Instability and Transition of Fluid Flows

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Module No. # 01

Lecture No. # 32

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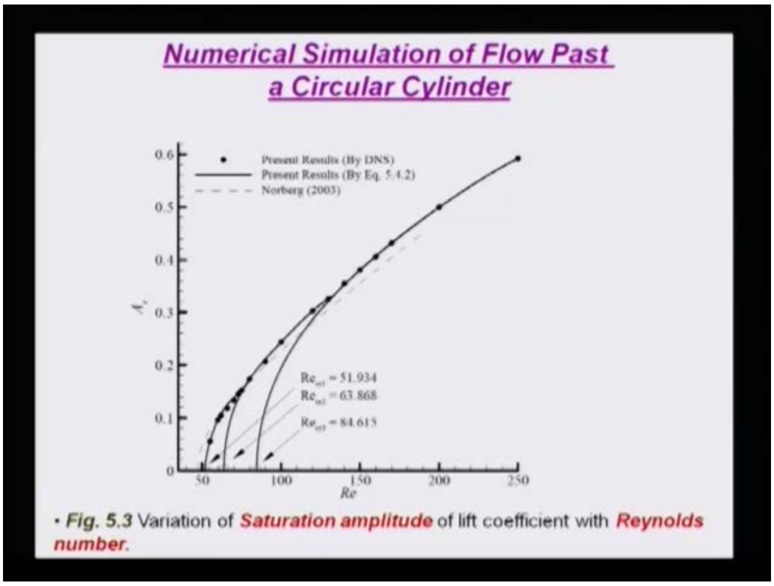

So, in the last class, we have been talking about multiple Hopf bifurcation, that we predicted by plotting equilibrium amplitude versus Reynolds number. And, if we do present our dilute numerical simulation results, what we notice, there are visual kinks. So, these are the locations where it would appear, that various bifurcation sequences have merged. For example, this could be the coalescence point of the first bifurcation and the second bifurcation. So, essentially, we are not completely throwing away the Landau model itself, all that we are saying, to use the same equation, we need to actually explain it in terms of multiple bifurcations.

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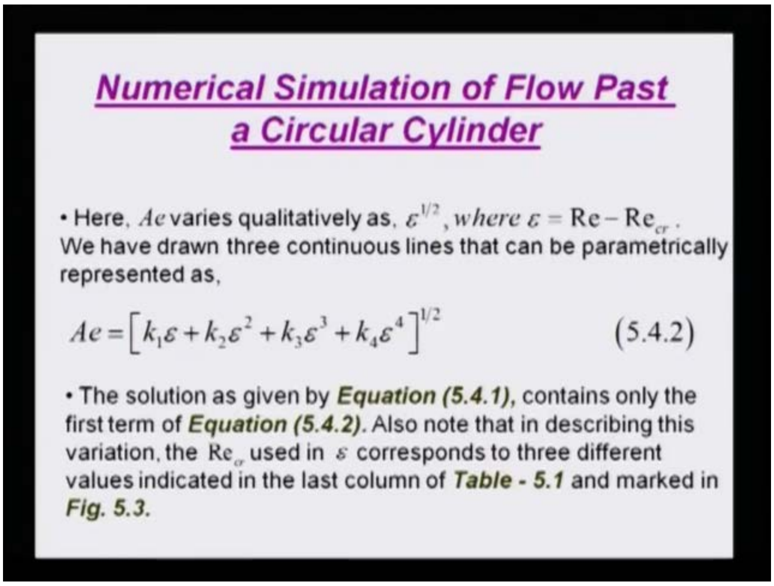

So, this is what we wanted to do and then, what we talked about that, we could perhaps try to fit it with a sort of a parameter in terms of epsilon; epsilon is nothing, but the departure of the local Reynolds number from the critical Reynolds number. And, if it is truly parabolic then, what should have happened, we should have had only a k 1 term; these other three terms should not be there. But let us see, what happens, when we do it like this, so that, the A e square is given in terms of quartic and then, we have the data on those three segments.

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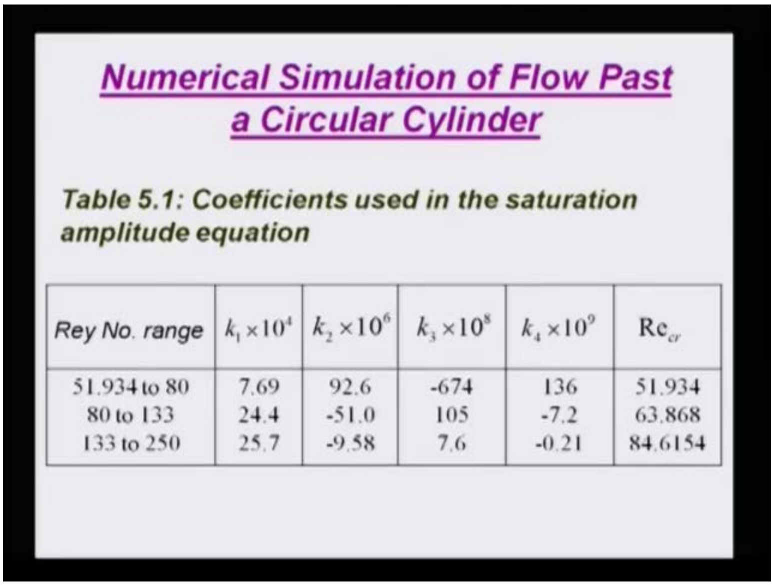

What we do is, try to fit these coefficients k 1, k 2, k 3, k 4 and then, we see what happens. So, we have basically, three ranges of Reynolds number starting from, let us say, about 52 to 80, then, 80 to 133 and 133 to 250. k 1 is in the range of 10 to the power minus 4; whereas, k 2 is in the range of 10 to the power, well, like, minus 5 or minus 6, depends on which range you are looking at and these are much lower in value. For example, this goes as 10 to the power minus 6 and while this one is even smaller by one order magnitude. So, ideally, what you can see is, predominant dependence on k 1 and lesser dependence on this. So, this tells us that, even this kind of empirical fit, will lead you to that same kind of Landau model.

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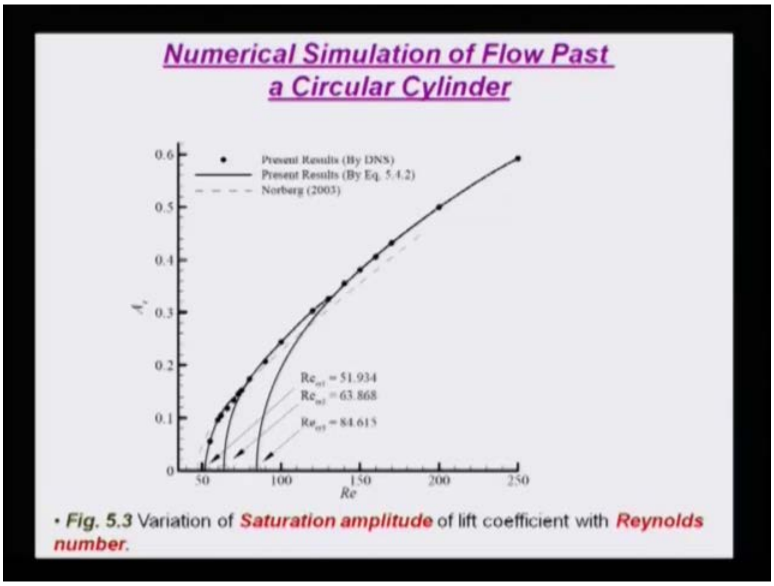

• Fig. 5.3 Variation of Saturation amplitude of lift coefficient with Reynolds number.

