Instability and Transition of Fluid Flows

Prof. Tapan K.Sengupta

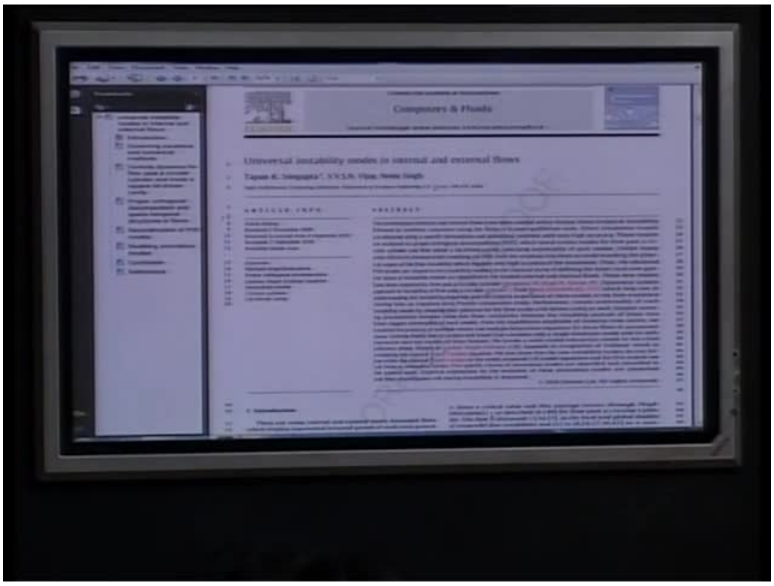
Department of Aerospace Engineering

Indian Institute of Technology, Kanpur

Module No.# 01

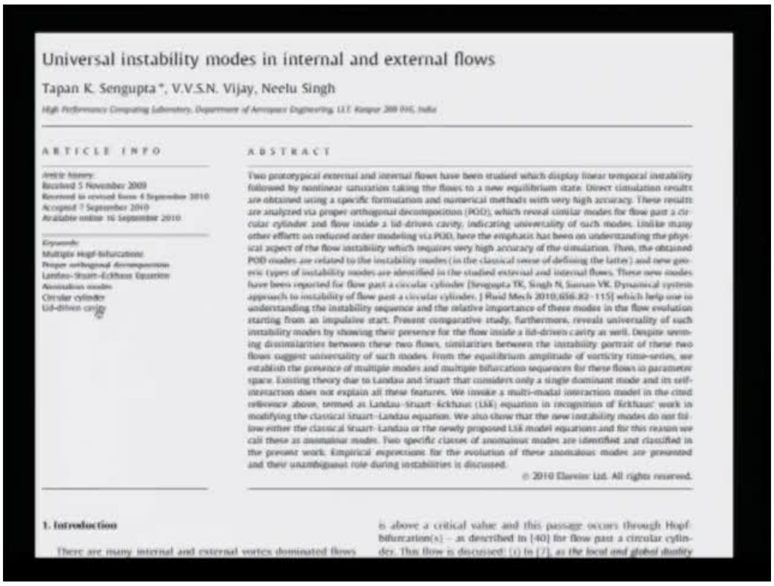
Lecture No. # 34

(Refer Slide Time: 00:21)



We are going to discuss about POD analysis that we had done. I am afraid, you will have to go through with me, the small letters here. What we are trying to do here is, to use POD as a tool to investigate instabilities. That is a new direction that we have given to the subject over the last couple of years and I discussed about, specifically, flow instability for flow past a cylinder. It is a bluff body flow that you have studied.

(Refer Slide Time: 00:58)



Universal instability modes in internal and external flows

Tapan K. Sengupta *, V.V.S.N. Vijay, Neelu Singh

High Performance Computing Laboratory, Department of Aerospace Engineering, I.I.T. Kanpur 208 016, India

ARTICLE INFO

Article history:
Received 5 November 2009
Received in revised form 4 September 2010
Accepted 7 September 2010
Available online 16 September 2010

Keywords:
Multiple Hopf-bifurcations
Proper orthogonal decomposition
Landau–Stuart–Eckhaus Equation
Anomalous modes
Circular cylinder
Lid-driven cavity

ABSTRACT

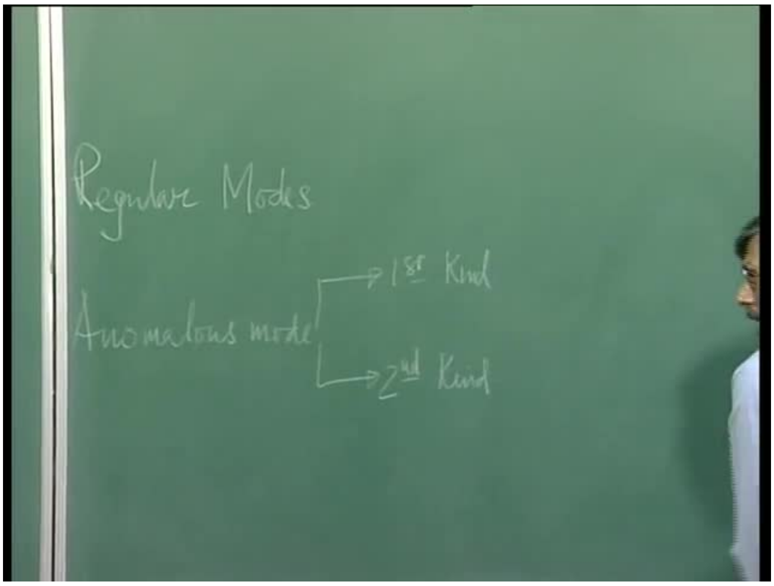
Two prototypical external and internal flows have been studied which display linear temporal instability followed by nonlinear saturation taking the flows to a new equilibrium state. Direct simulation results are obtained using a specific formulation and numerical methods with very high accuracy. These results are analyzed via proper orthogonal decomposition (POD), which reveal similar modes for flow past a circular cylinder and flow inside a lid-driven cavity, indicating universality of such modes. Unlike many other efforts on reduced order modeling via POD, here the emphasis has been on understanding the physical aspect of the flow instability which requires very high accuracy of the simulation. Then, the obtained POD modes are related to the instability modes (in the classical sense of defining the latter) and new generic types of instability modes are identified in the studied external and internal flows. These new modes have been reported for flow past a circular cylinder [Sengupta TK, Singh N, Suman VK. Dynamical system approach to instability of flow past a circular cylinder. J Fluid Mech 2010;656:82–115] which help one in understanding the instability sequence and the relative importance of these modes in the flow evolution starting from an impulsive start. Present comparative study, furthermore, reveals universality of such instability modes by showing their presence for the flow inside a lid-driven cavity as well. Despite seeming dissimilarities between these two flows, similarities between the instability portrait of these two flows suggest universality of such modes. From the equilibrium amplitude of vorticity time-series, we establish the presence of multiple modes and multiple bifurcation sequences for these flows in parameter space. Existing theory due to Landau and Stuart that considers only a single dominant mode and its self-interaction does not explain all these features. We invoke a multi-modal interaction model in the cited reference above, termed as Landau–Stuart–Eckhaus (LSE) equation in recognition of Eckhaus' work in modifying the classical Stuart–Landau equation. We also show that the new instability modes do not follow either the classical Stuart–Landau or the newly proposed LSE model equations and for this reason we call these as anomalous modes. Two specific classes of anomalous modes are identified and classified in the present work. Empirical expressions for the evolution of these anomalous modes are presented and their unambiguous role during instabilities is discussed.

© 2010 Elsevier Ltd. All rights reserved.

1. Introduction

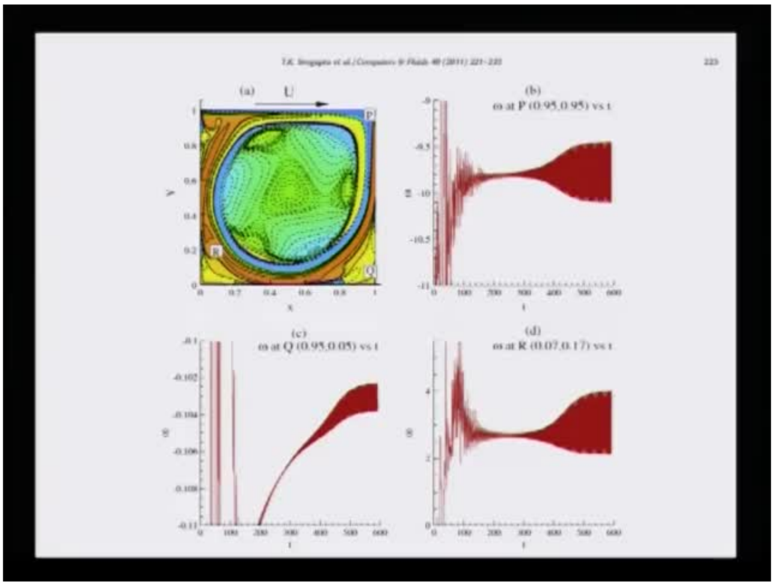
There are many internal and external vortex dominated flows ... is above a critical value and this passage occurs through Hopf-bifurcation(s) – as described in [40] for flow past a circular cylinder. This flow is discussed: (i) in [7], as the local and global duality

(Refer Slide Time: 01:21)



Now, we are trying to ratchet up the difficulty to one level more. Today, we are going to talk about another flow; it is an internal flow. So, we have discussed about an external flow; today, we are going to talk about an internal flow, and then show that, the modes that we talked about, we talked about three classes of modes, if you recall; we talked about the regular modes; then, we talked about those anomalous modes and there, we talked about the first kind and the second kind, for flow past a cylinder. Now, we are going to see, are these generic enough to be visible in other flows also.

