Instability and Transition of Fluid Flows

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Module No. # 01

Lecture No. # 31

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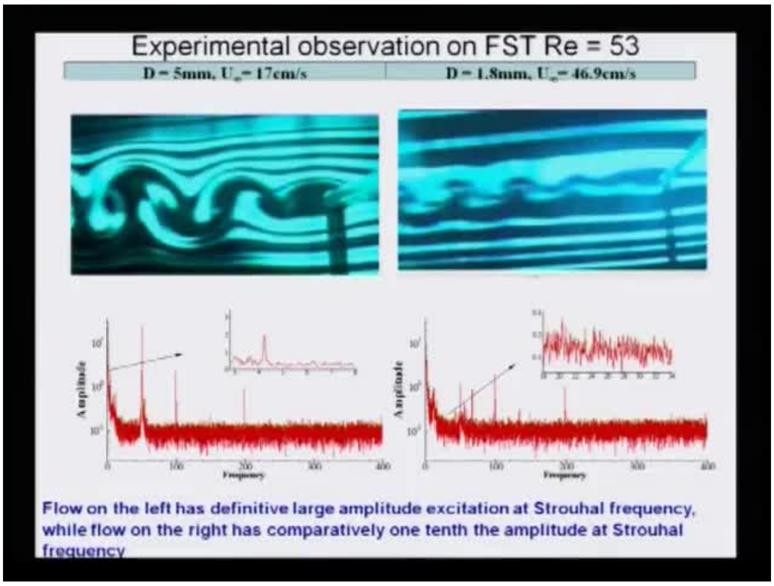

So, we are discussing about instabilities for bluff body flows and one of the redeeming feature of this particular flow is, here, you see temporal instability, quite unlike, stream line bodies we had discussed so far, where we mostly confined ourself to spatial instabilities. Let me just show you one of the figures, that we saw in the introductive part of the course, where we showed you some results, experimental results conducted here, in one of our tunnel. What we were interested in studying, is the role of background disturbances on instabilities. So, in the context of temporal instability, it is rather important to see that, the, how background disturbances affect flow instabilities. What we did is, we kept the Reynolds number in a post critical value, something like 53, which is quite compatible with the background noise that is prevalent in that tunnel. This Reynolds number were achieved for two cylinders of two different diameter, one with 5mm diameter; the other one is 1.8mm diameter. The reason that, this is a high diameter cylinder, allows you to run the experiment at a very smaller tunnel speed of 17 centimeter per second. When you reduce the diameter, you run it at 46.9 centimeter per second.

So, if I now try to, sort of do an analysis of what has gone on, and compare this two experimental results, what we did was, we removed the models and run the tunnel without any model, at those speeds, 17 centimeter per second and 47 centimeter per second, and obtained a time series; it is simply a hot wire trace and then, you can do a fast Fourier transform, so that, you can find out the spectrum of the noise in the tunnel. So, what we have shown is, the amplitude versus frequency of the noise. And, what we know also, from lots of experiments that have been done over the years, people have collected the data and they found out, what is the Strouhal number, the characteristic frequency of flow past a cylinder for this Reynolds number.

So, the Strouhal number is quite a given quantity. So, what we did was, tried to find out, what is the magnitude of the disturbance at the Strouhal number. And, for this lower speed, you can see that is the peak, the peak corresponds to the Strouhal number. And, there we see, the amplitude is roughly about more than 2; whereas, when we did this experiment at higher speed of 47 centimeter per second, and we tried to find out, what is the amplitude at Strouhal frequency, and that has been zoomed there in this inset, and you can see, that value is about one tenth of this. And, that difference of background disturbance is very clearly revealed in the wake pattern of these two flows.

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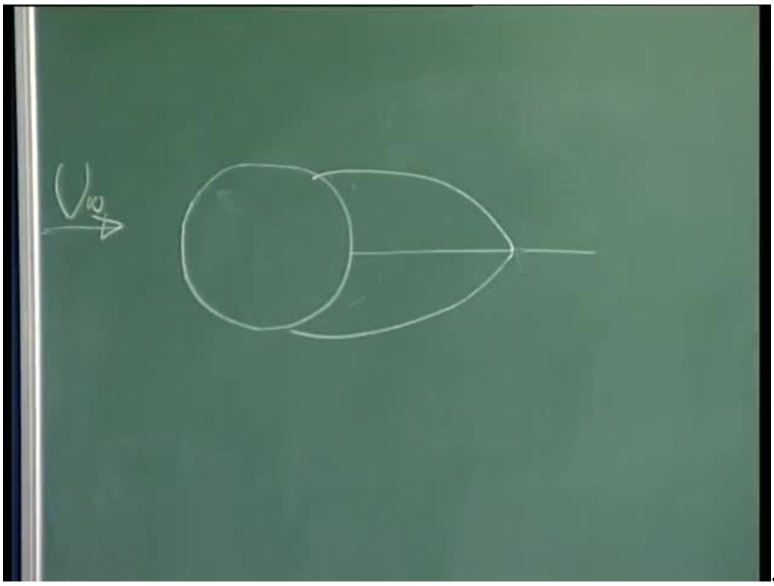

In this case, you can see very clearly, well-defined wake vortices, typical of Karman Vonal vortex trait; whereas, what do you see here, you see a kind of a attached bubble. This is something that, we were also talking about the experiment done by Fritz Homann. What was done in that experiment was, we mentioned that, lubricating oil was used as the working medium. There also, a similar flow feature were observed. It is something like this, that, if I take a flow past a cylinder, then, what happens is, as I keeping increasing the time, I mean, if I am solving the Navier-Stokes equation, and if I am doing the experiment also, perhaps, I will see it; but in fact, ((torapidly)) in computing, we can see it in much more better way; you can visualize it.

What happens is, initially, you see a sort of a symmetric bubble forms; that bubble keeps growing with time and after quite some time, the bubble takes some kind of equilibrium form; but then. what happens is, if it is a post critical Reynolds number, then, what happens is, one of the bubble starts growing asymmetrically with respective to the other; and when it grows in sufficient strength, that side, the vortex is shed from that side; while this one gets emaciated, while that was growing, but then, now, this cycle reverses; this one starts growing and that remains quiescent and this is what you are seeing here.

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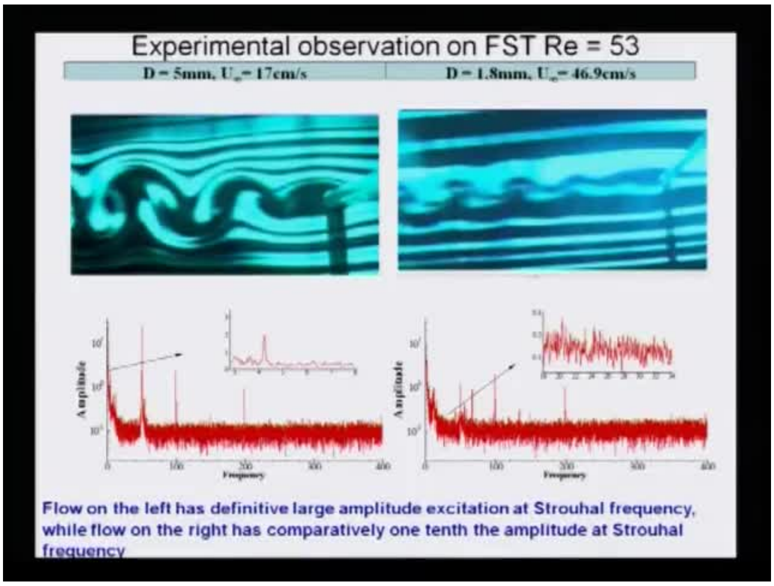

However, if I now have a flow, which is kind of noisy, but not to the great extent, that is what is characterized by free stream turbulence.

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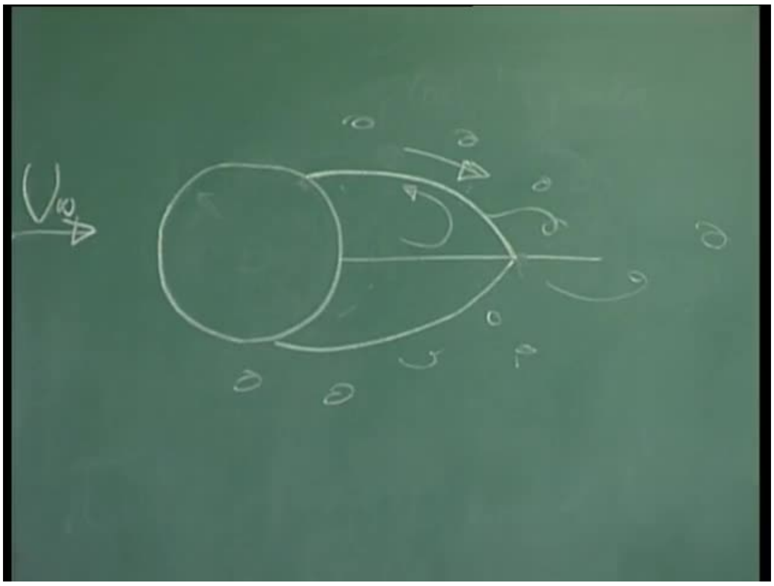

So, this noise is kind of flat across a large frequency. There is no preferential larger value at that. So, this is, what you are seeing is, basically, the receptivity aspect that, if I have a disturbance at the Strouhal frequency, I see the common vortex trait; if I do not have, then, I get this. Now, this flows also have large eddies like this. So, what happens is, this is what, this is a shear layer. If I look at it, outside, this streamline could be like this and inside, I have a re-circulating bubble and that might be going like this. And, if I look at this interface, now, this interface, this is something which we have studied; where, the Kelvin-Helmholtz instability. We studied that. And, what we found in Kelvin-Helmholtz instability is that, if I have the velocity in the opposite direction, that tends to destabilize.

