Instability and Transition of Fluid Flows

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Module No. # 01

Lecture No. # 35

In the last class, we talked about the various attributes of turbulent flows. We catalogued some of the characteristics of turbulent flows and one of the attribute that we talked about was that it is not a molecular phenomenon. You can see it in the macroscopic level. So, you can write down the continuum equation and that would still be valid equation for turbulence.

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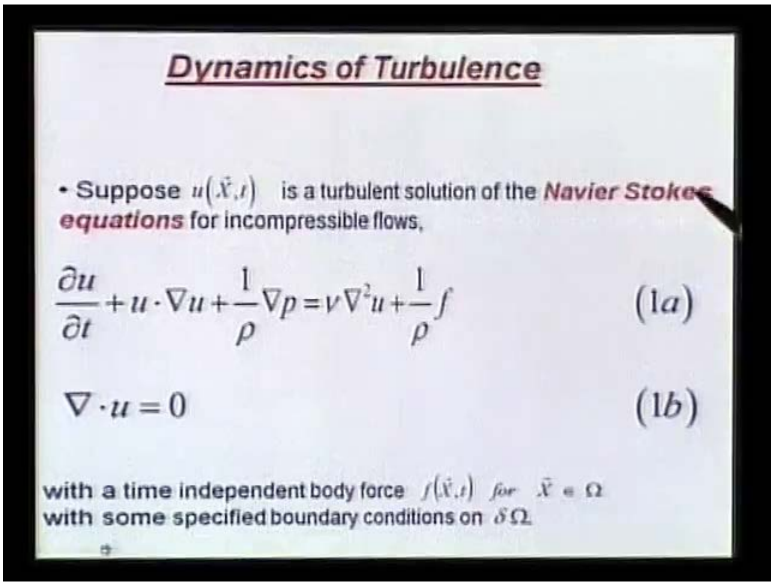

So, that is why, this is all is understood that if you look at Navier-Stokes equation, that should explain turbulence adequately. So, suppose we are looking at a turbulent flow at low speed, so that in compressibility holds, then these are your momentum in mass conservation equations giving rise to Navier-Stokes equation. What we have done here in writing the momentum conservation equation? We have added this time dependent body force term which is given as a function of position X and time t for all the points in the domain omega and the boundary condition is given by, boundary is located at del of omega. So, this is what we have. So, this is your problem statement.

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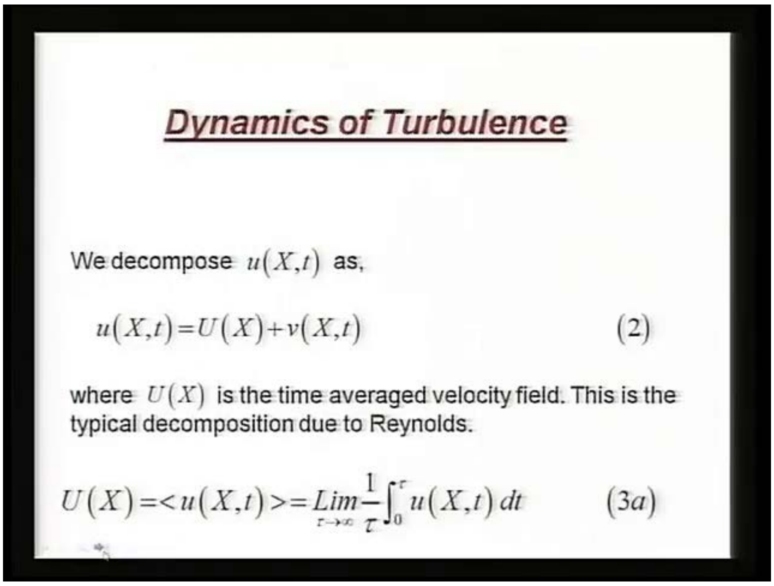

Now, once you have this, we need to find out how the solution of these equations can describe those features of turbulence that we talked about. One of the earliest attempts in this direction was made by Reynolds itself; what he proposed was that this space time dependent velocity field is decomposed into two components; one is the time independent part or the time average part and the other one is a kind of a fluctuating component.

So, when we are talking about a time averaging operation, this essential is indicated by this angular bracket that would involve, you take the signal here; integrate over a large time, and of course, divide it by the time span and in the limit tau going to infinity; whatever you get, that is what you are calling as capital U of X.

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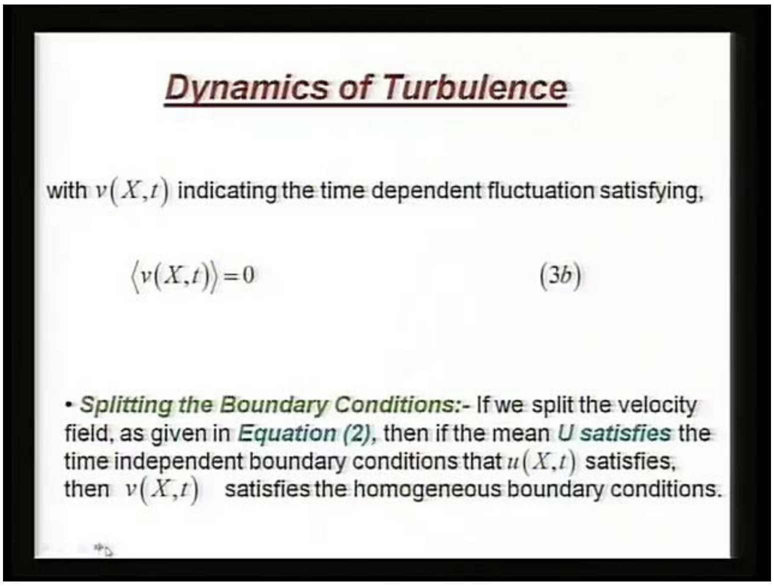

So, this is your mean flow or time independent flow part. This was what was originally done by Reynolds and it is called double decomposition because you are taking the full flow field, splitting into a time averaged part and a fluctuation part. Additionally, what Reynolds did? Reynolds basically said that this time dependent part is truly random. So, what does it mean? Truly random means if you take its time average, it will come to 0.

So, this is the way that when you split a stochastic quantity into two parts is what we will call as a Reynolds splitting. Now, let us look at the corresponding boundary conditions. How do we apply those boundary conditions for the velocity field? Now, we have split the velocity field in terms of a time average part and a truly random part. Then, suppose this turbulent flow is generated by a flow condition where your boundaries are not articulated, so the boundary motion is prick loaded. That means what? That the boundary conditions are going to be time independent.

So, if that is so, if you are looking for such a flow, then this the mean U could satisfy this time independent condition because that is the way we have split it. What will then remain of for v? v is the fluctuating quantity. So, all boundary conditions is taken up by your capital U. So, v ends up satisfying homogenous boundary condition. So, this is the story, but you know there is submit of philosophical (( )) that we can indulging at this stage. We are talking about turbulence to come out when we have not done anything to the flow because the boundary conditions are time independent, but still some of the flow has become time dependent. We at least have some clue where it has come about; it could be because of flow instabilities; flow instability is something which we have seen and lead to that.

But looking at it from the receptivity angle, we also know that if we eventually have gotten into some kind of a time dependent motion, there must have been some kind of an excitation which is time dependent. But this is for a long time held view that turbulence comes about out of nowhere. That is what this boundary condition in this splitting is indicating. So, now, we should keep this in mind; then with the fluctuations satisfy homogenous boundary condition and the total boundary condition is time independent and that is totally taken up by the mean field.

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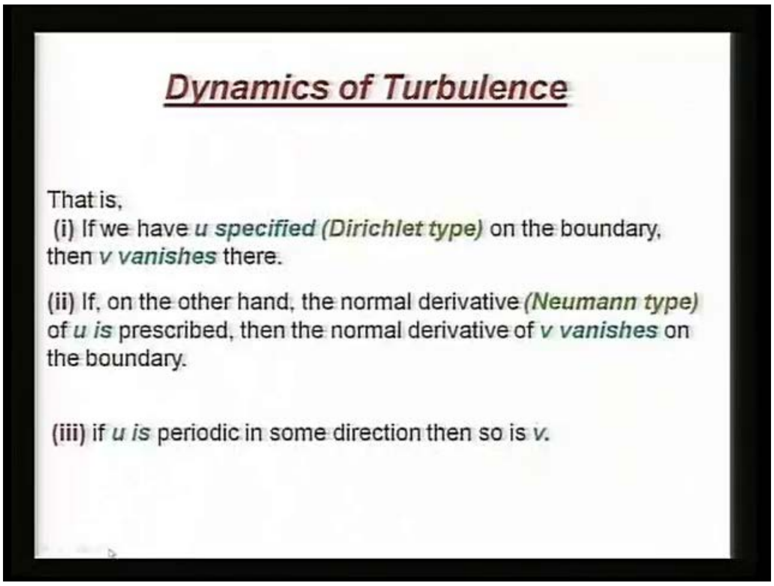

So, there are these possibilities. if we have u specified, that is a Dirichlet type of boundary condition, on the boundary, then v vanishes there. If, on the other hand, if I prescribe some kind of normal derivative for u, then the normal derivative of v will vanish because whenever the normal derivative is given for the total field, that is also again taken over by capital U. So, v ends up always satisfying homogenous condition in terms of the variable itself in the form of Dirichlet condition or in terms of derivative condition, the Neumann condition.