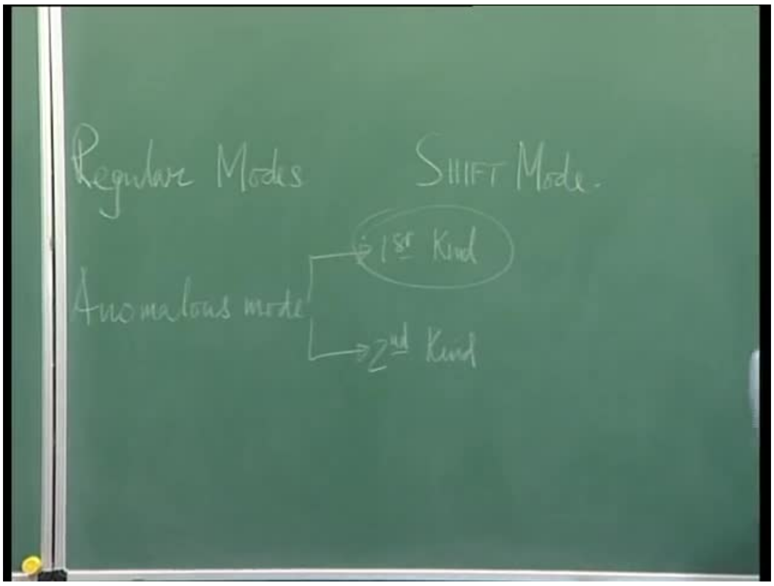
(Refer Slide Time: 02:57)



So, what we have done, we have taken to study the same thing for an internal flow. This is the, basically, a lid driven cavity problem. So, a lid driven cavity problem is shown here. What we are going to show you here, a picture that will demonstrate what is going to happen here. We will go through the same process of solving the Navier-Stokes equation and get those solutions, then, obtain the instantaneous flow; then, you obtain the mean flow and then, basically, calculate the disturbance flow. And, once you have the disturbance flow, you obtain those Galerkin projection that we are seeing, and this is driven cavity program is. What you have here is, a basically, a square cavity here. The flow is set up by moving the top lid from left to right at a constant speed u and then, we focus our attention on three representative points marked here is as p; p is a top most right corner point and q is this bottom most right point and r is over here.

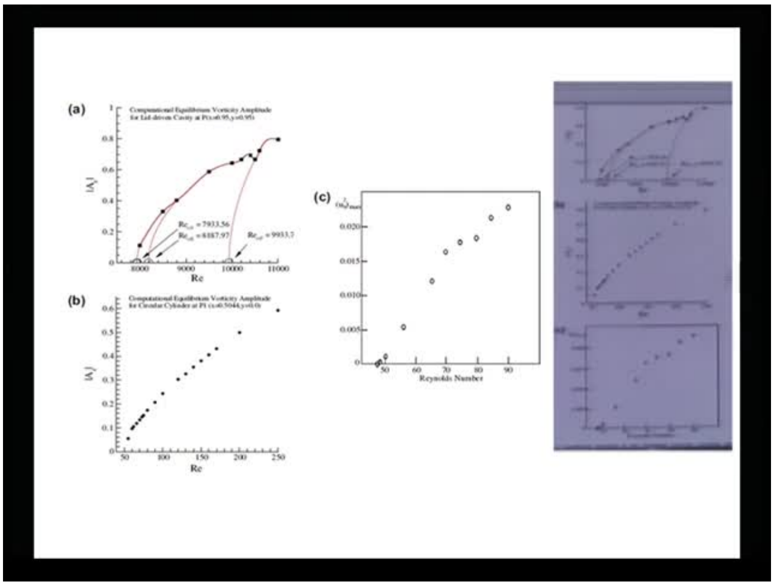
Now, what you notice is, depending on where you focus your attention, you see a different time evolution of this disturbance vorticity, as a function of time. However, the common feature is almost the same; that you start off with a semi-quiescent stage which is perturbed by linear instability and that linear instability is modulated by nonlinearity. That is the typical picture that you saw. What is the essential difference between this flow and the flow past a cylinder? Here, the initial flow is very chaotic; in case of a flow past a cylinder, flow was completely steady. So, we saw the instability was building up on a steady flow. And here, the linear instability is building up on a very transitory flow; it is, it is showing very massive transients. If you look at the other point q, q is one of those corner vortices, that is (( )) vortices that we call, and even there, you are seeing, the disturbance growing with time.

(Refer Slide Time: 05:03)



But at the same time, the mean is shifting; recall, we talked about those shift modes. This was what we called as a shift mode. So, here also, we are seeing the same kind of thing, that the mean is shifted, but overriding that mean, you have the fluctuation that grows linearly and nonlinearly saturates. And, this is this point, which is outside this corner vortices, which also displays the same thing.

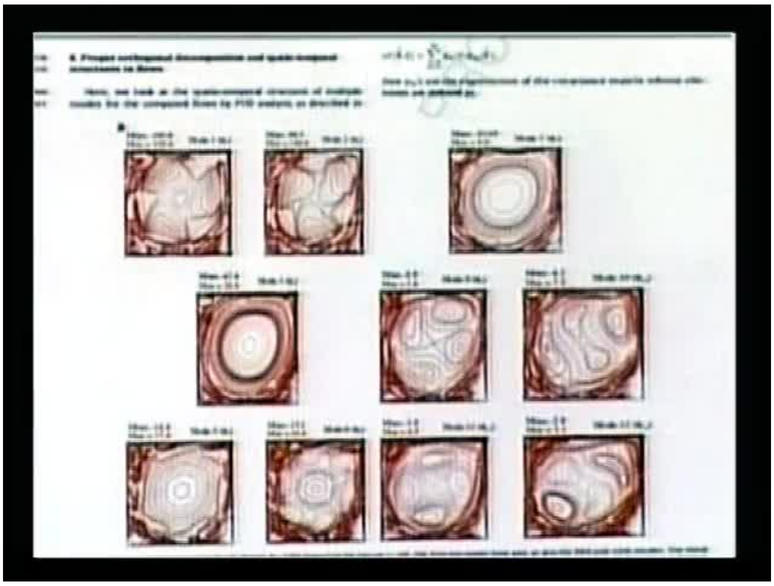
(Refer Slide Time: 05:50)



In a sense, what we are talking about here is the same type of behavior that we saw. Let me show you, what we are going to see. If we now obtain the equilibrium amplitude and plot it as a function of Reynolds number, then, what we are going to get is the equilibrium amplitude versus Re for this lid driven cavity problem. And, it is not a purely parabolic profile as Landau equation would have demanded, but instead, you are going to get this equilibrium amplitude to fall on various polynomials. And, that in concert with what we have talked about earlier, we talk about multiple Hopf-bifurcation and different Reynolds numbers are noted here. As you can see, the first one is around 7933; the second one is at 8187 and the third one that we have shown here is 9933.

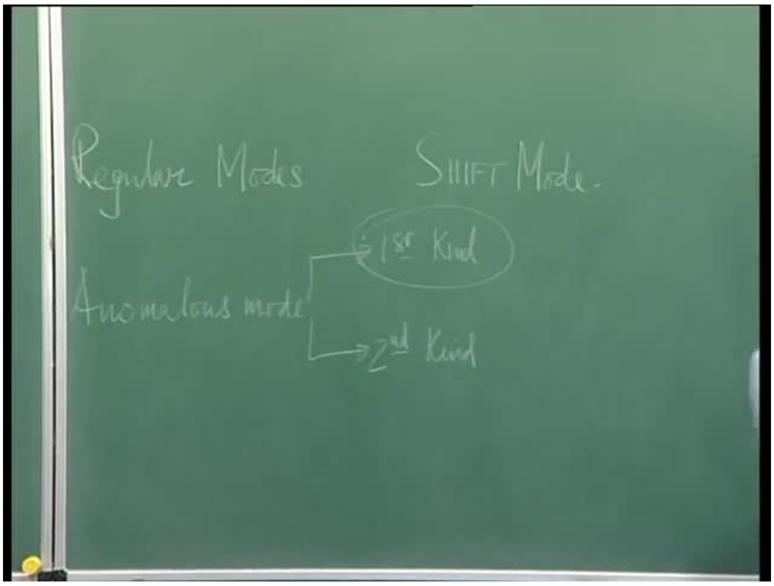
So, essentially, for flow past a cylinder, that is depicted here and the corresponding experimental results by Strykowski is shown here. You do get to see a, exactly similar phenomenon, that it is not purely given by the Landau equation; it falls on various branches of bifurcation; and, this lends credibility to what we have been talking about this, in terms of universality of this kind of instability modes. And, this was the result that I told you about Strykowski's experiment and there is one set of data here, another set of data here.

(Refer Slide Time: 07:57)

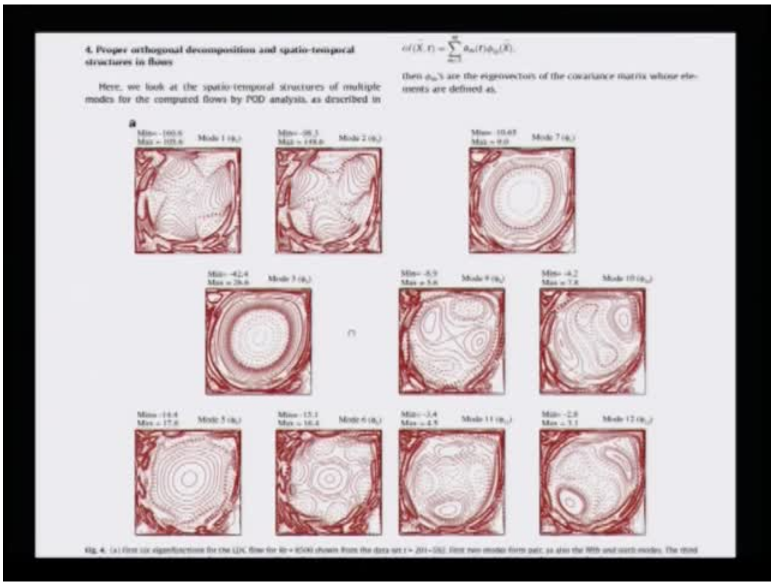


So, this is not something that is construct of a computations; it does come out from experimental viewpoint also. If you look at the Eigen modes for this driven cavity problem, you can go through that. Remember, we have shown the disturbance vorticity in the Galerkin set up. It is time dependent amplitude a m of t times phi m of x. So, space time dependence has been split up and if you perform the POD analysis, this Eigen functions are plotted here. So, these are your Eigen functions. This is the first mode. This is the second mode. They appear pair-wise, as you can see, the solid line and dotted line will toggle you between the positive and negative vorticity; and you can see that same structure you talk about, there is a core and there is a main satellite and there are the secondary ones.