So, here also, the same thing can happen that, while the computation in the absence of any noise, then, with very good quality numerical methods, we will show this kind of a steady periphery of the bubble. However, because of this presence of this disturbances, that interface develops a Kelvin-Helmholtz instability. And, that Kelvin-Helmholtz instabilities will cause some of this vortices to be stripped off from this. And, so, what you see is, essentially, some kind of a weaker eddies of this kind. So, please do not confuse by saying, this is a common vortex trait; this is a weaker form of common vortex trait. It is entirely a different phenomena altogether. The origin here was, alternate shedding right from surface; whereas, this picture corresponds to the case, where vortices are stripped due to Kelvin-Helmholtz instability from this weak bubble.

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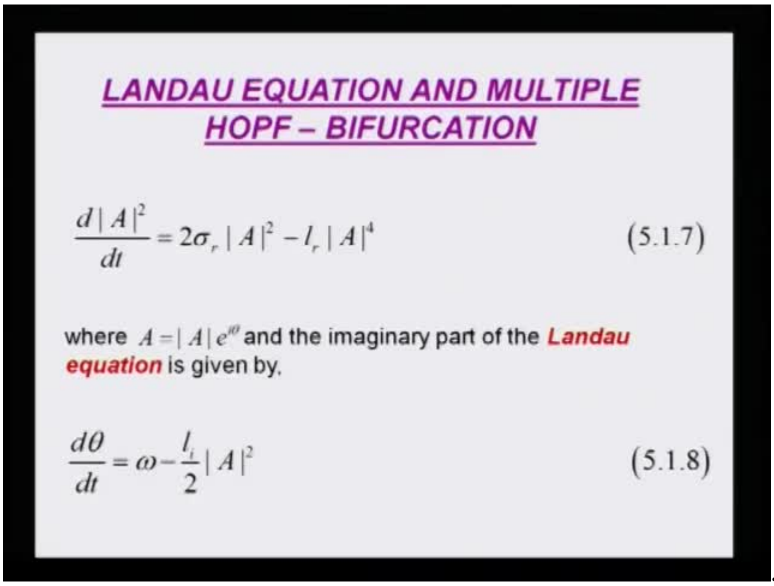

Now, if you go back and look at those results, those were shown by, those were shown in Fritz Homann's case, something that I would like you to take a look at, there you would see that, even the results those were produce for 65, there you would have seen that, this is more of this kind; you have a attached bubble at the end, and the end of the bubble is fluctuating and creating undulating wake. So, this is not like, what we would say, in the classical sense, the Karman vortex shedding. Now, when you do have Karman vortex shedding, to explain that, we, in the last class, we talked about the development of Landau equation and this was the evolution equation for the amplitude and this is the evaluation equation for the phase angle.

So, that actually, will give you a sort of a frequency of shedding. And, this frequency of shedding is rather interesting, because what it shows, that, this frequency of shedding of the non-linear dynamical system is equal to what is given by the linear system, modified by this part. And, this part has come about from the Landau equation. And, what it shows that, this frequency is proportional to the amplitude. That is a important thing, one of the important aspect of non-linearity amplitude dependence. And, of course, it also depends on the imaginary part of the Landau coefficient.

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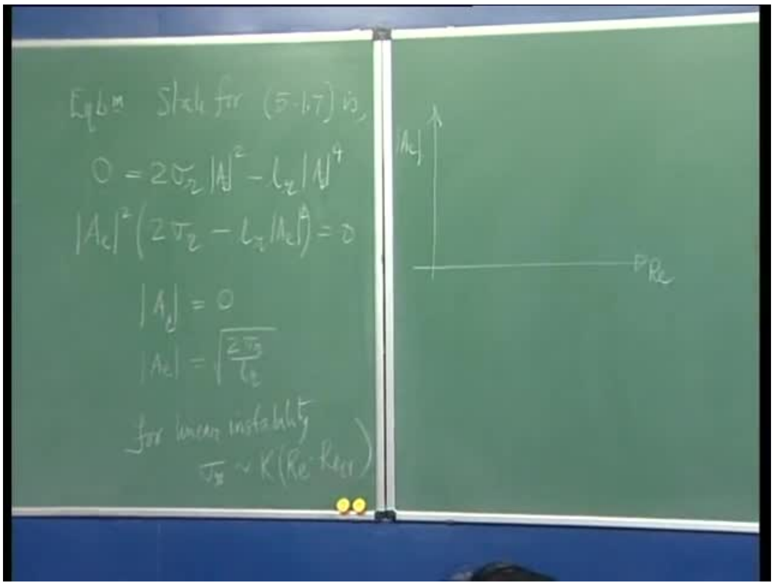

Now, if I look at the amplitude evolution equation, what I can do is, I can study an equilibrium state. So, equilibrium state of 5.17 is what; that would given by, when I put time rate equal to 0, is it not. And, that is what we figured out. So, that would be this, 2 sigma r mod A square minus l r. And this, this corresponds to a equilibrium state. So, I will write it as e and this of course, gives you two sets of conditions. So, one of the condition is, of course, this and the other condition is given here, that is equal to A e into square root of…And so, what could I do is, I could plot, on this side Re and on this side, I can plot A e. How should I see the variation? This comes from a linear instability part.

What have we? For linear instability, what I find that, in the post critical case, it goes as some K times Re minus Re critical.

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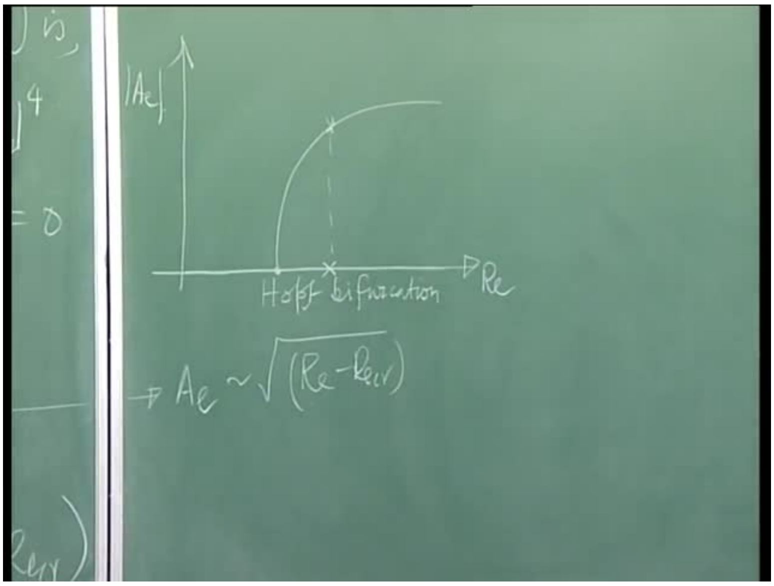

So, if I do that, what does this tell us? This tell, tells us A e goes (no audio). So, one of the solution is A e equal to 0. So, that could go on here, one branch of the solution. The other branch would be starting from Re critical, and that will be a parabola. And, we said that, this is your Hopf bifurcation. You can have two state. This is a beautiful, simple example of Hopf bifurcation where, beyond the critical Reynolds number, we can have two state, either this or this. So, what we have just now seen in the experiment, this would not be revealed from this picture; that the Kelvin-Helmholtz instability part, is not given by what we have been talking about, in this case. And, that is where we need, figured out that, equilibrium amplitude is something different than what we are talking about. This is strictly about Karman vortex study. So, please do keep that in mind.

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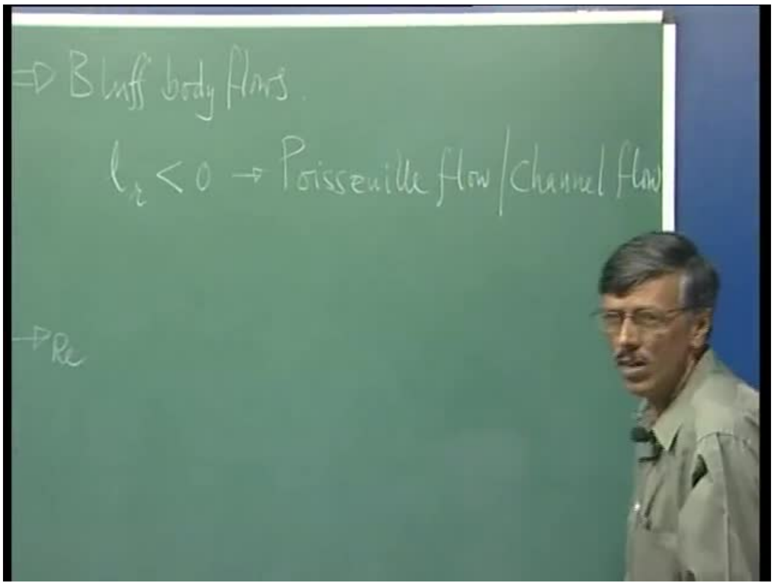

So, here, we have it, that we have seen truly the effect of non-linearity. Also, I would like to reiterate that, this case that we are talking about, we are talking about the cases for which l r greater than… This is typical of bluff body flows. We have, similarly, a case of l r less than 0; this, you actually see in Poisseuille flow or channel flow. What do you notice here? You notice that, in this case, we will not be spending time, but I am just simply mentioning the fact that, if l r is negative, what the non-linearity does there.

