Instability and Transition of Fluid Flows

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Lecture No.# 33

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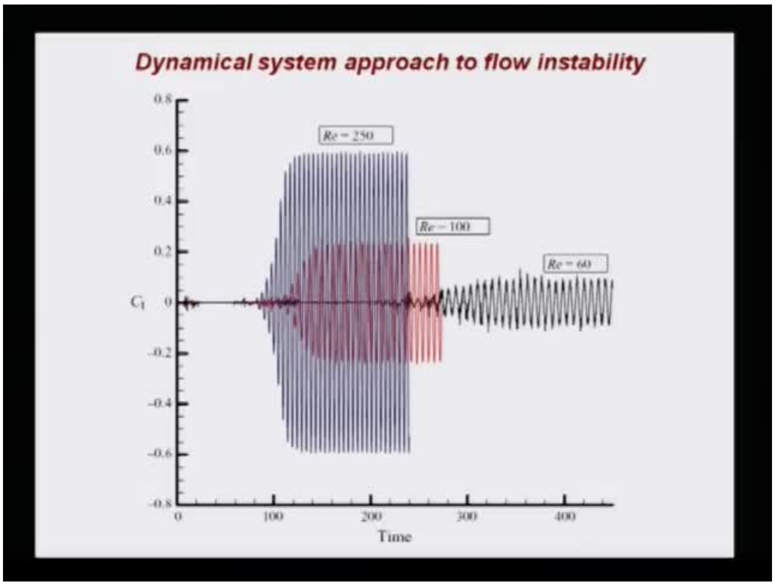

So we were developing a dynamical system theory for flow instability using DNS results and looking at the flow past a circular cylinder, we have shown here, the computed lift coefficient variation with time, for three different Reynolds number, 60, 100 and 250. There are certain features that immediately come to the fore. The number one is that, you see that, with increase in Reynolds number, equilibrium amplitude increases.

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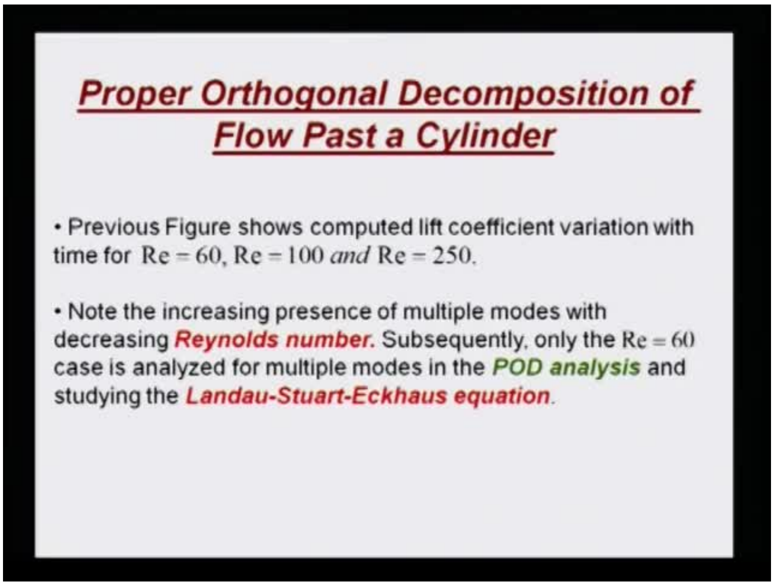

The other thing that you also notice is that, there are certain interesting features. For example, this saturation amplitude variation with time is very minimal for higher Reynolds number, but while you look at Re equal to 60, although one would expect from Landau's theory that, you would have attained an equilibrium amplitude, but it is not so. You can see a quite a bit of jagged edges and it is a, something specific about this particular lower Reynolds number, that is where you see most of this variation. So, we will like to make some comments, which we have stated here that if we increase, well, if we decrease Reynolds number we see more and more number of modes present. If it was just a single mode, then, we would have seen a very nicely developed homogenous amplitude variation. For example, for Re equal to 60, we saw that, even in the equilibrium stage, you have a significant variation of the amplitude; that implies that, there are more than one modes present is it not. So, that is what we are saying, presence of multiple modes are noted with decreasing Reynolds number.

For this reason, we will focus our attention to the results at Re equal to 60, to see what this multiple modes are doing and we will also develop what is called as a Landau-Stuart-Eckhaus- equation. We kind of resurrected this study, a year and half ago, to see that, unlike what Landau-Stuart- equation is, that talks about only a presence of a single dominant mode; now, if we assume there are more than one modes, then, how do we account for it. So, we developed a kind of a variation, which was originally proposed in a monograph by Eckhaus, but largely forgotten; but we want to see, how we can make use of that.

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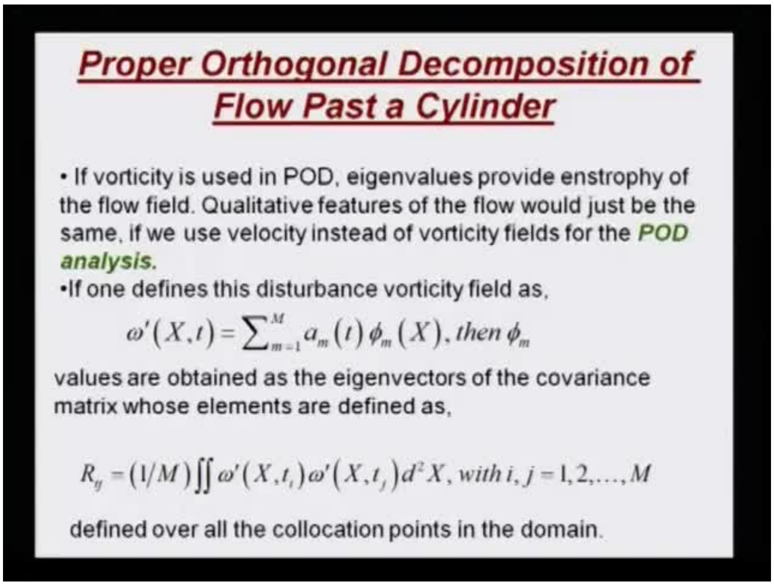

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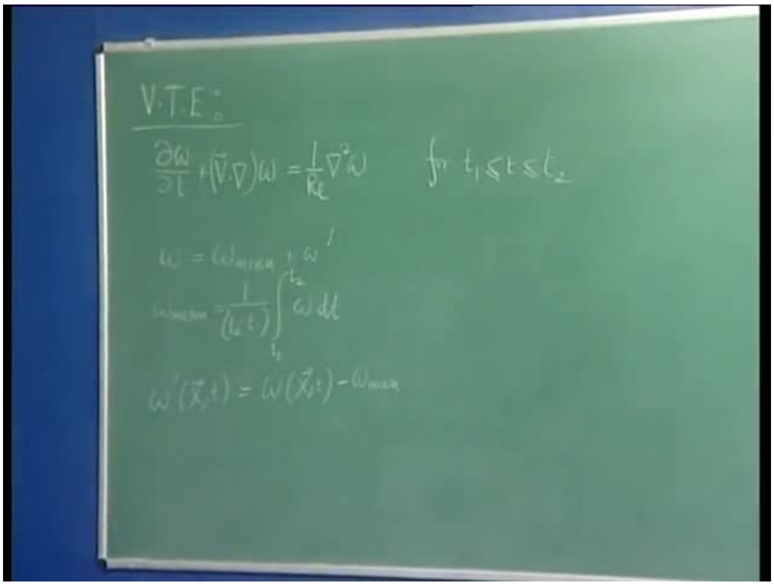

Now, to make use of that, we are basically, talking about the variation of the instability modes with time; that is what your Landau equation is, but we have started discussing about POD - proper orthogonal decomposition and this is the way that, we go through. I have just jotted down the steps. Let us say, we solve the vorticity transport equation in a time horizon span between t 1 and t 2. Then, we can spilt the instantaneous value of the vorticity into two parts; a mean or an equilibrium part plus a variation. The mean part is defined as some kind of a time average. So, this is the time interval over which the data has been obtained. And, we take this and obtain this, omega mean. Having obtained the omega mean, it is straightforward to obtain the disturbance quantity which is nothing, but the instantaneous realization minus this mean.

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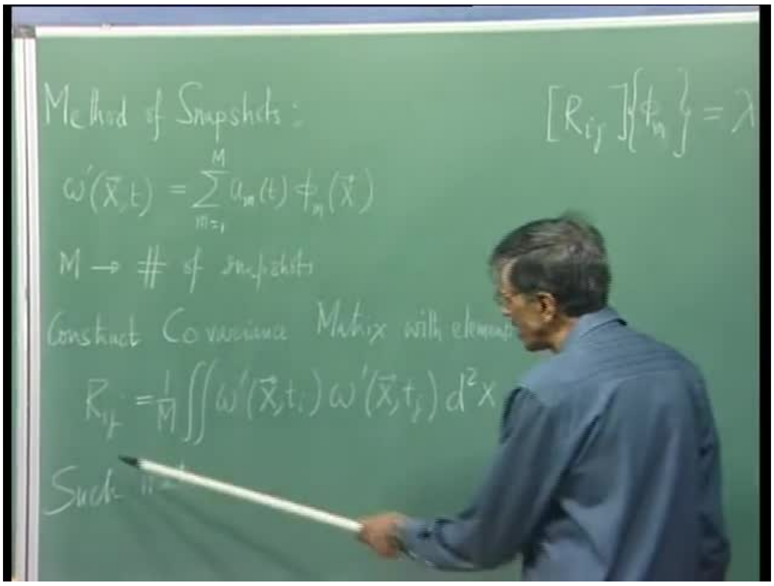

So, let us, now, briefly talk about what that method of snapshot-wise which was developed by Sirowich. What we do is, the disturbance vorticity that we have obtained here, represented in a Galerkin (( )). So, what is it, is some kind of a splitting the variable variation, with respect to space and time, into a time depend amplitude times a space dependent function. So, what we are doing, we are representing the total disturbance quantity in terms of, let us say m such snapshots. So, we take pictures. We have the simulation, either you have done it through experiments, better be, you do it with numerics, because there, you will have more control. Then, what you do is, you try to construct, what is called as a covariance matrix of this disturbance field. So, that element of that covariance matrix is called here as R ij. So, that is obtained by taking the snapshot of the disturbance field at time t i and the snapshot taken at time t j. So, what we are going to do is, basically, we are taking a product of the two functions at this two times, integrate over the whole domain; your x vector indicates your, the domain.