So, this is in a sense that, what we get; we can get those three sets of values merging onto the three points of Hopf bifurcation. And, in this, we have also put in a co-relation which was produced by Norberg. Norberg actually used, a set of experimental results for his PG thesis work, in a wind tunnel which is characterized by a turbulent intensity of about 0.06 percent, and based on his experiments, he has produced this kind of empirical fit. So, these are not truly experimental data, but it is a kind of an empirical co-relation. And, that, kind of gives you a value which is close to 49 or so. So, this is the kind of thing that, we would see. So, now, what I would like to do is, move over to find out, what the role of this background disturbances going to be.

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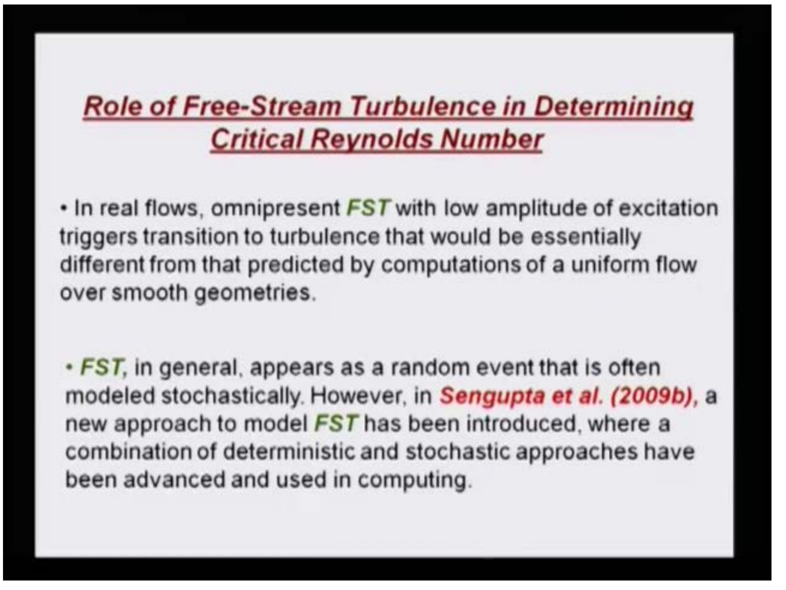

So, what is the background disturbance that we can think of, let us say, in a wind tunnel? That would be the free-stream turbulence. So, we state very clearly that, if we are looking at real flows, we will have these, always present free-stream turbulence; it could have a low amplitude of excitation, but that may be good enough to alter the bifurcation sequence. Free-stream turbulence, by definition, actually, people have tried to view it as a random event and people do try to view it and modulate stochastically. One of the most popular way by which, people in computing have used it is, treat it as some kind of a random number and then, impose that random disturbance to the inflow part of a flow. We have actually departed from that point of view, that free-stream turbulence has a definitive origin and what is that origin; if you understand how we design experimental devices, you would know where the free-stream turbulence is coming.

What is it that you are trying to do? We are trying to mimic the actual flow, where may be the body would move in a quiescent flow. Or what we are talking about a quiescent flow, we need not necessarily be quiescent; there could be still some kind of a convection of vertical disturbances. Whatever that is, in the wind tunnel, what we do, we keep the model of the body fixed and create a flow. And how do we create a flow? We create a flow by having some kind of an ear screw; we have some kind of a circulatory device and that creates a flow, which converts it into a rectilinear flow. Now, in the process what happens? Since the flow has begun with some kind of a circulatory motion, the remnant of that circulation is there. In designing wind tunnel, what we try to do is, we try to break down these larger vortices into smaller ones and that is what you see, you have all those honey combs and screens doing it for you.

So, basically, what we are talking about, in terms of background disturbance, what we called are those remnants, as those large eddies, which are originated in the size of the propeller that is driving the ear stream. But through the action of the screen and honey comb, we break it down to smaller ones; so, that is what we get as a free-stream turbulence. So, having obtained that point of view, we try to really model the FST. And, we understand the different tunnel will have a different designs; so, we will have a different FST environment. For that matter, talk about the same tunnel, if I try to run the experiment at different speed, then, what is happening? I am rotating my fan at a different RPM, and that is why the FST is going to be different. This is something people did not quite appreciate, but what we did say that, we need to view FST as a combination of two components; one is kind of a deterministic component, that comes from the fan size and its speed, so on and so forth.

In addition, when we have these smaller eddies being created by the action of these honeycombs and the screens and the contractions sections, which all modulate then, that gives you some kind of a statistical appearance, which people have been talking about. So, let us now look at what we can do.

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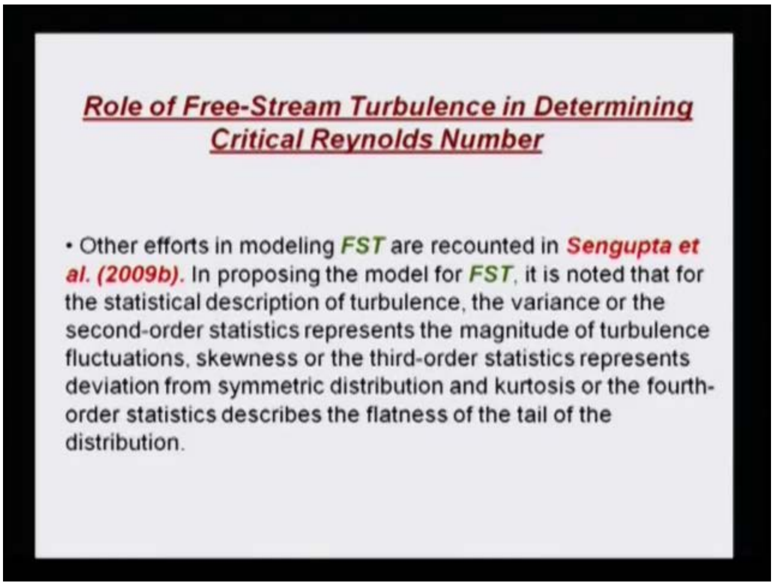

In modeling the free-stream turbulence, we note that, the statistical description of the turbulence gives us various moments.

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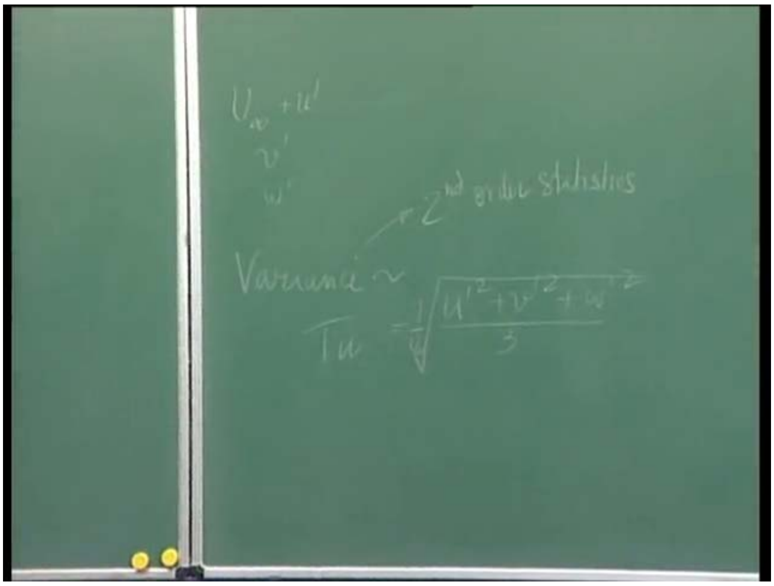

One of the most common moment that we usually use is, let us say, I am trying to create U infinity. So, in the tunnel, what we end up doing is, some kind of u prime, v prime, w prime and where this u prime, v prime, w prime are those random stochastic component. Then, I could define the variance; what about the mean, mean is straightforward. These are truly random quantities, the mean will come out as U infinity. The variance goes as given by this turbulent intensity that we talk about, which is nothing, but u infinity square, v infinity square, plus w infinity square divided by three, and then, I will take a square root of that; I will non-dimensionalize with respect to U infinity. So, that is what we are saying. So, this is what, this is a second order statistics. And, this is what you will find in the literature; all the time, people talk about this, as if the single number characterizes the tunnel. But as I explained to you, the same tunnel working at different speed, if you give different value of T u; so, it is quite a misnomer to characterize a tunnel by a single number like T u.