(Refer Slide Time: 05:03)

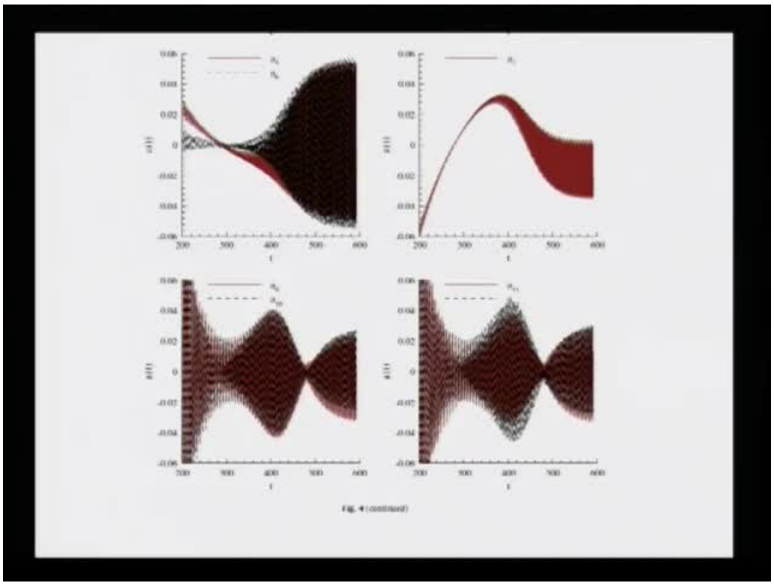


(Refer Slide Time: 07:57)



This is your anomalous mode of the first kind, the shift mode that we talked about, which appears singly; this has got no pair formation. Then, of course, that we mean that, if I am calling it 3, 4 would be the missing one.

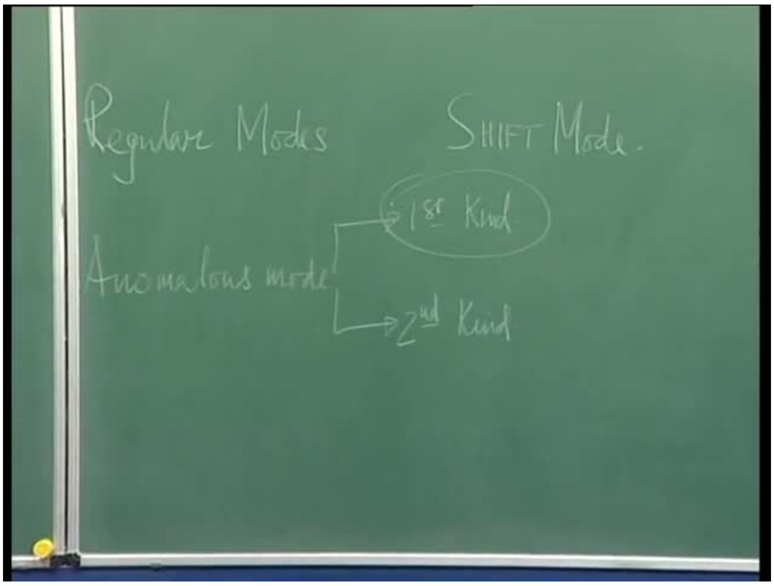
(Refer Slide Time: 09:43)



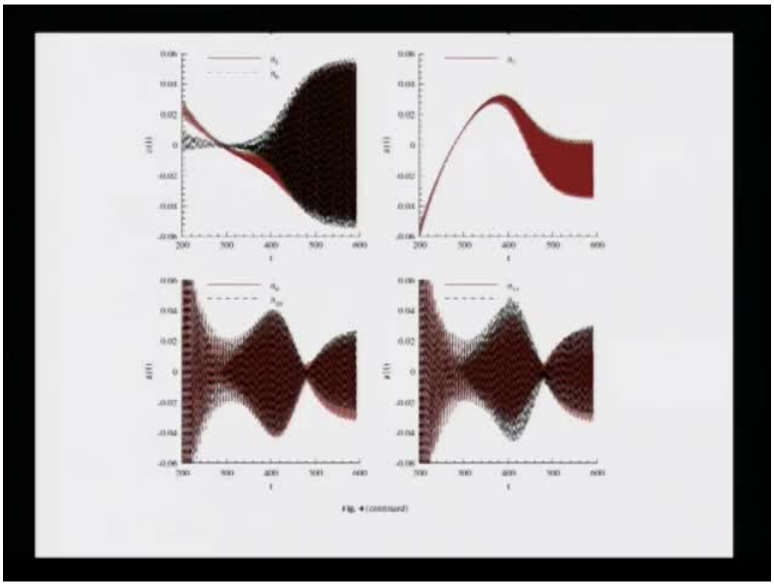
So, this we have fifth and sixth; again we have a seventh isolated one; this is also shift mode or anomalous mode of the first kind or t 1 modes that we talked about. Now, these are two pairs, but they belong to this t 2 kind. And, how do we know, you have to plot the amplitude functions. We will have to plot a m of t and once you do that… Here are the pictures a 1 and a 2, as you can see, is plotted here. They do, indeed, follow like Landau equation; that is why you call it a regular mode. And, this is your mean zero line and you are seeing the fluctuations. We have the third mode here, which was that shift mode, which appears singly, and you can see that is, that it does not start off from 0 like a flow past a cylinder, because of the stronger transient nature of the flow, to begin with. So, it starts off with some value, but it does go and settle down eventually there. This is another regular mode, pair. You can see, during the transient state, they do not have the pure orthogonality a kind of, that you expect from POD modes. POD modes are supposed to be orthogonal, but they do not remain so.

So, it is very instructive for one to understand, what is going on here. Our aim is to take a very stochastic system and project it on a deterministic basis. And, if we do that, if we are getting a perfectly deterministic basis, those have to be orthogonal; 1 and 2 seem to show that. If you look at 5 sand 6, to begin with, they are not orthogonally related; despite that, as instability takes over and nonlinearity saturates, you do actually end up getting a orthogonality. So, in contrast, we have this shift mode, or the anomalous mode of the first kind, the third and the 7, that is what we are seeing here. And, I told you about the ninth and tenth, and they are, (( )). So, ninth and tenth and eleventh and twelfth modes, they look like this. And, this mode amplitude will grow, as you would see, in a wave packet and this is what we are seeing here. So, that is the way to go about. You do the singular value decomposition of the correlation matrix; you get the Eigen values, that also gives you an idea of pair formation etcetera; then, you look at the Eigen functions, that gives you some additional information about it.

(Refer Slide Time: 05:03)



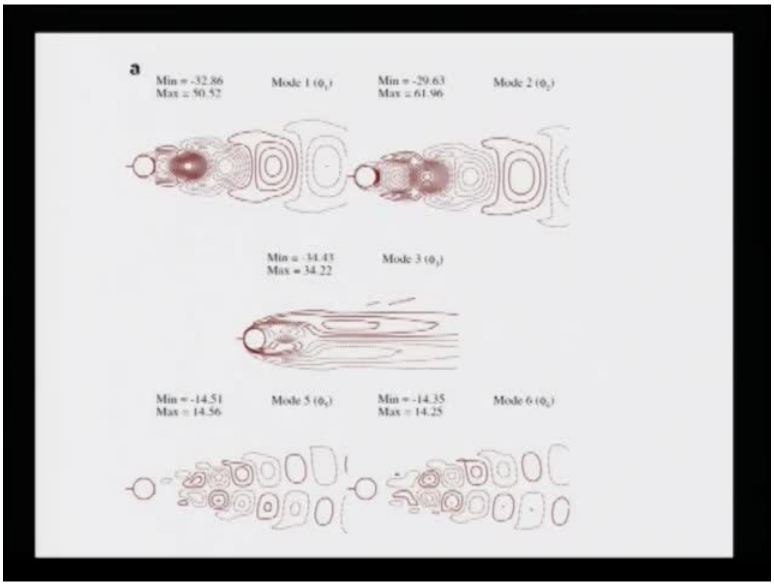
(Refer Slide Time: 09:43)



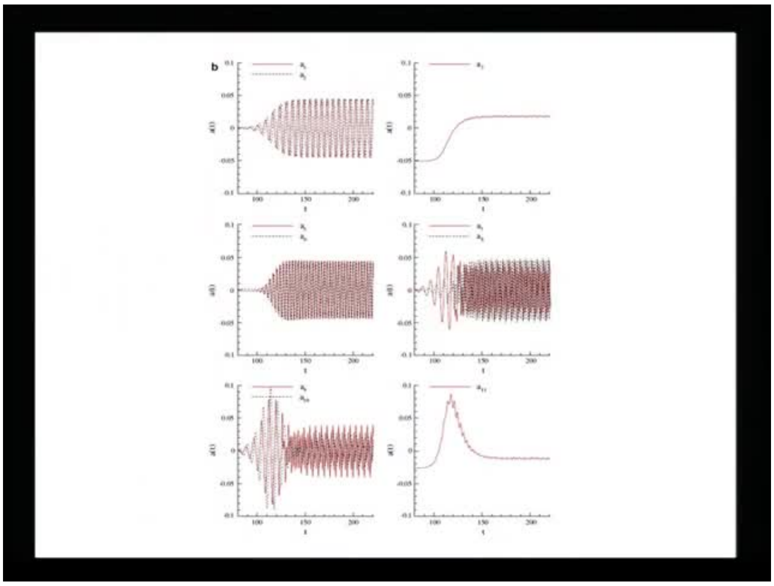
But whether they are truly going to give you this sort of classification into regular and anomalous modes or not, will be very clearly brought out, when you take a look at that amplitude function a m of t. So, this is a three step procedure; you cannot just simply look at one set and comment about it. One thing you must realize also, though that people in performing POD have not been cautious enough, you are doing direct simulation here. You may be doing the same thing in an experimental frame work. But then, when you are trying to do POD, what is its sampling rate is going to be? That is a very vital question and that is why I think, rest of the world has really not done the home work properly. Because, if I am doing, a kind of, a sampling of a time dependent function, then, I have to do it properly, to capture the appropriate frequencies. Most of the time, people may be doing high fertility calculations, but when it comes to POD modes, they have been niggardly; they take far fewer frames. And then, the moment you decide upon the sampling rate of POD analysis, that restricts you to what you are seeing. And, this kind of simulations have been done to pick up this kind of frequencies.

If I would have taken, here, we have shown it from 200 to600. So, here, the time interval is about, sampling rate is the order of about 0.01 and 0.02, that kind of a thing. So, you can imagine the volume of data, that you will have to handle to do a POD and if you do not do it, you will not pick up all this frequencies.