What I am showing you here is in terms of a 2D simulation; that is why we are talking about the elementary volume or elementary area as d 2x, but you could extend the same idea, for three dimensional flow field as well.

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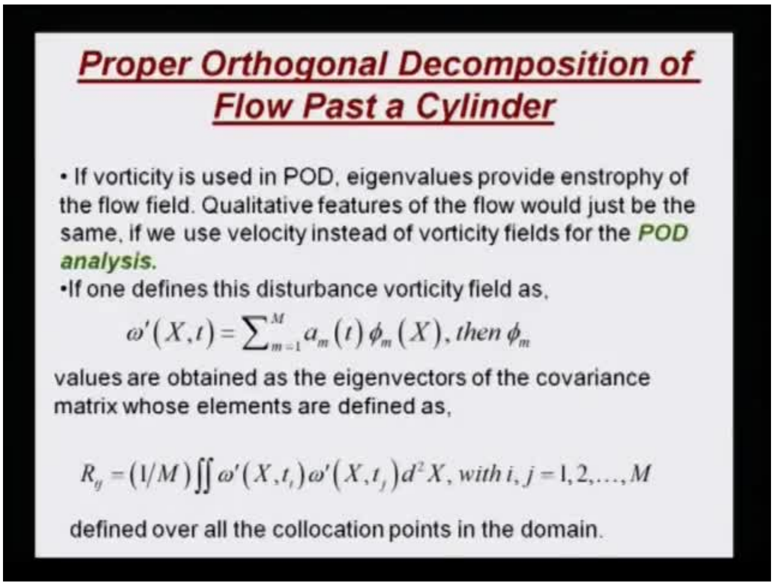

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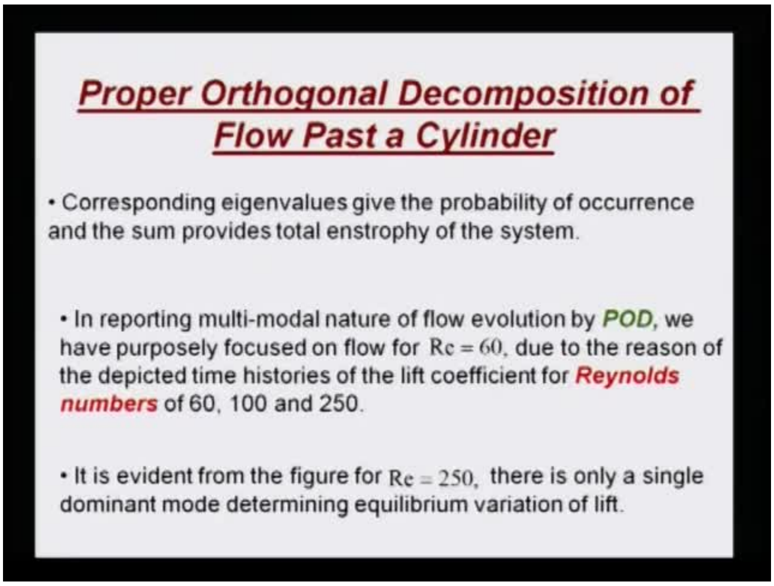

Well, you could trivially also do it for one d. now, what happens is this R ij is obtained by application of Hilbert Schmidt theory, in calculus of variation, which we are not going to do, but which states the following that, if I have this (( )) of time dependent amplitude times this, that space dependent function, actually works out as a Eigen function of this equation. So, this is that covariance matrix; this is the Eigen function phi m; lambda denotes your Eigen value. This is a fairly well developed theory. So, what we are going to do is, we have obtained the disturbance field; from there we will construct this. Once we have this, finding out the Eigen value of a matrix is very easy. These days, you do not even have to, probably write your own; there are many readily available tools, available to calculate the Eigen values and Eigen functions. So, that is what you would be doing. So, solving this equation, you will simultaneously be obtaining lambda, as well as the Eigen functions.

What does this Eigen values give us? These Eigen values, tells us the probability of the occurrence of those Eigen functions. And, if I am, all, sum up all this Eigen values, then, that will be the, providing the total quantity that is under investigation. What is the total quantity under investigation? So, this is like, basically, a square of vorticity here; that is what we call as the enstrophy.

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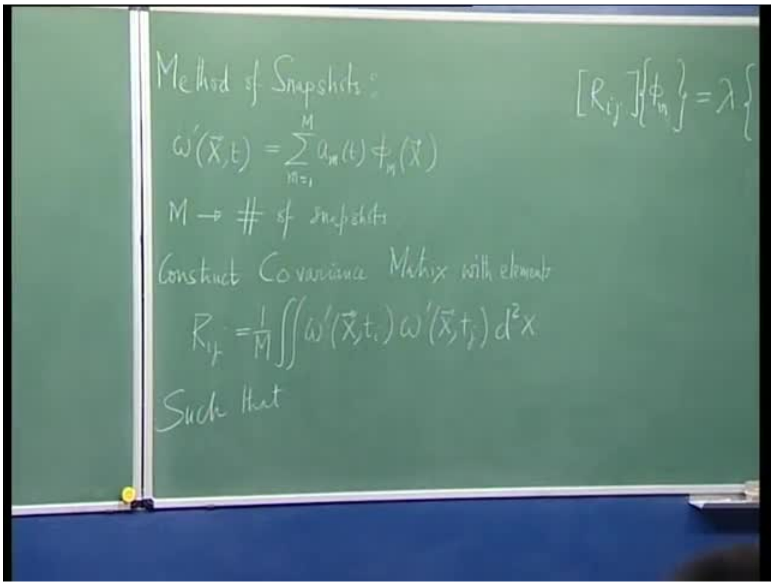

So, enstrophy field, that will be obtained in terms of the product of this functions. So, lambda is a very good measure of the enstrophy content of the disturbance field. So, this is how we are going to live with; we are performing the analysis, POD analysis by focusing our attention on the flow, for Re equal to 60 and I explained to you, what is our main incentive; we are trying to establish why we get multiple Hopf bifurcation; multiple Hopf bifurcation due to what? Is it due to, say, qualitatively different instabilities and different Reynolds number, and when this instabilities occur, they are still dominated by this corresponding single mode, or what we saw in terms of the time history of the lift and what we saw for Re equal to 60; there was presence of multiple modes. So, may be all this multiple modes are present and they selectively dominate in different ranges of Reynolds number and 60 Reynolds number case was very specific, where we saw a visual signature of such multimodal presence. So, that is what we are going to do. That, by and large, if you look at, instead at the highest Reynolds number, that we have shown at 250, we get predominantly a single frequency; while in case of 60, Re equal to 60, we have multiple frequencies present.

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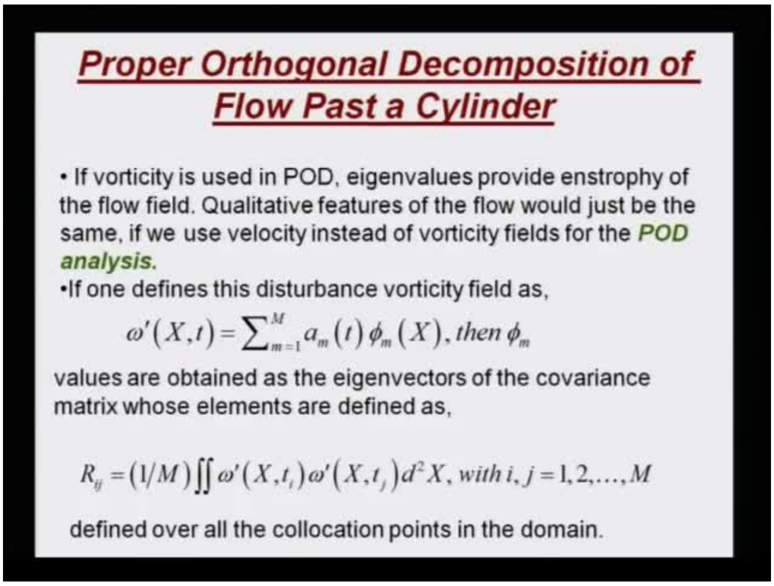

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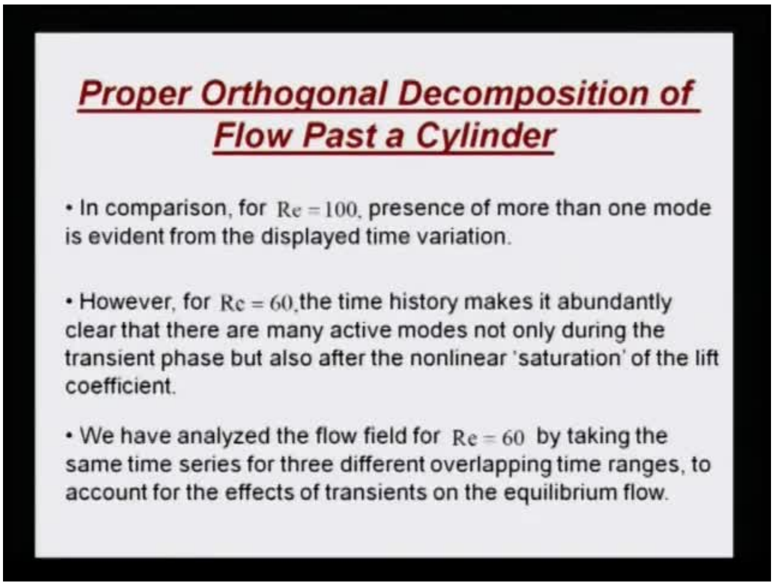

So, that is what we are talking about here. You can see, if it was dominated by a single mode, you would have had a, say a nice envelope, not a jaggedy envelope that we are seeing here. So, that being the motivation for focusing your attention for the results of Re equal to 60, we will go ahead and we will see what we can do about it. Even when we take the results for this Re equal to 60, what we are going to do is, basically, study the same data, but over different time ranges. So, t 1, t 2, I will purposely choose, over different time ranges. Why we want to do it, because, you see, study of POD as a tool has been gone off on a particular route to exploit its, I would say, engineering usage. For example, like, let us say, you are listening to a piece of music and it occupies some amount of memory.

Now, what I could do is, I could take that signal and I can do a, kind a of a model decomposition and if I have, let us say, 1000 modes and I see that, most of the signal noise is contained within first 10 modes, then, I can do this model decomposition and only store ten modes. And, what happens? Instead of, let us say, 10 megabytes, I can get the same quality of signal; it is only with very good acoustics or very discerning ears, people can distinguish between this 10 modes and 1000 modes. But by and large, in a commercial scenario, this 10 modes will do well, and this is what people have been doing, in one of the applications; data compression; massive saving.

