Instability and Transition of Fluid Flows

Prof. Tapan K. Sengupta

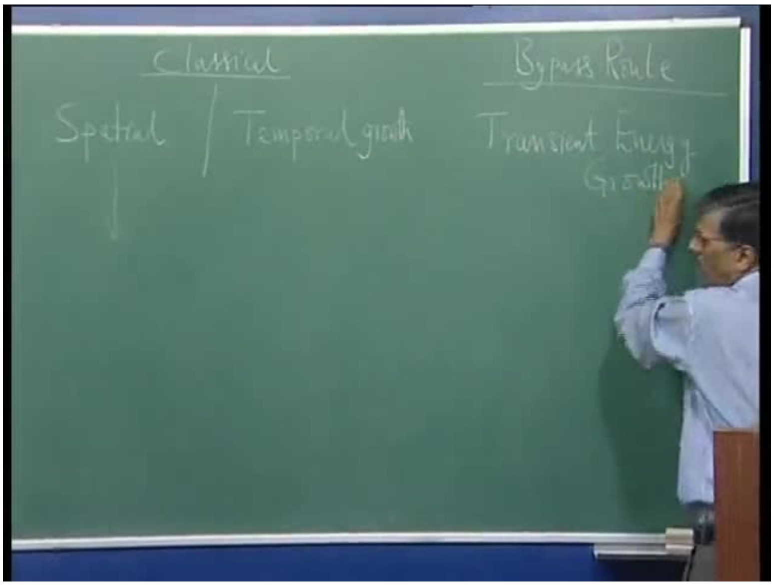
Department of Aerospace Engineering

Indian Institute of Technology, Kanpur

Module No. # 01

Lecture No. # 28

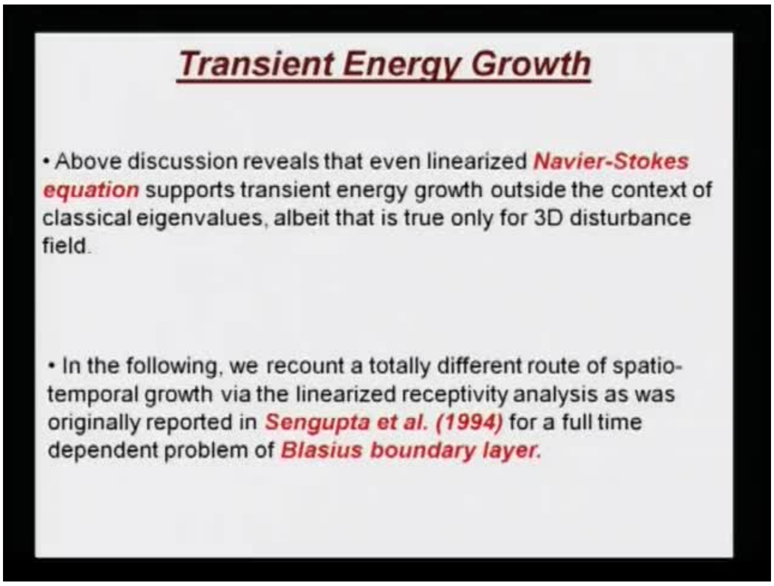
(Refer Slide Time: 00:45)



