. However, what we are talking about? Let us say, we perform the integral in theta and phi direction and then it becomes what? It becomes a one dimensional quantity.

So, basically, then, looking at a spherical shell like this, we have performed all possible variation in theta and phi, that how we are getting this spherical shell. And what is the volume of that shell? The surface area is 4 pi k bar square; width is d k; so, that is that.

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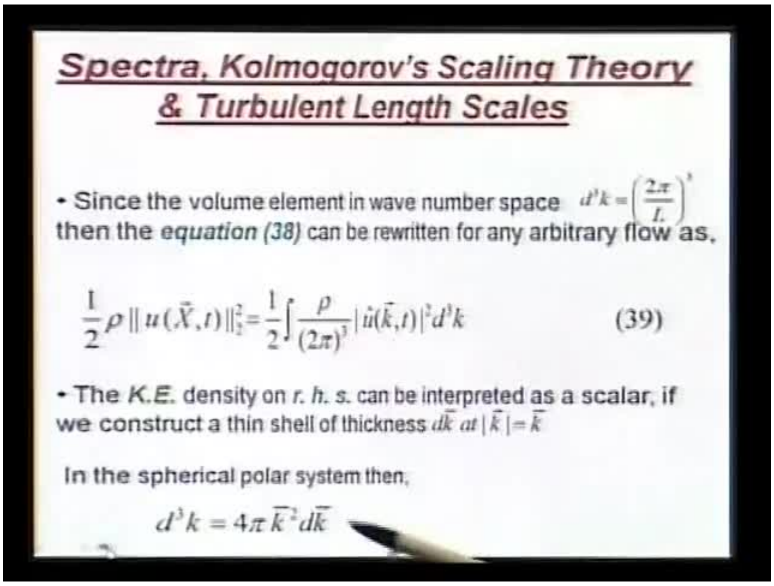

So, that is what you are seeing, that if you migrate from Cartesian frame to a spherical polar system, then this elementary volume you could write it in kind of in terms of a single quantity. And what is k bar? Actually, k bar is the modulus of the wave number vector because you can take k bar and can project in 3 component that will give you along x, y and z direction.

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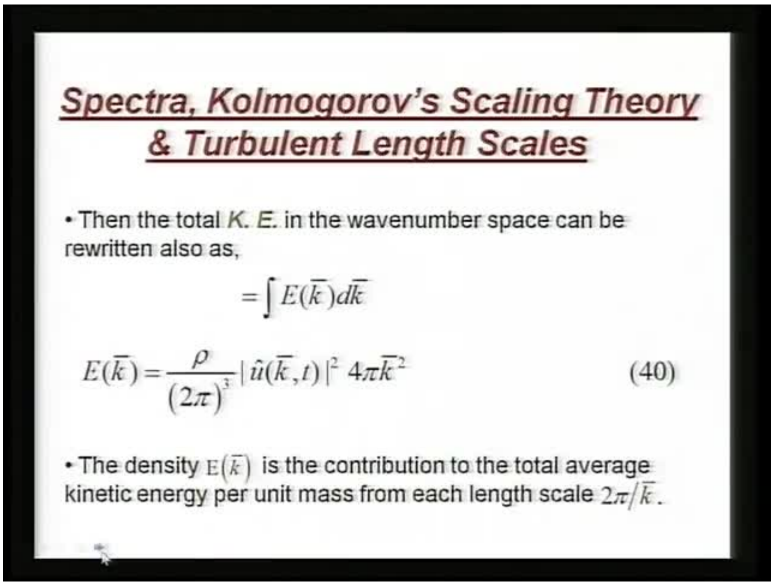

So, basically, it is another way of looking at the quantities in a spherical polar system and then the total kinetic energy that we have got. What we are going to do is now we are going to integrate over the remaining unknown independent variable k bar.

So, I will integrate that over all possible k bar I will get the total energy so if I write it like this and if you have seen that previous expression that I had written for e of k that involved d cube k remember if I substitute it here (Refer Slide Time: 33:34). So, I will get this quantity times 4 pi k square that should be your my e of k bar because I have to just simply perform that k integral; so, that is what we are going to do. So, from each length scale, I can identify corresponding k bar, I get the corresponding contribution of the energy spectrum.