Similarly, people have been also using POD as a tool to control flows. If I understand that, this 10 modes are most important, then, I should be able to control those 10 modes, instead of worrying about 1000 modes. I can develop strategies to control this. So, what has happened is, people have been talking about, mostly from these aspects; exploiting their engineering usage. Thereby, people are not too particularly interested about seeing what happens during the transients, like spatially, in the context of fluid mechanics, you have initially a very large transient, and then, the flow settles down to some kind of a statistical stationarity. So, if you are interested in looking at, only the statistically stationary position of the signal, there may not be a much of a motivation or incentive to look at the transients.

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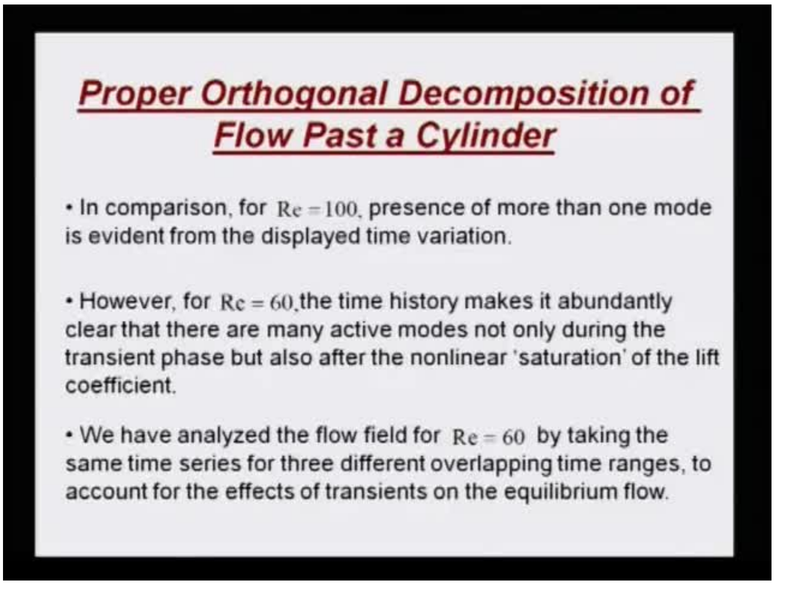

However, as I explained to you that, our point of view is totally to decide for the physics of the phenomenon of instability. So, we want to study, how we get to that statistical stationary state; that information may be embedded to the system, what happens during the transient. For that reason, what we will be talking about, we will take the same time series and we look at three different time intervals and to find out what is happening.

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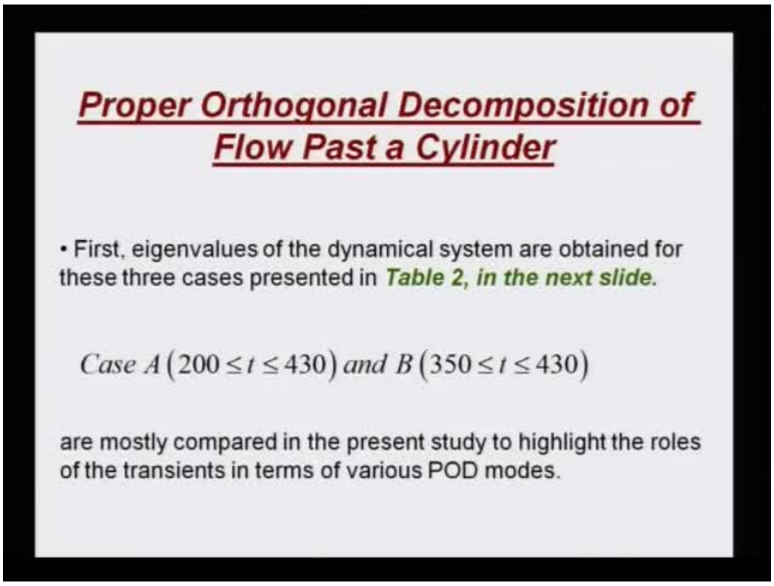

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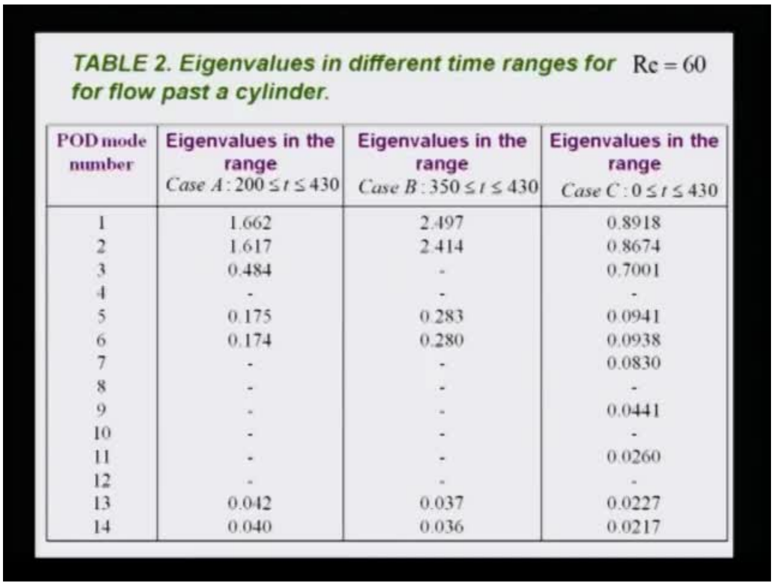

TABLE 2. Eigenvalues in different time ranges for $Re = 60$ for flow past a cylinder.

POD mode number	Eigenvalues in the range $Case\ A: 200 \le t \le 430$	Eigenvalues in the range $Case\ B: 350 \le t \le 430$	Eigenvalues in the range $Case\ C: 0 \le t \le 430$
1	1.662	2.497	0.8918
2	1.617	2.414	0.8674
3	0.484	-	0.7001
4	-	-	-
5	0.175	0.283	0.0941
6	0.174	0.280	0.0938
7	-	-	0.0830
8	-	-	-
9	-	-	0.0441
10	-	-	-
11	-	-	0.0260
12	-	-	-
13	0.042	0.037	0.0227
14	0.040	0.036	0.0217

The results should convince us that, we would be having quite a bit of a success. So, let us, basically, focus our attention to this flow past a cylinder at Reynolds number of 60. We have actually done a simulation from, let us say, 0 to 430; that I will call it as case C. So, that is the full trace, time trace. What we can look at, we can look at a subset of it, 350 to 430, where you may have arrived at, at that statistically stationary part; so, that is your case B. So, by 350, all the transients would have gone, but then, we can also take a case, which I call it as case A, which will have some bit of late stages of transients plus the equilibrium state. So, let us look at this kind of things, and then, we can perform this analysis, obtain this lambdas. And, this table actually shows you the lambdas, for this three different time ranges. When I look at the full time series, then, I see the Eigen values given by this. These are raw Eigen values.

Now, you may notice that, there are some missing spots. I will explain why it is so. Historically, the early practitioners of POD, in this particular field, spatially about flow past bluff bodies, what they noticed? They notice that, you get this Eigen functions, they kind of occur in pair. And, this pair formation is quite significant. However, sometimes, in the early part of this decade, a (( )) Noack and his group from Germany, they discovered that, talking about this modal behavior, in terms of these pairs, is not adequate; there is something that sticks out, which they called as the shift mode. So, what it means is that, if I look at the behavior of the dynamical system without the shift mode, they will be just about varying about a 0 mean, but when you look at the experimental data, we will be seeing, as if there is a kind of a mean shift. So, there has been a shift in the mean flow field. To explain that, they modeled what is called as this shift mode. However, in contrast to that effort of Noack, what we are doing here, we are not doing any kind of modeling; we are taking our full DNS data and then, just performing the POD analysis, as it has been proposed here.

What we notice that, whenever we have the shift modes, they do not occur in pair; they occur individually. So, what I find here, this third mode is one such isolated mode, which does not form a pair. That is why, we leave a blank here. Then, we have another pair followed by a single mode, another single mode here and so and so forth. And, you also notice that, when you have the pair formation, the Eigen values kind of have a similar magnitude. So, looking at this data set itself, you will make out, which is forming pair with what; like the first is forming pair with the second; this one is forming pair with this and so on and so forth; while this last two, thirteenth and fourteenth, also form a pair.

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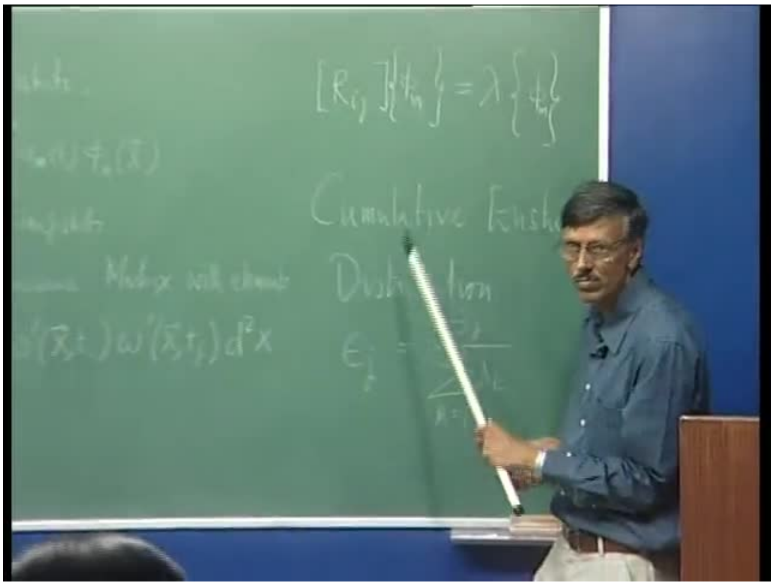

Now, this is for the full time series. Suppose, I look at only the equilibrium state, look at the data set from 350 to 430; then, what we notice, it is only a story of three pairs; nothing else. And, you can ask me that, why did I put all these blanks here. That is because of the essential nature, because this is the most predominant one. So, that better come there and you also know that, there is this hierarchy of this Eigen values. Higher up the position of Eigen value, more energy it contains; because, what we could do is, we could look at a cumulative enstrophy distribution. That basically gives you, which I may call by, let us say, number epsilon j, which would be written as some lambda j divided by summation of all lambda k, k going from 1 to m. You understand why it is m, because we have taken m snapshots; it is a m dimensional metrics. So, I will have m Eigen values.