It actually increases the instability, because the time rate of change of amplitude is augmented by this term. So, that is, what was the whole intention of Landau in proposing this equation, that it can explain super critical stability; that is what it is doing. The non-linearity is causing a stable, to helping you get a stable amplitude and that is your Karman vortex shedding; whereas, if I look at plane Poisseuille flow, l r is negative and there, we can talk about subcritical instability. So, this Landau equation is quite a nice model equation to explain both.

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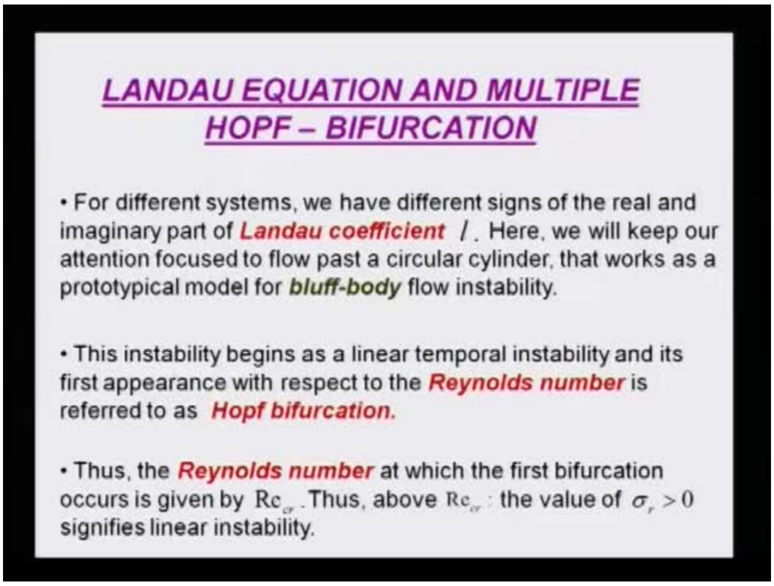

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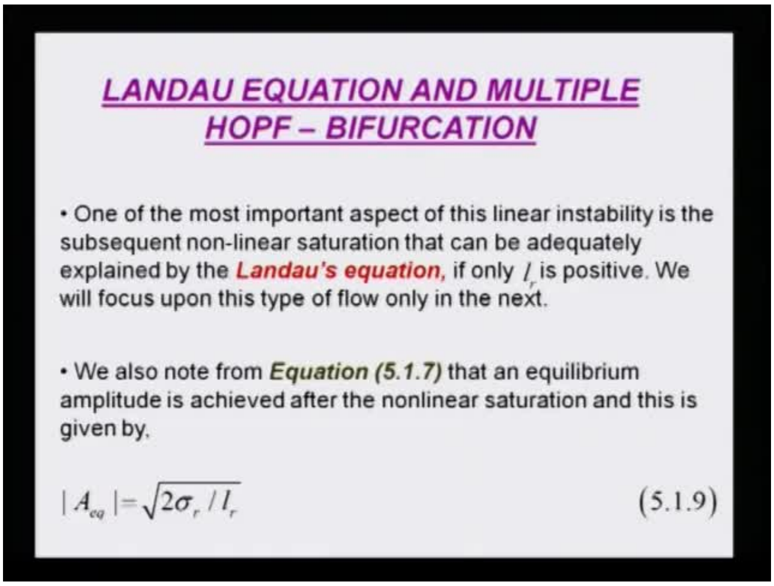

So, we have talked about all of this. So, there is nothing much, that we need to add to that. We did also talk about the equilibrium amplitude as given there; we have shown it. So, we could talk about an equilibrium Strouhal number. You know, there were lots of attempts in 70s and 80s. People have spoken about describing this non-linear action in bluff body flow instability by measuring Strouhal number. It is a easy thing to measure. Because, unlike the amplitude, you will have to put in a probe and I told you about Kovasznay's experiment in late 40s, when you put in a hot wire probe, you saw the vortex shedding disappeared. We talked about that, subsequent word done by Strykowski and Sreenivasan, which investigated that, putting in a smaller cylinder in the wake of large cylinder, and controlling the vortex shedding completely for Reynolds number less than 120; it was shown experimentally in late 80s.

However, that started with the original observation by Kovasznay; when you put in a probe in the wake of a cylinder, the wake pattern changes. So, basically, to understand this non-linear action by putting in a probe and measuring is rather difficult; but if I put it far downstream, far downstream, because Strykowski and Sreenivasan, mapped the region where you have to put this control cylinder. So, if I put it beyond that region, then, I can get it; then, I can study the wake. Problem is two; one is, of course, the amplitude is too low. So, you would not know with reliability, what is it, that you are studying. And, the second thing is about, it is much easier to measure the frequency. I will get the oscillation and once I have the time trace, I can do a Fourier transform and I can obtain the Strouhal number.

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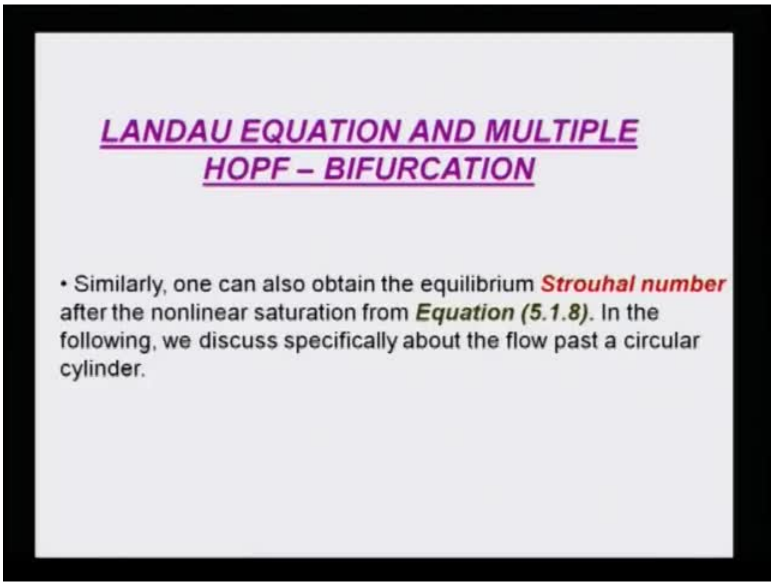

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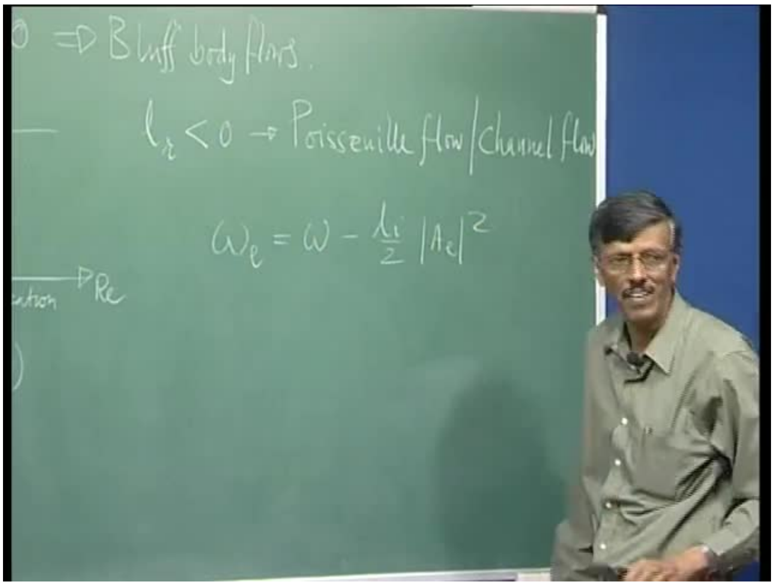

So, this is what we are talking about. The Strouhal number is the equilibrium Strouhal number, from the other equation. And, that equation was what? That equation was omega equilibrium should be equal to what you get by the linear theory and this was l i by 2 and this is that.