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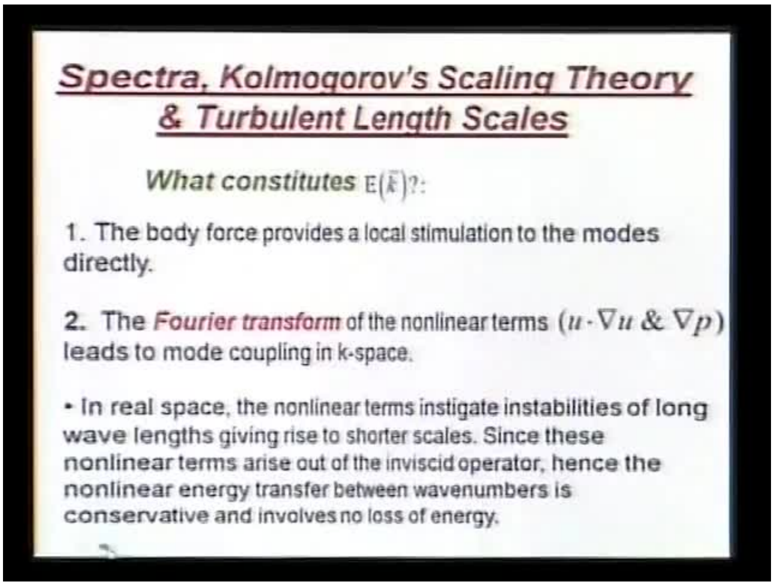

Now, you may like to know what constitutes this E of k bar. See, going back to our original equation, not this, the original equation which also included the body force; the body force can directly stimulate a particular length scale; at whatever scale that is happening. This is what we are going to study now and we are going to show what the non-linear terms do the… non-linear terms actually provide a kind of a coupling among different modes in case space. In the real space, when I look at the non-linear term, what do they do? They are responsible for all kinds of instabilities; that is what we have studied instabilities of long wave lengths which also give rise to, where the dispersion relation and also nonlinearity will give rise to smaller scales. Since this non-linear terms arise out of the inviscid operator, there is the inviscid operator, convecting acceleration term. And so what happens is this non-linear energy transfer takes place between two wave numbers is going to be conservative because we have seen inviscid mechanism; no loss is involved.

So, that is what, we are saying that we have to look at the non-linear contribution. If you want to study, we do not need to really look at the viscous part; we can just simply look at the inviscid operator and see what this non-linear contribution is going to be.

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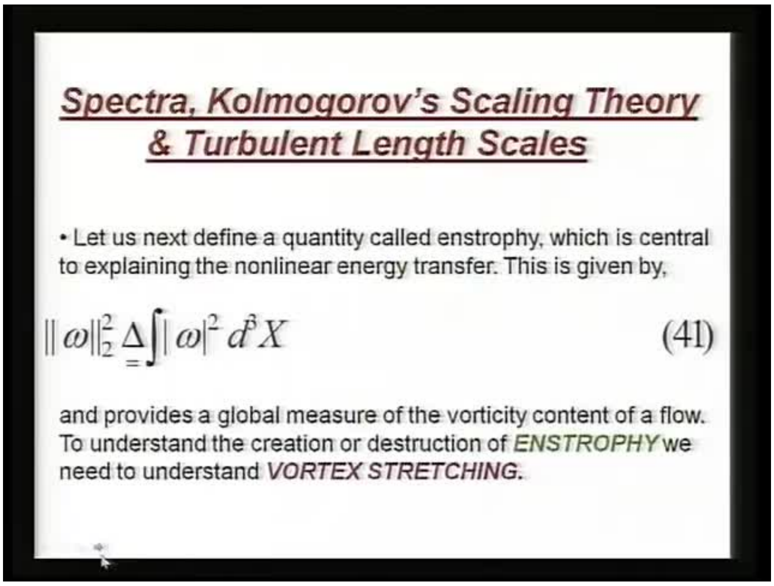

So, we are trying to understand what? This non-linear energy transfer mechanism is to help us in understanding that. Let us define a quantity which we will call as the Enstrophy. Enstrophy by definition would be nothing but mean squared; vorticity integrated over the whole domain; that is what we are defining it as that omega square integrated over the whole volume; that is our enstrophy.