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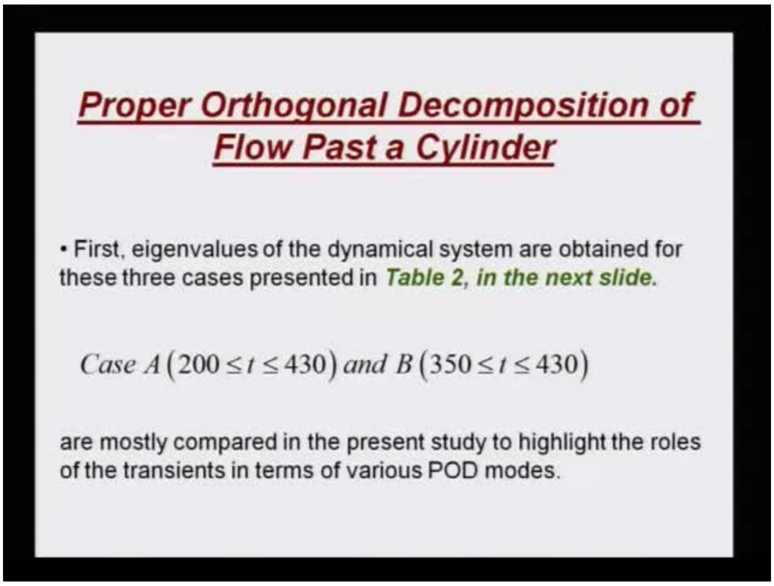

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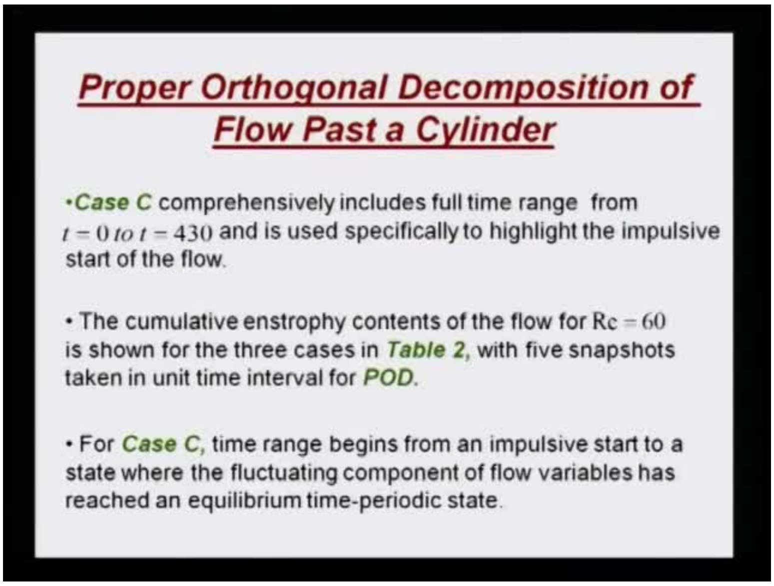

So, that is what we are going to do. So, when we look at the equilibrium state, we only have three pairs. When we look at this particular case A, which is a mixture, and mixture of late transient and the equilibrium state, we obtain a pair here; we also obtain a shift mode and then, we have a another pair here, another pair there. Now, what we need to look at here is, how we get this POD perform, that, when I look at case C, I am also including the impulsive start of the flow; at time t equal to 0, the flow was considered inviscid; that was our initial condition. And then, we saw how the flow picked up it is viscous action, so, that is what we are doing here. How do we perform this POD? How densely we take the data? What we notice here that, here we have taken a 5 snapshots, taken and you need time interval. So, if I am talking about 0 to 430, I have taken a snapshots of 430 into 5.

So, you can see, it is more than 2000 snapshots are taken. This is significantly different than, what people have been doing before. Even we did some of those POD studies for bypass transition, as I showed you earlier, when we were looking at vortex induced instability by a periodic vortices. What we did there, we took only about 10 snapshots or 20 snapshots, in a interval of 10, but here, we are talking about taking 5 snapshots in unit time interval. This is important. This is important because, irrespective of the accuracy with which you are doing your direct numerical simulation, the sampling rate of the snapshots will tell you, what is the frequency range, you are able to display with the POD. If you take too few, then, within the (( )) limit, you will be getting only very few peaks.

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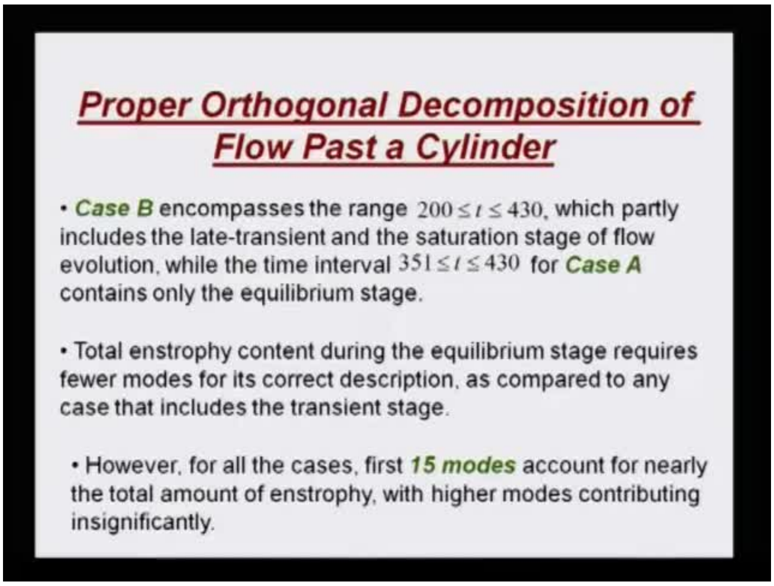

So, that is why, sometimes, even in recent times, I think we are even doing PODs, where we are taking what, about 40 snapshots in a unit time interval, 40. So, you can see, and please do understand that, taking insufficient sampling rate will always mislead you into drawing wrong conclusion. So, it is better, that you go on the side of caution and have more data than less. So, this is something that we need to understand. Now, we have already stated some of this things that, for example, the case B encompasses the range from 200 to 430, which partly includes the late transient and as well as the saturation stage of the flow evolution; while this particular time interval, 350 to 430, that is a case A, it only contains a equilibrium stage. And, the total enstrophy content during the equilibrium stage requires very fewer modes, for it is correct description, as compared to any case that included transient. In all these things that I have shown you, here, most of the time, we have accounted for the total enstrophy content, in those retained modes, to that level of about 95 percent.

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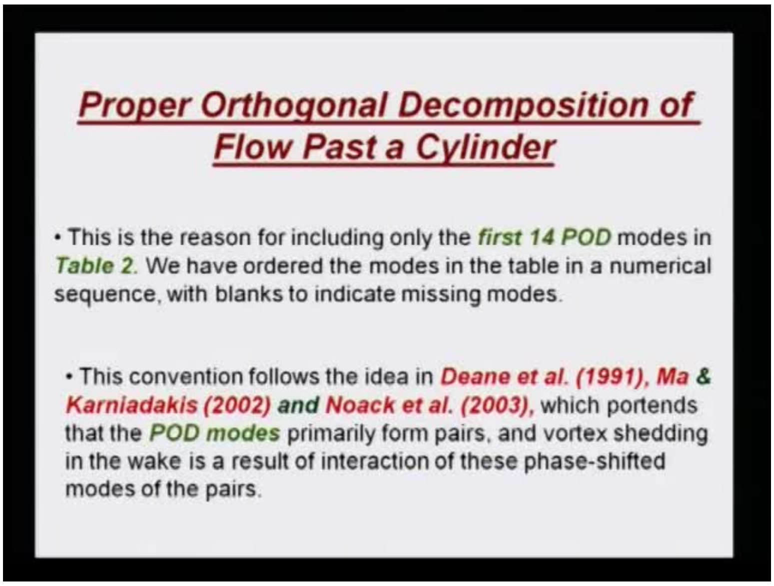

So, what happens is, for the case A, those 6 modes accounted for 95 percent. That is why we did not go any further. And now, if I look at all the cases that, that is what we are saying that, if we take, go about to first 14, 15 modes, then, we account for the most of the enstrophy and the higher modes keep on contributing, incrementally smaller and smaller quantum. So, that is what we need to understand. This convention of identifying modes pair-wise, was used by this group Deane, Karniadakis and Noack et al. This is the work that I was talking to you about. Noack's work is one of the most interesting and outstanding work in the last one decade, and…Well, all of them thought, you need to worry only about the primary modes. If there are non pair-forming modes and Noacks' only talked about one such mode; it is one such mode. But, you saw the table that we projected.

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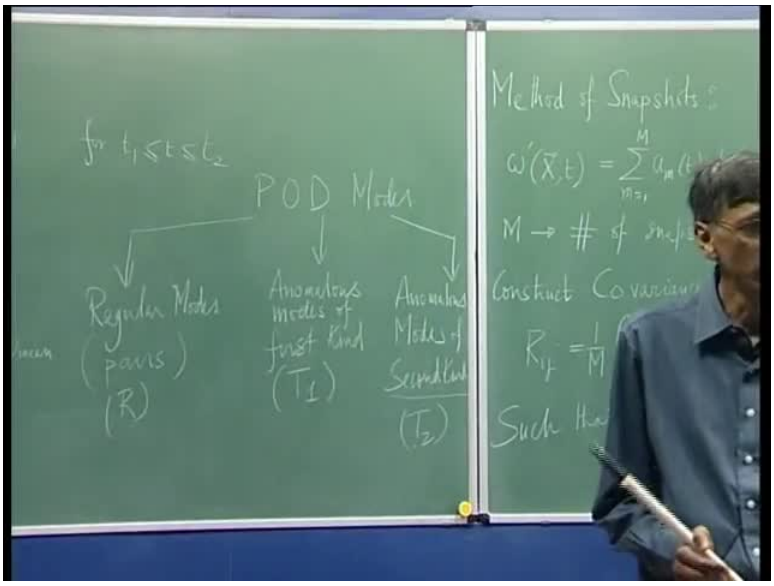

When we took the data from 0 to 430, we had many isolated modes. In fact, in that 14, we had about 4 such modes. So, we do not call them, these modes as shift modes; we have classified the POD modes into the following group. One is what we call as the regular modes; while regular in the sense, what everybody thought originally would be, they are, basically, caused by pairs. Then, we talk about those modes, which do not form pair; this we call anomalous modes and here, in anomalous modes also, we subdivide into two groups. Anomalous modes of a first kind, which will call as t 1 mode; this we call as the r mode, and we are going to talk about a third classification, which is also anomalous modes, but they do form pairs.