(Refer Slide Time: 15:05)

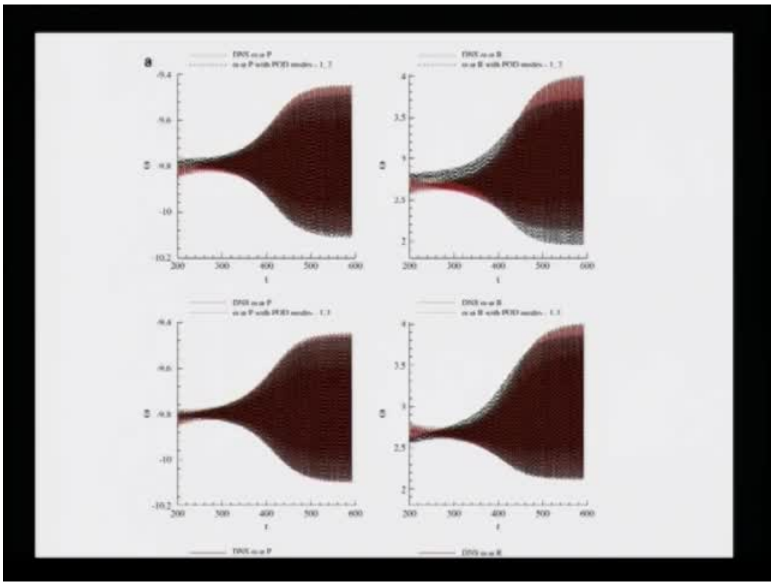


(Refer Slide Time: 15:18)



So, you must pay heed to the requirement of resolution and this is a very vital issue, that we must talk about. Now, these are some of the things that, we have seen for our circular cylinder. So, there is no question about it and these were also the, sort of a mode shape that we have seen for flow past a cylinder. We have seen that, the regular modes here, 1 and 2; then, we have the shift mode; then we have another set of regular mode. This is a very interesting thing that, in the end, you get, to a regular mode, but in the transient phase, once again, they are dissimilar; the individual component, they are dissimilar. Same thing over here. Should we call it a kind of a t 2 mode or it is a kind of a admixture of a t and r mode. We need to understand that, these elements are the basic ingredients. Whatever we are seeing at the end of the day, is basically, a kind of a mixture of all this things that we see.

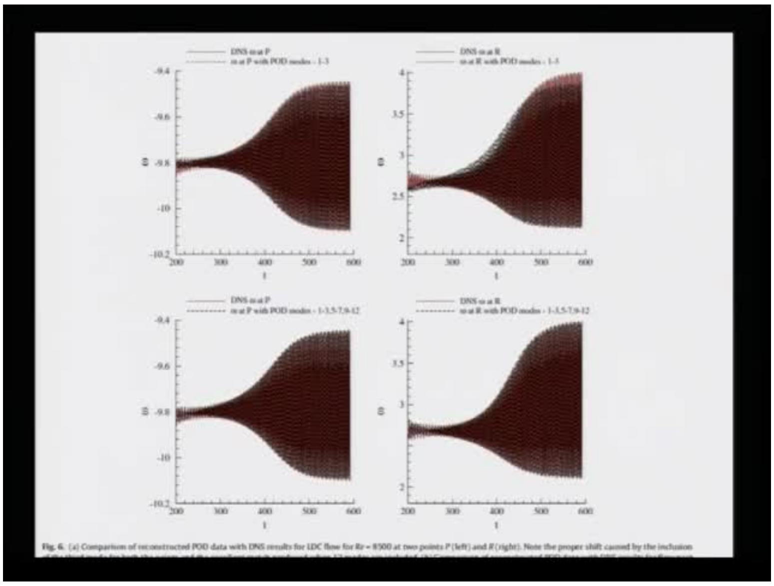
(Refer Slide Time: 16:24)



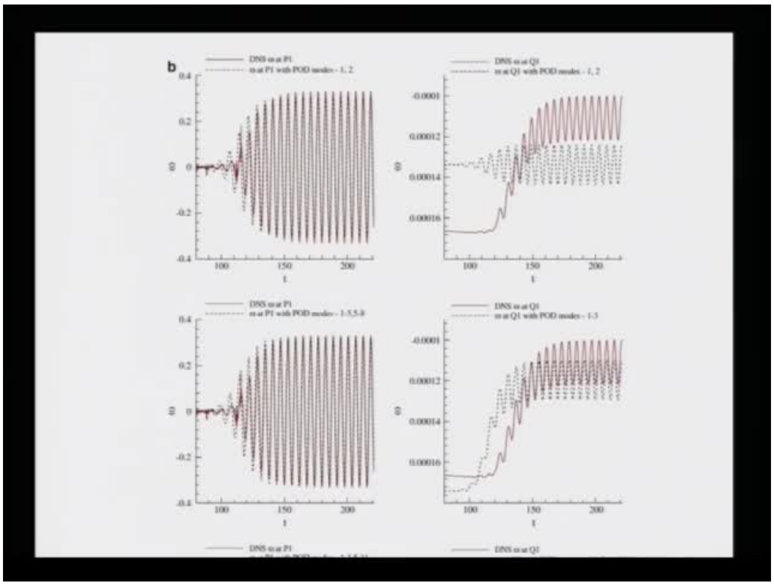
Now, this is what you are going to get, when you look at the DNS of that internal flow. And, here also, you can notice that, despite the talk of universality, there is some bit of difference in behavior of the successive modes at different locations.

But by and large, you can really capture the events that we are noticing, essentially that linear instability, followed by non-linear saturation and mode behaviors belonging to either of these three generic classes.

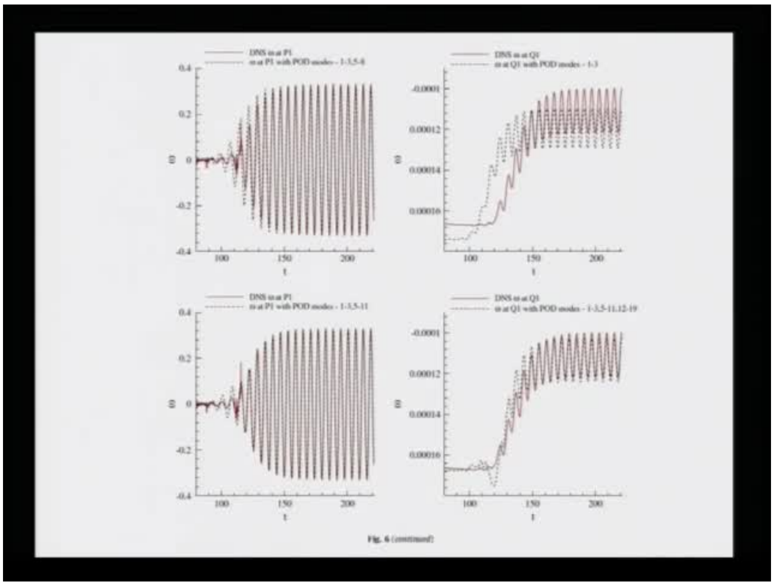
(Refer Slide Time: 16:24)



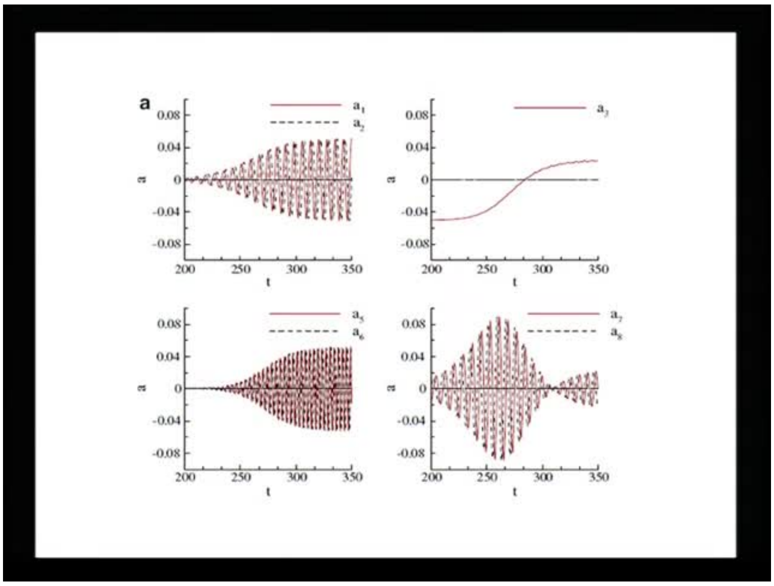
(Refer Slide Time: 17:25)



(Refer Slide Time: 17:33)

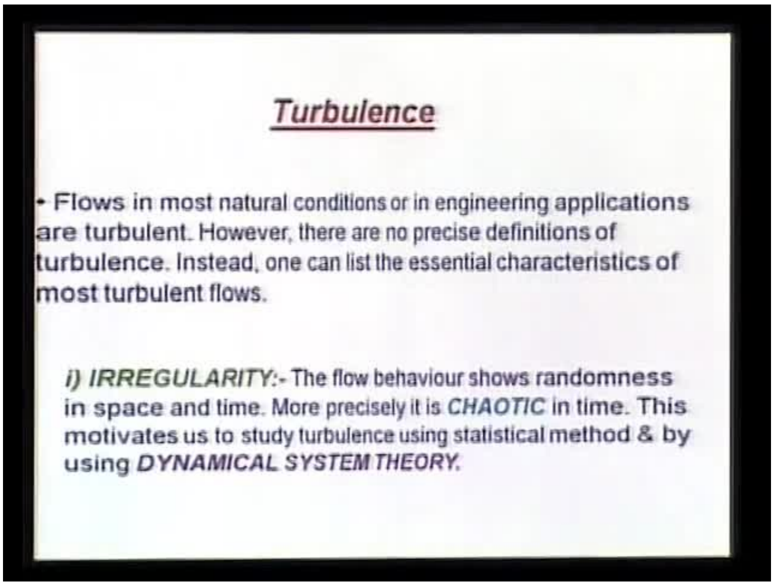


(Refer Slide Time: 17:38)



I suppose, one can go ahead and do all this. These are those mode shapes for cylinders. I think, what we are essentially looking at, this is a mode shape for flow past a cylinder at Reynolds number of 60 and again, you can see, this two are those regular modes; this is the t 1 mode and this is t 2 mode and so on, so forth. So, you can study this, as sort of a, universal building blocks.