This is a global measure because we are integrating over the whole domain, tells you about what is a total vorticity content in the flow. To understand creation or destruction of enstrophy, we need to understand the role of vortex stretching mechanism which we have alluded before, but let us spend a little time understanding it and somewhat little more detail.

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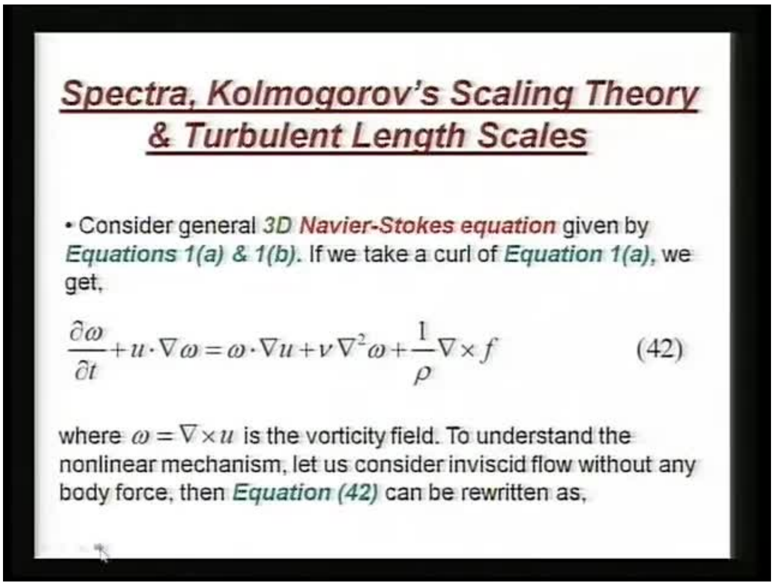

Now, if I look at the 3D Navier-Stokes equation and take a curl of it, I will get the corresponding vorticity transport equation. Here, every term is involved; even if the body force is non-conservative, this will be non-zero. If it is conservative, this will be 0, but you can also see the viscous term. And this is the term that we are talking about as the vortex stretching term. Omega by definition is nothing but the curl of the velocity field. Now, if we consider the inviscid flow and we also ignore any non-conservative body force, then this equation can be significantly simplified.

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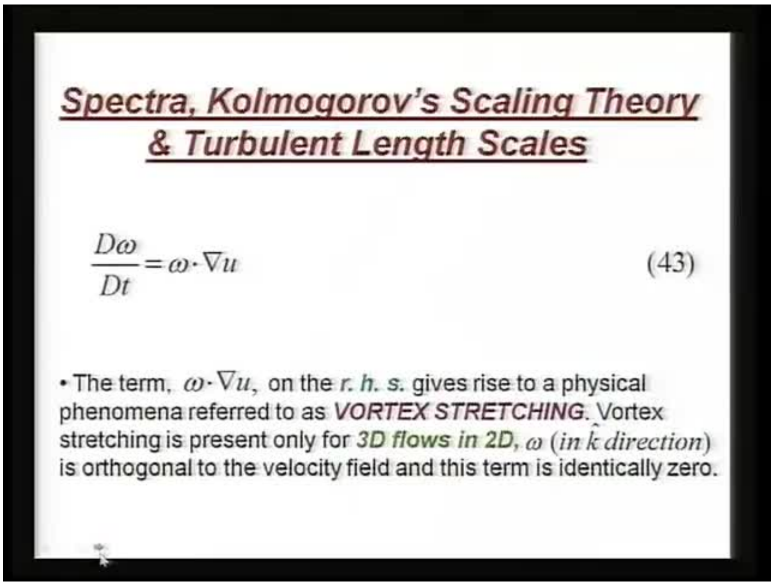

So, basically, what we are talking about. We are talking about the vorticity transport equation for 3 dimensional flow, which is inviscid and which is not effected by any non-conservative body flows. So, this is your equivalent vorticity transport equation. This is what we are calling as vortex stretching term and we have already noted that this is present only for 3D flows; because in 2D flows, these two operators are orthogonal omega and the gradient operator. So, this is identically equal to 0. So, this particular term that we see on the right hand side of this equation has the role to play for the 3 dimensional flow only.