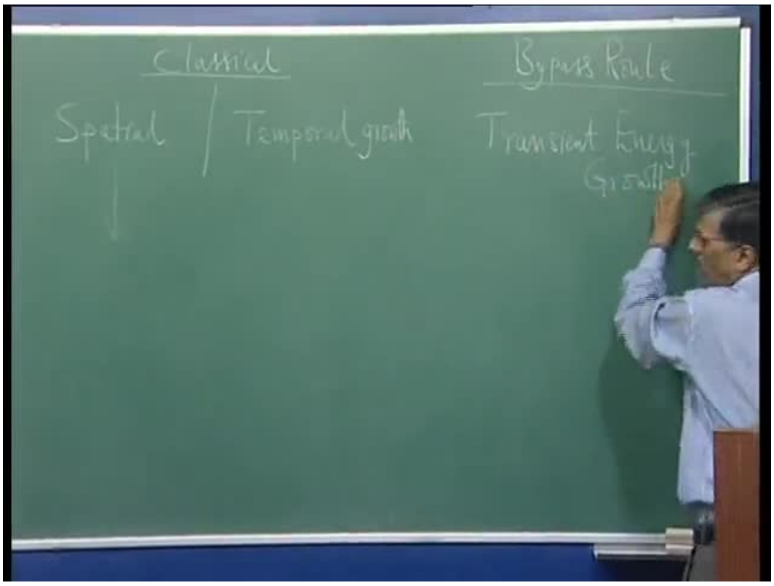
So, we had been discussing about disturbance growth in shear layers, and we had generally classified the problem into two aspects that for the sake of mathematical simplification, that the growth is either in space or in time. And, there were some examples given. So, we have been looking at disturbance growth either as a spatial problem or as a temporal problem and one of the example that attracted us most was this external flow, where we saw through a lots of tests and trials that there are situations in external flows where you do indeed see disturbance growth in space.

Now, that explained, most of the time, you know, like, I had explained to you about the paper by Trefethen and his co-authors, who pointed out that for shear-driven flow, it looks ok, but even then, it was said that Blasius boundary layer was not a very good example where spatial growth was seen. So, people did talk about, from this classical approach where we either look at spatial or temporal growth; there was this other part, where we did talk about the bypass route, classical and bypass route. In bypass route, of course, we accepted that things can happen simultaneously in space and time.

(Refer Slide Time: 02:35)

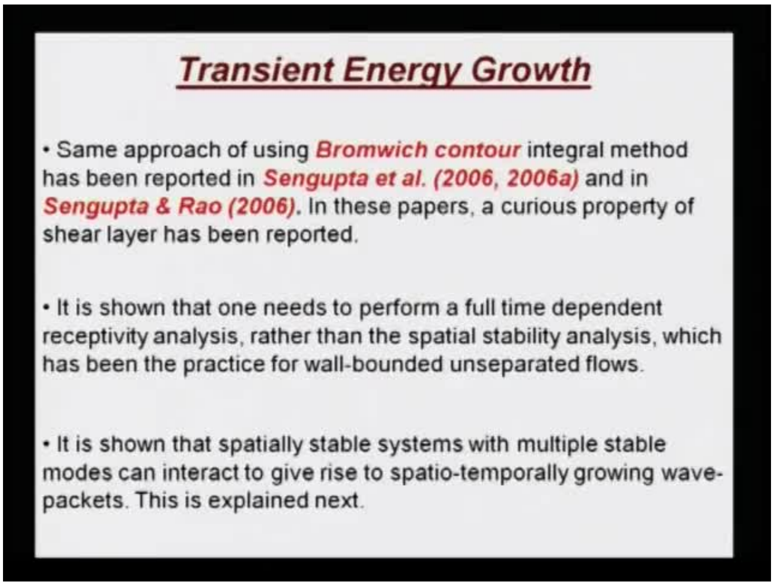


(Refer Slide Time: 02:44)