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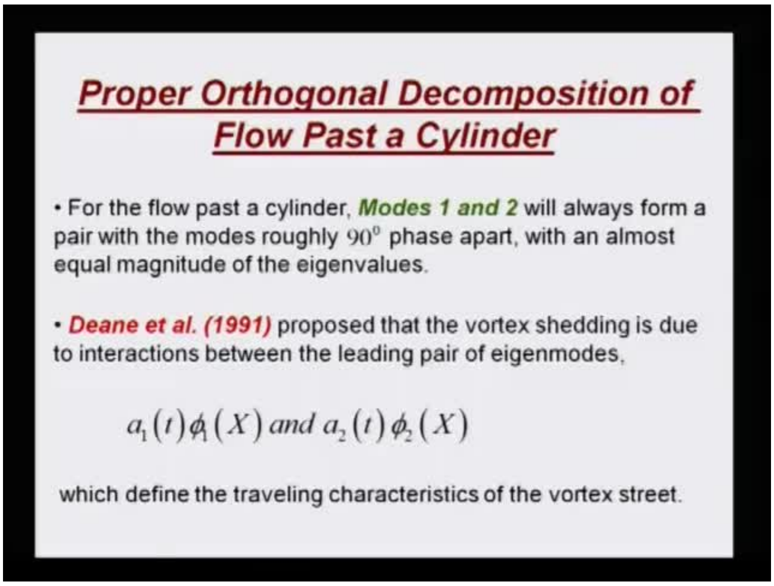

So, this t 2 mode that we are talking about, they also form pair. Why we are calling these as a anomalous mode is simply for the reason that, their time variation, when we relate this POD modes to the corresponding instability modes, they will not follow Stuart Landau equation; that is why we are calling anomalous. Whereas, regular modes, we can show, as we have seen from the c l plot that, it starts off with a linear instability and saturates; that is your Stuart Landau equation. So, this regular modes follow that kind of variation, whereas, t 1 and t 2 modes will not do that. And, what is the difference between t 1 mode and the shift mode? Noack predicted only one single t 1 mode and he assumed, that mode is present at all time.

Now, we are going to see results that, this anomalous mode of the first kind, they appear singly all right, but number one, they do not appear as one; there are more than one modes, like what you would see in case C, when we take the data from 0 to 430, and then, secondly, we can also see that, they do not necessarily remain invariant with time. This anomalous mode of the first kind, they are strict function of time. So, if I look at the flow past a cylinder, this is what is always observed; modes 1 and 2 will form a pair and the modes will be roughly 90 degree phase apart. Now, this might raise a query from your side, why it is roughly 90 degree and not exactly 90 degree? While we are talking about orthogonal decomposition, so, what does that orthogonality mean?

Orthogonality would imply strict 90 degrees; but you recall also, what Kosambi started off with his motivation that, you have a Stochastical dynamical system; you are projecting it into a deterministic basis, which are orthogonal. So, if I look at some of those modes, which form pair and the phase difference is not perfectly 90 degree, that shows that, even this pairs are contaminated by stochasticity. So, that is the main issue. We need to understand that. This work done by Deane and ((quarters)) they proposed that, what you see as the vortex shedding, is due to the interaction between the leading pair of Eigen modes; that is what it is; a 1 phi 1 and a 2 phi 2. And then, they said, the way they interact, that defines the traveling nature of the shed vortices.

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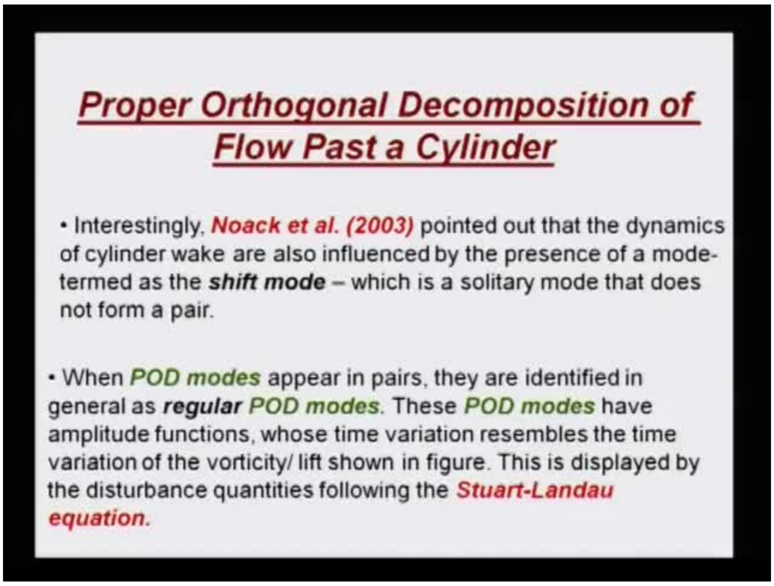

This is what I have been telling you that, Noack et al were the first to deviate from the earlier work and POD by stating that, the dynamics of the wake is determined by the presence of a mode, shift mode, which is a solitary mode, does not form a pair.

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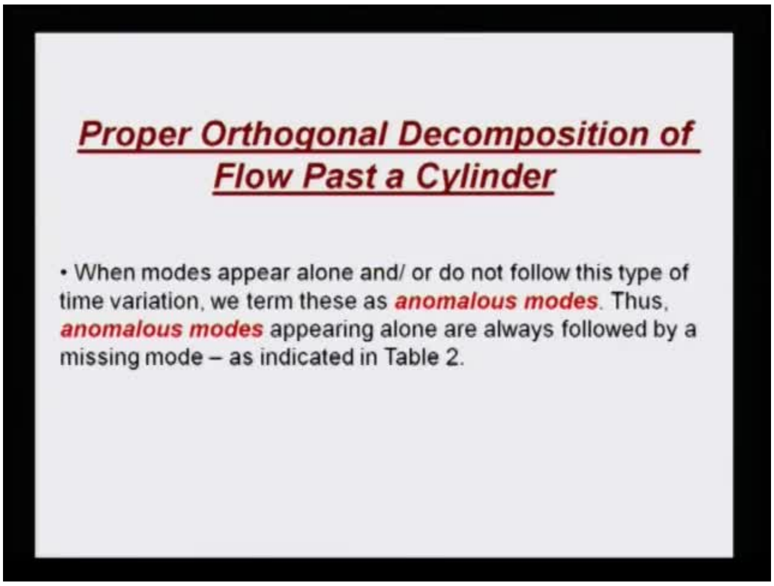

So, this is what we said already that, if I look at the POD modes, and they are pairing, then, we will call them as regular modes, when they follow Stuart-Landau-equation. If they occur in pair and do not follow Stuart Landau equation, then, I will not call them as regular mode; I will call them as the t 2 modes. So, this is the taxonomy. So, this is your definition of anomalous modes. Modes appearing alone are, or do not follow the time variation given by Landau equation, call them as anomalous modes.

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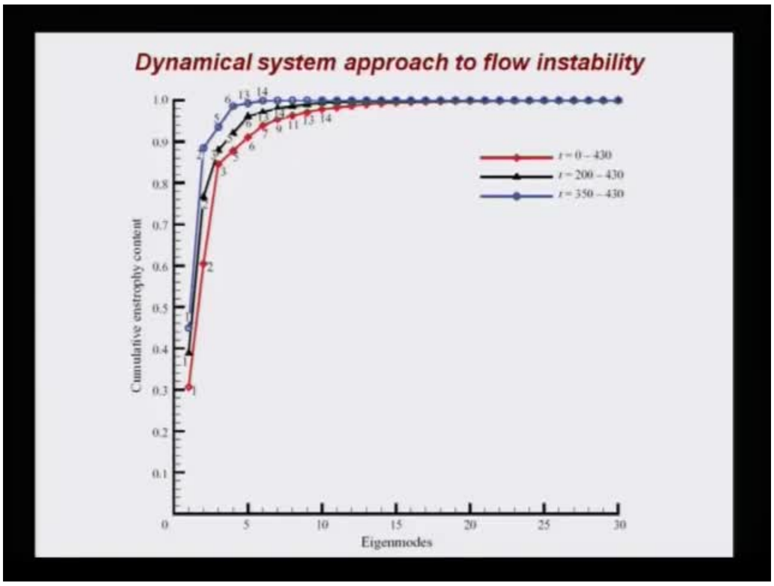

So, whenever I will have a anomalous mode which appears alone, the next line I will put a dot; that is what I did when I plotted in the table. Whenever I saw a single mode, I left a dot, blank and then, I did the counting. So, when I say, there are 14 modes, that does not mean, the, all the 14 elements are filled up; there would be say 1, 2 present and 3 is there, 4 is missing; then, I could again have the fifth mode, sixth missing and so on and so forth. Now, if I calculate this lambdas and plot them; what am I doing, I am actually summing up this epsilon js slowly; so, for the three dataset, I am showing you the result. The red curve corresponds to the full data set from 0 to 430.

So, the first mode contains about 30 percent of the total; the next one also contains somewhat 30 percent. So, this two together, accounts for 60 percent of the total enstrophy. The third mode takes you there, up to about, let us say, 84, 85 percent and then, slowly it goes up. And, if I want to look at this, this is almost, say 99 percent. This, when I take the full data set, it approaches there, requiring more and more number of modes. Whereas, if you look at the pure equilibrium state, where we have taken from 350 to 430, then, you see we reach that 99 percent, is in the first 6 modes only.