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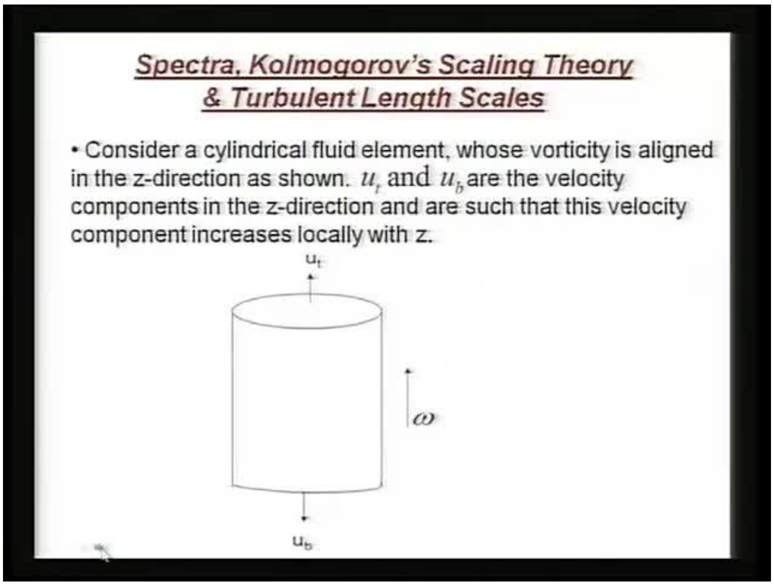

Now, think of a cylindrical fluid element whose vorticity, let us say, is aligned in the z-direction. So, that is what we are showing; omega is in this direction. So, let us say, the top of the element has a velocity u t and the bottom part has a velocity u b.

So, what it is doing actually then? It is elongating the fluid element. So, what is happening is if I take the velocity as a linear function of z, that means if z increases, v also increases; so, then it is going to be stretched in this z direction. So, this is what we want to see.

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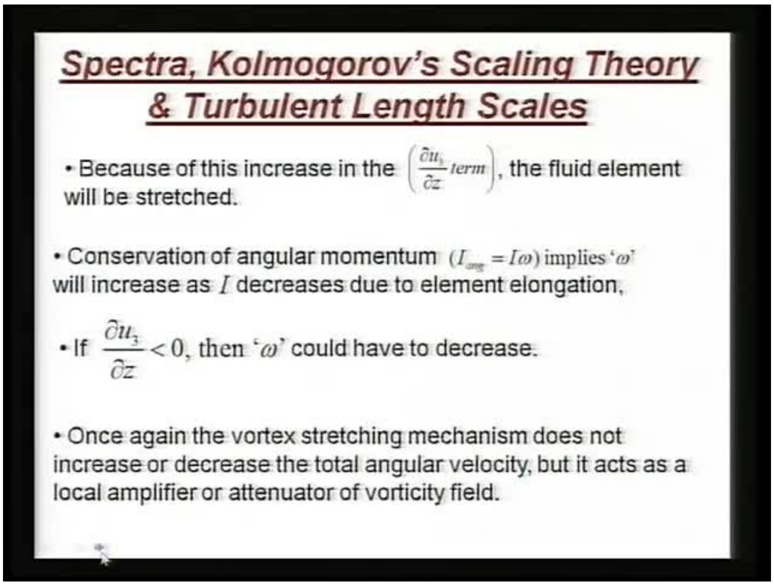

So, you called it a vortex stretching. So, we are talking about a situation where we are increasing the velocity in the z direction. So, what it does? This is now going to be a positive quantity; that is what we are saying. So, as a consequence the fluid element will be stretched. If the fluid element is stretched and we are talking about inviscid dynamics; so, there are no losses. So, everything has to be conserved, including the angle of momentum.

And what is the anglular momentum? I times omega. So, if I angular, well wrong choice of variable, but it is angular momentum is this that is proportional to moment of inertia times the angular velocity.