Now, if we are talking about a total flow field that is periodic, then that periodicity would come about from the geometry of the problem and that would be totally taken up by both the mean as well as the fluctuation because you cannot say like the periodicity is completely taken over by the mean. Then, if v is not periodic, of course, in the total field you cannot have periodicity; so, v also has to be periodic; so, please do remember that these are the three possibilities for the fluctuations.

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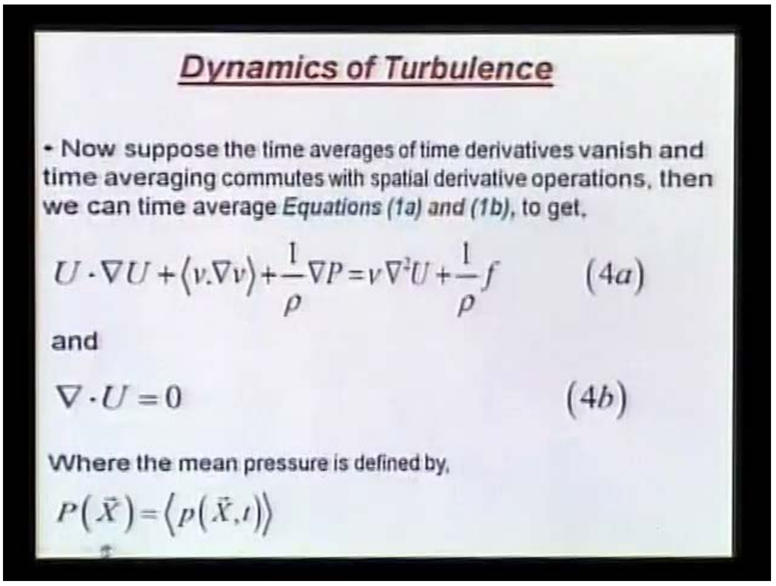

Now, having described this, let us go ahead and look at our Navier-Stokes equation with the help of that double decomposition we talked about. If you do it, we are going to get an equation of this kind. Well, how does it come about?

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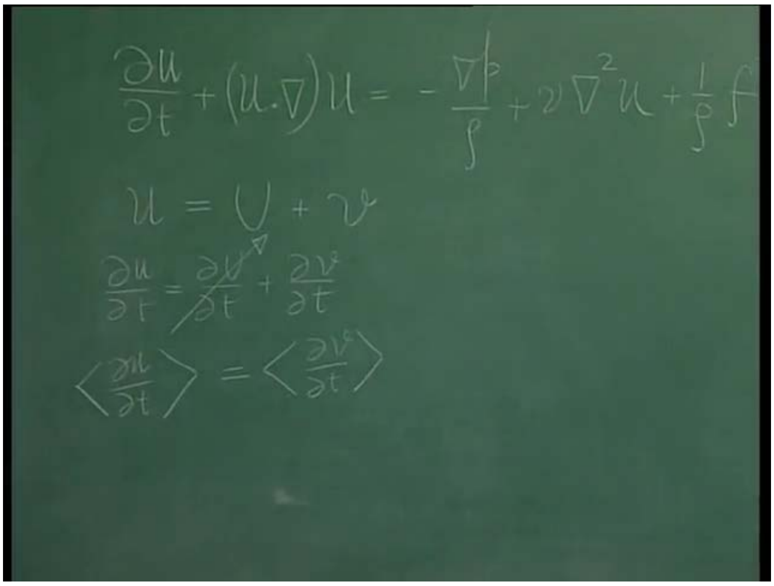

We have this condition. We are writing the Navier-Stokes equation. The momentum equation in this form with the body force term kept here. Now, what we have done here basically? So, this is that u dot del. Now, u itself we are writing it as capital U plus v. Please note that unlike what we did for instability studies these fluctuations, but we are not talking about small fluctuations in turbulence. The fluctuations are of the same order of magnitude as the mean itself. So, that is why there is no need to put some epsilon or anything; it does not hold.

If I now substitute this kind of decomposition in this, what will happen to this term? (Refer Slide time: 10:18) This will be base. So, by definition, of course, this is 0 because u is the average called time averaged quantity. So, it is not a function of time. So, that goes off. Now, if I average this equation, so that means what? I will be averaging individual term, one at a time. So, if I do that, we are talking about this. (Refer Slide time: 10:58). So, this will be this.

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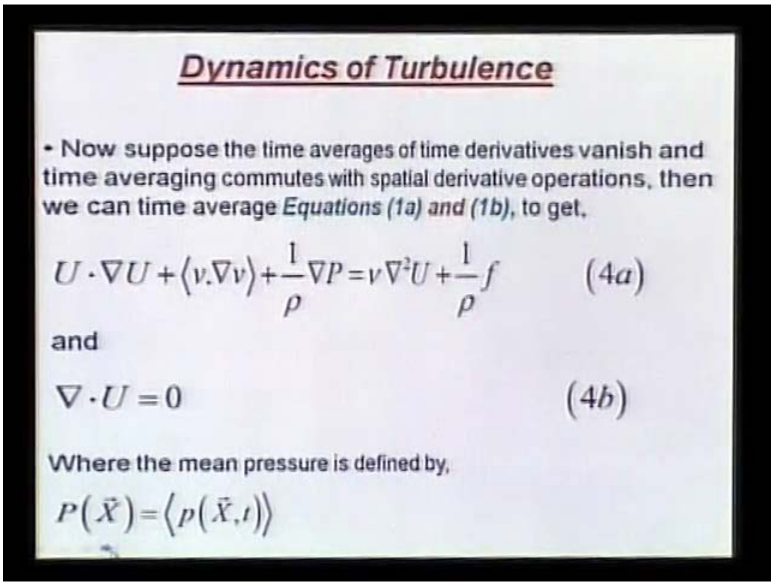

Now, what happens is we make a supposition that the time averages of time derivatives vanish may seem troubling; computing course we do not go that deep, but here we can afford to explain it little better.

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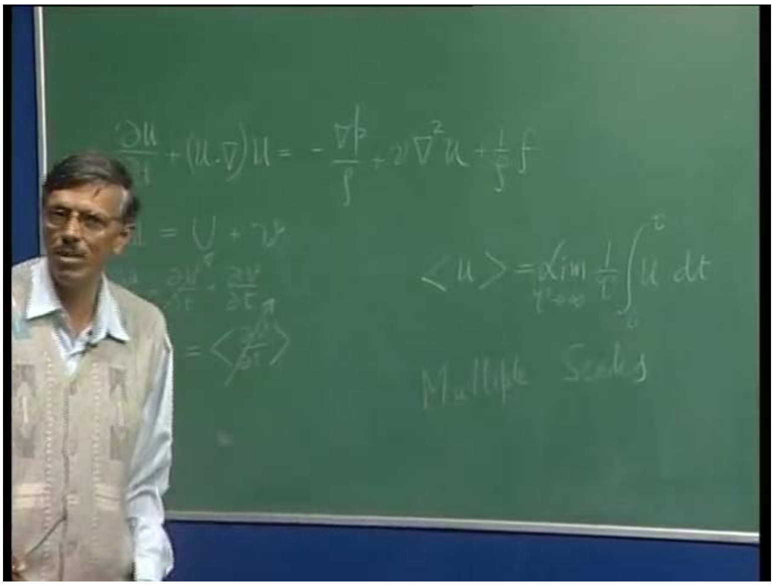

Why because when I am talking about time averaging operation, what was the definition that we talked about? This is a limit tau going to infinity one over tau integral u dt and this is going from 0 to tau; that is your definition of time average.

Now, what is this averaging operation? Averaging operation is over a very large time. If I am talking about a very large time, what is the corresponding frequency? Frequency is going to be very small whereas, the fluctuations that we associate with turbulent flows are characterized by very high frequency oscillations; that is why we call it chaotic or random. So, what happens is even though we are looking at a time dependent motion, there are two types of time variation: one is happening over a very large time; that is what this time averaging operation is suggesting, and another is that very high frequency fluctuations that is characteristic of hydrodynamic turbulence.

So, these two frequency scales are widely separated. If that is so, we could treat them independently. This also is the gist of what people talk about theory of multiple scales; is not it? We are having different types of time scales; one is happening over a very small time scale with very rapid fluctuation; another is slowly varying. In fact, that is one of the reason that people have gone ahead and started talking about unsteady RANS equation. We have not talked about RANS equation. Now, we are seeing, what is the philosophical justification of talking about an unsteady flow.

So, this is what we are talking about; the supposition that, if we are looking at the time average quantities associated with the time derivative, this fluctuation - the derivative where would it come from? It would, of course, come from this high frequency fluctuation; that is what rapid time variation is about, whereas this operator, angular bracket operator is happening over a large time. So, that is what, we are saying that this term will have to go to 0. You do not have to really struggle very hard to think about it; think of even a simple harmonic variation of say v.