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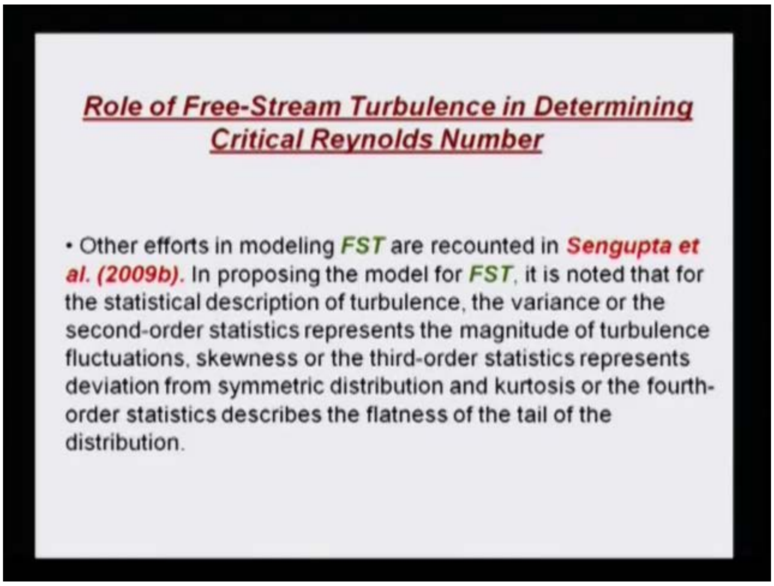

Now, what happens to the other statistics? We can talk about the third order statistics, which is called the skewness. If the statistics was truly combination of infinite number of sources, they would have all worked independently; then, through central limit theorem, we would have seen this probability distribution function of these variations would have been like a pure bell cup, like Gaussian, perfectly symmetric. However, it does not remain. So, as I told you, there is a distinct bias, because of the design of the tunnel. So, that is what, this skewness basically represents. The skewness represents a deviation from the symmetric distribution.

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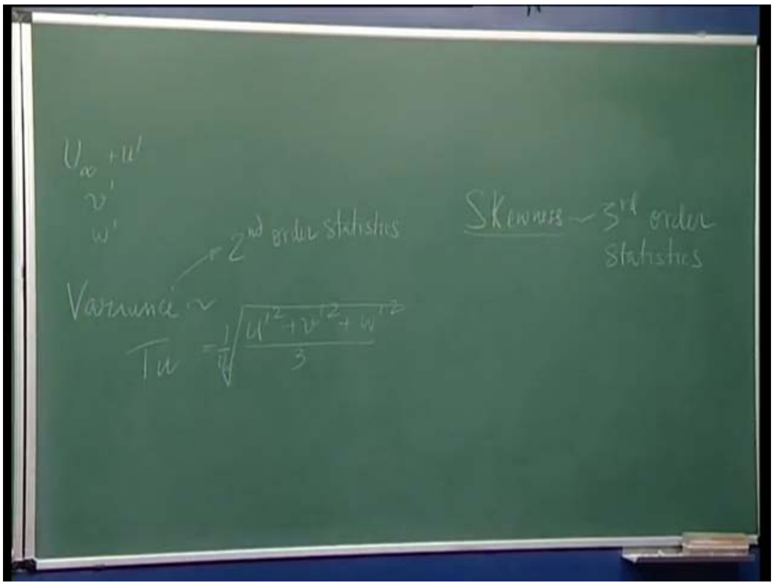

So, my PDF is not symmetric; it is asymmetric; and that is given by the skewness. So, this is your third order statistics. And, it has been actually shown long ago by Bachelor and Thompson that, even if you are looking at, say homogenous turbulence, as it is created in an experimental scenario, this skewness is contributed by the instability of the flow.

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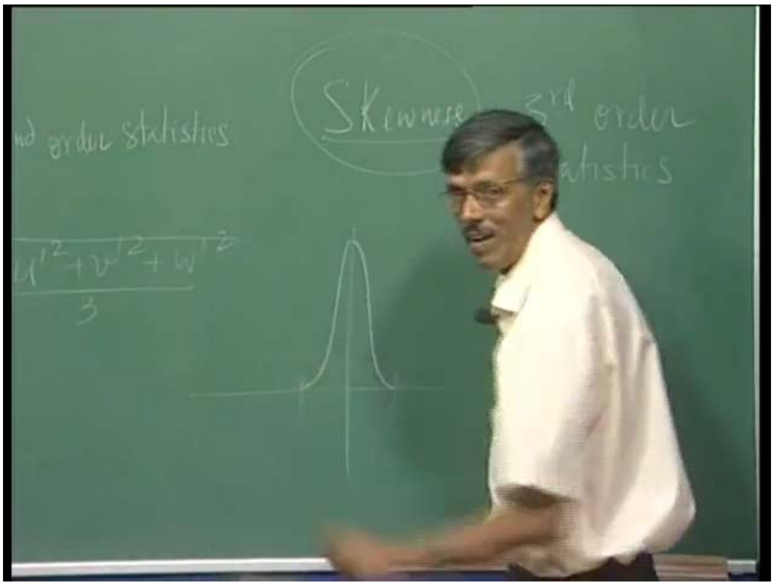

So, there is this seed of instability, that is there in the skewness itself. And, if I create a, kind of a very well designed tunnel, then, I would have a U infinity here and then, that is, to the variance etcetera would be such, the all the moments would be such that, I actually would like to get it like this; a perfectly symmetric distribution like this. But as I told you, it will not be symmetric, and there would be one more factor that, we are saying it is going to a band limited to a small number; and now, it becomes 0 and remains so; but that is not what you get.

What you are going to get is, some kind of a tail at the high frequency; that is given by the fourth order statistics or kurtosis; this is what is called as the flatness of the tail of the distribution.