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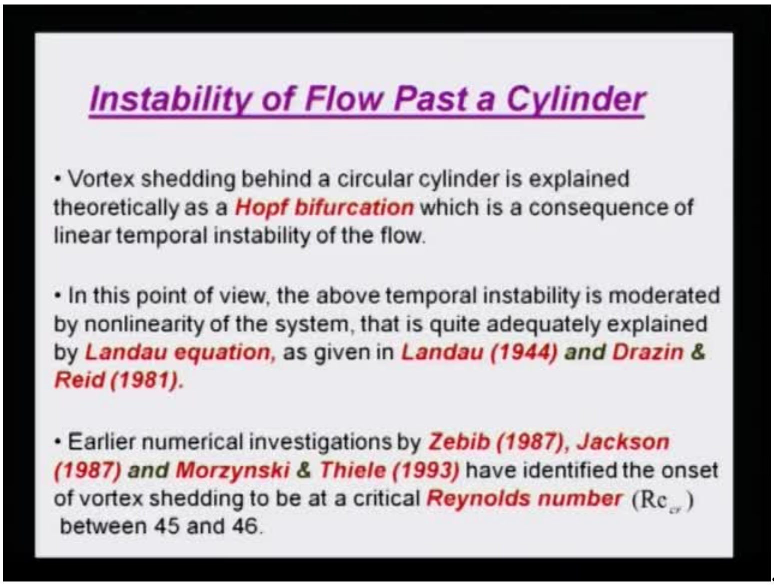

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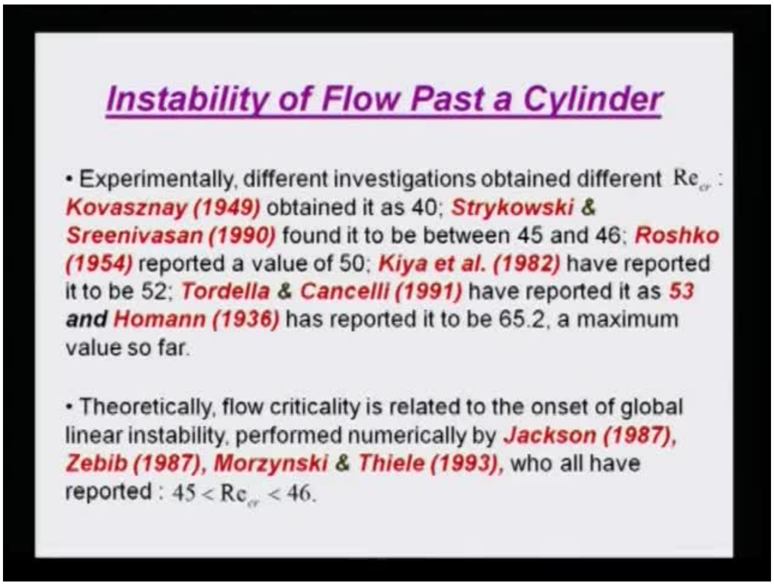

So, now, one can collect the data and lot of people, in fact, emphasize on this aspect. Now, this is something that, we need to really talk about; that earlier numerical investigations by various groups identified the vortex shedding at a critical Reynolds number of 45 and 46; but this was indeed, not the case that, we reported of with respective Homann's experiment. And, we did talk about all of this, the whole range of values experimentally produced. And, this is what it is. The tunnel experiment that I showed you, in beginning of this lecture, on that particular tunnel, I think, you can even get vortex shedding at a Reynolds number as low as 40. So, there is no need to really think anything particularly about this particular work. And, this is a completely, a theoretical approach, pedagogic approach strongly dependent upon the numerical method that they have chosen.

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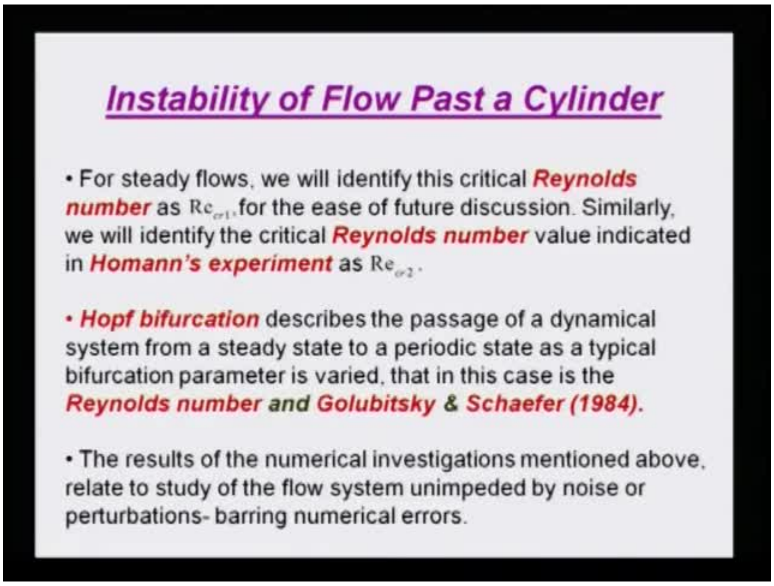

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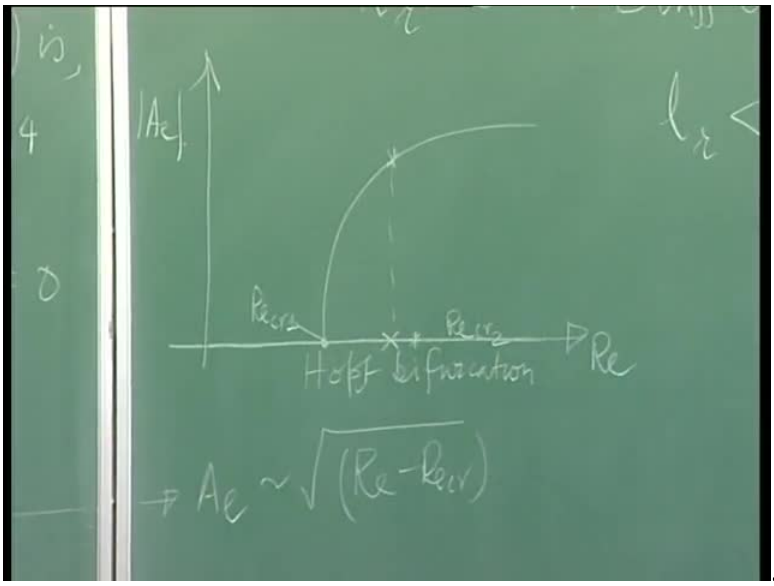

And, what we talk about, then, when we look at the experiment done by Fritz Homann, that does not display Re critical 1. This value we are talking about, Re critical 1, the first Hopf-bifurcation; instead, Homanns experiment identified a critical Reynolds number somewhere here, which is far ((no audio)) off what this is. So, if this is about, between 45 and 46, this is about 65 or so. So, that is what we are talking about as Re critical 2.

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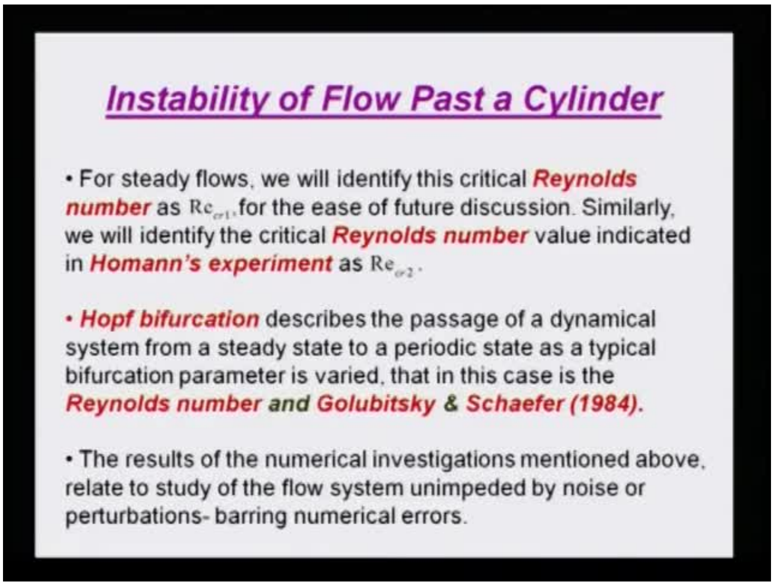

Now, basically, you should view those computations reported by Jackson, Zebib and Morzynski and Thiele as one, which basically talks about the transfer function of the dynamical system. That shows, this Re critical one. Now, the experiment that we saw, is a case where the system is ready, but the input is not there; that is what we showed by showing that empty tunnel noise. And then, you do see a different scenario altogether.

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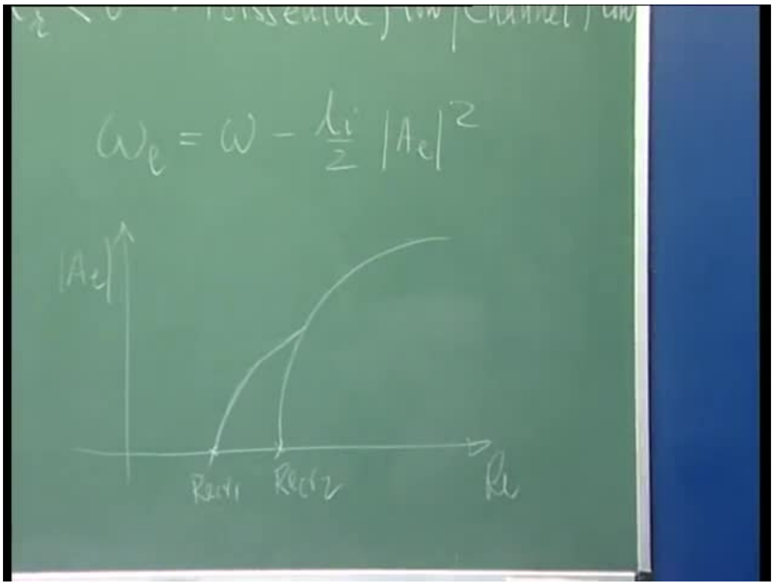

So, it may so happen that, if I do this experiments, measure the critical amplitude versus Re, people have noticed, I do not know whether I have a new graph for you to see that, but this is quoting from the experimental result of Strykowski's thesis, PhD thesis, it is noted that, it has something like this; this you do not have a parabola. So, you have a kind of parabola here, and you do tend to have, let us say, another parabola here. And, this kind of a scenario, where you may have bifurcation at Re critical 1, and again a virtual bifurcation at Re critical 2, is what you see actually in experiments; that is what was shown in Strykowski's thesis experimentally.