Now, what will happen? If I take an element and I stretch it in the z direction, what happens? I comes down; if I comes down, what happens? Omega has to increase because this is laminar conserve. So, we are saying, that if I have a velocity field which causes the fluid element to stretch, that also causes the vorticity in that direction to increase; intensity. So, this is one mechanism by which vorticity increases and it can increase if this term is positive; it can decrease if this term is negative. But irrespective of whether it is increasing or decreasing, if I look at the enstrophy term, what will happen?

It will always increase. See, that is why we purposely introduce that quantity which is nothing but the mean square term. So, whether the element is stretched or compressed, enstrophy is going to increase and that will tell you what is the rotationality of the system.

Now, we are very clear here that this vortex stretching term that we are looking at does not increase or decrease the total angular velocity because of the conservation. We are looking at in one direction it is increasing. What happens to the other direction because totally we are again looking at inviscid mechanism, I omega has to remain conserved. So, if that I is all pervading, that is not changing; so omega also will not change. So, that is exactly what we are saying here, does not increase or decrease the total angular velocity, but it simply acts like a local amplifier or attenuator of the vorticity field.

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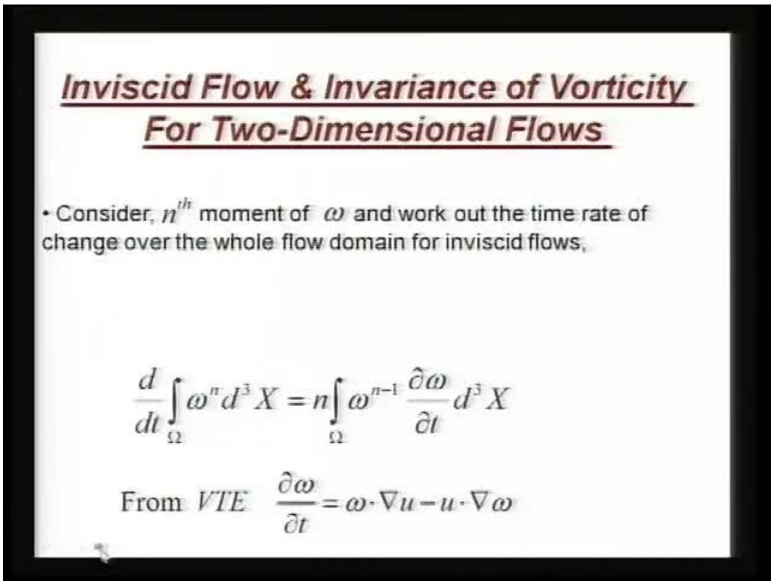

So, let us generalize this result by looking at the nth moment of the vorticity field. So, what I am trying to do is I have omega rise to the power n integrated over the whole domain and find out it is time rate of change. So, if I do that, I can bring it inside; then I will get n times omega to the power n minus 1 del omega del t integrated over the volume, and this we have already obtained from the inviscid vorticity transport equation. So, this plus this is the convective term; this is that vortex stretching term (Refer Slide Time: 43:20). So, I can replace this del omega del t by these two terms.

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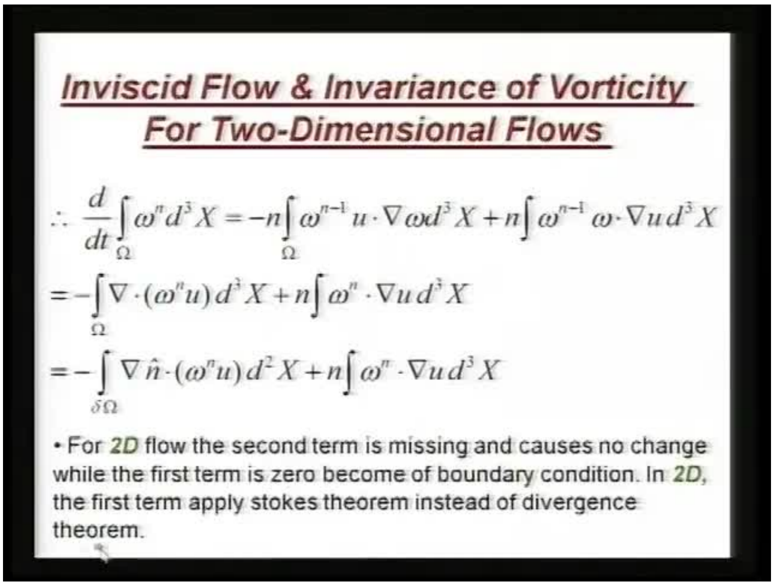