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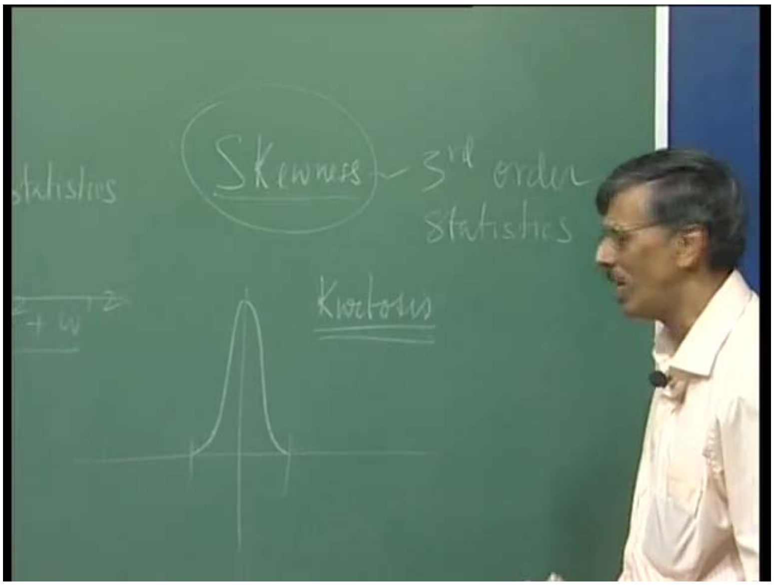

So, we need to really worry about all this. So, while skewness gives you some kind of instability, the kurtosis tells you what happens at high wave numbers, high frequencies; kurtosis tells you about that.

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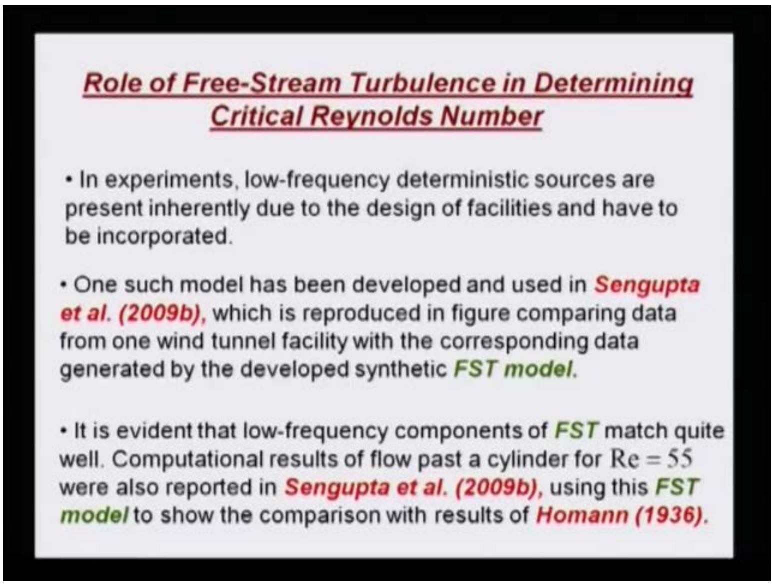

So, you know, computer generated random number cannot be a true substitute for free-stream turbulence. This is what we did realize and we did model it. And, in experiments as I told you, they are all low frequency deterministic sources, because of the way the fan is positioned, because of the way the honeycomb, the screens are; that, creates a kind of a deterministic source.

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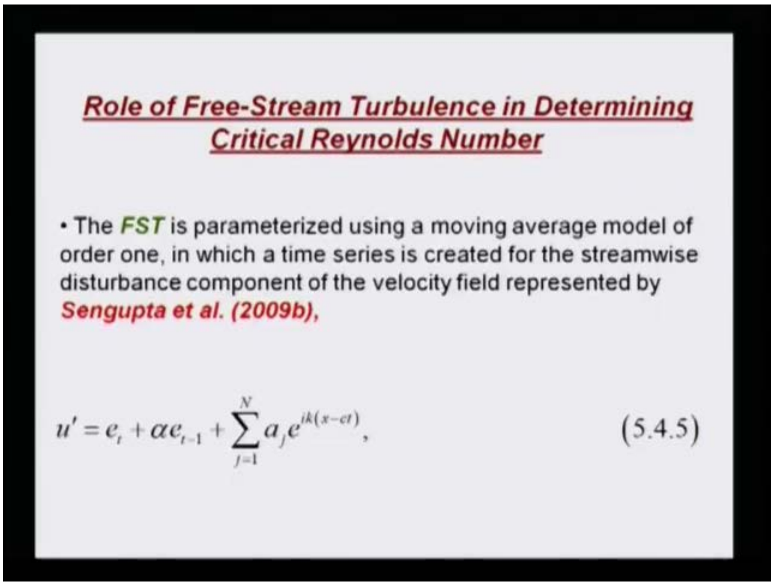

So, if I am trying to develop a model of turbulence with respect to some experimental data I am trying to simulate, then, I must introduce that deterministic source. This is what we need to do. So, what happened is, we did some measurements in one of our smaller tunnel and then, we developed a free-stream turbulence model. And, what did we do? We wrote down one of the disturbance component, given like this. It has some kind of a normal distribution. So, what happens, the random number that we generate in computer, they are basically uniformly distributed from minus 1 to plus 1.

So, from that uniform distribution, you can convert it into a normal distribution. What is a normal distribution? It is a kind of a Gaussian. So, it has a 0 mean and the way you have some second order moment. So, what you do is, you generate a time series, that is given in terms of the normal distribution at time t and the prior time t minus 1 and this is multiplied by some coefficient alpha that, we want to keep it because, we are going to mimic the second order moment and the fourth order moment; because, these are constituted by normal distribution, a combination of the two will always give you some kind of even moments only. So, with the help of that, so, we could fix the second order statistics and the fourth order statistics and the deterministic event of the tunnel itself would be explicitly kept; that corresponds to basically the, skewness. So, that was the whole idea of generating this particular turbulence model.

So, what happens is, we could do that, and that is the story of this particular FST model that we developed and what happened is, because we pick up the low frequency even from the empty tunnel data itself, it of course, matches quite well at the low frequencies or low wave numbers. In addition to that, we fix the constant alpha to match the second and fourth order statistics. And this is that model, that we have created, and we have computed this flow field by using that. And, this is how it looks like.

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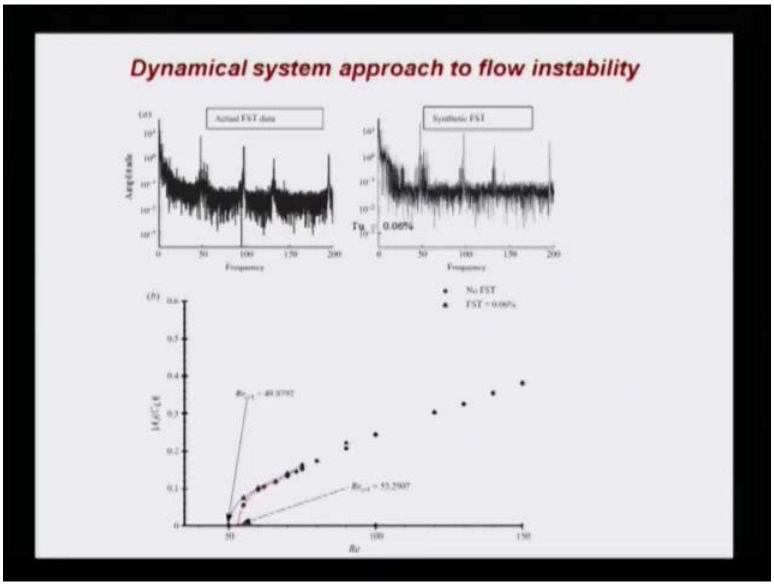

So, if I go to the tunnel, measure the free-stream turbulence, this is how it is going to look like. So, this is your experimental FST data, and this is, what we synthesize. So, this is your synthetic free-stream turbulence. And now, you see what happens, we use such a free-stream turbulence model, and then, see what we get. So, what we did, we took the value of FST, as given by this T u, which is given by 0.06 percent, exactly the value that Norberg quoted. And then, you see what happened is, the calculations without the FST goes like this, and meets there around 53.2907. The moment we switch on an FST model, with 0.06 percent T u, we go and get a value of forty nine point something. So, you can see, this is what actually people should be doing, and we have successfully demonstrated here that, indeed the background disturbances can, have create a Hopf bifurcation onset at an earlier Reynolds number.