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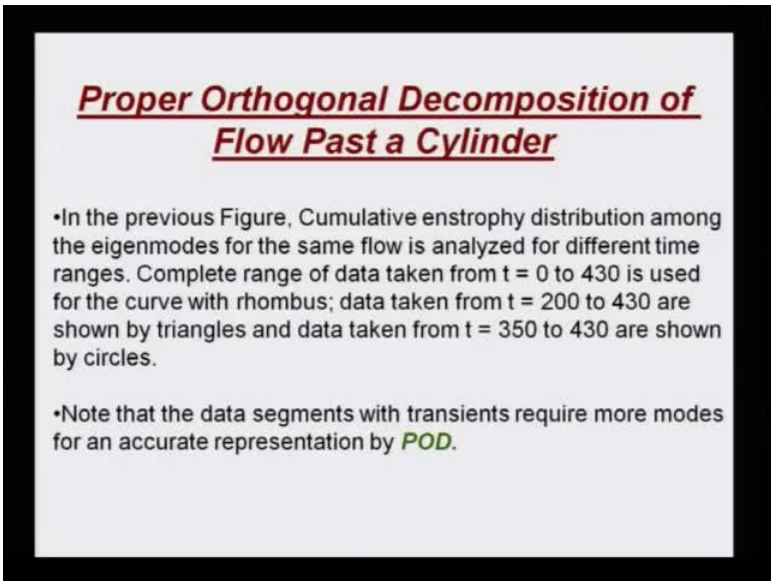

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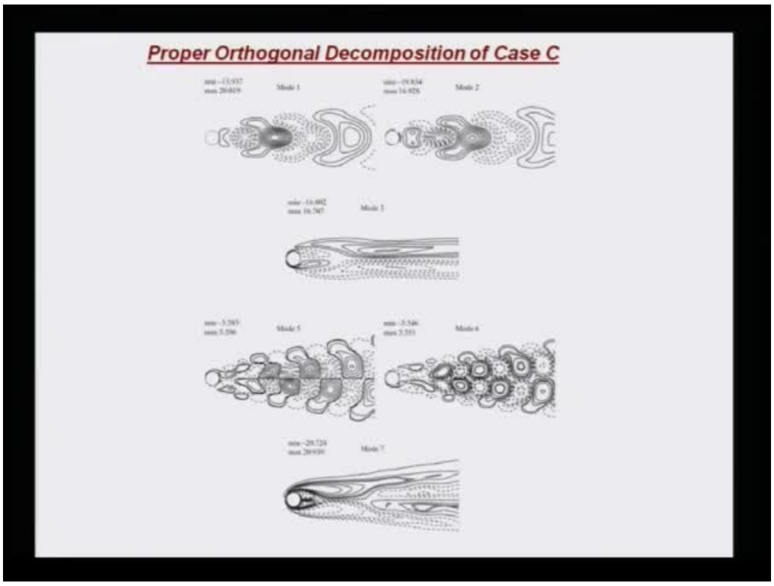

You also noticed that, what we obtained there, we just simply have one pair, 1, 2; then, we have another pair, 5, 6 and the last mode was 13, 14; because we said, look , whatever that the last one accounts for the 99 percent, that will be in the first 14 modes. So, that is what we are going to get; whereas, the intermediate data set between 200 and 430, we see, that is the way it is. So, you see, you having now, a very powerful tool to explain how the disturbance vorticity field is, in terms of this lambda. And, what we can do is, having obtained the lambda, I can solve this equation and obtain this Eigen functions. And, what this Eigen functions are? This Eigen functions are, functions of both space dimensions. So, I am going to get a spatial portrait. And, this is what we are showing you, the data, the 5 functions for the case of 0 to 430, when we have taken the full data set.

What you notice is, mode 1 and mode 2 looks like this. And now, you have no reason to suspect, why we paired them together; they look almost same. Now, you see that, why I said, that to draw a composite picture, looking at one subcomponent is not good enough; lambda gave us some information; Eigen functions gives a something more, that is very interesting. Now, we can see, why we paired them together. This looks like a complement. Then, you can very clearly see the complementary nature. These are essentially what, vorticity information as a function of space. And, the positive vorticity is shown by the solid line, and the negative ones are shown by the dotted line.

You can see a kind of a shift, of one with respect to other by 90 degrees; that is why, where you have, see, in this case as positive, there you will see negative. So, thus, this is a complementary picture, that is what you are doing. And, you see, this is the so called, the t 1 mode or the shift mode, the first shift mode, that is what we get. And, this shift mode is qualitatively different than what you see as a regular modes; that is why we would call it a anomalous modes; they are not the same as we see here or there. And, this is again a pair; this is again a pair and you can also see the maximum, minimum ranges are given; they are almost similar.

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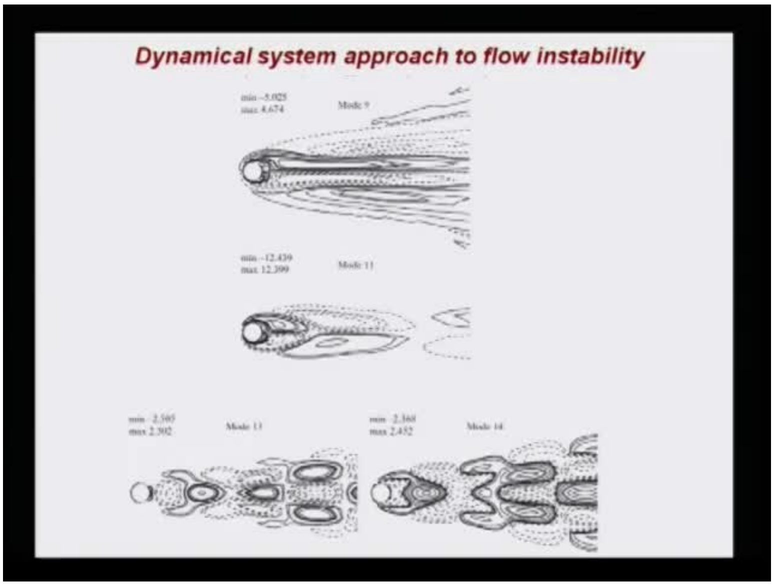

When we look at the pairs and there is a kind of a similarity in the vortical structures too. This is the fifth and this is the sixth mode; fourth mode is missing. Now, we have the next isolated mode; we are calling it as a seventh mode. So, eighth would be missing. So, this is the way, that we are going to see and you also understand, the way we plotted those, that, as the mode number kept increasing, the relative contribution kept on shrinking. So, do not be too much enamoured by interesting looking structure, because that may actually contribute to very little, but at the same time, we must understand that, we are doing a instability study. So, one of the cornerstone of instability study is, small cause having large effect. So, that is why, we keep showing, as much of the energy content, modes that is possible, enstrophy content, modes that is possible, because some of them may decide upon what is going to happen and you see, this is the spatial variations. So, the next mode is also an isolated mode, ninth mode. So, 10 is missing;

then, we have a, another isolated mode, which we call as mode 11. So, 12 is missing; then, we have the, finally, the pair 13 and 14.

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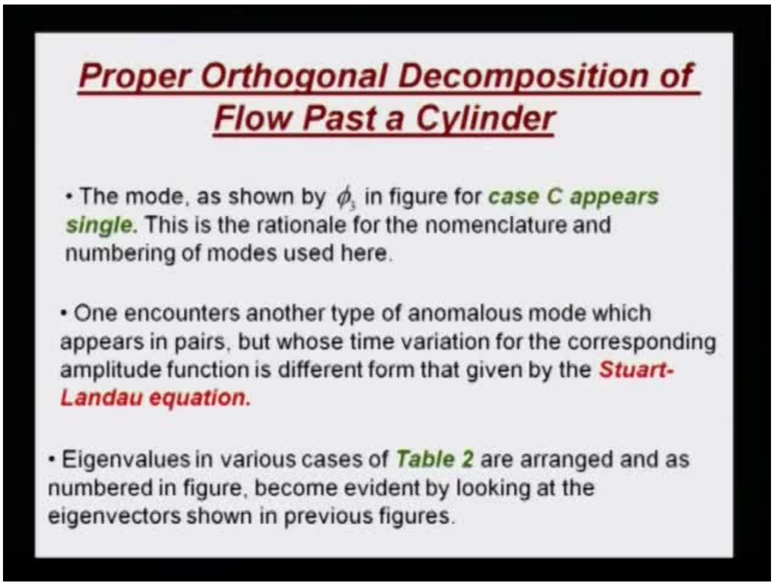

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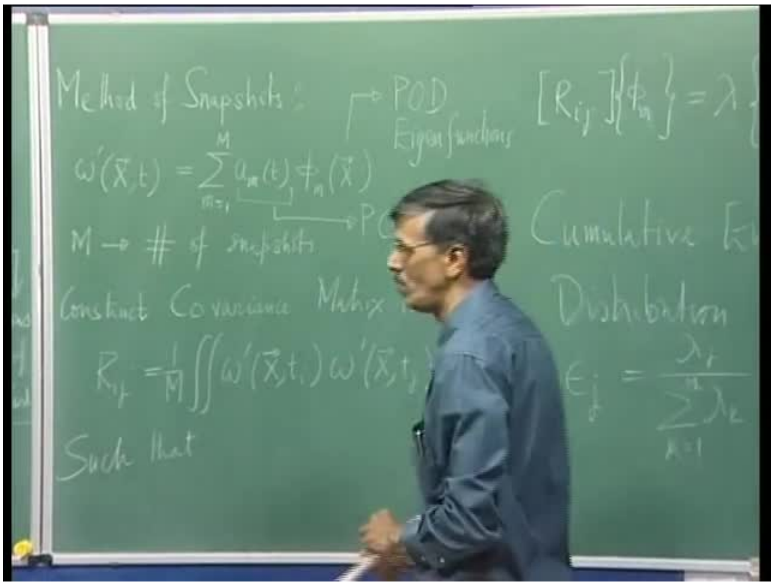

So, you have now, a fairly a decent idea, how we went about forming this composite picture. So, we will look at it, as we go along. So, we are not too interested about this, only the space variation; we would like to also, get in to the time variation; that is, that is important. And, what is the time variation? Time variation is given by this. So, what we say, this is our Eigen functions, POD Eigen functions and this is what, we will call it as POD amplitude functions.