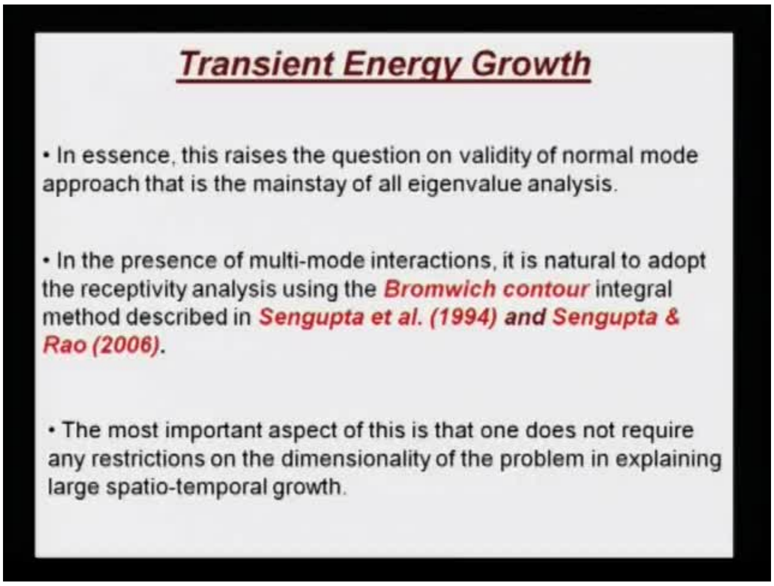
However, in those cases, where you do indeed see spatial growth, you also see at times that, there are simultaneous transient energy growths. While this transient energy growth was projected as one of the most rival route of bypass transition, what we started discussing before the break is that, even when we have strictly spatially defined problem, we do see associated transient growth. And, this following part, we are basically talking about a spatio-temporal growth which is observed directly via linearized receptivity analysis of Navier-Stokes equation. And, this is not something new, which we have done for quite some time. The only difference between what we studied here and the rest of the studies, in the context of spatial growth is, we treated the problem as a fully time dependent problem.

(Refer Slide Time: 03:56)



So, we introduced what is called as a Bromwich contour integral and in this Bromwich contour integral, we approached the problem in the same way as people have been doing using Orr-Sommerfeld equation, but with a difference that, we have to take the contours simultaneously in both the wave number and circular frequency plane. This has been followed up in recent times also and a very special property of the shear layer was noted that, when you do indeed perform a full time dependent receptivity analysis, rather than the spatial stability analysis, you do see that, there are multiple modes; there are more than one modes.

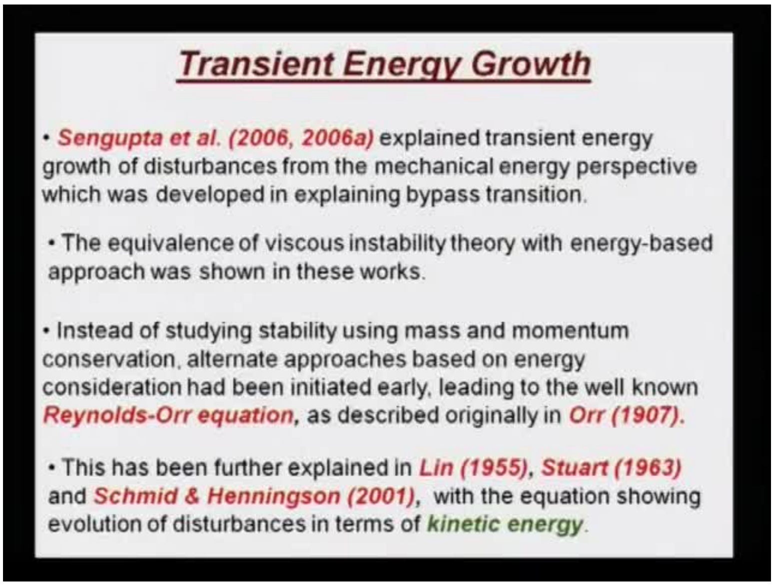
(Refer Slide Time: 05:23)