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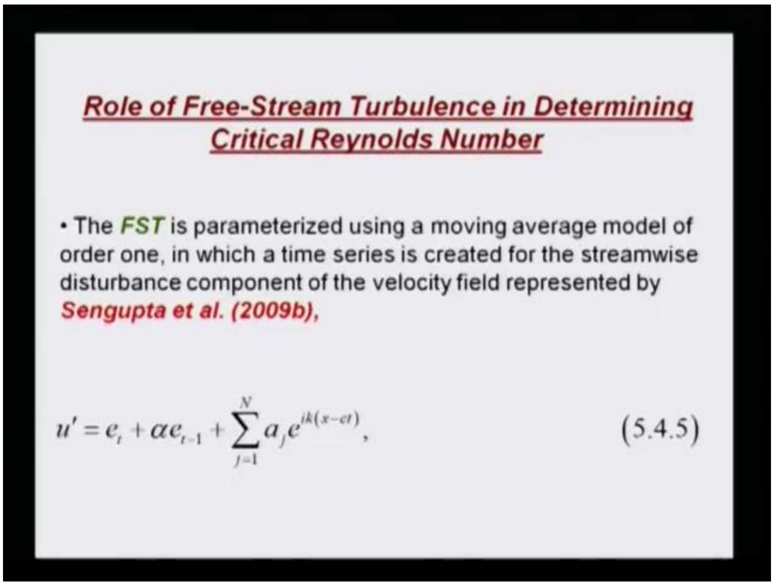

So, this we talked about, how we go about doing it, only thing that, I did not mention that, the c is the propagation speed of all those eddies coming with it. We did some exercise and we figured out that, if we keep it below U infinity, it is good enough. It is quite insensitive to exact value of c. So, you can just give a sort of a value of c, which is less than U infinity and that, should be all right.

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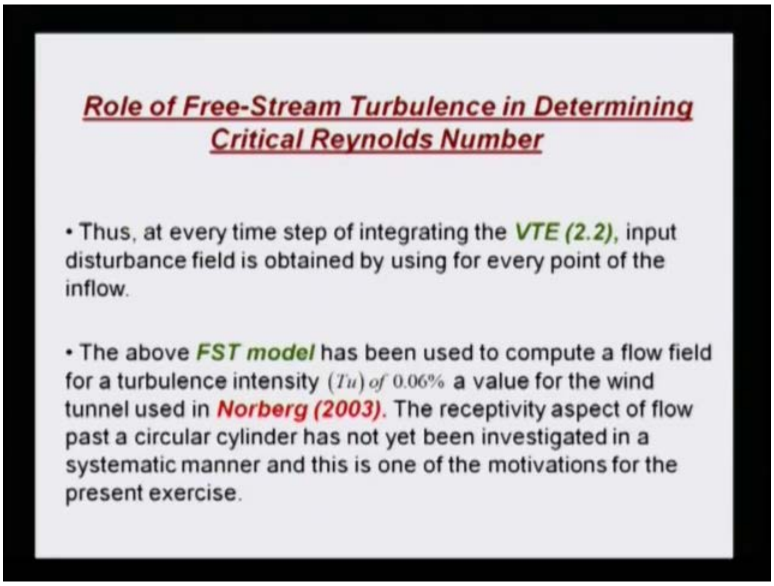

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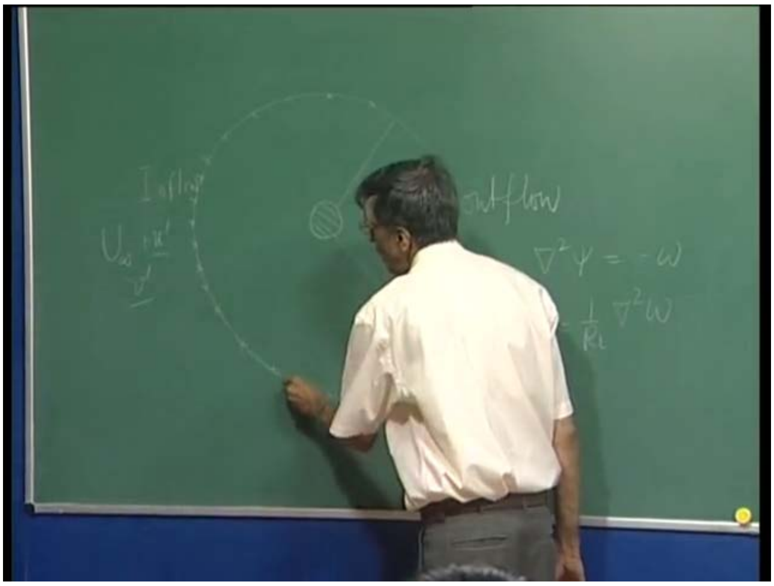

So, in essence, when, what we are doing is, the following that, if I have a cylinder, and then, this is what we compute, in a large domain. And then, what we do is, we identify a region, where we call this as the inflow and this is, what we call as the outflow. And in the inflow, what we are doing, unlike what everybody else does, we get U infinity and u prime, v prime; if you are doing a 2D calculation, we will only have two components, u prime and v prime; that, is what we put in there.

Well, if you have done a pretty good tunnel design, and the contraction is not too rapid, then, it is quite ok to consider that, u prime and v prime are the same statistics; kind of isotropy you can talk about. But if you want to also include some additional information you may have, like for example, in a channel flow, you know, v prime is going to be less than u prime, and then, you can put it in there and you can actually get also those nice wall normal structure that, you can put it. There are experiments available. So, we can do that. That is what we are saying that, in solving the Navier-Stokes equation, we try to solve it, in terms of a stream function vorticity equation.

So, this is your stream function equation and the vorticity transport equation is this. So, when we are trying to solve this vorticity transport equation, at every time step, we are going to add this u prime and v prime. And, this u prime and v prime, you are going to run your FST model at every time step, for each of the points. So, that equation, that, I have shown you before, in terms of e t and e t minus 1. So, in all the points here, on the inflow, we run the code subroutine and generate u prime, v prime, add it, and we can keep doing it for all the points. And, we did compute that case, that we showed you there with the help of this kind of model.

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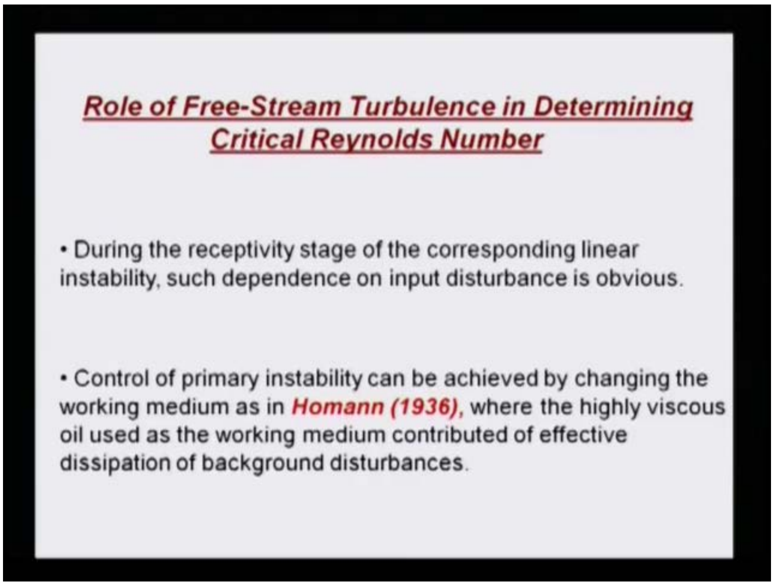

So, why should noise be important, despite what we saw in Landau's equation? Well, talking about the Landau's equation, we commented, what? That long term solution is independent of the initial condition; we eventually get into A e and that A e, what we found, comes out like this.