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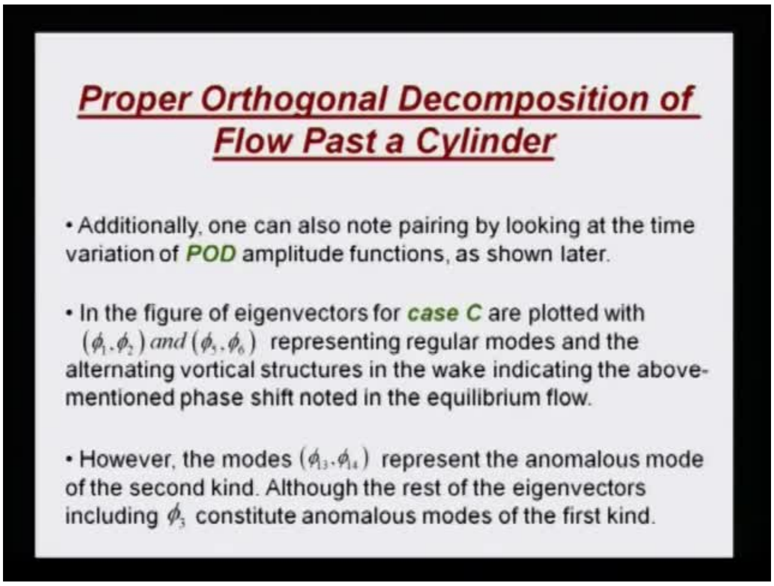

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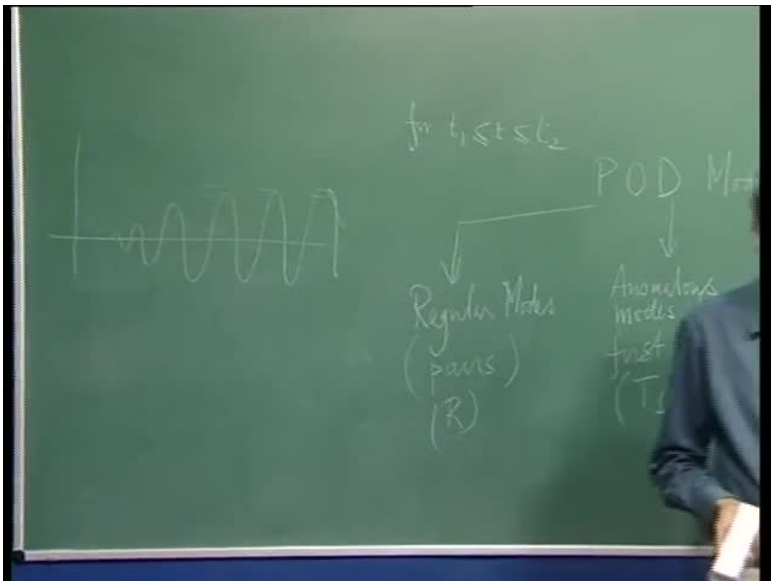

Now, what we just now seen, we have seen how lambdas vary, how phi js vary, but we still have not talked about the time variation. So, we need to do that. To do that, we need to find out what this POD amplitude functions is going to be. We have talked about all of this. We just want to point out one thing about the case C that, phi 1, phi 2, phi 5, phi 6 represents regular modes, showing alternating vortical structures in the wake and what we notice, though, this is 13 and 14 forms a pair, but their time variation is not like what we are used to seeing. What we are used to seeing is that, the sketch that we have shown before; the Stuart Landau equation would show this kind of a time variation, which will pick up from this and then, will reach this equilibrium state. So, that is your Stuart Landau mode. This 13 and 14, if I look at the time variation, then, they will not show this kind of linear instability followed by non-linear saturation.

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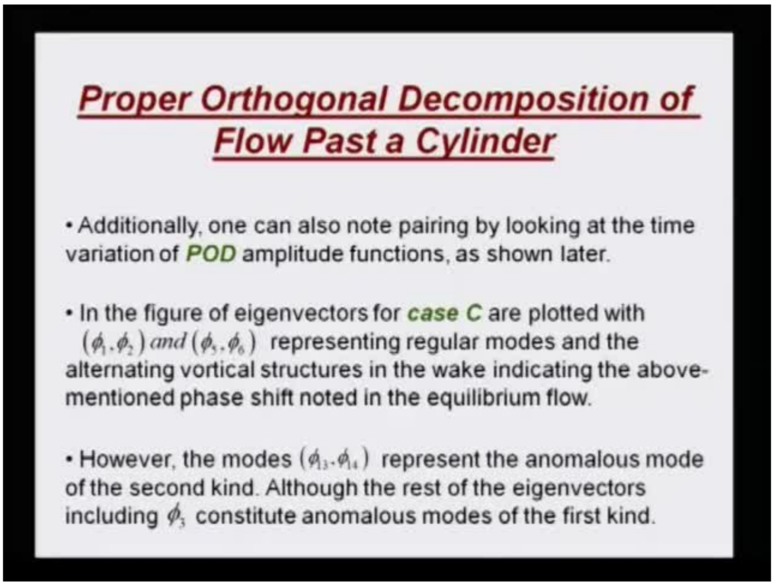

This is somewhat different. How do we know this time variation is? Well, that should come from this. So, we have to get this.

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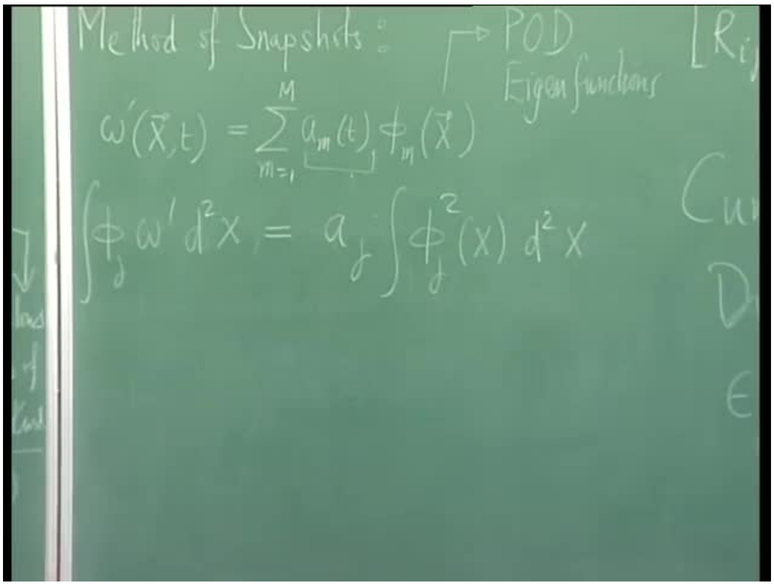

Now, if we call this functions as orthogonal functions, then, what I could do, I could obtain this a js, 8, a js are a m of t; how do we do that? How do we do that? They are orthogonal. So, if I have this, I multiply this by phi a m and integrate, then, what would I get. So, if I multiply, if I take this description, if I take this description and multiply this, let us say, by phi j and then, integrate over the whole domain, what do I get from this side? Only, I will get, a j and… So, I can get a j; that is it. So, this is the whole concepts. So, we can do that.

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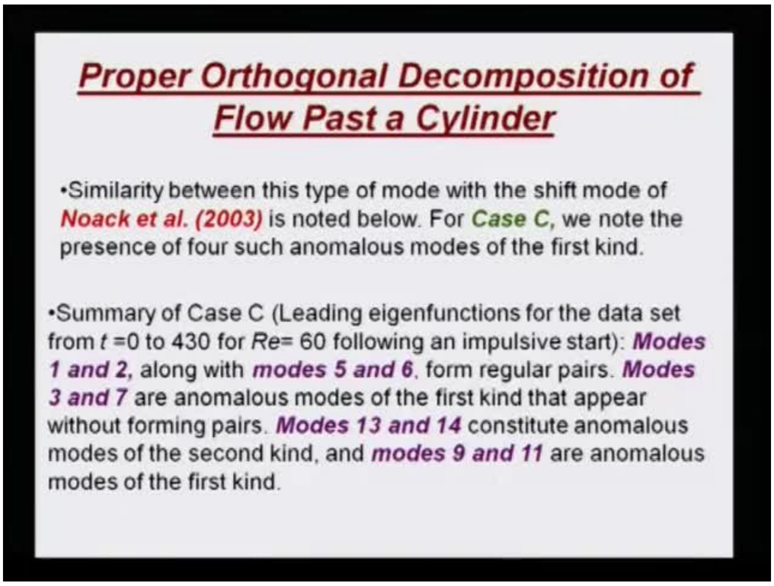

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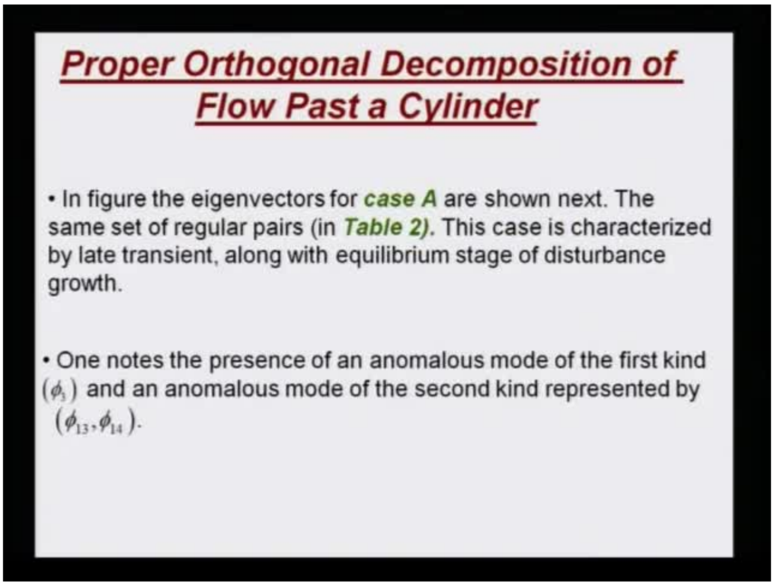

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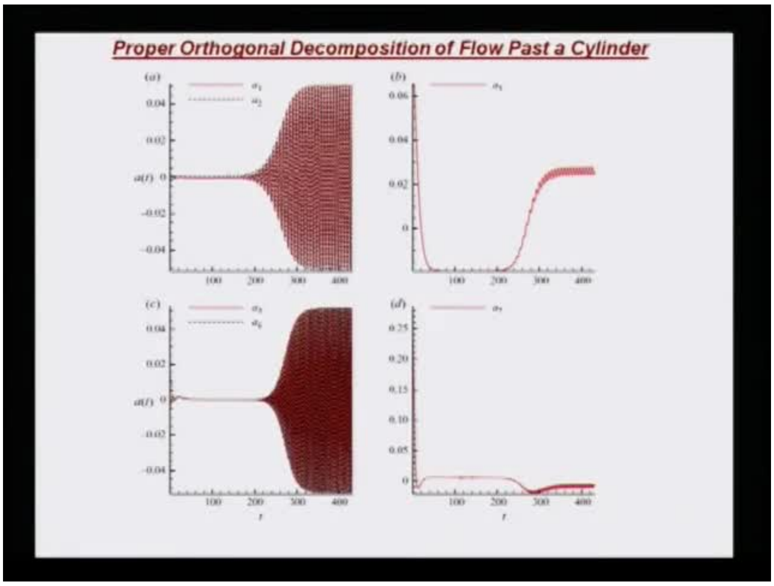

So, basically, if I want to summarize case C, which we have seen so far, in terms of the leading Eigen functions, mode 1 and 2, along with mode and 5 and 6, form regular pair; modes 3 and 7 are anomalous modes of the first kind, that appear without forming pair; modes 9 and 11 are also anomalous modes of the first kind; mode 13 and 14 are somewhat different and that difference is obtained by noticing this. I will show you what they would look like. These are those a js that we have; that we can obtain from here. So, this is your a 1 and a 2 plotted together and you see, they form such a nice pair; they complement each other; time shifted, that is what you are seeing.