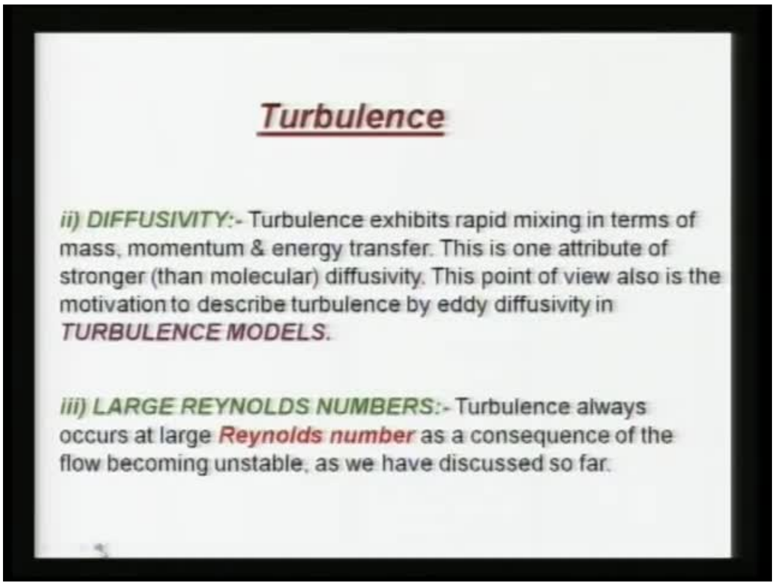
(Refer Slide Time: 18:41)



So, I will close the discussion here on POD analysis. I suppose, you have seen quite a bit of variations. Now, we go to the last phase of our course and that is basically, talking about turbulence. So, if you look back, flow that we see in nature or in engineering devices, most of the time they are going to be turbulent. Despite the fact that, this part is considered to be a special topic in studies of fluid mechanics, but it is a, sort of a interesting thing, because this is not the exception; this is the rule; all flows are turbulent, but very little attention is paid to it. There are various reasons for it, because, we still do not understand what turbulence is. And, there are no precise definitions of turbulence. There are lots of people who can rattle away strings of sentences, turbulence is this, turbulence is that, etcetera; they essentially, tell you about the characteristics of most of the turbulent flows. I am going to talk about some of those characteristics, one by one.

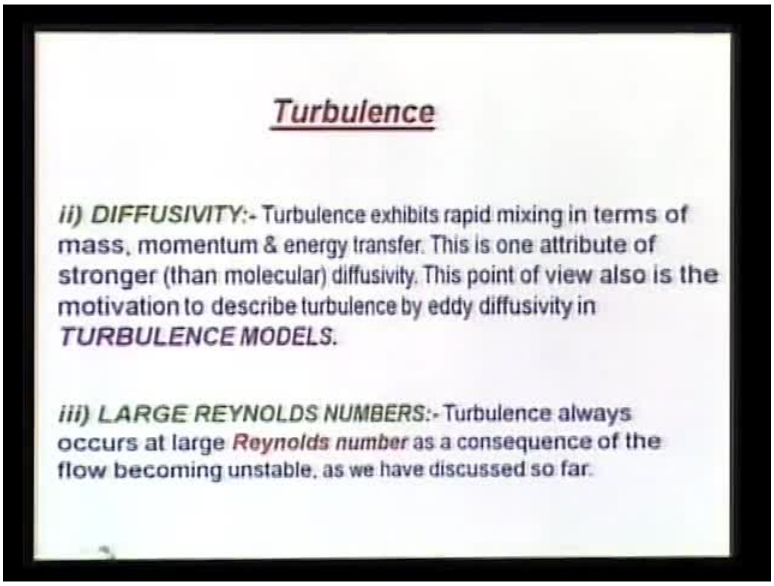
But please, do not misquote me by saying, turbulence is irregular flows; just one of it; it is not going to be adequate. The flow behavior, if you look at it, whether you are looking in space or in time, you would see that, there would be a sort of apparent dichotomy in showing that, there is a lack of coherence. What you see is, basically is, some kind of a randomness, or it is, if I look at its time variation, it is going to look like chaotic. I purposely wrote this word chaotic in the sense that, chaos dynamics was a topic of intense studies in, starting from 70s, upto probably, even now; today, lots of people try to study lot of aspects of chaos dynamics, although it started off in some of the work done by fluid dynamicists in trying to explain turbulence; it does not fulfill its promise in defining turbulence; because, what happens is, irrespective of this randomness and chaos that we try to characterize turbulent flow with, people know, turbulent flow, in its underlying features have some kind of deterministic structure. And, you are really ready to appreciate that point of view, because that is what we have done with our POD analysis that, we will take a completely a stochastic dynamical system and project it on to a deterministic basis.

(Refer Slide Time: 22:02)



So, those deterministic basis are called coherent structures of turbulence. That is why, people may like to study turbulence using statistical method or they may like to study in a dynamical system frame work, like what we have just now done. So, irregularity is one of the feature of turbulent flow. There is this other feature, its diffusivity. If you go back, when we were discussing about bypassed transition caused by those convecting vortex in the free-stream, what did we see in those flow visualization picture.

(Refer Slide Time: 22:34)



The dye that was coming out nice and straight got diffused. There was a significant increase in mixing or diffusion. So, whenever you see turbulence, you must understand, there is a enhanced mixing process, that is characterized mathematically by the process of diffusion. So, diffusivity is very important in turbulence. They represent rapid mixing in terms of all quantities of interest. There could be mass diffusion; if I add in a sort of a dye and it gets mixed to the adjoining fluid, it is a mass diffusion. At the same time, when I see a flow and the shear layer developing, then, flow is undergoing transition and it becoming turbulent and I see thickening of the boundary layer.

So, the mixing is occurring over a larger wall normal distance. That is what we are talking about enhanced mixing of momentum. And, when mass and momentums are transferred, you have to understand, that would lead to energy transfer too. What you need to understand that, if you did not have the turbulence, then, the boundary layer, the shear layer would have grown and that mechanism was given by kinematic viscosity, the molecular viscosity. So, molecular viscosity, is like one layer of fluid going over the other and then, they are transferring momentum in orderly fashion, from one to the other, in a macroscopic sense; but in a microscopic sense, you can think of, in terms of those molecules, or the constituent particles exhibiting rapid variations; but in a macroscopic sense, they are still very regular.

However, when you have turbulent flow, even at the macroscopic level, you are going to see a massive increase in diffusivity. So, if I try to characterize my turbulent flow in terms of a diffusivity coefficient, that coefficient of diffusion has to be significantly higher than what you get in laminar flows. So, this is one attribute which is very stronger than your molecular diffusivity. This is something very important. And, unfortunately this point of view also is the motivation, to describe turbulence in terms of a diffusivity coefficient. That is what is, goes by the name of any diffusivity, that is modeled most of the time and turbulence models.

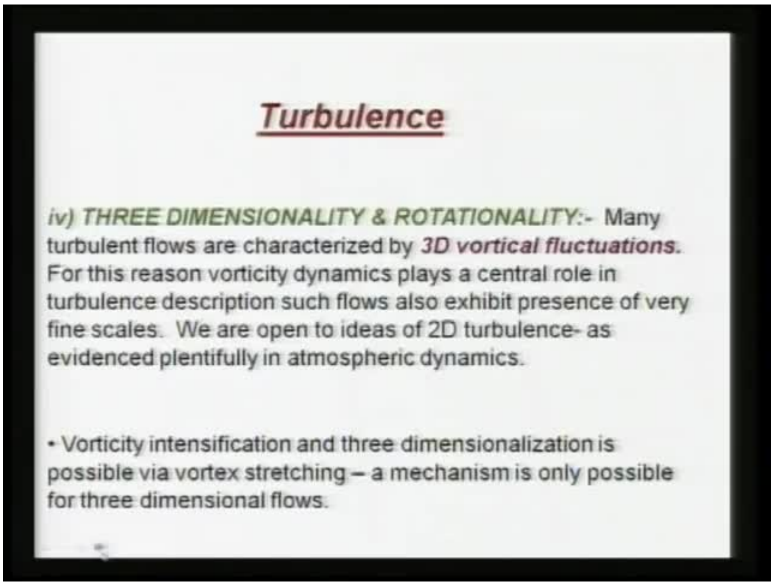
But do not try to understand, like, in the name of engineering fluid mechanics, lots of people understand turbulence in terms of coefficients of turbulence models. That is far from the truth. This is one of the attribute of turbulence, that you do have enhanced diffusivity; that eddy diffusivity is associated with the eddies, you are going to see in a turbulent flow, is a multiple presence of eddies of different sizes. So, you have a hierarchy of sizes of eddies. So, this is what is called as eddy diffusivity. So, different size eddies interact with each other and we have seen from the Navier-Stokes equation that, the non-linear term and the convection part, helps you do that through the process of cascading.

We will talk about it and get a detail as we go along. There is the other thing, that is very typical of turbulent flows. You would hardly ever get turbulent flows, which is occurring at low Reynolds number. Whenever we are talking about turbulent flows, we are talking about turbulence that is generated by some mechanism, and which is also self-sustaining. Remember, that experiment that we have seen, when we dragged a single vortex, what happened, when we removed the vortex, things again came back to normal; would I call that a generic turbulence? No, because that is more like, your forced vibration, but when we are talking about turbulence, we are more like, talking about a natural vibration.

So, it is self-sustaining; it should be there and that kind of self-sustenance comes about, when your basic equilibrium flow is unstable. So, instability is the core of everything. See, that is why in this course, (( )) title transition and turbulence; I had spent most my time, talking about the transition aspect. With the twin object, one is, all this phenomenon are instability driven, and number two is, to basically fight the ghost of the commonly helped perception that, turbulence is universal. So, it does not matter how you get there; it will all be given by single model, which is not true. We firmly believe that, they are all multiplicity and the point of view of the turbulent flow itself and what end product you get, depends on which route you have traversed, and which are the routes, these are the routes of instabilities.

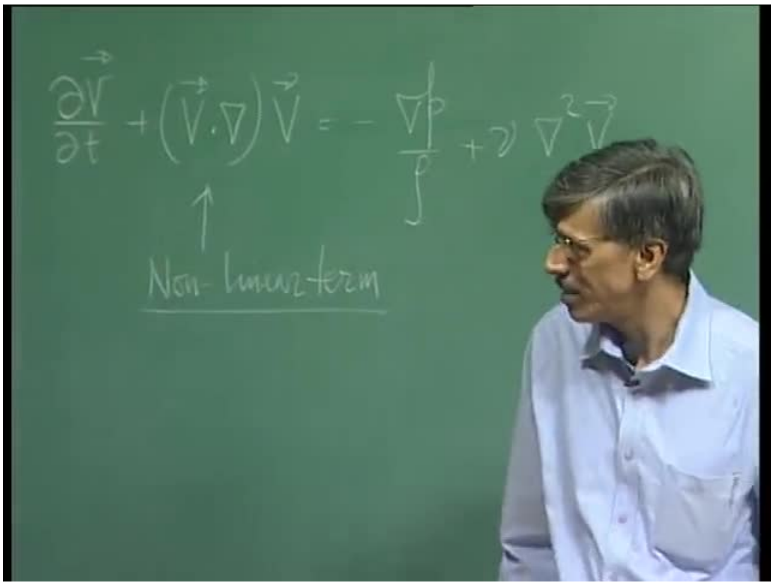
So, the instabilities will be characterized, only when you have large Reynolds number. If we have low Reynolds number flow, we can excite the flow, but it may not sustain itself. Once I remove the source of disturbance, it quietens down; but if I go to high Reynolds number flows, which are unstable, then once I get it started, it will be there or I do not have to get it started; the small back ground disturbances are good enough to trigger it.