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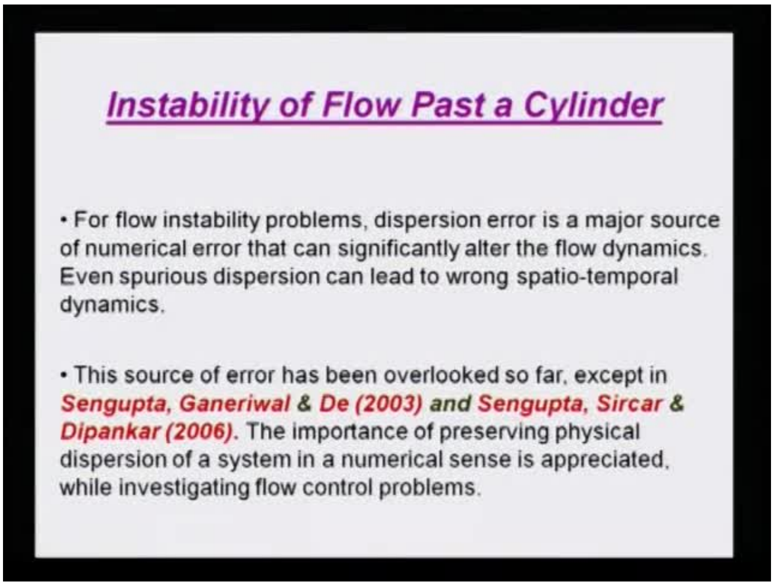

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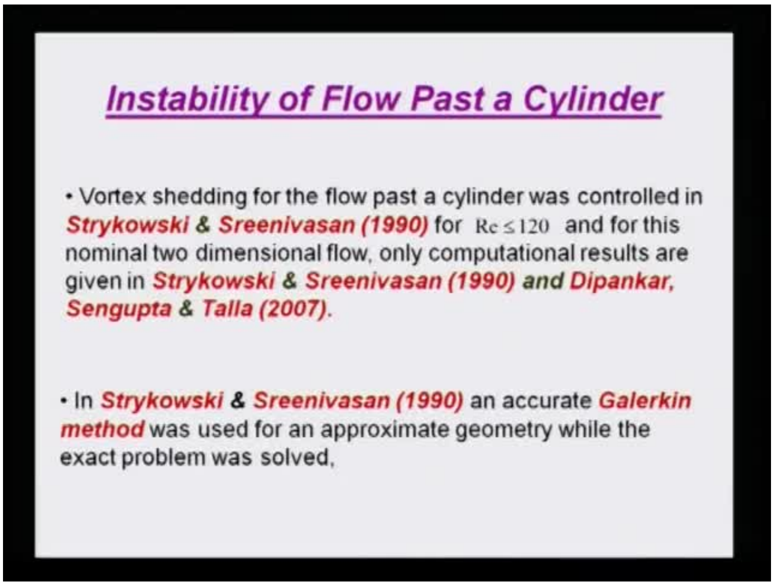

So, what one can talk about is the following that, in Homann's experiment, the use of highly viscous liquid stops the flow of the background disturbance altogether. So, you do not get this; the first critical Reynolds number is bypassed; instead, you go to a higher value of Re critical, where the flow is susceptible to even a smaller levels of disturbance, and that gives rise to that. Now, when it comes to numerical error or computation, one must also understand that, you have to develop numerical methods, which faithfully follows the physical dispersion relation. Most of the cases, when it is not done with care, numerical solutions are laced with dispersion error. And, that can really alter the flow dynamics qualitatively. And, we did look at these issues and developed numerical methods and in this particular work, well, I think it is not here; there was this particular work, where we tried to reproduce that flow control experiment of Strykowski and Sreenivasan. That is what we were saying that, by putting in a small control cylinder in the wake, they could delay vortex shedding all the way up to Reynolds number 120. And, the flow was nominally two dimensional. There were some computational results produced there.

But I suppose, this was where the most recent and more reliable results reside. We computed this flow, including actual geometry, unlike what was done in Strykowski and Sreenivasan, where they had difficulty with grid generation, and the method was less reliable in that sense, that you did not simulate the control cylinder; instead you took a cluster of points, where you put an equivalent effect of putting some body there. So, that was most like a modeling effect. And, in this particular paper, in Journal of Fluid Mechanics, we did show what was reported experimentally and that is basically, a testimony of developing numerical methods which are more correct.

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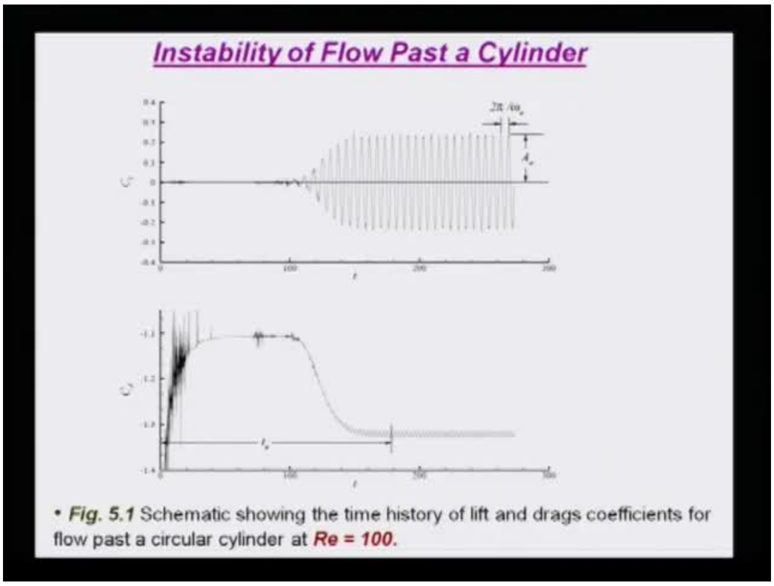

• *Fig. 5.1* Schematic showing the time history of lift and drags coefficients for flow past a circular cylinder at ***Re = 100***.

This, we did show that, if we compute the flow for a circular cylinder at Re equal to 100, this is how we see it that, initially we do not have shedding; and the c l lift coefficient remains more or less flat, barring this very high frequency oscillations that you see in c l versus time plot; that is seen even better, if you plot the drag coefficient versus time. Those fluctuations are present here too. And, what happens is, despite those high frequency fluctuations, the flow remains kind of insensitive; after sometime, slowly, you start seeing this instability as given by linear theory picking up. And, these are the ones, this initial exponential growth is governed by the linear instability and this non-linear saturation is quite nicely modeled by Landau equation, right. And, the drag coefficient actually, initially, after the transient reduces, then, again it increases; please understand, this is done in a negative scale.

So, the drag increases and then, we have, the drag variation also gives a kind of a frequency, which is not same as the frequency for the lift. Why, because in each complete cycle, the lift varies; whereas in each half of the cycle, the drag keeps varying too. So, what will happen is, the drag variation frequency will be twice that the frequency that you see in lift. So, think about it again, why would you do that? Why you would see that, because, the full cycle gives you a one Karman vortex shedding, but in that full cycle also, the drag varies; the drag varies about its mean and so, that is why this frequency is twice than this frequency.

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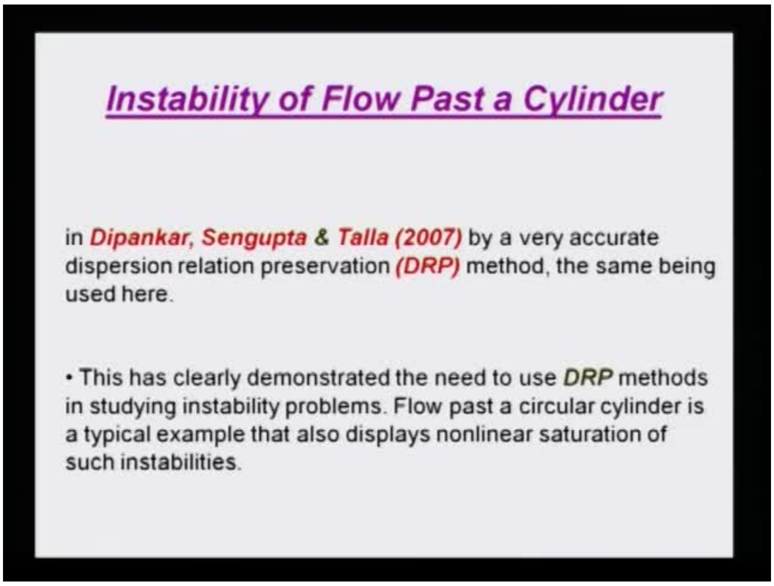

So, this was a post critical Reynolds number calculation result and we have already talked about this dispersion relation preservation method, which we have used and we should actually use all these dispersion relation preservation method in studying instability problems, because, we have by now convinced that, instabilities are governed by the dispersion relation. So, our numerical dispersion relationship match a physical dispersion relation; this is quite must.