If it is so, then what we are talking about? We are going to write about del v del t here; del v del t will be something like i omega v at the amplitude and then we are going to do this operation 1 over tau and tau going to infinity. And if I take this thing, what happens? Over one cycle, it is going to give me 0 contribution. So, even if I take to a very long time scale, I may have at the most a fraction of a time scale because every time scale will cancel each other out and then if I am getting that contribution in dividing by tau and that tau is very large, it does not give you anything.

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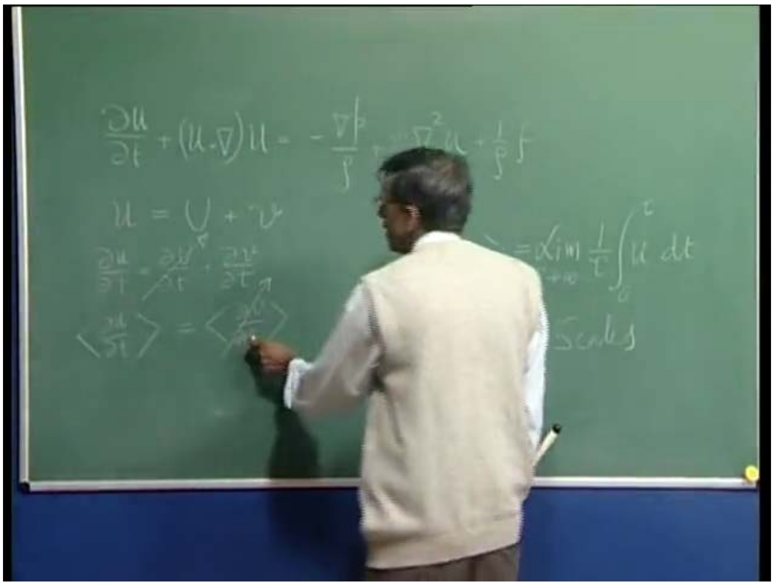

So, now, you understand why this term does not survive also; that is what will happen that when I am time averaging this equation, I am not going to get any contribution coming from this. Convinced?

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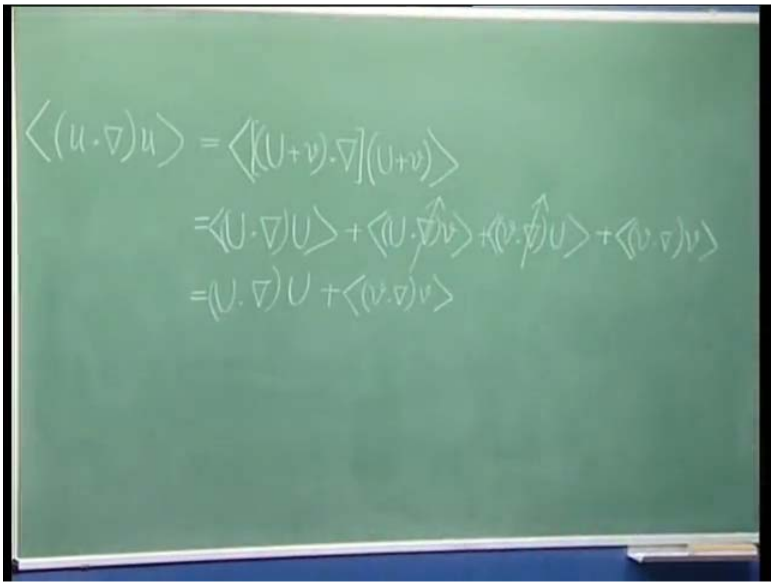

Now, look at the other term. We have to be doing this. Well, we will just split it into this part. So, we are talking about here U plus v with a dot product nebula; this is operating on U plus p; that is what you are doing. So, if I do that, I am going to get here one set of term U dot del U; this is this term (Refer Slide Time: 17:20). Then I will also have a term which is like this - U dot del v; this term, then I will have v dot del U, this term plus v dot del operating on. Now, it is very easy for you to suggest me that these are time independent; so averaging operation will do anything; so, this transmit itself as it is. What about this term? This is a time independent quantity; this is the random quantity; so if I do the averaging operation, again this will go to 0. So, this goes to 0. What about this? This also goes to 0; this does not; this remains like this v dot del v.

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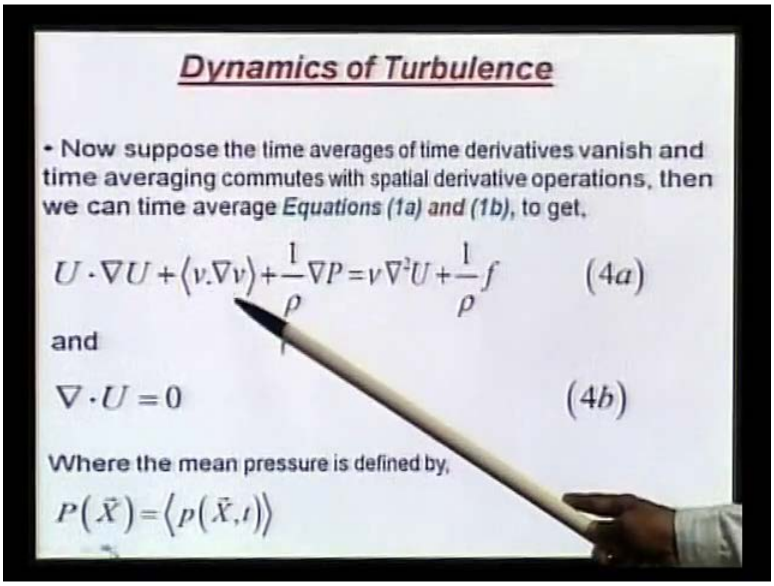

So, of course, in this equation this first bracket has gone missing, but you can think of v dot del is the operator operating on v; so, that is that.

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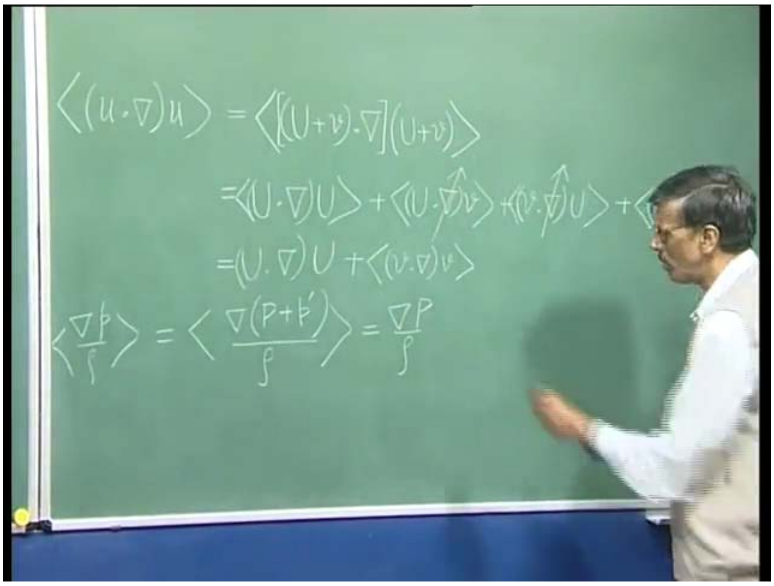

Now, what has happened is I could, similarly, write this term del p by rho. If I do this, what will I get? del of this p, instantaneous p that I am writing is this plus some kind of fluctuation; let me write it as this. So, this fluctuating pressure would be a function of position and time. So, any time dependency of pressure would be embedded in p prime and it is incompressible flow. So, we do not allow any fluctuation and rho; so, that remains as it is. And you can see this is the time independent part. So, this will give me nothing but grad p by rho. What about this term? The fact what pressure fluctuation is also random and if I am doing a time averaging operation, this also should go away. So, we do get that.

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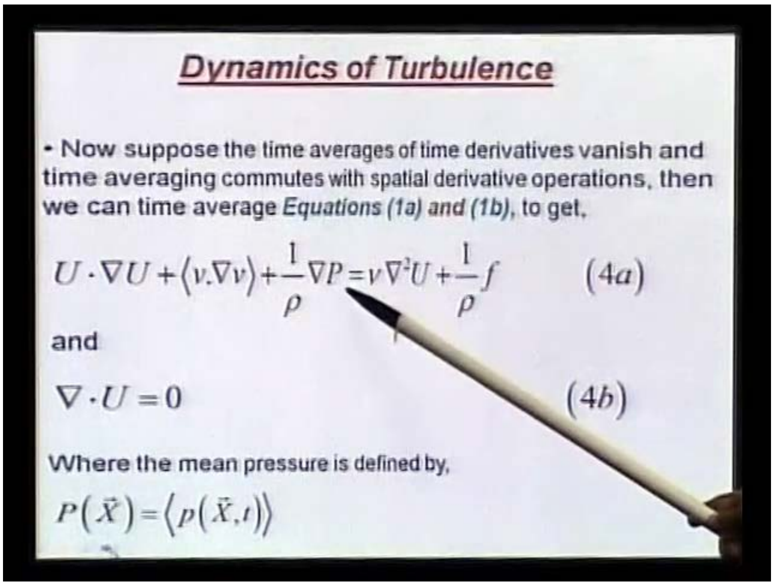

This pressure term will just simply give us the gradient term p by rho; that is what we get. So, a convection term has given us two terms; one is directly coming from the mean; another is coming totally from the fluctuation. This is the mean pressure term. This is the viscious term and this is the term that we are going to get from the viscous term (Refer Slide Time: 20:51) this term So, that term is also very easily understood.