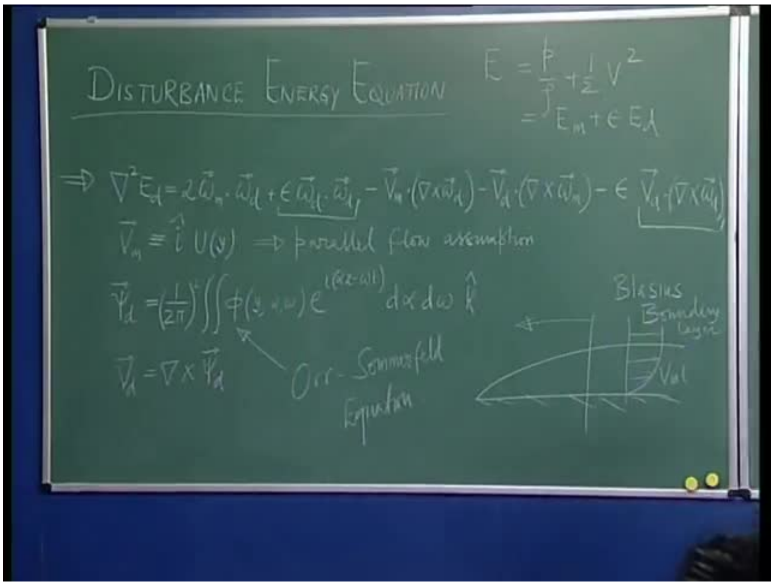
While the spatial stability theory looks, focuses upon only the dominant least stable mode, but in this approach, in Bromwich contour approach, you do track the presence of multiple modes. And, what was noted, this curious observation that we are talking about that, this multiple modes, at the onset time of the problem itself, can give rise to transient growth, which will grow in space and time together. And, this is what we had started discussing about. Well, when you do have some such thing happening, multiple modes interacting with each other, then, you start questioning the validity of normal mode analysis; because in a normal mode analysis, you only study one mode at a time, without any recourse to having various modes interacting.

When you have multiple modes interacting, then, of course, Bromwich contour integral becomes a very natural choice, and the most interesting aspect of this is that, the results that you obtain is not restricted to only two dimensional flow or three dimensional flow. If you recall, the earlier work on transient growth by various groups who talked about non-normal modes, they showed such a mechanism only for three dimensional flow. So, let us try to see, if we could explore this in the context of two dimensional disturbance field itself and then we can notice this.

(Refer Slide Time: 06:38)



(Refer Slide Time: 06:58)

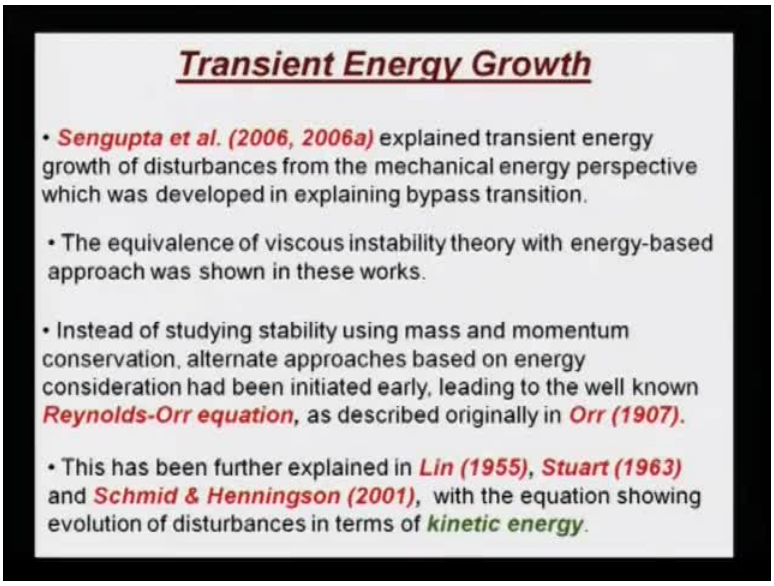


Now, we do actually adopt the transient energy growth route, starting from what we have developed in describing bypass transition in terms of disturbance energy equation, that I have written it down for your reference, which we have done in the previous set of lectures, which showed that, if I define the energy itself, as like a Bernoulli's head, that we have talked about before, then, I can split this energy into two parts, the mean part as well as a disturbance part, and here, we are talking about the disturbance energy. So, if I split this into a mean part and a disturbance part, then, we did show the governing equations for disturbance energy is given by this equation.