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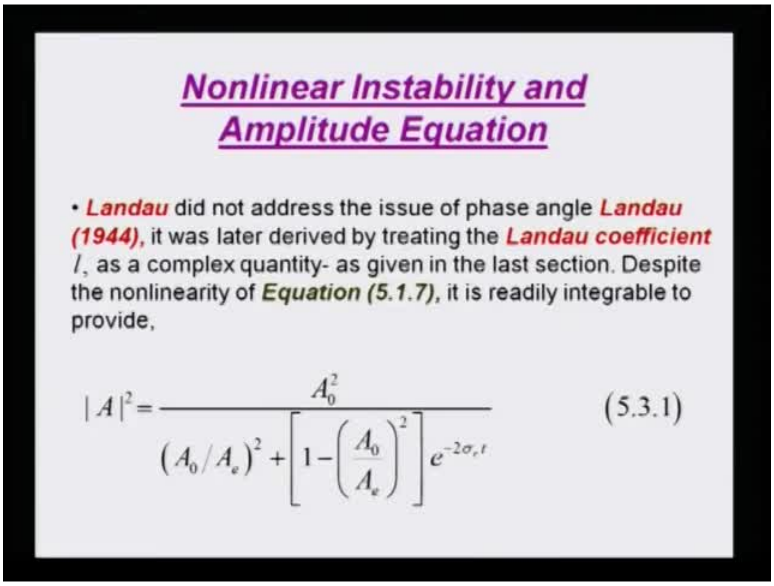

Let us look at the solution of that evolution equation for the amplitude, that we have gotten. This is what we have written. I suppose, this was the reason that, Landau was particularly interested in this model, because A e we have shown here, is like this. So, in that ordinary differential equation, although you have a equation for A square, but there is a A to the power 4 terms. So, it is a non-linear equation; despite that, it admits exact solution and that solution is given here. And, A naught represents the value of A at t equal to 0 and A e is as given here.

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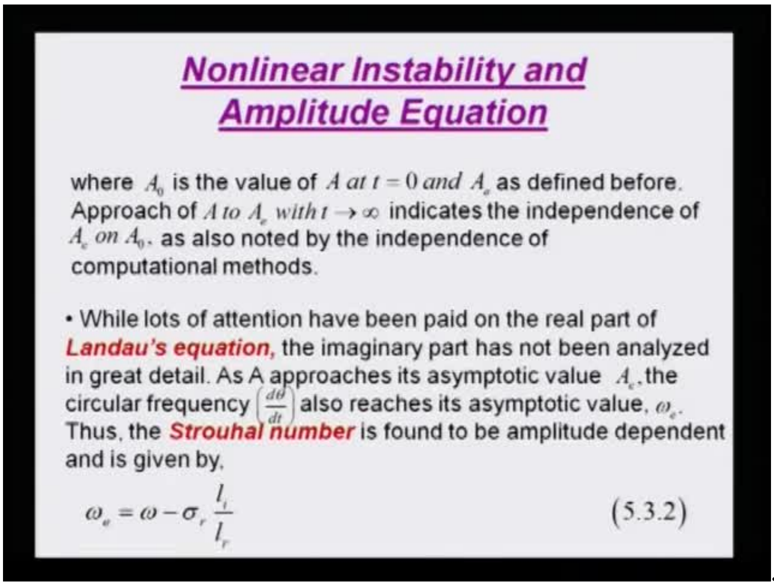

So, there is no such problem. So, you get the time variation like this. Now, in a post critical scenario, of course, sigma r is positive. So, with time, this is going to decay; e to the power minus 2 sigma r t. So, for a very large time, what will happen? This part will cancel out and what will happen, this A naught square, A naught square will cancel out, and A e square goes upstairs, and that is exactly what we saw in the figure, that you achieve a equilibrium state. So, this is the beauty of Landau's equation. And, in the bottom, we talked about, that while Landau himself paid all his attention in the amplitude equation and he claimed that, the phase is indeterminant, which is not true, we have seen; the phase is given by this and that is that equation. And, as A approaches its asymptotic value A e, the circular frequency actually comes out like this. It is interesting, what you notice that, the equilibrium amplitude is totally determined by the linear stability property. That is what it is - sigma r plus i omega is the linear exponent, but it also depends in the Landau coefficient alone.

Again, this is interesting, because this Landau coefficient does not depend on the initial condition; it is again a system property. So, although we wrote earlier, in terms of, like this, but now, suppose, I replace A e square by what we have there, that will be 2 sigma r by l r, then, we are seeing that relation. So, it shows that, it is totally determined by the system (( )); does not depend on amplitude. Unfortunately, though lots of a experimental investigation has been carried out, thinking that the Strouhal frequency is a function of amplitude, which is not true. By the way, there was this experiment done by a French group in 80s, Provencal, (( )) and (( )); what they did was, they did a similar experiment like what we have shown here for Re equal to 53. They took three different cylinders in the same tunnel and they found different critical Reynolds number for different cylinders; and they attributed it to three dimensional effect.

There was this systematic study done by Williamson in 80s, C H K Williamson, and Williamson has written lots and lots of review paper on this topic. If you look at annual review of Fluid Mechanics over last 20 years, we will see at least two or three such review paper by Williamson. And, Williamson has demonstrated experimentally, by doing very careful experiment that, three dimensional effect, instability effects shows up for a Reynolds number which is above something like 180, 200, in that range. So, Provencal and his groups' reporting of critical of Reynolds number for three different cylinders giving three values, all less than 100, implies what; implies what we talked about, because they were doing the experiment in the same tunnel; different cylinders, but they are doing at different flow speed. So, this is something very amazing that, even today, if you look at many experimental results, people talk about free stream turbulence of a tunnel, as if it is a fixed albatross hanging around the neck of the tunnel; it is not so. This free stream turbulence is a property which has a spectrum and this spectrum keeps changing with the operating condition. So, this is what we are trying to emphasize over the last few years that, please do take a look at the spectrum and you will see that, the spectrum keeps changing with speed; and that is exactly what Provencal's group also did. And, that was another sort of a supporting evidence, where you could see the effect of noise, the spectrum.

Now, instead of using this equation, that we have written here, it was Koch, (( )) Koch who suggested that, Strouhal number is given by what you get from the linear instability; the system latches on to the linear instability mode; proposed a kind of resonance, and that is more or less correct; that is what we have also seen that, at the Strouhal frequency, if I have a large enough amplitude, we will get that. So, that is something we need to understand that, the Strouhal number is related to what you get out of linear instability.

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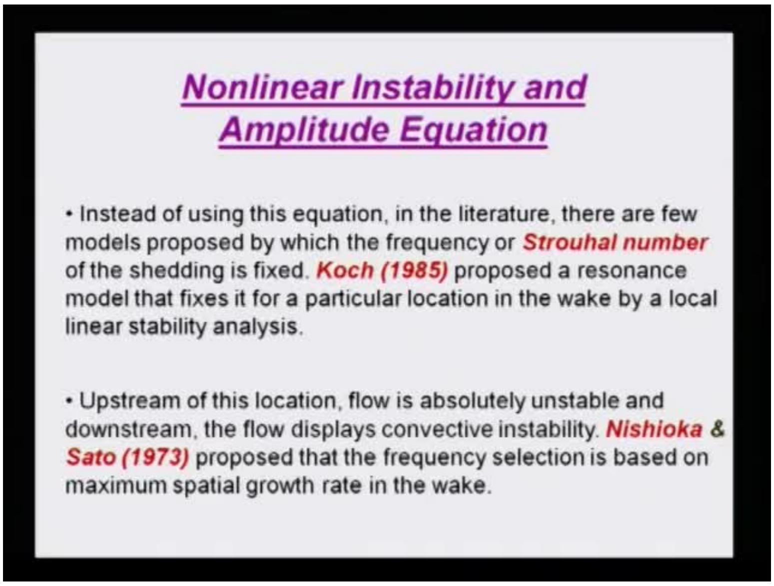

So, let us understand what Koch was essentially trying to say. This is important, because we need to understand, what really happens in a flow like this, in a bluff body flow. See, when we saw the flow past in flat plate, what happened? We created a fixed frequency excitation at a point and then, it created all those waves which convected downstream; you called it a convective instability. What happens in the wake of a cylinder? This has been perplexing people for a long time that, if I put in a probe here, somewhere in the wake, then, I will see the signal increasing with time. So, there is a space very near to the wake of the cylinder, where we see temporal instability; that means, where the disturbance grow with time. Once it grows with time, that gives rise to formation of this separation bubble, the recirculating wake. And, we have no linear saturation that takes us to the equilibrium state, but that equilibrium state, give rise to this vortex shedding. That is perfectly fine, but once the vortex is shed, what does it do, it convects away. So, this is what people have been talking about that, you must have some location in the wake, where the linear instability is predominant and that is fixing your Strouhal number.

Note that, earlier Nishioka and Sato, they also have come to a similar point of view. They said that, if you are looking at this particular location, flow is absolutely unstable;

what is absolute instability? Absolute instability here is something like, you are located at a fixed location and the disturbances are growing in all directions; that is something like your temporal instability also. So, there has been a kind of a, I would say, a model thinking, in the field, where people have interchangeably used the word absolute instability versus temporal instability. So, if I have a temporal instability, well, it is there at a fixed location; from there, the disturbances are going to grow in time if I keep measuring; but then, do we see such a thing at all? We see, eventually, some disturbance growth and then, that is thrown out; that is exactly what we are seeing in the bluff body wake.