So, that is what we are going to do. Now, if I do that, what do I get? Well, I do get this 2 ets of terms. Now, this is easy; the second term is easy because this is going to be n times omega to the power n; that gradient. Now, this part I have written in this form; if you open this, then what will you get? You will get omega to the power n and then you will get del dot u term, but del dot u is 0. So, that is why this term has been written like in this form.

So, again we note that this term that this term (Refer Slide time: 44:22) is coming from this stretching term. So, this stretching term is missing for 2D flows. And what happens to this? I am going to perform this following integral. So, I can use the divergence theorem and write it interns of a surface integral; gauss divergence theorem. So, basically what I have done? I have taken the quantity; find out it is normal component that is on the domain boundary del of omega and that I do is surface integral.

Now, this quantity; this quantity itself, if I look at the whole volume, what may happen? If I go very far away, the effect of the body will not create a non-zero omega. Again, on the body, omega will be non-zero, but u will be 0 because of no slip. Let us understand that.

So, what will happen is this first term will be most of the time equal to 0. And please do understand we are talking here about inviscid dynamics. So, we are not even talking about creating omega at the wall. Even if i talk about creating omega at the wall, I will have to corresponding it talk about no slip condition, but if I talk about inviscid dynamic, then u will be nonzero at the wall; that omega is not getting created.

So, basically that is what we say here, the first term is 0 because of boundary condition and if we are talking about 2 dimensional flows, then instead of doing a volume integral, I will be doing a surface integral and then this term will be a surface integral. I can use the stokes theorem and convert it into a line integral flow. So, the analogy is very simple; either I use the gauss divergence theorem or the stokes theorem; in either case, I will find that the first term will go to 0 and for the 2D flow, the second term also additionally 0. So, what basically it says that in 2D flow, any order moments that you look at for the whole flow field does not change with time for 2D. For 3D flow, what happens is only thing that comes about, comes about from this stretch in time. So, this nth moment increases due to this vortex stretching for 3D flows.

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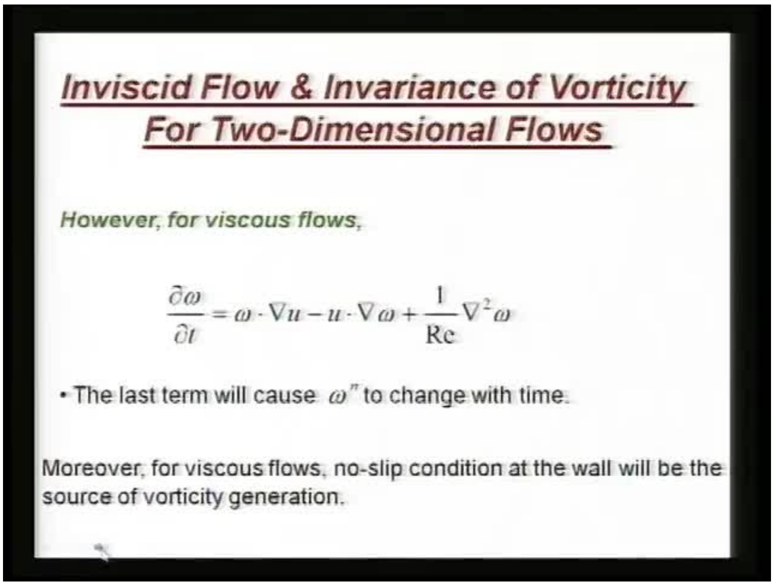

So, to stretch it, to extend it to viscous flows, we would have to basically replace del omega del t by not only these two terms, but also the viscous diffusion term. his term also will cause omega to the power n to change with time. So, that will be additional source of change and moreover for viscous flow to no-slip condition of the wall will be the source of major vorticity production. So, this is what we need to understand.