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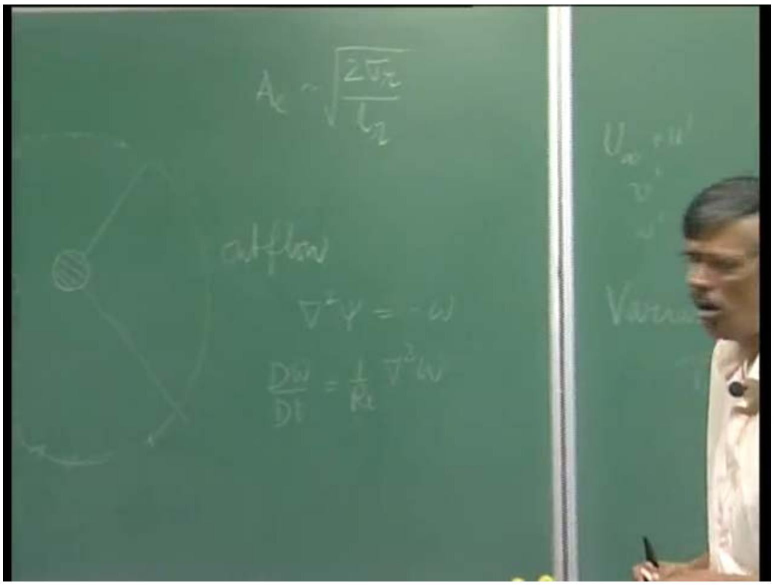

So, what happens, we have the flow in the following sequence that, we have an onset of linear instability that linearly saturates and gives you this. But if we are talking about the linear instability, we know linear instability has the receptivity aspect; that is what we have talked about. So, that is the reason that, we must have strong dependence of input disturbance, the way we are going to get the instability itself, because, that is how it begins. So, for every Reynolds number, this quantity itself, sigma r, depends on the background disturbance. So, that is why we should have that.

Now, that was also the reason that, Homann could do that. Homann's experiment was one such, brilliant experiment, where this control of primary instability was achieved by a working medium itself; because, it was very highly viscous fluid and it could dissipate those background disturbances, and, you could sort of, delay the primary instability indefinitely. Now, I think, we now have a fairly decent idea what happens to bluff body flows. So, there is not much. Now, we need to know a little more about it, and what we need to know more about it is, achieved by using the computed results.

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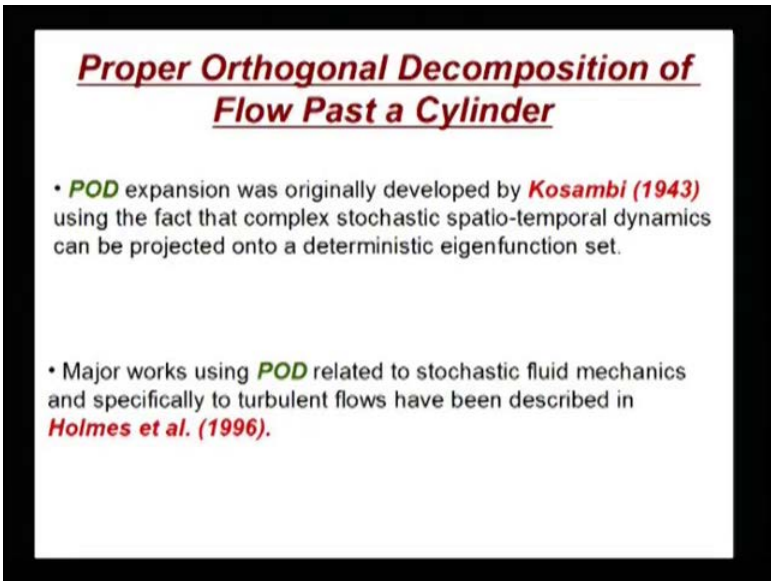

Now, we have the ability, to compute the flow, with and without disturbance and then, we can sort of, post-process the data; and what do we do by post processing; that is where we use a technique called proper orthogonal decomposition technique, which was pioneered by Kosambi, and which has been later on, sort of, very much popularized by professor John Lumley and his group; and that is, related to the stochastic fluid mechanics, specifically in understanding turbulent flow and that is given in that monograph of Holmes, Lumley and Berkooz.

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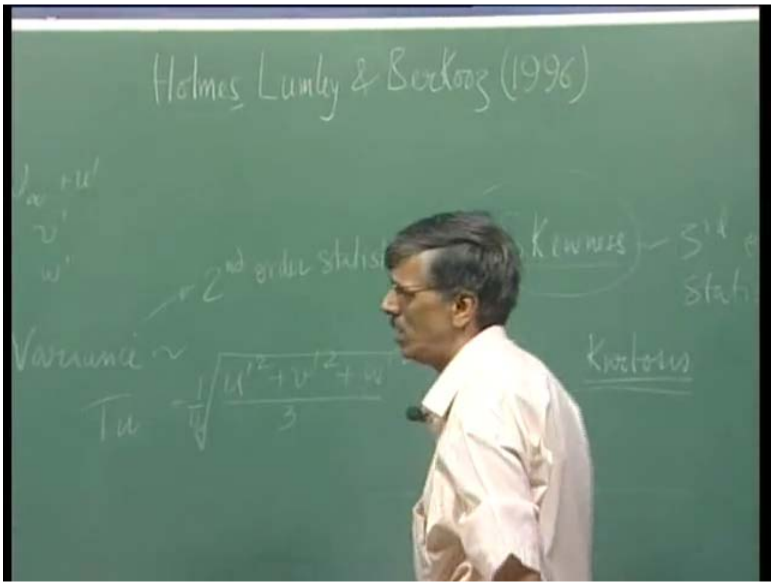

So, you must be aware of this reference Holmes, Lumley and Berkooz. This is a monograph. I think it was printed in 1996 by Cambridge University Press; one of the brilliant book. See, Kosambi was a mathematician. His idea was very simple that, this debate is going on whether this physical world is deterministic or random or not. So, he said, like in many dynamical system, when you look at the response field, it may look stochastic, chaotic, but then, underline that stochasticity, there might be some deterministic structure involved. He was so far ahead of his time, to think of it not necessarily with respect to turbulence, but he was talking about a general dynamical system. So, he said that, can I take this stochastic system and project it on a deterministic basis.

What will the property of this deterministic basis? They will be kind of a linear independence. So, they are going to be orthogonal to each other. So, it is like taking your orthogonal functions. Very simple example is your trigonometric functions, like the Fourier transform, that we have already seen; that I could take sine and cosine. So, each one of them are going to be orthogonal to each other. So, that is what is being attempted. So, that is what he was trying to understand that, even when you are looking at complex stochastic system, where the dynamics is given by space-time variation, you can actually project the whole thing in terms of a deterministic basis.

So, where do you start and where do you get off, that is the major question. That question also he answered. That for example, if I am looking at, let us say a mechanical system, like a fluid dynamical system, I will go on taking these orthogonal functions up to a level, which more or less defines the major events of the flow. So, in a fluid flow, it could be, let us say, the energy. If I can decide it, say explain 98 percent or 99 percent of the energy, I am done. So, that was the whole idea behind what Kosambi started and which was brilliantly exploited by Lumley and his colleagues.