(Refer Slide Time: 29:49)



So, this is something that we understand that, large Reynolds number is a necessary condition, not a sufficient one, to tell you that, all large Reynolds number flow will have to be a turbulent flow; but the propensity of setting up turbulence, is very much there, when the Reynolds numbers are high. There is the other aspect of turbulence that, we must keep in mind. This has to do with the way we look at Navier-Stokes equation.

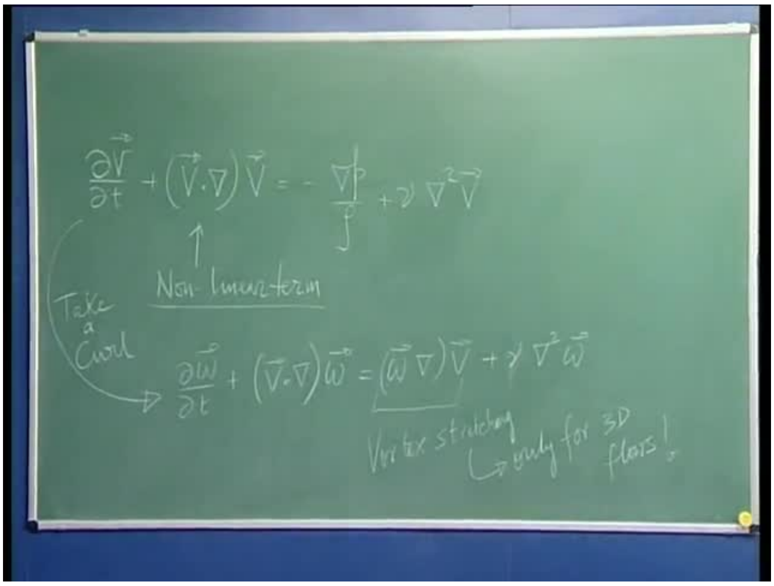
(Refer Slide Time: 30:16)



If I write down the Navier-Stokes equation, then, we can understand the basic processes. The basic processes are very clearly understood. This is the local acceleration at any point, if I look at what the time rate of change of events occurring, that is given. This is your convective acceleration. This is the way, let us say, the pressure is driving the flow; you may have it or you may not have it. Of course, this is the viscous diffusion. Of all the terms that you see, this plays quite an interesting role; this is your non-linear term.

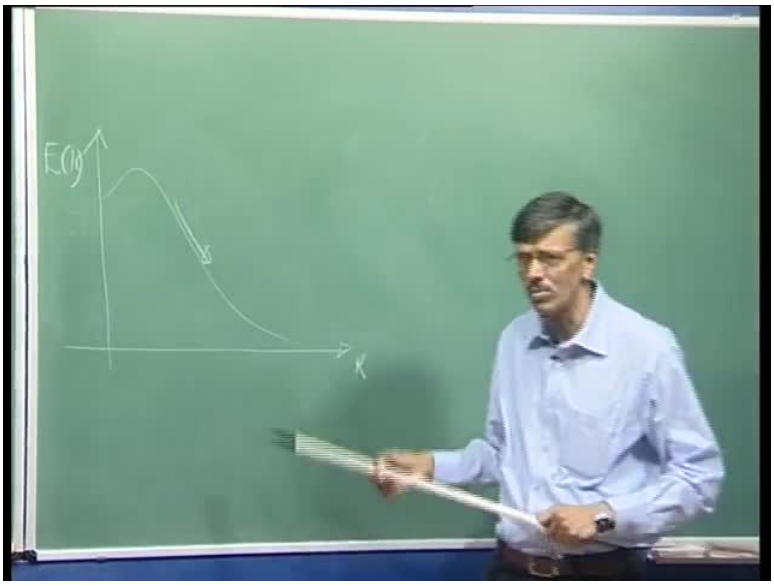
If you are a mathematician, you would say it is a quasi-linear term, because the highest derivative is second and that is why it is linear; but this is a non-linear term, that plays a significant role in determining the dynamics. We have already seen that, all instabilities do start off from here. If you recall, we have written down the Navier-Stokes equation in rotational form. I can write it down in terms of v cross omega and gradient of v square by 2. If the flow is irrotational, that term will go away. So, what the non-linear term is doing, just simply redefining the pressure, the p by rho plus half v square. That is what we see, what we are talking about the role of rotationality.

(Refer Slide Time: 32:54)



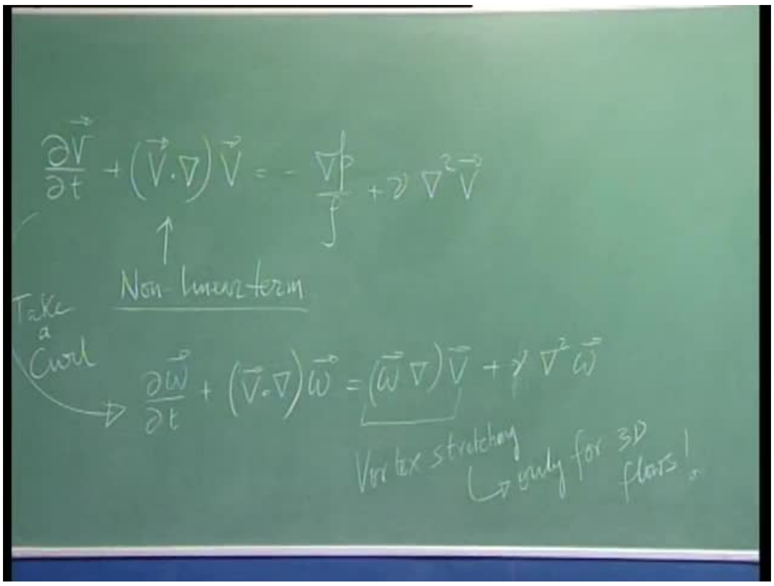
So, if I do not have rotationality, nonlinearity does not differ much; it just simply redefines the pressure. So, rotationality is central in understanding turbulence and three dimensionality is written down when you take a curl of this and then, you get the vorticity transport equation. So, you take a curl and then, you get the vorticity transport equation. How does it look like? So, this is vorticity transport equation. Look at this and understand what each of this term is doing. This term is rather important. This is called as vortex stretching term. You can very clearly see that, this term will be identically equal to 0, for two dimensional flow, because your flow gradient will be in x y plane; omega is in the normal direction, so, z direction; so, omega dot del is 0 identically. This vortex stretching is only for 3 D flows; for 2 D flows, it is identically equal to 0. You can understand that, vortex stretching will only be there for 3 D flows and not for 2 D flows. And, this is what has the perplexed our fluid dynamics community for last half a century.

(Refer Slide Time: 35:50)

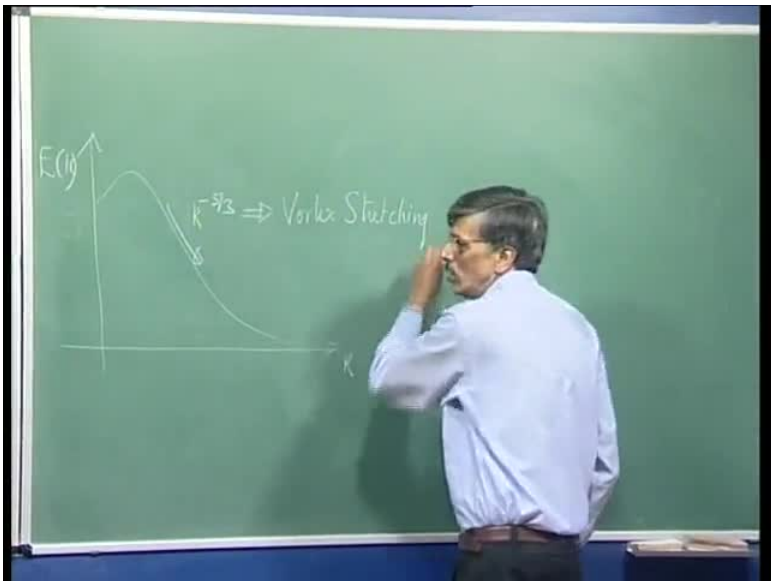


So, this tells us, when this is going to be important. So, vortex stretching will not be there for two dimensional flow. What does vortex stretching do actually in a real flow, that you would understand, when you look at the energy spectrum. And, if I plot the energy spectrum of a typical flow, inhomogeneous flow, it would be like this. This way, that the energy is being transferred from small k to large k, one of the mechanism that can do is vortex stretching.

(Refer Slide Time: 32:54)

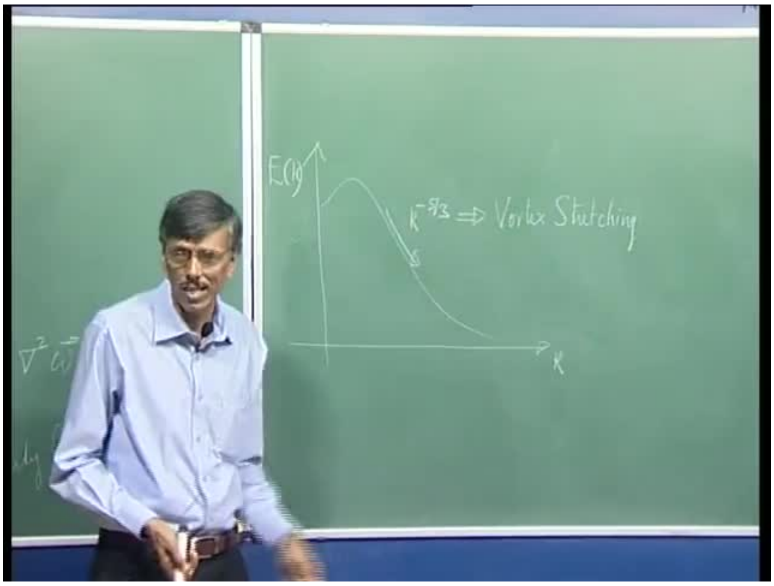


(Refer Slide Time: 36:24)



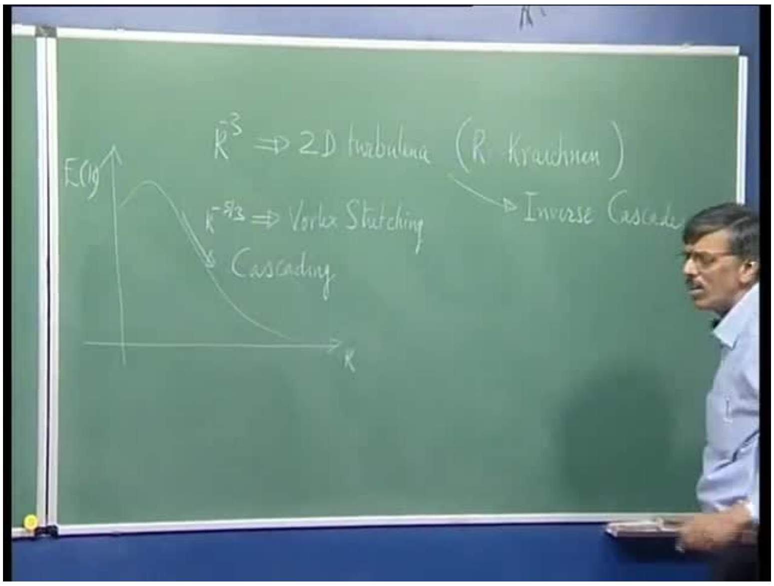
So, this, when you see this slope as k to the power of minus five third, this was shown by (( )) that, that can only happen corresponding to vortex stretching. So, that is what we are talking about that, if we look at turbulent flows, they are always characterized by three dimensional vortical fluctuations. Vorticity dynamics plays a central role in all fluid mechanics, more so, for turbulent flows, because, we have defined the turbulence as an ensemble of eddies of different kinds.