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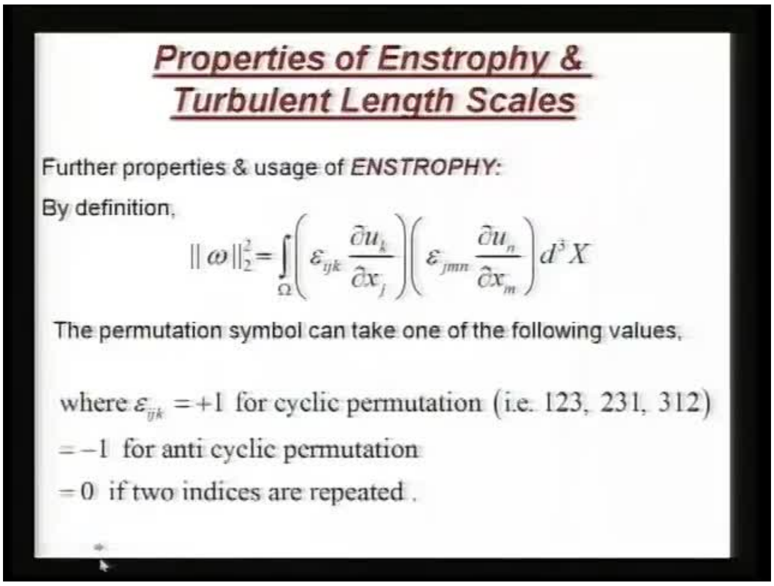

Now, look at some additional properties of enstrophy. This is way the enstrophy is defined. Those give all tensorial notation; so we have this permutation symbol here; epsilon i j k times del u k del x j; so, what does it mean actually? We are looking at omega i. So, this is your omega I; that is how it is defined because you are integrating over j and k. So, what is left is j omega i part; so, the same thing here. What is this? This is going to be omega j. Well, that is what we are doing and individually the property of this permutation symbol is the following that if I take a cyclic permutation, that means if go in natural, I takes i equal to 1, j equal 2, k equal to 3; that is a natural cycle. Or I could take i equal to 2, j equal to 3 and k equal to 1; that is also natural.

So, it is basically, you are going in a circle or I could take it 312 etcetera, then permutation has quantity. Symbol takes a value of k plus 1; if I interchange any of these two, so, I take, instead of taking 123, I take 132; then that becomes minus 1. And if two indices are identical, then this quantity is 0; that is the meaning of the permutation symbol.

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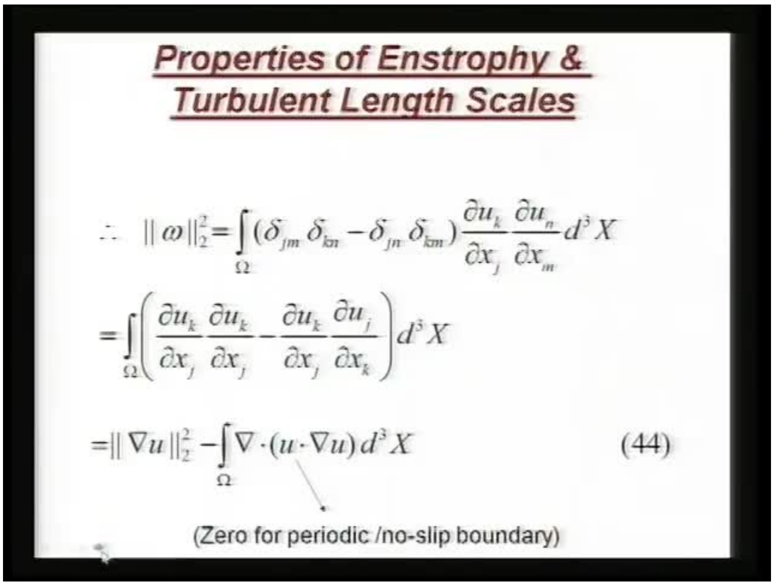

So if you adopt this way of looking at enstrophy, you can also write down the enstrophy in terms of this. Once again, you can notice that these are two products of delta function from kronecker delta; you can very clearly see that this multiplied by this will be non-zero, when j equal to m and k equal to n; that is what I have done here.