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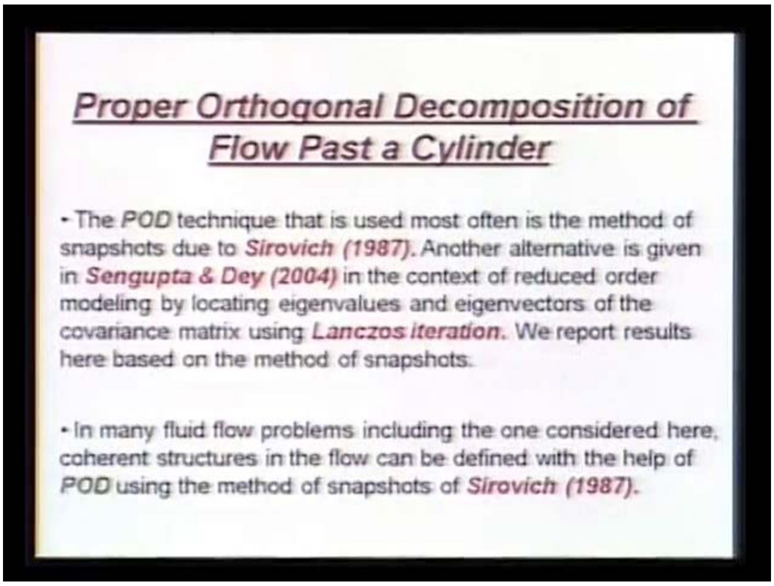

Now, POD as a technique was further given a boost by Sirovich. Sirovich said, look, how do I do this decomposition; it is like, basically, it is a continuous system. So, if I look at the continuous system, then, what I could do? I could take a snapshot and I have all these points and then, I can decompose it, the corresponding sort of a discrete description; I can define it, because eventually, what you do? When you actually compute, you end up solving some ax equal to b equation; the dimension of a determines your discretization. But this discretization, theoretically speaking, should have been infinite dimension, because it is a continuous system. But what we are doing in computing, we are discretizing and making it a finite number, damp; but even that finiteness, in a computational sense, can become very bothersome.

So, that was one way of doing it and we did look at one such way that, if we look at the spatial discretization and then, try to find out the Eigen values and Eigen vectors of the spatial discretization by a iterative process, due to Lanczos; that is one way of doing it; you get a pretty good picture. In fact, the picture that you get, we found out a very interesting property; that if I take a large domain and I do it in two small part, and I get the picture, and I then put them side by side, they just match seamlessly.

So, what is it that we are doing? If I have, say, n by n dimension picture and if I do, n by 2 and n by 2 dimension, I just make it into, say two parts, then, what happens; your corresponding A matrix dimension has come down; and in terms of solution process, you know, the computing goes as that matrix to the order 3. So, if I can subdivide the domain into smaller domains and I can compute them separately and put the picture together, I get a full picture. So, that was something, that was attempted and it was quite, but still it amounted to lots and lots of work. However, much earlier, Lawrence Sirovich, he did propose an alternative way of doing it; say, do not look at the picture in terms of space, but look at in terms of time.

So, what you do, you grab lots of snapshots at different time intervals and from that information, because, what you are trying to project; you are trying to project a spatio-temporal event. So, you try to get that picture by looking at snapshots of different time and reconstruct the average picture, over that time interval. So, the basic idea is what, it is a stochastic system. So, we are trying to give it some kind of, in the decided time horizon, get a kind of a fit of this space-time variation by a deterministic picture, subject to the constraint that, this explains 90 percent of the energy or this explains the 95 percent.

So, method of snapshots is so much more beneficial, because, instead of talking about, let us say, millions of points, here we are going to talk about fewer snapshots; number of pictures in time. How many that could be? Like, Yogesh will tell you that, you take an interval of, let us say, delta t of the order of 0.01. So, in a unit interval, you take hundred pictures. And then, of course, sooner or later, you reach the limit of your process level and you stop there; but still, that is much better than what we have before. We will talk about little more in detail about method of snapshots and how to interpret the data, but this was a sort of a remarkable achievement by which this whole subject of POD technique got a boost and it was revived; that is all due to Sirovich.

So, in many fluid dynamical systems, it is not necessarily have to be one of turbulent flow; any unsteady flow, we can make use of it. And, in fact, that is what we are trying to do now; because, we are going to use this POD technique, to study instability of flow. And during instability, we have already seen that, it is going to be a space-time dependent phenomena. So, can we use POD for studying instability? That was the question, that we have been asking for the last few years and I think, we have now come to a state, where we have been able to develop a coherent picture of it and we will see some of these results.

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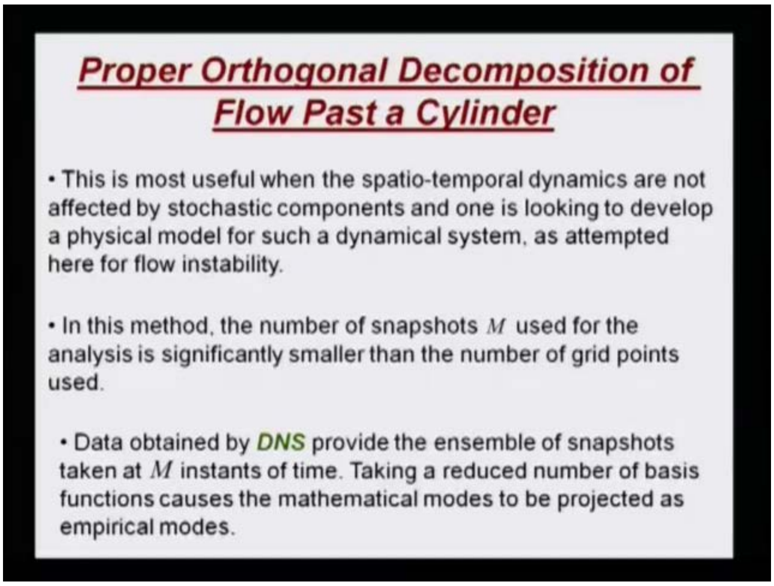

So, if I am going to look at dynamics of a system, where stochastic component is not very important, like flow instability, we are talking about. So, there are no such stochastic components involved. Then, what happens, with the help of the POD we can develop a physical model of the dynamical system. So, that is what we have been attempting, and in this method, what we are doing, we are taking a number snapshots, let us say M of them; and we use it for the analysis and this really is very easily done. So, what happens is, first and foremost of course, we will have to get results, which are meaningful.

One of the reason that POD had a little stuttered start is, for two reason; if you try to use it with experimental results, you are very much affected by the experimental results' accuracy. So, that has not been done very well. And, we also have not started talking about turbulence, which you would be doing shortly, then, you will realize that, when we talk about turbulence, turbulence is an end product, after the instabilities have taken over.

So, you may have, some kind of a sort of a saturated state, which is not very sensitively dependence of the fluctuations of this parameters of the system; whereas, when I am looking at the stability of the system, you know now, very clearly that the disturbances are very important. So, if I am going to use POD for unstable flow, that is going to be much more demanding than using POD for turbulent flow. This is something we have to appreciate that. So, when we are trying to use POD to study instability, then, we better have what we call as a direct numerical simulation, so, where we would be getting the solution without making much of error.