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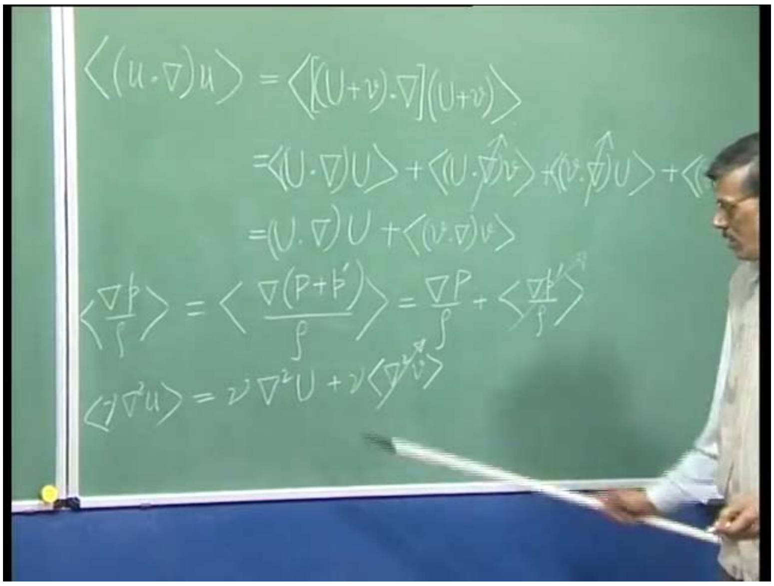

We are talking about nu times del square u operating and that will give us nu times this plus nu times… Once again you can very clearly see this will go to 0. So, only this will remain and that is what you have given.

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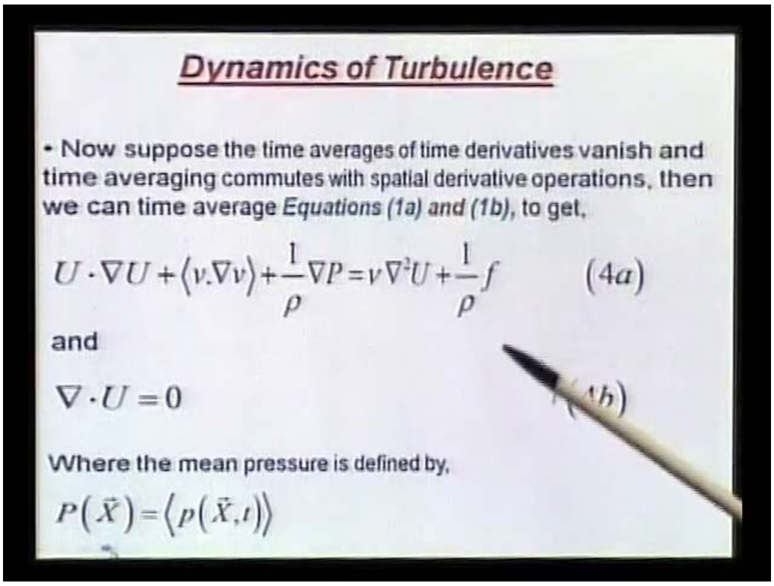

Well, I admit that we should have indicated this by a different symbol because that term its coming from here (Refer Slide Time: 21:48).

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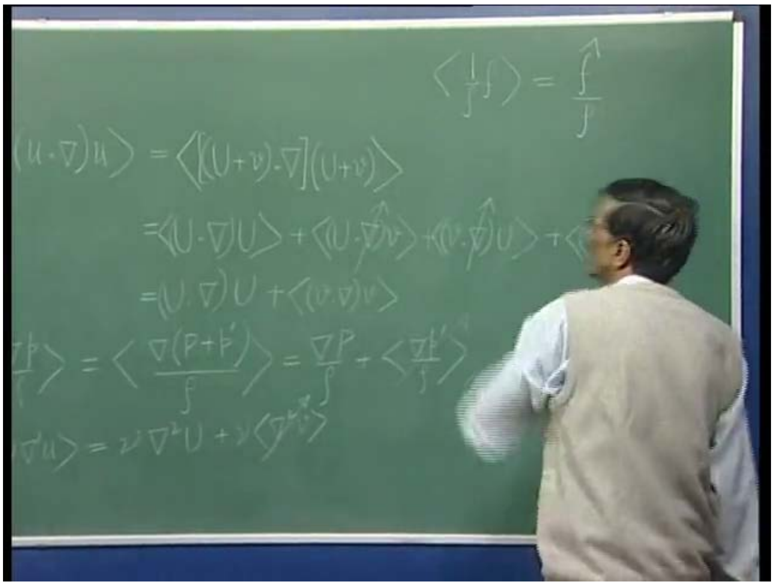

So, that that term gives us… we should write it like this - 1 over rho f time average term; let me call as time average term in terms of f hat; this is hat.

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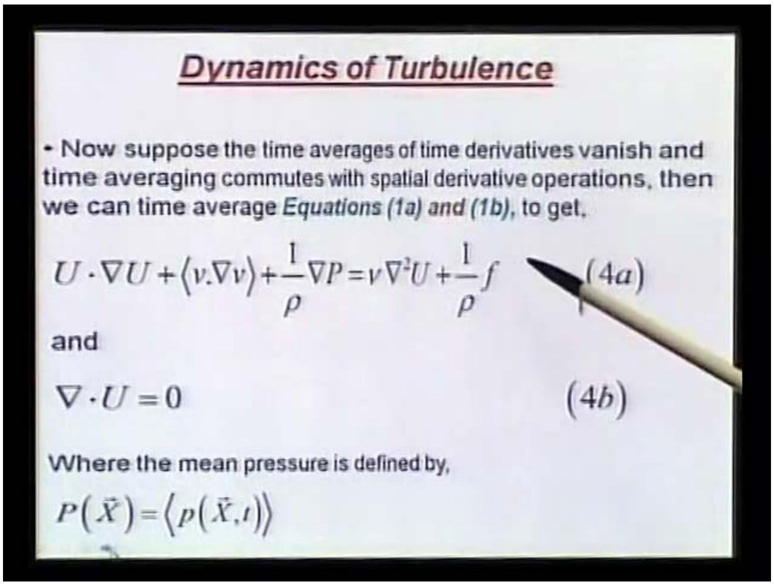

So, please do note that this is not quite right. I should indicate it by its time average because we talked about it. We talked about it that the body force is basically a space time dependent function. So, you need to talk about its time average. Now, this directly comes from mass convection; without any difficulty, we can understand it and the pressure is what we have defined here. So, there is no difficulty in understanding that.

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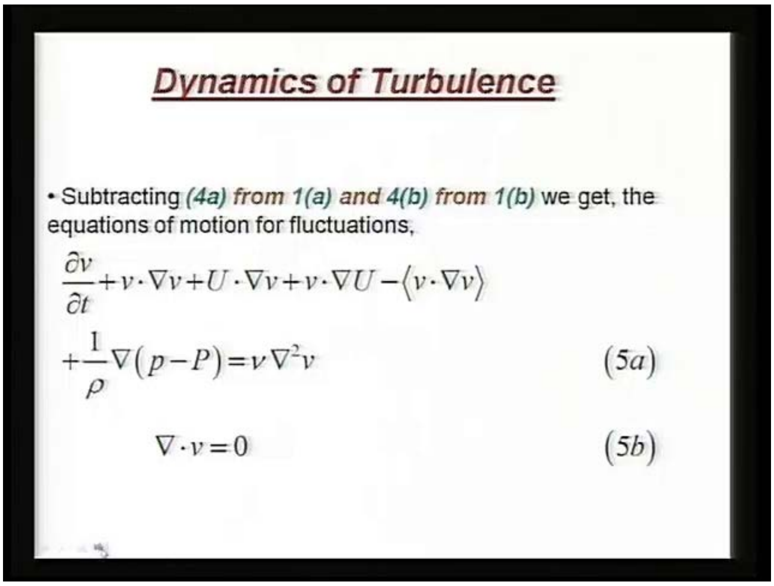

Now, let us see what we get. We have obtained the total equation and then we have also obtained the mean equation.