This is your a 3 and a 3 is, got a very interesting behavior. Initially, after impulsive start, it has a large value, then, it goes to the other side, becomes negative, then, it remains flat; and then, once the linear instability starts, then again, it picks up and it settles over there. And, although, you see, recall what Noack said that, this is going to be present at all time; which is not true; you are seeing there. It is present at all time later in the equilibrium state, but also, this is not steady. We are seeing this fluctuation and this fluctuation is something like a Strouhal frequency. This is characteristic of that. So, you will get that.

Now, a 5 and a 6 form this mode, that we can very clearly see. Then, a 7 was a anomalous mode of the first kind and once again, we are seeing the same thing. See, what happens is, when the linear instability is not there, they are, kind of flat, time independent; some nonzero values.

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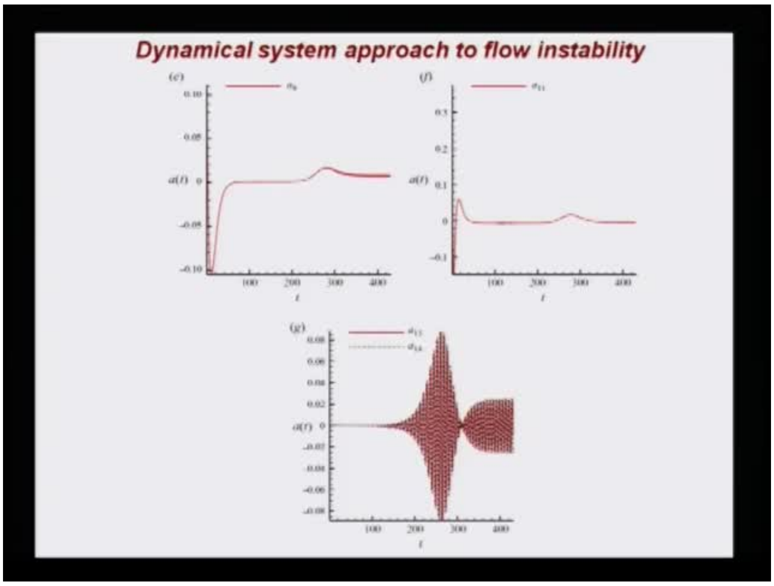

So, this value, being nonzero, had shifted up, call, I mean, really prompted Noack et al to call it as a shift mode; as if, it had shifted the whole thing by a data. Same thing about a 7, that you are seeing here. Let us go through and see the other modes and then, we will find out what we are talking about. The next two were, you said are going to be anomalous mode of the first kind, that is your ninth mode and eleventh mode. This is also something similar, that you have seen. And, as you can see that, as you come, go to higher and higher mode, their contributions are coming close to 0.

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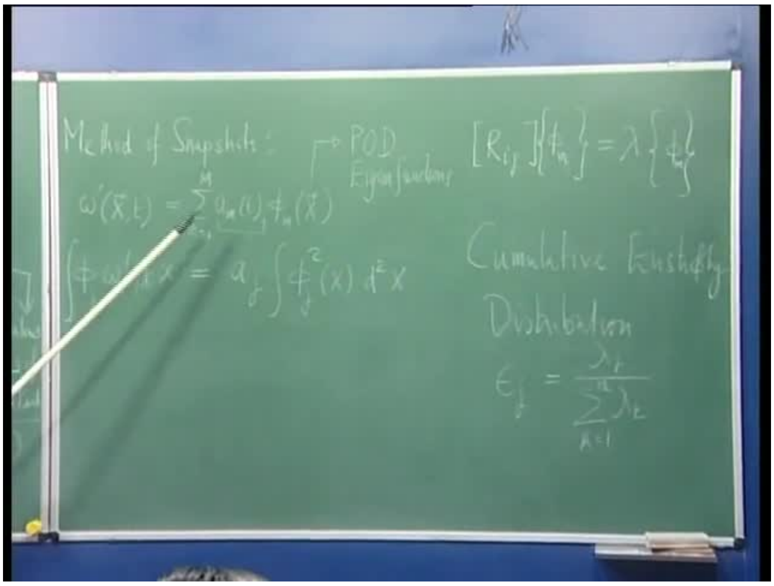

So, when I represent this, this contribution level should come from the magnitude of m. What lambda gives there, is nowhere present here. So, we make that observation that, lambda determines the relative contribution. But what we are noticing here, when I represent the disturbance function, the relative quantity contribution would come from m and that is what you have seen. They are very close to 0. As you go higher and higher, their contributions are smaller and smaller. This is the interesting bit. I told you that, the thirteenth and fourteenth form pair, but they do not go like, some kind of saturation; they appear like wave packets. They are like wave packets. So, they are present in, some time, then it quenches there, then, again it picks up; and, if you would have done the calculation, again it would have come down, what happens, so on and so forth.

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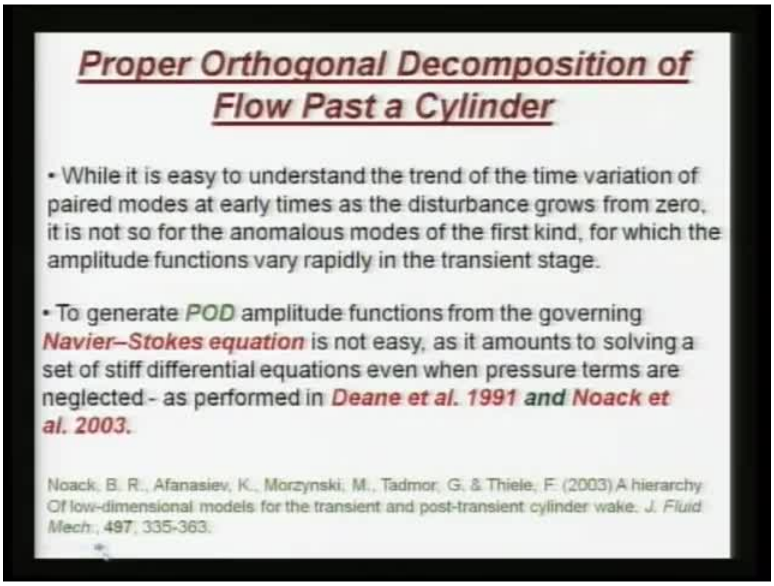

So, this is what we called as the t 2 modes. Now, you should, you understand, why we are calling this. In fact, in recent times, we have been looking at air fall aerodynamics. Yogesh is looking at that with great care and we are seeing some, different instability mechanisms have different foot prints; but by and large, this general classification is nearly universal. This is nearly universal. I will not be able to show it to you; I will give you the reference in the next class, but we have seen that, same type of properties.

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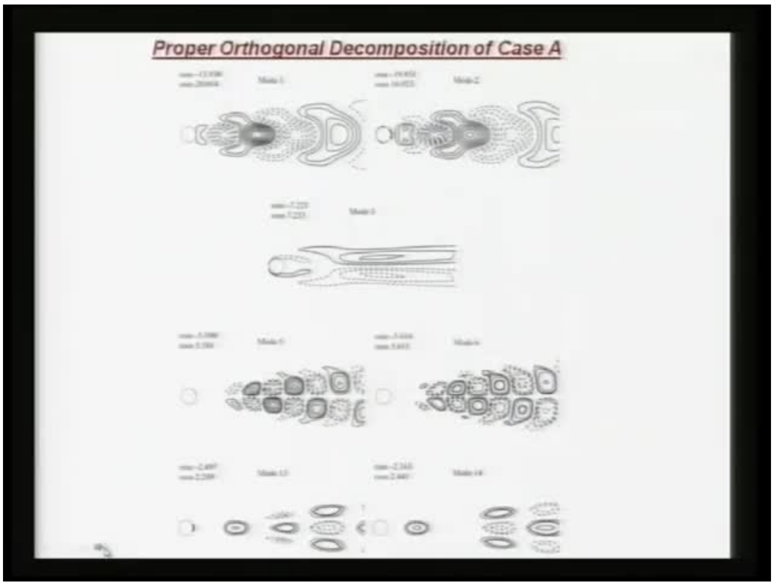

Now, this is another picture, what we have done for case A. What was the case A? Case A was from 200 to 430. So, what you are seeing here, again the leading Eigen functions forming pair; this is your shift mode; and fifth and sixth again is a regular pair; and then, there is nothing in there; you go all the way upto thirteenth and fourteenth, which is the t 2 mode, that we just now saw; similar behavior. So, this is what we are going to see. So, you understand that, there is a need to really, further explain, why and how this things relate. This is your case B, where we have just looked at the data set from 350 to 430, and you can very clearly see three pairs; that is it and that takes care of everything.

So, if you are trying to exploit the flow, I mean POD analysis, in terms of flow control, etcetera, and if you are only interested in the steady state, then, probably taking a small such data set, once you have reach the equilibrium stage is quite good. And then, you will have to only worry about this very few modes and you should be able to control them to your advantage. But when it comes to understanding flow instabilities, it is always better that you take the full data set, because you do not, you want to explain how you have arrived here; how you have arrived here. In the next class, I will explain to you, what is the source of these anomalous modes? Why do they appear? That is rather important and this we have seen.

So, let us make some observations, when we are looking at flow past a cylinder, we can easily understand the trend of the time variation of the paired modes at early time, as a disturbance grows from 0; it is not so, for the anomalous modes of the first kind, for which the amplitude functions vary very rapidly during the transient stage. This is rather important. So, basically, the shift that you are getting in the final state, the history lies there.

So, that is why, it is necessary for you to take the full set; otherwise, you will be not able to say, why you have arrived at that particular equilibrium state; that shift has occurred in a transient manner. And, to generate POD amplitude functions from the Navier- Stokes equation is not so easy. What we have done here, we have obtained the solution; we calculated the disturbance vorticity and then, we calculated a of j. What some people have done, they have tried to put this back into the Navier-Stokes equation and to derive an evolution equation for a js. So, what will happen? Of course, you have all this modes. So, the basic intention is to convert the PDEs into ODEs; because your, space dependence is given in terms of your phis.

So, once you have put in that information and integrated over the whole domain, then, you will get that. But there is a problem. If I am looking at, let us say, the Navier-Stokes equation, in u v formulation, then, I may have this kind of depiction for the velocities; what about the pressure? So, that is where some people have done some kind of a cardinal sin; they have actually omitted pressure terms and that is one problem. And, there is another problem, that will be taking about when we meet next time.