It is basically, a sort of a boundary value problem, because the energy is given by the Laplacean operator, which is shown here to arise from various interactions of the mean and disturbance field; subscript m refers to the mean field; d refers to the disturbance field. And, some of the terms are of the lower order, for example, this one and this one. So, even if we throw them away, what we notice that, the evolution of disturbance energy is given by how mean and disturbance vorticity interact with each other. How basically, the mean flow interaction, the ((helicity)) of the flow that is the curl of the vorticity field. And, also a similar complementary term, which tells you of the disturbance velocity, sort of takes a dot product in the ((helicity)) of the mean flow.

What is the mean flow, what we are studying? We are studying a very simple problem here. We are studying the prototypical flow, that has dominated this field, that we are talking about, say zero pressure gradient boundary layer, which was originally studied by Blasius. And, if we neglect this part, where significance growth of the shear layer takes place, then, we can treat this boundary layer to be almost like a parallel, without any growth and for such a boundary layer, I could talk about a mean velocity profile, which I am calling as V M and that is given by this; this is your parallel flow assumption. So, this is your parallel flow assumption. That is, what we are in essence saying, the shear layer does not grow with x very much and then, we substituted that in linearized vorticity transport equation to get the Orr-Sommerfeld equation that we know. That was the part, that defines this disturbance field amplitude, because this is governed by our Orr-Sommerfeld equation.

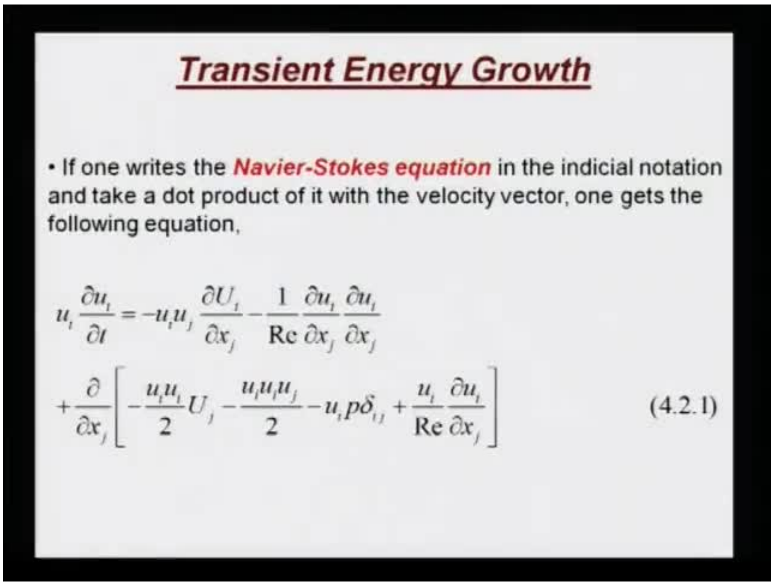
(Refer Slide Time: 10:56)



Now, what we are actually attempting to do here, is to show that, the viscous instability theory as given by the Orr-Sommerfeld equation and as given by this disturbance energy equation are equivalent. That is one of the thing, that we want to show, simply for the reason, it is a kind of a sense of completion, that I am discussing this with you; because, this equation that we have written here, is a product of the full Navier-Stokes equation. We did not have to do any approximation, any assumption, unlike Orr-Sommerfeld equation. Orr-Sommerfeld equation is a restricted set of equation, because, it assumes a flow to be parallel. And, it also, of course, neglects all the non-linearity, whereas, this equation, disturbance energy equation that we have developed, we do keep the scope for including the non-linear term, which has been underlined here.