So, here I have changed n to k and here j equal to… well I suppose yeah this del x n has become del x j. So, that is that part and this will part will give you this. Now, this is nothing but this quantity; is not it? (Refer Slide Time: 50:52).

See, we call it quite often,(()) we were writing the contribution coming from this as a nu times modulus of grad u square. So, that is because of this term. What about this term? This term is going to be 0. If I am looking at 0, for this is going to be 0, if I am looking at a periodic problem and if I am looking at the boundary contributions omega 1, no-slip boundary will ensure that.

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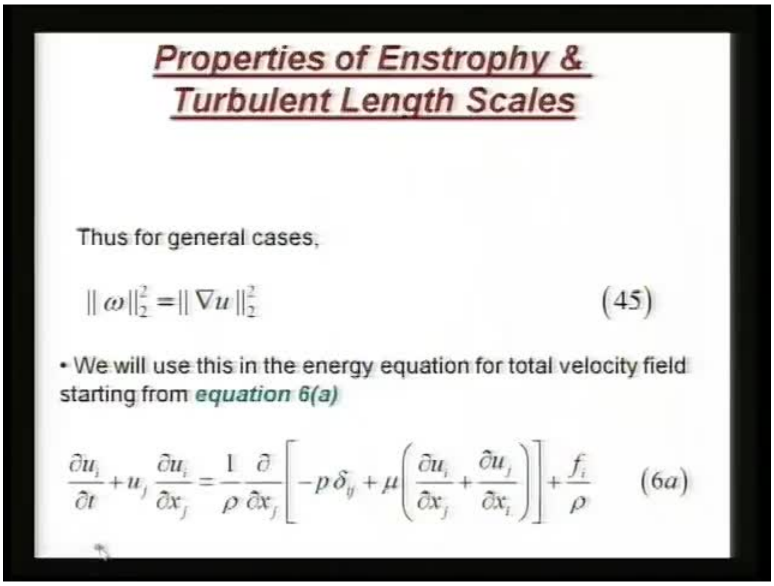

So, basically then what we are essentially talking about is the enstrophy is also related to the gradient of the velocity field by this. So, we can use this expression in energy equation for the total velocity field. Again, we start from, let us say, this equation that we have already written and Navier- Strokes equation including the body force term.

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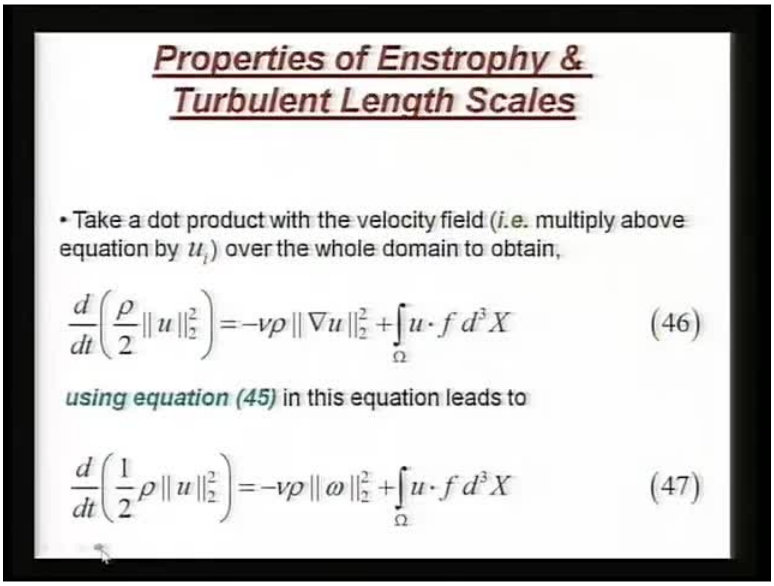

Now, what happens is that, equation that you have written in the previous swap, you have take a dot product of this equations 6 a with velocity field. Basically, I will multiply it by u I and integrate over the whole domain and this will give us this equation. So, I think what I would do here, I will just stop here today.

And in the next class, we will just wrap this thing up. We will see that how properties of enstrophy can help us in understanding whatever additional length scales are involved in this problem. I will stop here.