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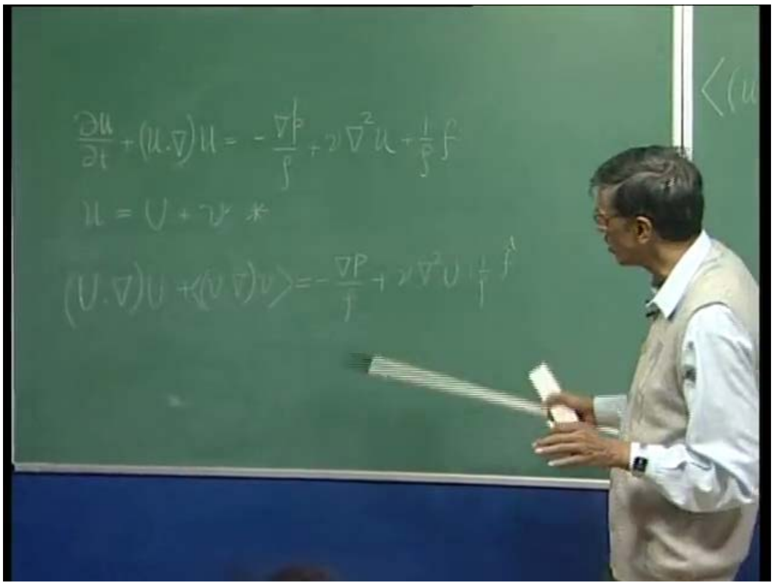

So, if I write down the mean equation, so I am going to get the following term here. From here, we have seen that this (( )) del U plus v dot del, this is that and here we are going to get del of capital P by rho and here we get nu del bar capital nu, and this I will write 1 over rho; I will call it as f hat.

Now, what? We have seen that we have split the total variable into this part. So, what I could do is I could try to write out a dynamical equation for v itself. What do I do? I just simply subtract this one from this (Refer Slide Time: 24:07). If I do that, what would I get? this Remember, this is a time averaged equation where this is the actual original equation without doing any operation; I mean this is that instantaneous equation.

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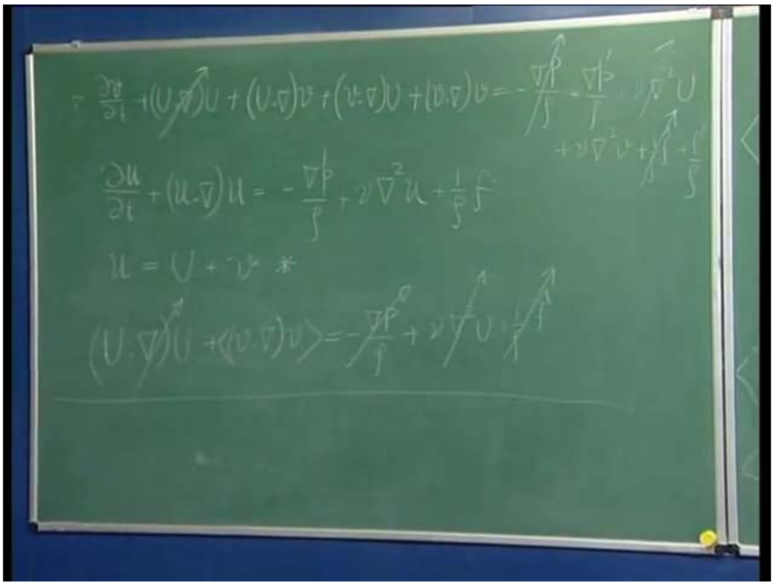

So, I can subtract this from this; then what do I get from here? I will get this term del v del t. So, I will get del v del t. Now, if I write this equation here without doing any averaging, this is what we are going to get del v del t. And from here, what do we get? U dot del U plus I have written those terms; o, I will just write them down once again and there is not much of a problem when doing that U dot del operating on v, then v dot del operating on U, and of course, I have v dot del operating on v and that is equal to minus del P by rho minus del p prime by rho.

And then nu del square U plus nu del square v; then of course, I will write 1 over rho f hat plus f prime by rho. So, if I now subtract this from there, which are the terms that are going to go away? Well, on the right hand side, this goes away with this. So, that is what we get. What do we get is retained when we then get to retake all these three terms. Please do understand that this term and this term are not same (Refer Slide Time: 26:56). This is retaining all its time variations. This is the corresponding time average term.

So, that is why both of them would be there. This plus this; both of them are there and these two are those terms which are here; this and that this and that they appear there (Refer Slide Time: 27:07 to 27:55). then of course, I could see that this one will cancel so I will get minus del p prime by rho or I could write instead of writing del p prime I will write del of p minus capital P. That is your p prime anyway and this part will cancel, and this part we have said, already they have cancelled remaining retaining this.

So, this is the term that we are going to get on this. What about this term? Now, do understand; the body force is not an attribute of turbulence. What is the turbulence that you talk about? Turbulence is an effect. What are the causes? All the turbulence is caused by all those inputs and one of the input happens to be the body force also; we are not talking about a body force which is random. We are talking about some deterministic but it could be time dependent body force.

So, if I talk about that, then what happens? There is no such thing as fluctuating component of a prime; that is why we did not write anything here. So, this is something that we understand, but I must also remind you that you can talk about fluctuating body force itself and this is what you do in Fokker-Planck equation in physics, but we are not going to go through that route. So, we will not talk about Fokker-Planck equation, but we will just simply say there are no fluctuating body forces.

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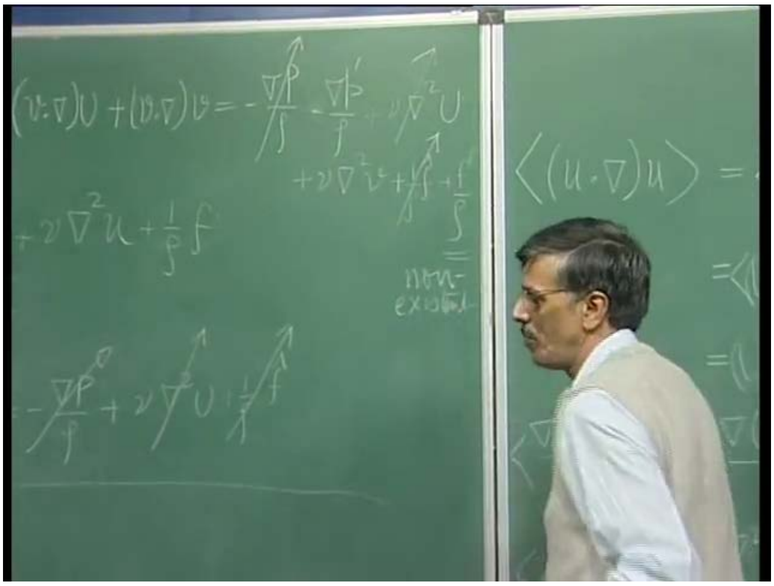

So, we will say this is non-existent. So, basically, then we have been able to write out a dynamical equation for the fluctuation and there is a corresponding mass conservation equation (Refer Slide Time: 29:57). So, these are the two sets of equation that we have written down for this. Previous equation that we had written, this equation 4a defines the time average field and that is why that equation is called the Reynolds average Navier-Stokes equation; it is a time averaged equation.

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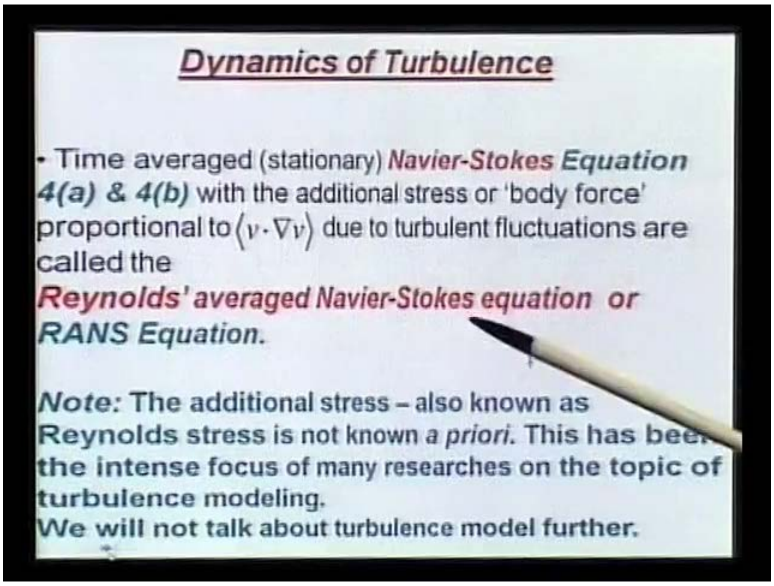

It was proposed by Reynolds. So, that is why we call it as Reynolds averaged Navier-Stokes equation or RANS.

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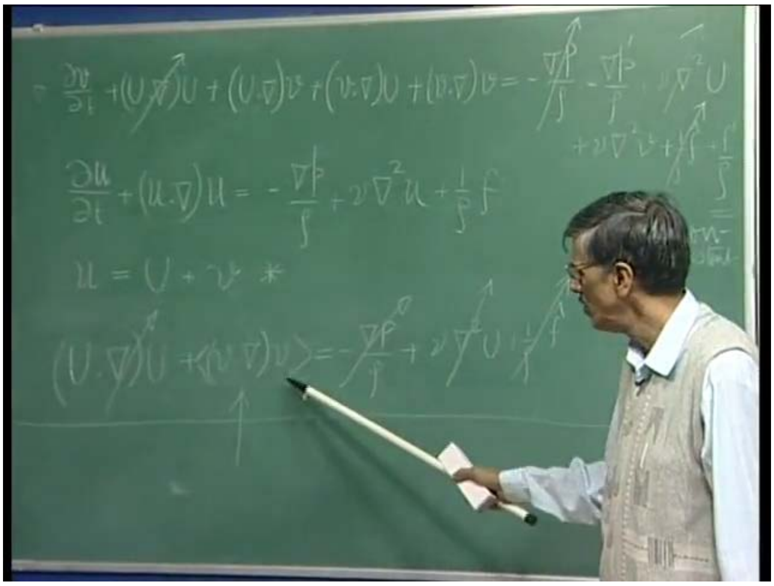

However, what we have noted in that equation, well we have it here; we have no clue to this term immediately; we do not know what it is because this is related to the fluctuation. What is it is physical nature?