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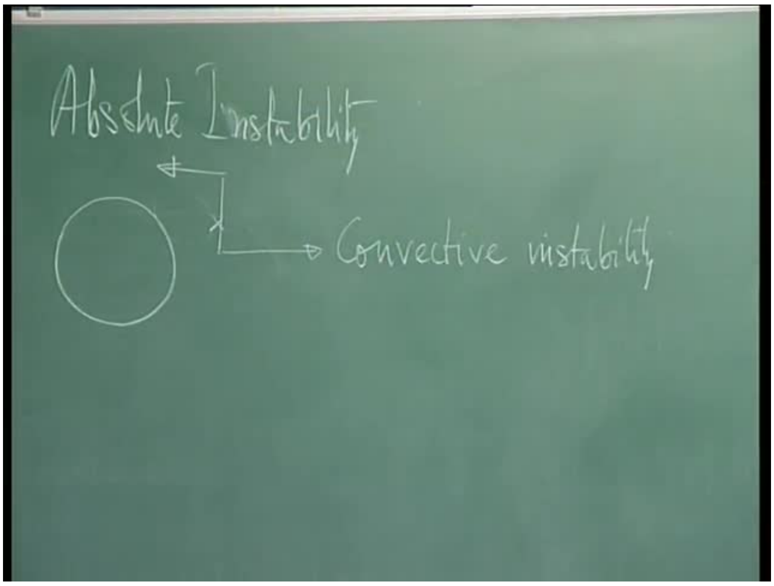

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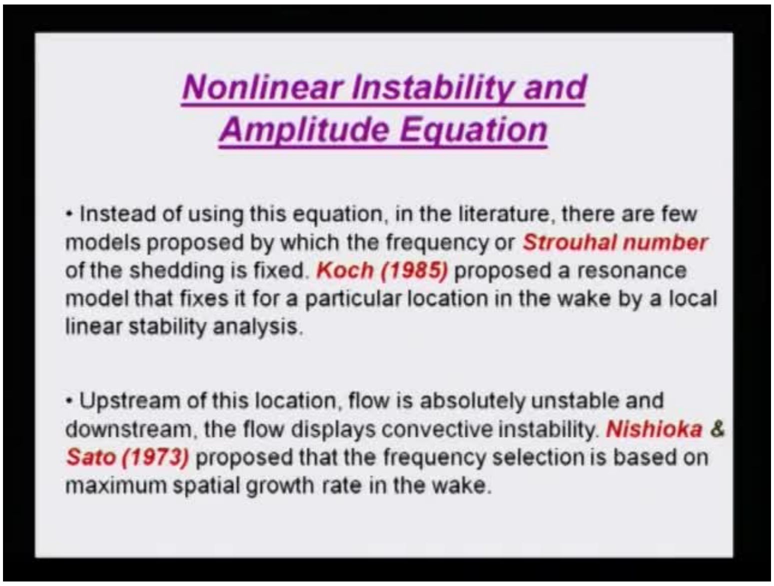

What Nishioka and Sato said that, if you are looking at it, that location, that we are talking about, upstream of which, if I look at it in this direction, then, we have absolute instability; and downstream of which, we have convective instability. So, in a sense, that kind of, sort of explains, what you see. And, what was Koch proposing in his work is that, at this location, you are getting the things started, because of some resonance mechanism; that is a resonance model.

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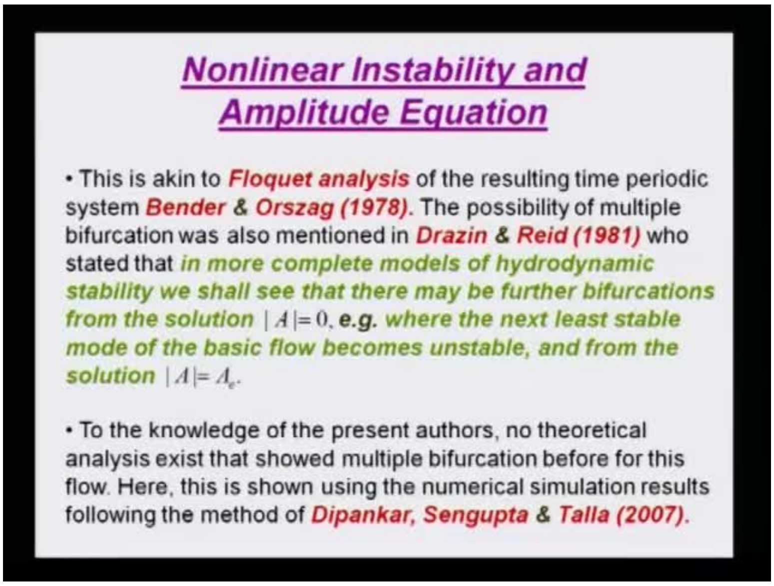

So, I suppose, this is what happens, what people have looked at. Now, let me also, ((audio cut)) what happens. We have a Karman vortex shedding, fine. Now, suppose, we decide to increase the Reynolds number further, then, what we are going to study. Then, that particular flow itself, could be susceptible to background disturbances and suffer instability. That instability analysis is done by what was proposed by Floquet. You can see this book by Bender and Orszag, that talks about instability of a time periodic system. It is a fairly, a decent way of looking at the many time dependent systems in electrical engineering, you know, when we are studying instability via ODEs. Now, if you get time periodic state, that state itself could be susceptible to further instabilities and that is what is done.

This possibility of multiple bifurcations was mentioned in Drazin and Reid, who stated that, if we take a more complete model of hydrodynamics stability, we shall see that, there may be further bifurcations from the solution; that is where the next least stable mode of the basic flow becomes unstable. That is precisely, what I am talking about; that suppose, I miss the bus here, I still have A equal to 0, I can latch on to here. And, that is what is being ((audio cut)) there, that is subsequently, later, higher value of parameter, that A equal to 0 would be again susceptible to instabilities. And then, you can get higher order instabilities.

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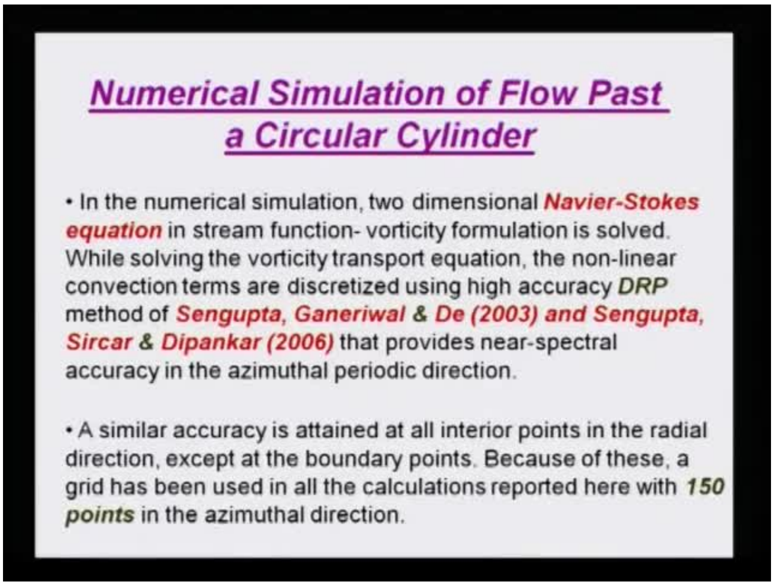

To the knowledge of some of us, we do not see, if there are any theoretical analysis, that has actually shown multiple bifurcation. This model was not studied till very recently. We did propose some multiple Hopf bifurcation model. So, that depended totally upon numerical simulation of two dimensional Navier-Stokes equation. We actually, used stream function vorticity formulation, because of its inherent accuracy.

The accuracy of the solution, one of the factor I told, is about dispersion relation preservation. In addition also, what we need to do is, we need to see certain conservation principles are also followed very carefully. Conservation principle of what, conservation of mass; if I solve, say the Navier-Stokes equation, in primitive variable, you know that, it takes a lot of effort to preserve the mass conservation correctly. That actually, is quite time consuming. The moment you use stream function as a dependent variable, you obviate the need for satisfying mass conservation. It automatically, analytically satisfies everywhere. And, since, we are looking at vorticity dynamics, so, studying vorticity itself as the other parameter, is a very attractive one and we do see the point. When we solve that equation, the non-linear convection terms, after we sort of discretized, using dispersion relation preservation methods, which provides, actually, near-spectral accuracy.

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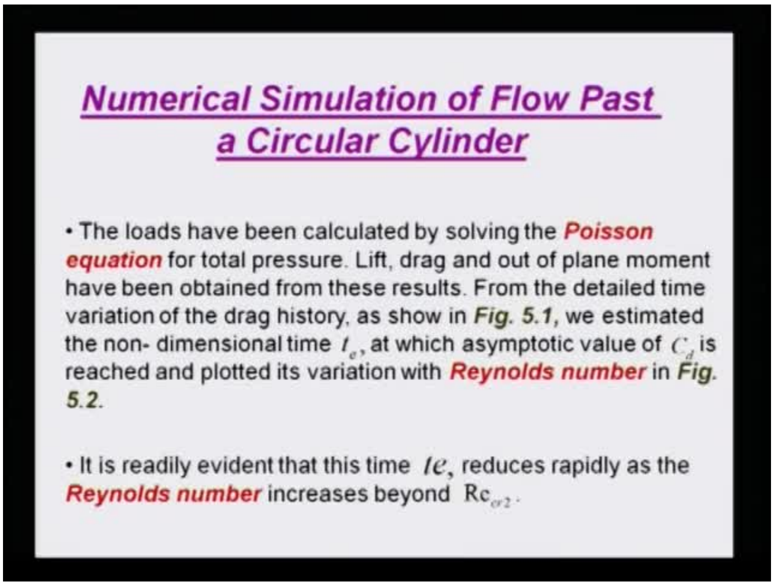

We have this accuracy so much high that, we can do calculations with a very few points in Azimuthal directions as well as in the wall normal directions. And, the loads, the lift and drag and the pitching moment, are obtained by solving the Poisson equation for total pressure. Now, what is the Poisson equation for total pressure? It is nothing, but that same energy equation that we wrote it down. So, basically, what happens in many of the calculations those are reported in the literature, people do talk about solution of Navier-Stokes equation, which satisfies the mass and momentum conservation.