(Refer Slide Time: 36:24)



So, eddies means vorticity, circulation. So, vorticities dynamics has to play a central role. And then, this non-linear process of vortex stretching, transfers the energy from small k to large k; that means, it is basically, siphoning off the energy from larger to smaller eddies.

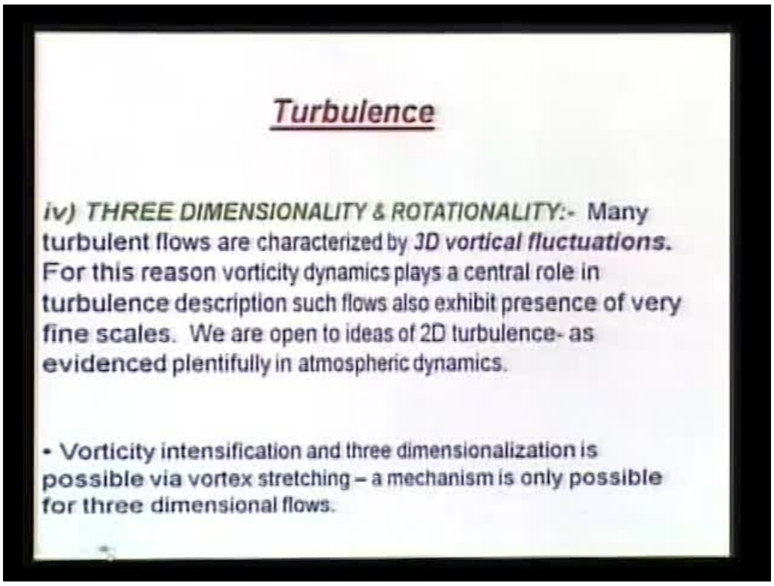
(Refer Slide Time: 37:46)



If I do not have this vortex stretching, I would not have a powerful mechanism by which the energy will be transported from larger to smaller eddies. But please do understand, a very large part of the sky is dominated by 2D turbulence. Nothing exemplifies better than the atmospheric dynamics. I will show you some results, as we go along. When you look at the atmospheric data, we will see that, this energy spectrum is not given by k to the power of minus five third, but it is given by K to the power minus 3.

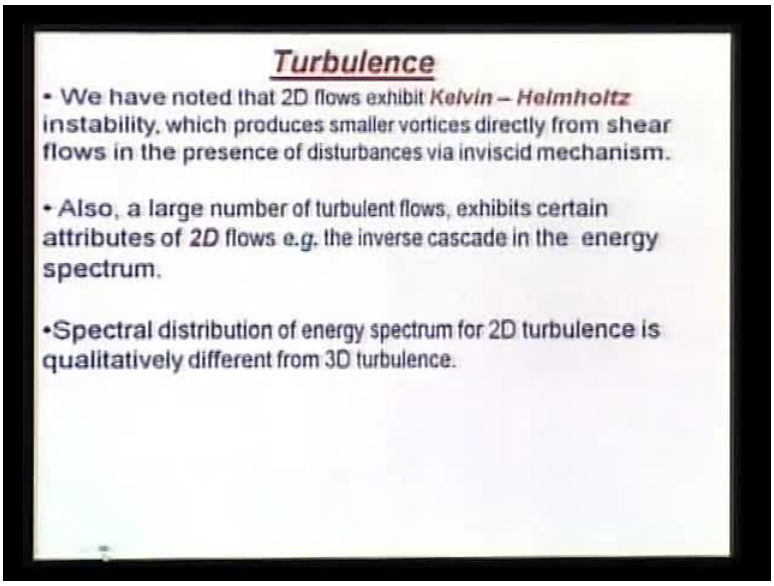
This is an attribute of 2D turbulence. All the people used to think, 2 D turbulence is a sort of anachronism; it was just studied for the sake of study; but lots of the people are waking up to the reality that, you can have 2D turbulent flows. And, Kraichnan did some, lots of pioneering work in this; one of the Post Doctoral student of Einstein working on turbulence was Bob Kraichnan. He had done lot of work. The 2 D turbulence is characterized by K to the power minus 3. It is also characterized by a physical process, where smaller eddies goes and merges together and forms bigger one. This is called as inverse cascading. So, this process that we are seeing here, is called as cascading of energy. But 2 D turbulence displays, what is called as inverse cascade. See, conceptually, this picture is not difficult to visualize, because we have small eddies joining up together; it happens all the times; it is nothing very sacrosanct.

(Refer Slide Time: 29:49)



But mathematically, it was shown that, when you have 2D turbulence, one of the attributes of that, is this. And, we are open to ideas of 2D turbulence, as evidenced plentifully in atmospheric dynamics will show that. Vorticity intensification and three dimensionalization is possible via vortex stretching, a mechanism that is possible only for 3D flows.

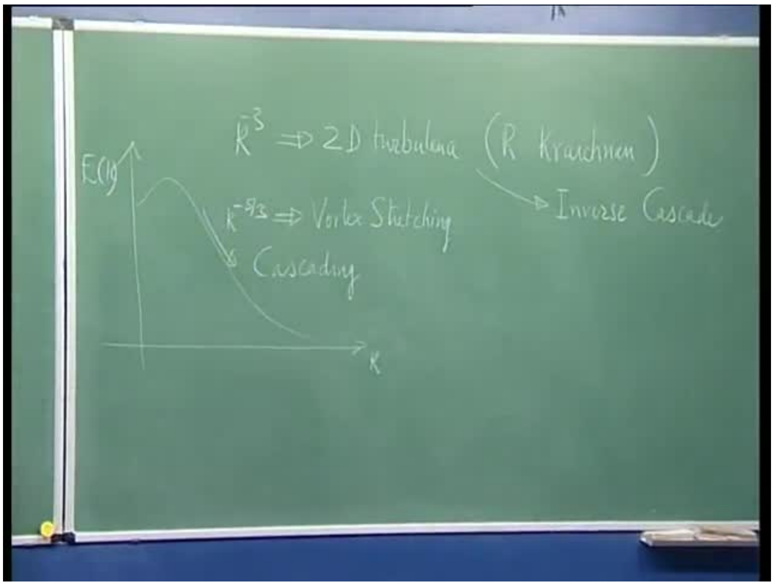
(Refer Slide Time: 41:00)



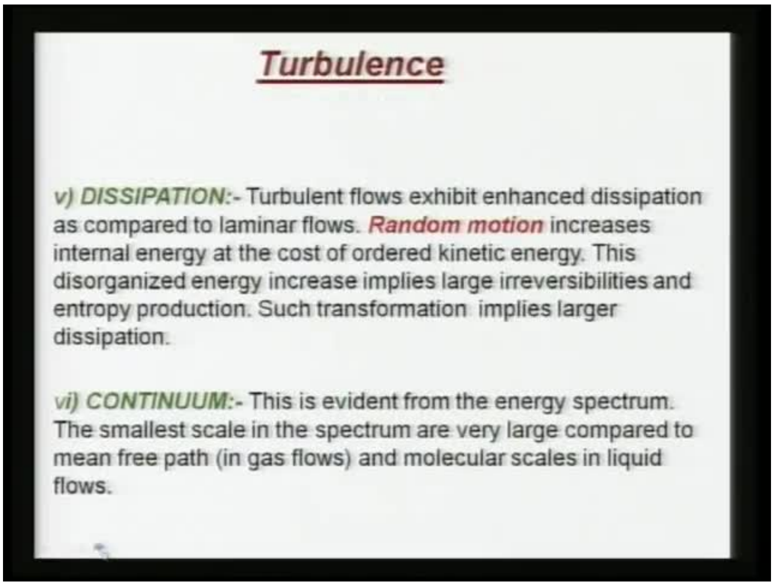
You have already noted certain things, where different scales can come about. For example, think of Kelvin-Helmholtz instability. I have a uniform flow on top and uniform flow at the bottom; there is an interface; this interface is disturbed by some background disturbances, and this interface curls up into vortices. So, that is, basically, a direct mechanism of creating vortices of various sizes. It was possible, more so, for a two dimensional flow than three dimensional flow; we established it. 2 D flows are more prone to Kelvin-Helmholtz instabilities than 3 D flows, remember.

So, sometimes, this 3D card is overplayed, but we need to understand, there are many instances; this is one of such things. And, what was so important about Kelvin-Helmholtz instability is, that it was an inviscid mechanism. So, you do not need to have the presence of elaborate viscous mechanism to show up the instability; you can directly get it from a two dimensional flow; you have the shear. So, shear is synonymous with viscous phenomenon, but we are talking about the disturbance growth mechanism. That could be completely inviscid mechanism. That is one thing, we must understand.