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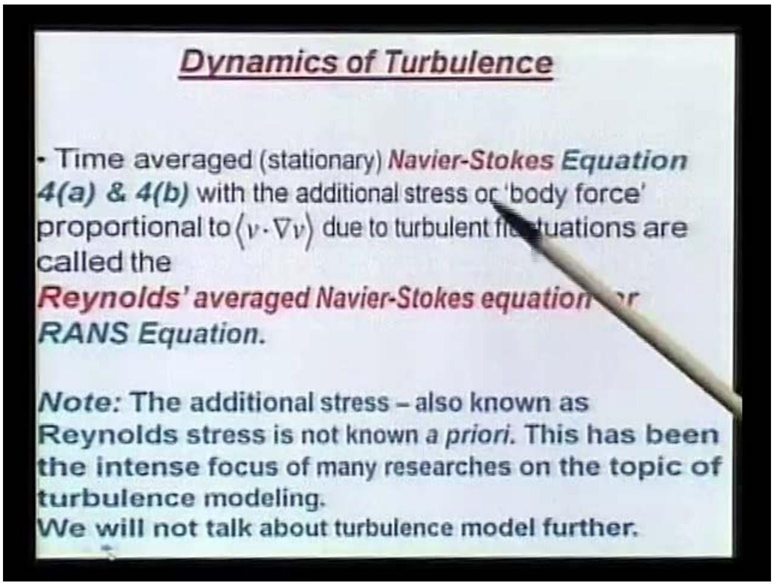

Here, this is like here a stress term, stress term that we are familiar with; even in laminar flow we have this stress term; so here also we get this. This is nothing but vi vj kind of term. So, that is what it is. So, these are some additional stress terms or you could even visualize it as some kind of applied body force.

So, you can term that as either an additional stress term or a body force term that arises due to turbulent fluctuations. This additional stress is known as Reynolds stress. So, this is what was originally suggested by Reynolds. However, we confess that apriori we do not know what this is and this has been the intense focus of many research work going on, on the topic of turbulence modeling. In turbulence modeling we try to model this term because this is not known a priori. So, we do have to supplement it with the help of some additional phenomenon logical information or some idea is based on theoretical fluid mechanism. How? I am not going to talk about turbulence models any further.

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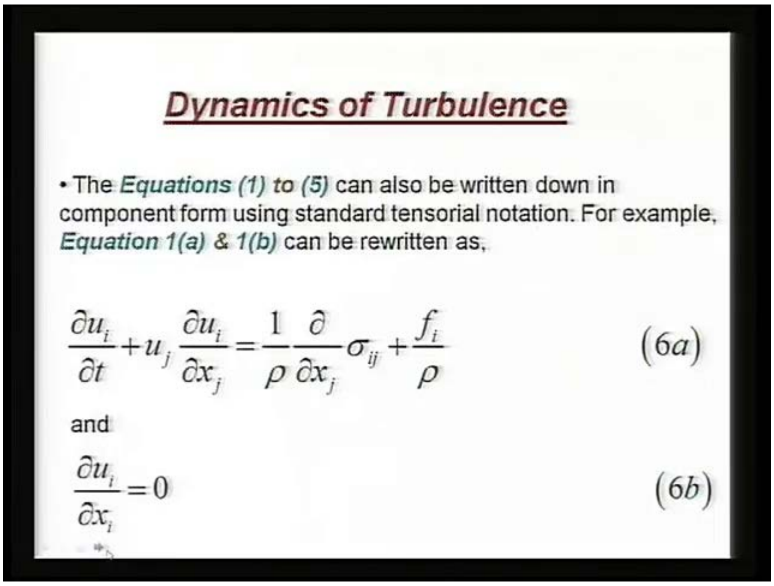

So, what we have now, we could also write down the same equation. Here, we have assorted to vectorial form where we used vector calculus, but you can also use tensor notation to write it in terms of this. So, you can very clearly see local acceleration, the convective acceleration and this is a stress term. If you recall before Navier-Stokes equation was written, that this equation was invoked and this is attribute it to Cauchy; so, this is also called the Cauchy equation.

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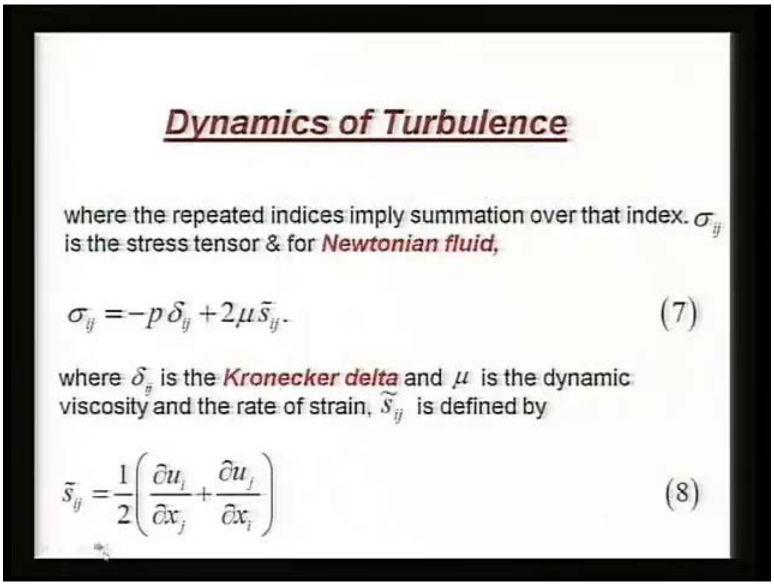

What Navier and Stokes did which many people do not understand that these two scientists, they related the stress with strain rate; that was that; without that this remains as if it is an additional unknown and this is of course, the body force term and this is the mass conservation terms. So, that is what we have. We could write in this form. We will use both these notations because it becomes easier for us to handle. We are familiar with the fact that whenever indices are repeated, that imply that you sum over that index that stress tensors might be written for a Newtonian fluid like this.

So, the stress tensor has two components: one is when there is no fluid motion; so that is given by the hydrostatic component and this is due to the motion and this motion is for a fluid which has a specific characteristic which was originally suggested by Newton himself. So, any fluid flow which satisfies this paradigm is called the Newtonian fluid. What it essentially does? Basically it does tells you that the stress is related to strain and you are so much used to hearing that stress is proportional to strain, you automatically tend to think that nu is some kind of a proportionality constant which is really not strictly adequate or correcting to do.

Why? Because here we have a tensor of rank 2; this is a tensor of also rank 2; at the most or at the least nu should be a tensor of rank 4. So, basically it is like a dot product that you are taking. So, nu matrix is a fourth order tensor; taking a dot product to give you a another second order tensor. That is what you do; in a vector equation you just do not write one vector is proportional to the other; you always see that Navier-Stokes equation is that when you are writing say del v del t, local acceleration term that comes as a sort of a contraction term from the convective acceleration term that involves a dot product of a stress term say vector.

So, vector is a tensor or rank 1. So, that is what you get; any way, we will not go about it; we will go with the crowd and say stress is proportional to strain. That is precisely what Newtonian did himself. He took the simpler example of quite flow and then said I apply this strain and this is the stress appears. So, these are proportional and this is proportionality constant; it is not. There is a very beautiful theory developed from theory of elasticity which talks about how to relate stress and strain, and this is what is called as constitutive relation.

So, the constitutive relations are necessary to rid propose a relationship between stress and strain. And in developing those constitutive relations, you can invoke certain properties some of which comes naturally for physical variables in terms of symmetry isotropy etcetera. That brings down the number of unknowns is that tensor. How many components would we have for a fourth order term? That would be 3 to the power 4; that is the 81 components, and with all this concepts of isotropy and everything you can actually bring it down to only 2. And in fact, you are familiar with the stokes hypothesis which even brings down this 2 quantity into 1.