Theoretically speaking, theoretically speaking, satisfaction of mass and momentum for incompressible flow, guarantees a satisfaction of the energy, analytically; but when I do it numerically, there is no such guarantee. However, when you actually calculate the load by solving the Poisson equation for total pressure, that is nothing, but a statement of energy conservation. So, basically, in this approach, what we are doing, we are, in a sense, also satisfying energy conservation; and this is something that needs to be a fairly well understood.

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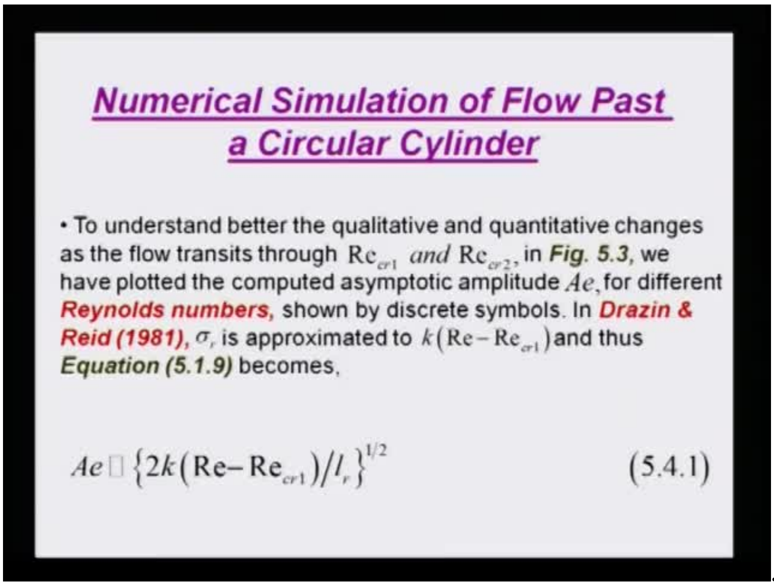

Now, well, of this part, we will not talk about this equilibrium, time, etcetera, but this, what we need to find out is, let us talk about this multiple Hopf bifurcation feature that we are talking about; that is what we will come to, shortly. I will show you the actual figure.

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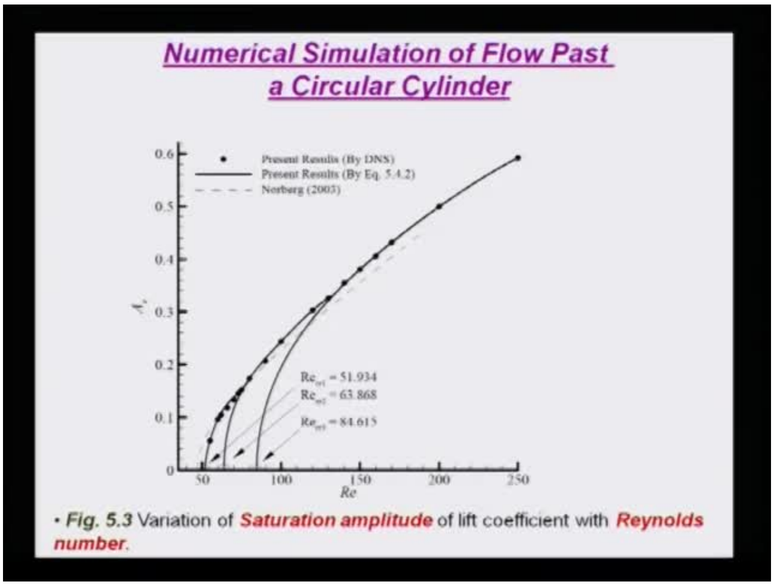

If we plot A e versus Re, we expect parabolic variation, as given by that equation I explained, if it was true. If it was dominated by a single mode and single bifurcation, we should get a single parabola; but suppose, there are multiple bifurcations, then, what happens? I will show you, if I have this figure; here we have that figure. This is what we get. If we do numerical simulation and put in also, a sort of a correlation, that was a proposed by Nordberg, shown by the dotted line, this is what we get, when we actually, numerically solve the Navier-Stokes equation. This discrete points are computed solutions. And, what we are seeing here is, basically, three different qualitative parabolas. Like this later part, as if, if I would analytically continue, it goes and hits the axis here; and there is a second part, which is there and there is this part.

Well, how do you know there are three parts? You will see those kinks; we are noticing there are two such kinks. So, we are saying that, this could be the first bifurcation, meeting the second bifurcation somewhere here, and this is the meeting point of the second and third bifurcation. So, this is an example, where you can see, it is very much there, inbuilt in your governing equation, Navier-Stokes equation, and what you find, is a very interesting thing.

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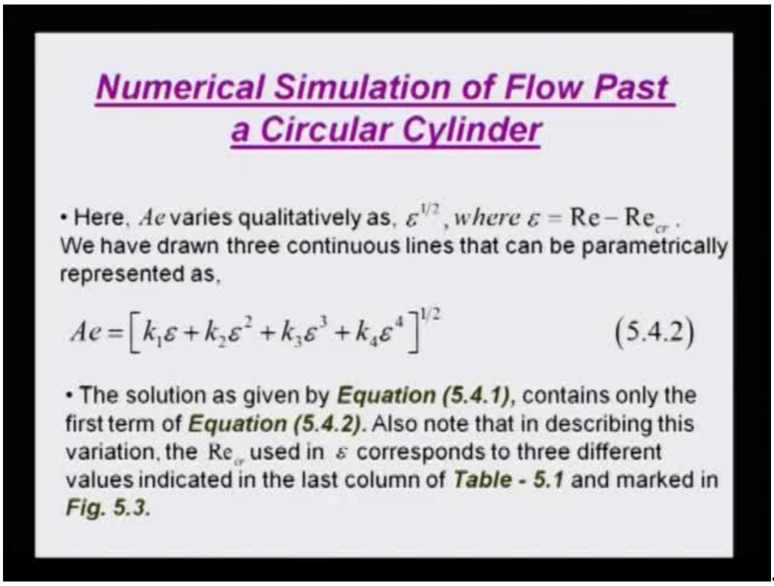

The first Hopf bifurcation, according to the numerical method we have chosen, that indicates a critical Reynolds number of, about 52. The second one is about 64; the third one is about 85. Now, what I can talk about here is that, these calculations are done without any background disturbances. The moment we add background disturbance, we could actually, bring it further down. So, this is something, that we should keep in mind that, multiple Hopf bifurcation is not a really a figment of imagination. So, what we tried to do actually, if you are interested, it is some kind of a empirical modeling; what we try to do is, try to fit in a curve A e as a function of epsilon; epsilon is the departure of the current Re from the Re critical. And this, we try to do it in terms of a higher order polynomial and you can try to fix those constants k 1, k 2, k 3, k 4.

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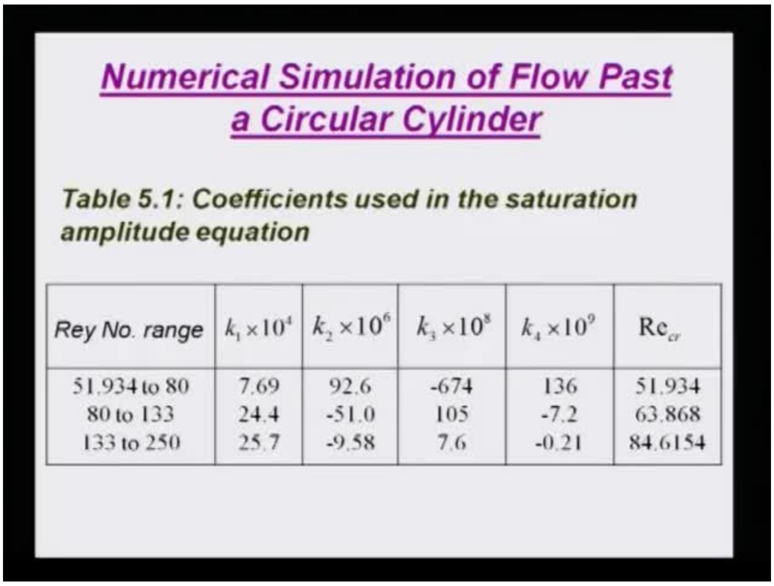

Rey No. range	$k_1 \times 10^4$	$k_2 \times 10^6$	$k_3 \times 10^8$	$k_4 \times 10^9$	Re_{cr}
51.934 to 80	7.69	92.6	-674	136	51.934
80 to 133	24.4	-51.0	105	-7.2	63.868
133 to 250	25.7	-9.58	7.6	-0.21	84.6154

And, what we have done, we have seen in that figure that, we have three different distinct parabolas. So, in this three different loops, we could actually obtain this multiplicative coefficients and this is what we get. By and large, what you can notice is, basically, what are these? Tthis is like your cubic and quartic term, because, you see, they have been magnified 10 to the power 8 times. So, what it shows that, even though you are trying to fit in a higher order polynomial, this dependence on cubic and quartic, is rather weak. So, essentially, what you are seeing is that, basically, the original model which was supported by Landau, that we should have a parabolic variation. But it is only, the fact is, there are three different parabolas; in this range, the first parabola. This is the range of validity of the second and this is the third one. I think I will just stop here; we will pick up from this point onwards and see the other aspects of bluff body flows.