(Refer Slide Time: 37:46)

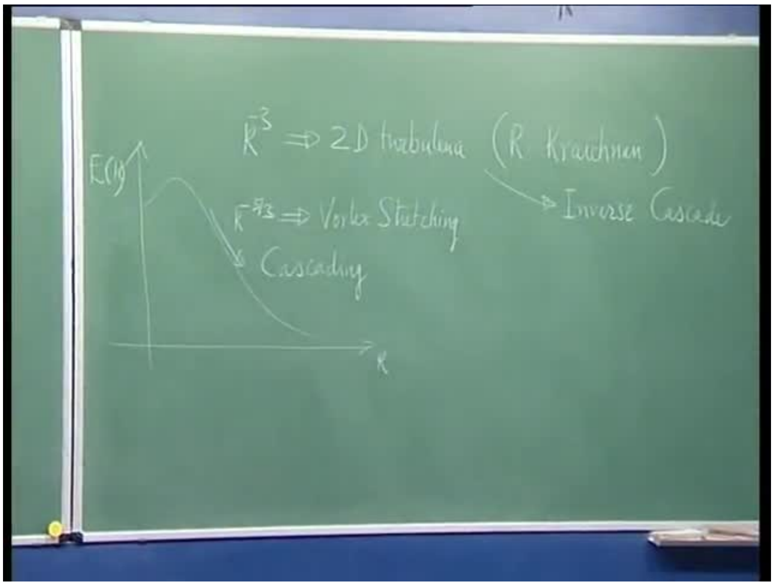


(Refer Slide Time: 43:04)



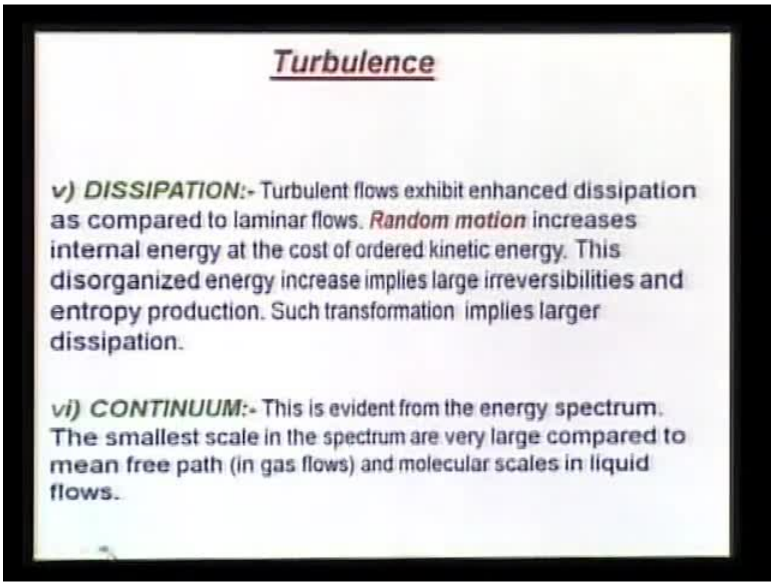
We already talked about inverse cascade. So, we do see that, and we have already said that, spectral distribution of 2D turbulence is qualitatively different than what you get in 3D flows. And, one of those central theme of this course, as we have talked about, is the phenomenon of dispersion. Dispersion is a very strong mechanism by which different scales can be created. Remember, we talked about those train of vortices going over shear layer. So, I may be defining a length scale which is given by the spacing of the successive vortices.

(Refer Slide Time: 37:46)



However, because of the dispersion relation, I could create a whole host of length scale. So, do not be too much, be taken up, by thinking only about vortex stretching and nonlinearity; those instabilities are linear instability. So, the linear mechanism itself can give you a whole host of length scales.

(Refer Slide Time: 43:04)



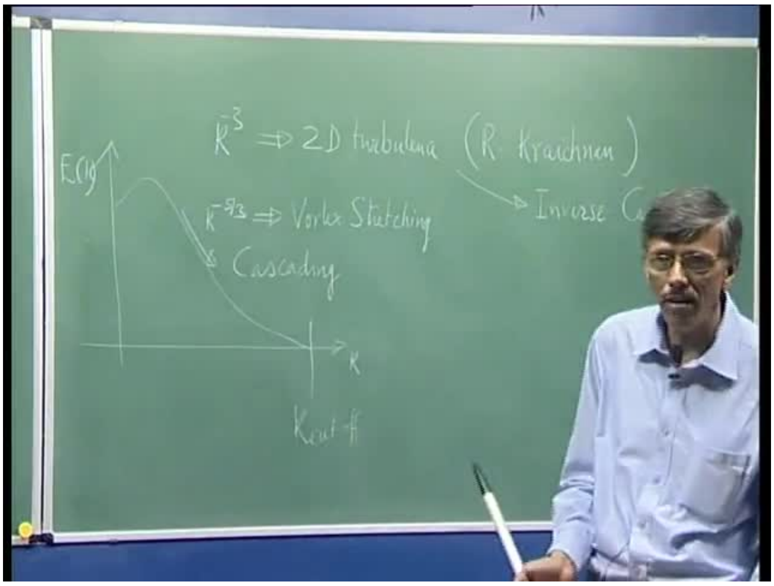
So, we must be aware of this. Now, this is a phenomenon logical observation that, turbulent flows are more dissipative. You ask an airline operator; they will complain till their face turned blue that, they spend all the money, because flow is turbulent. They would love to have laminar flow; that is because, we have very high dissipation in turbulent flows. We have very high dissipation and that is not very unusual for us to understand; we already have talked about diffusivity. If I have larger diffusivity, I will have the larger dissipation. Physically trying to understand what happens, if I have a laminar flow, I will have a thinner shear layer. So, this transition from a no slip wall to the fully developed flow, is occurring over a thinner section. However, if I now set in motion, turbulence, this will be can thicker.

I just create a very stronger motion in a macroscopic scale, which is random. Turbulence is randomness in a macroscopic scale. You have randomness at the microscopic scale even for laminar flow, but turbulence is special that, it brings in the randomness at the macroscopic level. That is what is something, that we need to understand.

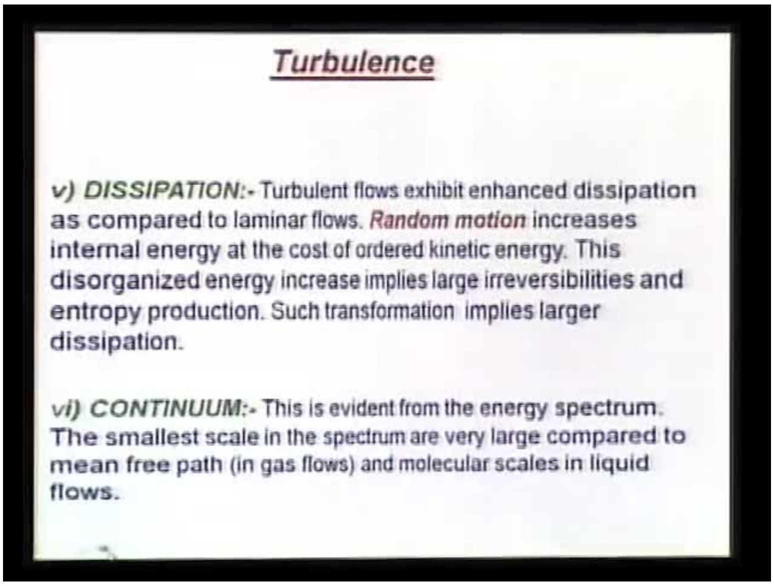
The moment I remove orderliness, I bring in randomness. Then, from your thermodynamics knowledge, you will say, internal energy is increasing. So, earlier, flow was spending all its energy going along the stream-wise direction; now, we have created motions in all possible directions. So, you have actually, lost lot of ordered kinetic energy and got it into internal energy. This disorganized energy increase, implies large irreversibilities; that is why your entropy increases. So, such transformations, needless to say, will imply dissipation

So, in fluid flow, orderliness is laminar motion, carrying motion which is coherently directed, but the moment you bring in turbulence, you have a additional source of entropy production, loss of orderliness, that leads to larger dissipation. Having said all of this, and we have already emphasized this, turbulence occurs at macroscopic scale. Do not ever be deceived by thinking that, it is a molecular phenomenon; it is not. You still view the flow as a continuum; you do not have to approach the study of turbulence by looking at microscopic level.

(Refer Slide Time: 47:39)

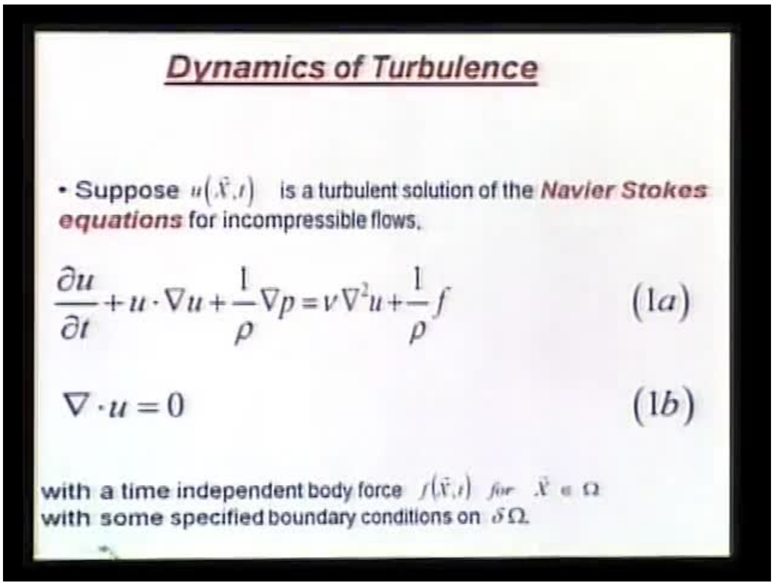


(Refer Slide Time: 43:04)



So, you look at it as a macroscopic level; the flow is a continuum. If I look at the energy spectrum, it just goes, somewhere there, it just stops. So, it has a finite cutoff wave number. So, if I call that as k cutoff, this k cut off corresponds to a length scale. That lengths scale is many times larger than mean free path in gas flows or the molecular scale for liquid flows.

(Refer Slide Time: 48:24)



So, they are not something, manifestation of your flow phenomenon at the molecular level. It is basically happening at the macroscopic level. So, let us try to understand the dynamics of turbulence and this is what we would be talking about in greater detail in the next class. Today, I will just simply say that, now, whatever the attributes that we have talked about, let us say, the velocity field, if I write it as u, it is going to be a function of space and time; you cannot have a steady turbulent flow.

So, please do get away from the habit of calculating turbulent flows by time averaged equation, and think that, the flow is steady. So, that is something, we must understand that, u will be a function of x and t; governing equation will be given by the, if I am talking about incompressible flow, it will be given by momentum conservation and mass conservation. We will start from here in the next class. f that you are seeing here, we are purposely kept a body force, which could itself be time independent or there could be other sources of bringing in time dependence from this equation. We have seen instability is one such mechanism. We saw that, a unidirectional flow coming and then, it is becoming time dependent; but a body force will keep it as it is, for the whole domain. And, non-steadiness can come from the boundary conditions also. If the domain is omega the boundaries del of omega.