So, what really happens is, we perform DNS, then, we collect the set of snapshots, which you call as the ensemble and then, taking this reduced number of snapshots, we come out with basis functions, which are directly related to the number of this snapshots. So, that is what we are doing. And, this gives you a kind of mathematical modes, which give you a very good representation of the dynamics of the system.

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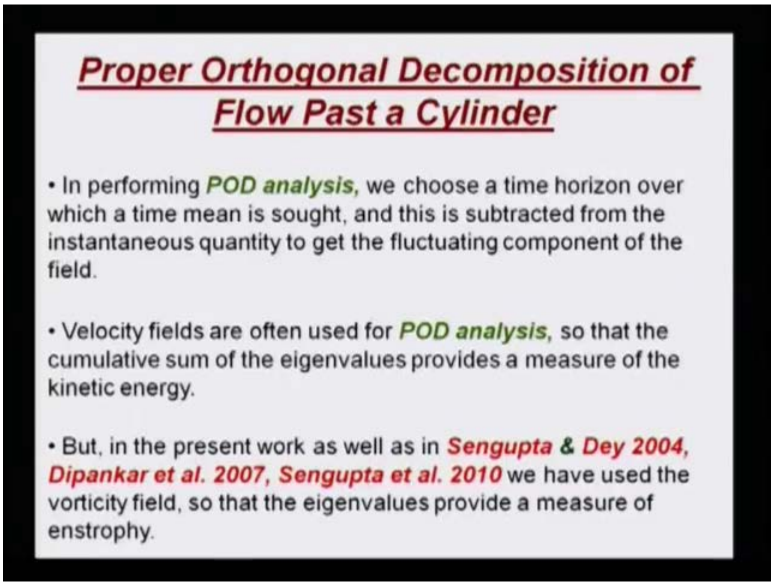

So, what we are going to do now, to perform this POD analysis, we will choose a time horizon, over which we will get some kind of a mean or the equilibrium flow. So, what we do is, if I choose, say from t 1 to t 2, I take a kind of a time mean and then, once I have it and I also, through this DNS, I am going to have an instantaneous realization. So, I can subtract one from the other, so, then, I can get so-called disturbance flow. And, on this disturbance flow, I can embed the POD technique; then. I will see how this disturbances are growing.

You see, that is how we have approached the study of stability in various forms. And, what is very satisfying in this approach is, you are doing it with the full non-linear equation, without any empiricism, without any restrictive assumption. So, instability analysis based on POD, will be a much more complete description of the system than anything. So, if I use velocity field to define the disturbances, then, what we are going to get? We are going to get an estimate of the kinetic energy. And, if we are going to use vorticity as a disturbance field, then, what we get is called as the enstrophy.

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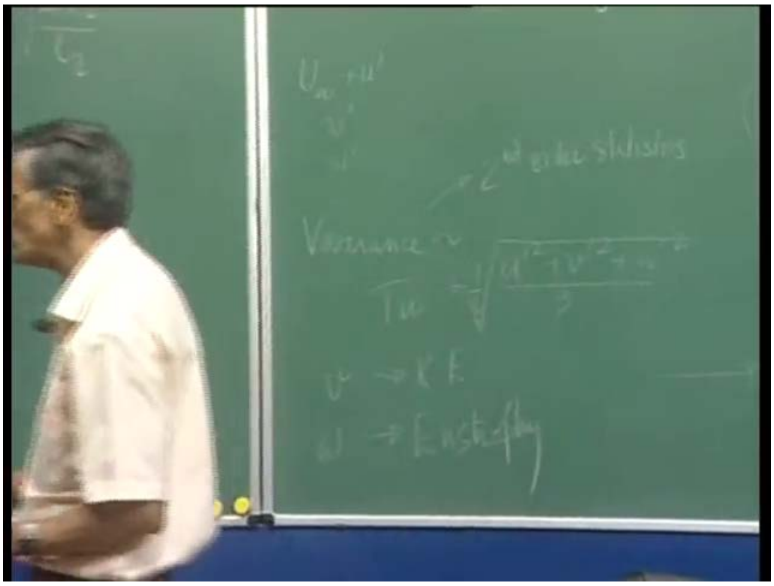

So, if I am using velocity, I get kinetic energy, and if I use vorticity, I will get what I call as enstrophy.

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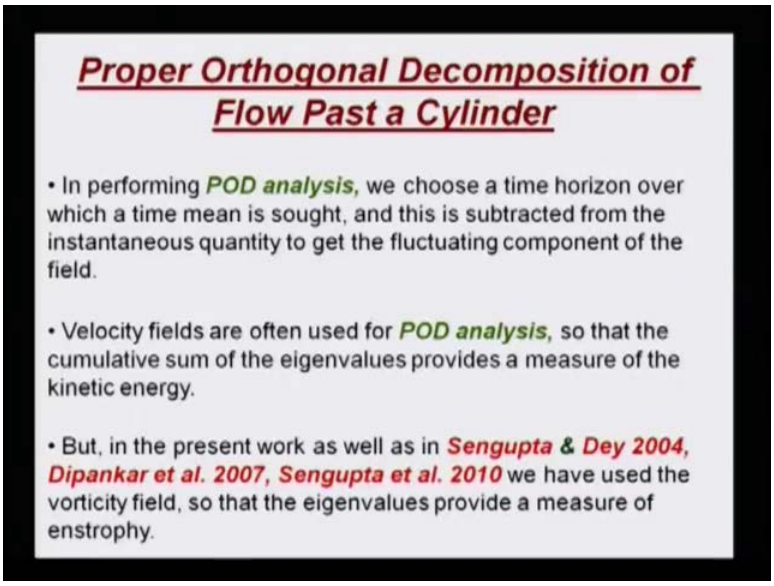

So, we basically prefer working with the vorticity field, because, that is what fluid mechanics is all about; it is all about vorticity dynamics.

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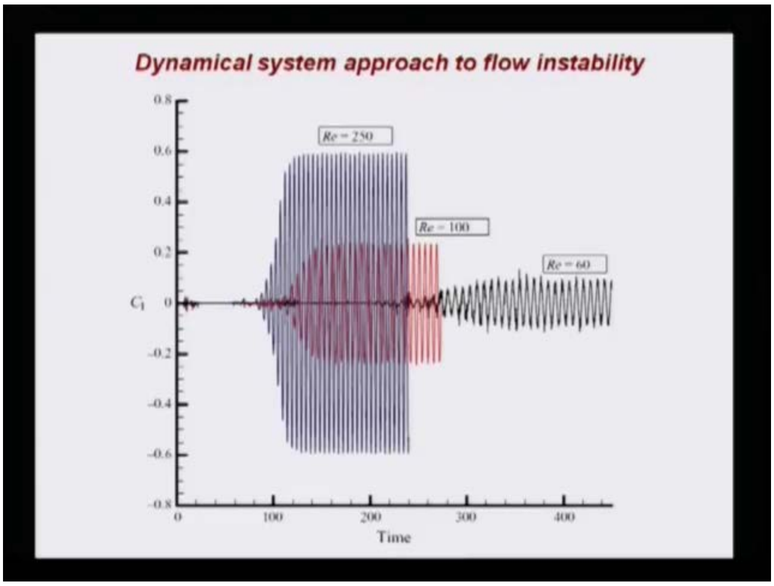

So, you work with vorticity, you work with enstrophy and that is how we are going to do it. And, I am just going to stop here by showing you this result. We have done this stimulation of flow past a cylinder at three different Reynolds number, 250, 100 and 60, and I am going to describe it, but you can see a very curious feature that, this is more colorful than this. I will stop here. We will begin from there in the next class, that is tomorrow.