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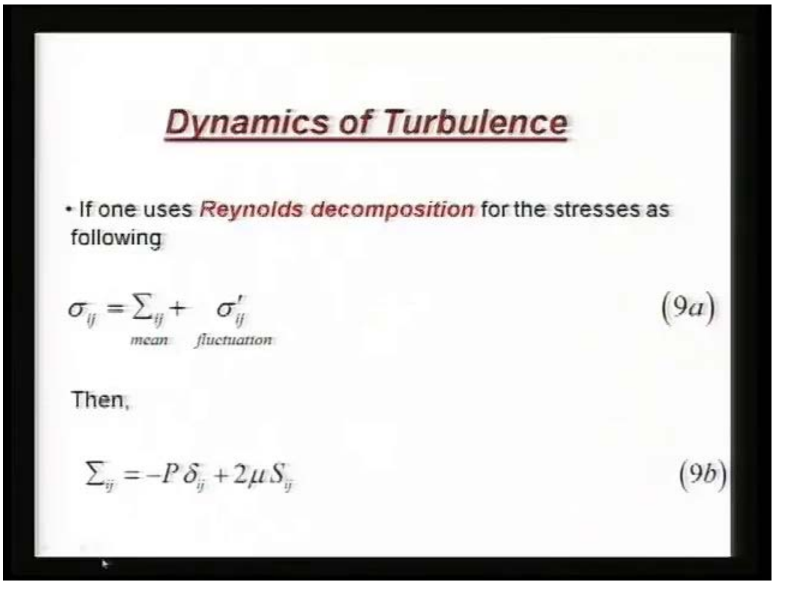

So, we are familiar with that in basic fluid mechanics. We will not go through that; S tilde ij is your weight of strain and that is defined as this. So, this is something like a average angle at a interstice of a… so what happens is we have a stress term; we can also perform a Reynolds decomposition in terms of a mean component and a fluctuating component where the mean component itself will have the hydrostatic part plus a mean strain rate. So, we will expand it little further. So, you can see that the total stress would be split into mean and fluctuation; the mean would be given in terms of the hydrostatic component and in terms of the mean strain rate.

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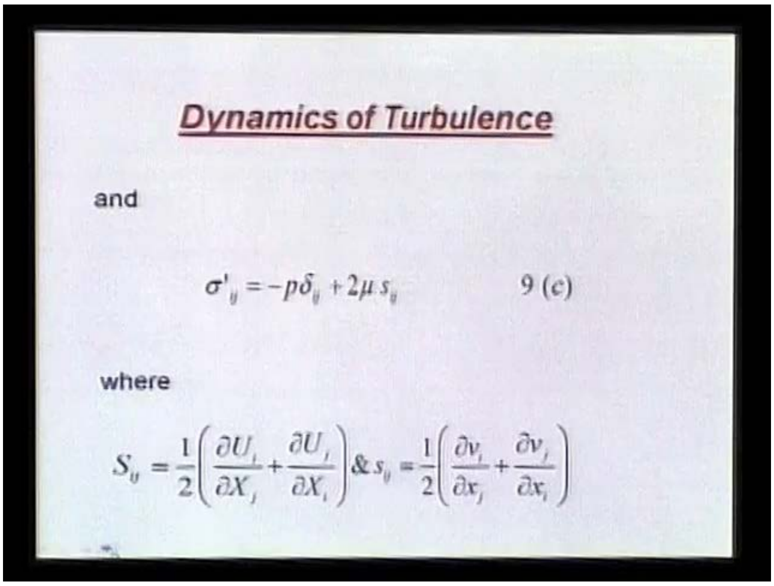

So, the mean quantities are all defined in terms of capital letters. So, capital Sij represents a mean strain rate. So, this mean strain rate - we should be able to define it in terms of the mean component of the velocity as indicated here; below that, the mean strain rate is given as the average of the angular rotation at this corner of a line; that is a small fluid element.

And the fluctuating stress part also would be contributed by a fluctuating pressure and a fluctuating strain rate. So, the fluctuating strain rate would be given in terms of the fluctuating velocity component. So, that is what we get. There is a sort of a mistake here. This would be also lower case x; it is basically the same independent variable space variable (( )).

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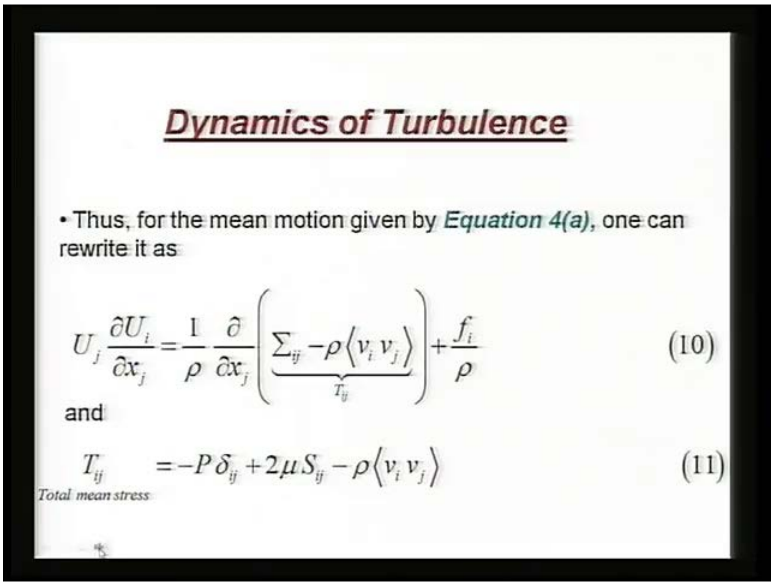

Now, what we need to do is having done that we could write down the mean equation that Reynolds average Navier-Stokes equation also in this particular form. So, this is your convective acceleration is equal to 1 over rho. Now, please do understand that in writing this term we have put the Reynolds stress term inside and this mean stress this 2 together is what we call as the total mean stress Tij. So, this is a tensor of rank 2 which is contributed by hydrostatic pressure, the mean strain rate, and the Reynolds stress and this is the corresponding body force term that we have. So, this is an alternate way of writing out the RANS equation.

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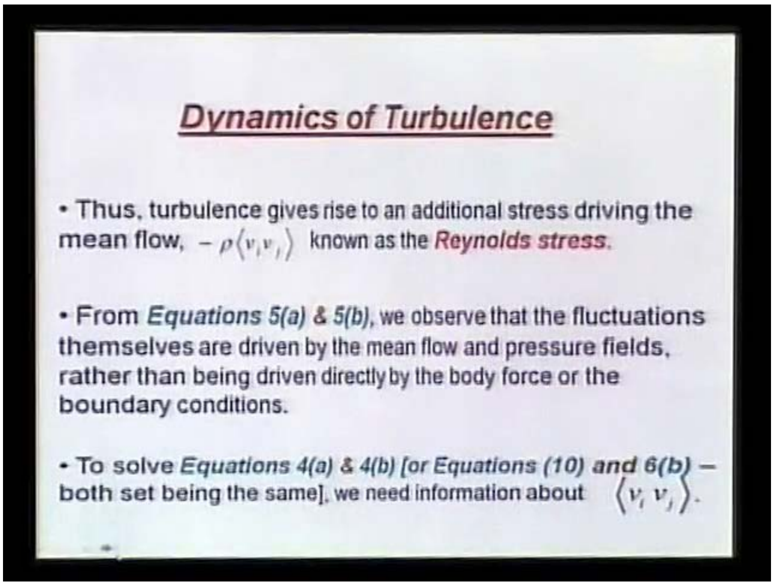

So, we find that because of turbulence we had these fluctuations which we called as v vector or in tensor real notation vi components. So, this dynamically has a dimension of stress term so rho times vi vj and we have taken a time average; this is what we call as a Reynolds stress term; we have already noted.

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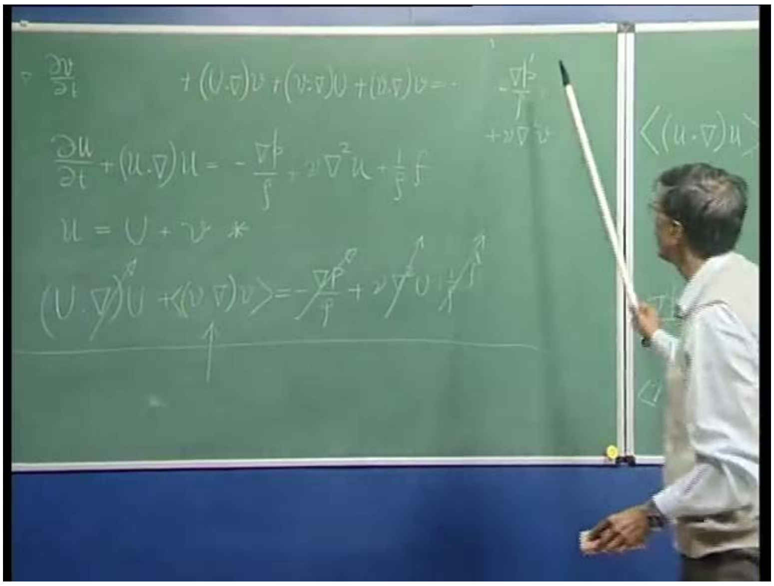

Now, when we write down the governing equation for the fluctuation that is written here, so, if I remove all the terms which are not there, so this, of course, goes away; this goes away; this goes away; this goes away; this goes away, but this is what we have. So, this is your governing equation for the fluctuation; that is what we are talking about here.

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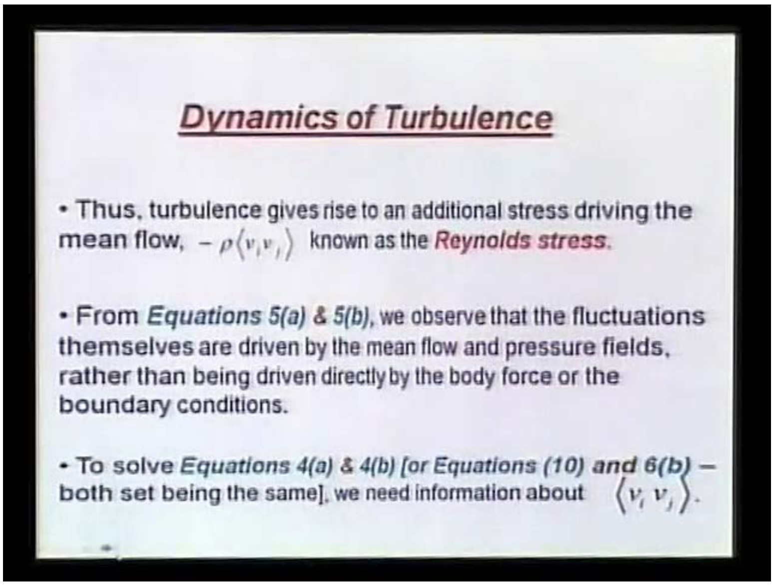

And the fluctuation, the dynamical equations of fluctuation is driven by what? The mean flow and also by pressure. Because of the way we proposed the body force not to have a fluctuating components, the body force does not come into picture at all. So, that is what we have saying here that the fluctuations are driven by mean flow and pressure field rather than being directly driven by the body force or boundary condition. Why not boundary condition? Because we have seen already that boundary conditions for v are most of the time homogenous; whether it is a Dirichlet or Neumann type, it will be homogenous.

So, boundary condition really does not drive the fluctuation. So, please do understand; this is very very crucial that in a turbulent flow, what we see as a fluctuation, that fluctuation is driven by the mean flow itself. I have also, of course, the applied pressure gradient body force per say does not give rise to turbulence. In fact, that is one of the reasons that when we talk about what are waves, they are the whole thing that comes about because of the (( )) term.

That is like your body force that does not directly contribute to turbulence. Tturbulence has to be contributed by the mean flow, the instability of the mean flow and the pressure gradient that is implied. So, do understand this that whether we solve the RANS equation in the vector form or in the tensor form, we need information about this term. There are no other records; we have to get some information about Reynolds stress.

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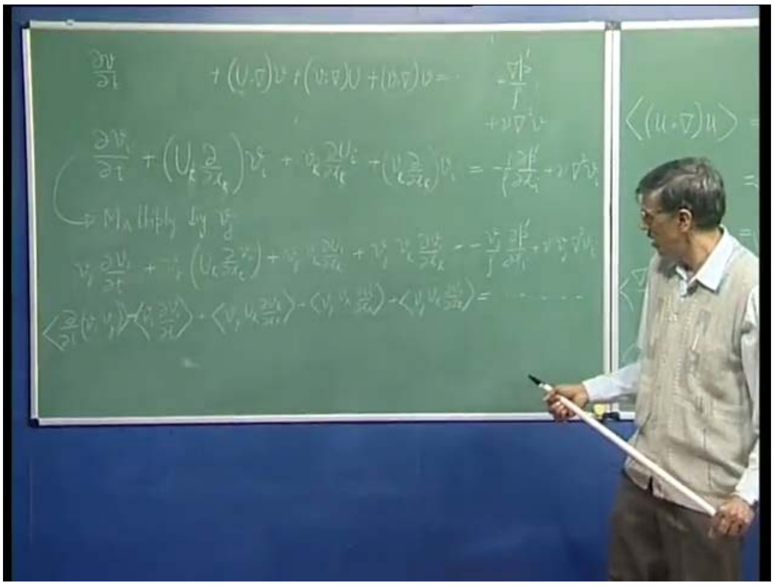

Now, how do you get the Reynolds stress equation? It is a basically second order quantity; we have written down the equation here for the fluctuation itself. So, I could write it in terms of in a tensor notation form. So, what we are talking about then let me just clean up this bit and then we see, if we write it in terms of tensor notation, then I would perhaps write let us say the ith component; then I will write del vi del t plus Uj del del xj operating on vi; that is this term.

What about this term? This term also I will write vj del del xj operating on, say we are writing it for the ith component, so here also I will write here as Ui. What about this term? (Refer Slide Time: 46:26) Well, this term is going to be vj del del xj and this is operating on vi; that is that and on the left hand side, I will have, basically I will write this as 1 over rho del p prime del xi. We are writing the ith component equation and from here, we will write nu times delta square vi. So, this is your ith component of the fluctuating quantity.

Now, let us try to write out an equation for Reynolds stress. So, what I do? I multiply this equation by let us say vj. So, basically what I am going to do? Well, let me just, to avoid conflict, let me write here this repeated index by some other notation, so that we do not have a conflict here. So, what I do is I can multiply this vj and then we time average because we want to get an information on the time average quantity. So, if I do that, of course, I am going to get this as vj del vi del t; then we have vj and Uk del del xk of vi. So, that is what we have.

And from here, we will have vj vk del Ui del xk and here we will get vj vk del vi del xk and here we will write vj by rho del p prime del xi and here we will write nu times vj del square. Now, what we are going to do? We are going to time average this equation. Now, what we are going to get? This term can be written in terms of del del t of vi vj minus some term; so I can get that; so I probably can contrive it to write it like this del del t of vi vj minus say vi del vj del t. This I could write it like this; is not it? I could do this similar things, but when I actually now time average this equation, again we will talk about this term will give us what?

This term will not go away; that will tell you how the time average of vi ;vj changes its time; so we will have to talk about this time average minus this time average and this will be let us say the mean flow is given to us, but still will have to be performing this vj Uk del vi del xk. Now, this also would coming here like this vj vk del Ui del xk. But look at this term; here we have vj vk del vi del xk. So, I will have those other terms written on the right hand side. So, in writing down an evolution equation for the second moment; so vi vj is the second moment. If I do that, I see immediately that here I have a triple product term coming here.

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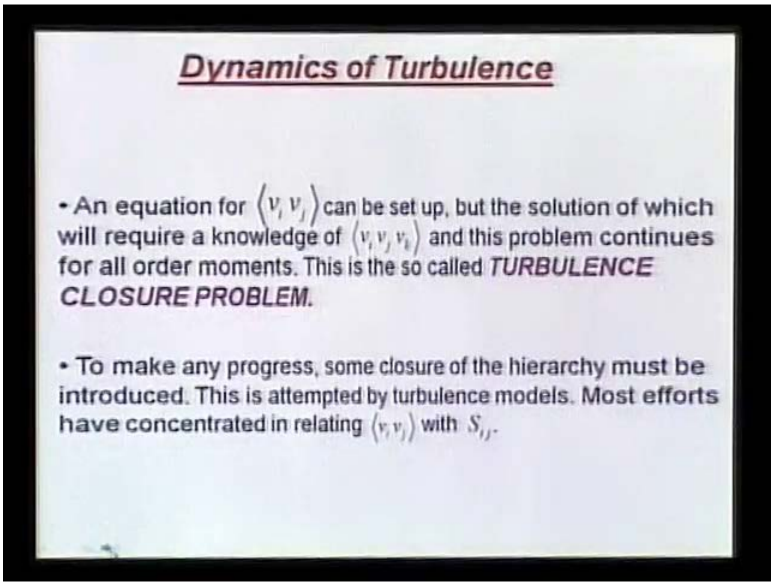

So, what is happening here that in writing a equation for the second moment, we need information on the third moment. So, it is not very happy situation that when I am looking for the first moment equation, that is what our RANS equation was; is not it? So, in writing the RANS equation, we had understood the need for modeling; the second moment that is a Reynolds stress equation and when I try to write down an equation for Reynolds stress I need to get information on triple correlation term.

So, this is what is called as a turbulence closure problem. At whatever level you look at, you will have always something which is not known at the higher level. So, you know for long long time, specially people emphasizing on the experimental aspect of turbulent flows, they kept on measuring various moments of turbulent flows with the hope that after some level those higher order correlation terms are not going to be important. So, you could stop at somewhere and then you do not have this problem of turbulence closure problem because it gets over. Suppose, at this level suppose I could have said this term is 0, then my problem is solved. I have been able to write out an equation for triple correlation term.

But it is not possible. In fact you would note that there is a major activity in turbulence modeling that people do write down this equation for Reynolds stress directly and try to solve this equation, but please do mind remember that in writing those equations for this triple correlation term, they need to do some kind of modeling. So, it is not apriori that you are going to do; completely get an analytic solution; you will have to do some kind of empiricism, some kind of modeling or take help of some experimental input to close the problem. But otherwise, turbulence remains stubborn because of this closure problem; it never disappears. So, that is what we note that if we want to make any progress, some closure of the hierarchy must be introduced. This is what is attempted in most of the turbulence model and what is interesting is people do not want to go into third order terms itself, but be happy with the Reynolds average equation itself and then try to relate Reynolds stress with the mean strain itself. This is something that we need to touch upon next, when we meet tomorrow. Stop there.