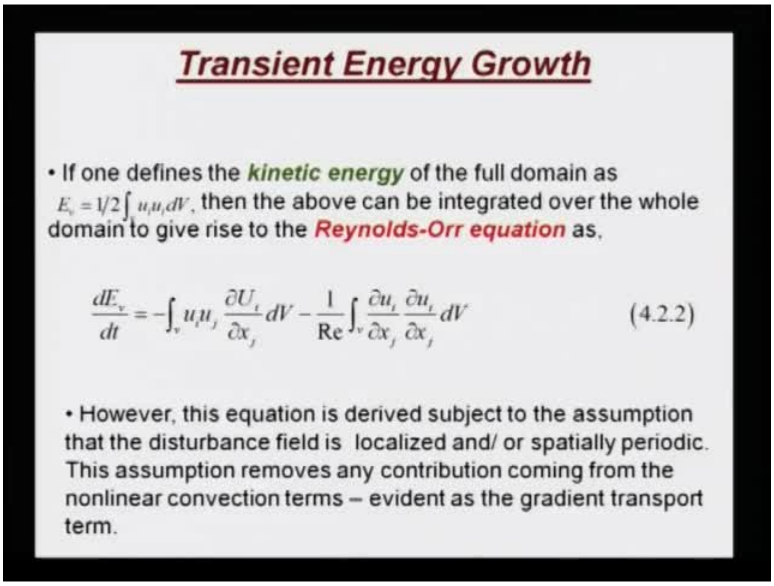
We can keep them or we can study the linearized part, but then, if I am looking at the linearized part of this disturbance energy equation, that must have all the seeds of the Orr-Sommerfeld equation and that is what we are talking about. Now, there are this historic legacy of studying energy propagation equation for flow instability. There were a couple of earlier attempts by Reynolds and Orr, who studied the kinetic energy only; whereas, we are talking about here, not only the kinetic energy, but the pressure head too, and in the absence of any other body force, this constitutes the total mechanical energy.

(Refer Slide Time: 13:13)



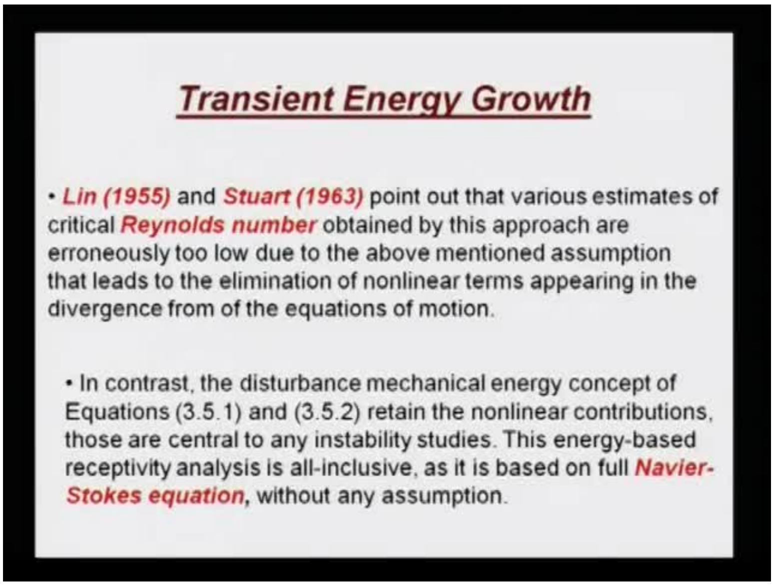
So, this is somewhat more inclusive, than what was attempted there. Well, it was, been discredited later, in these following work, where they say that, of course, the results that you get out of such analysis, the critical values of the Reynolds number, etcetera, they are indeed, very low. So, what you do is, you start off with the momentum equation and take a dot product with the velocity field to get this equation. And, this forms the basis of Reynolds-Orr energy equation. So, we did see that, it has a time rate of kinetic energy here. What is this? This shows how the Reynolds stress interacts with the mean shear. This is the viscous dissipation term and this is the gradient transport, because it is written as a sort of del del x j of a term.

(Refer Slide Time: 14:15)



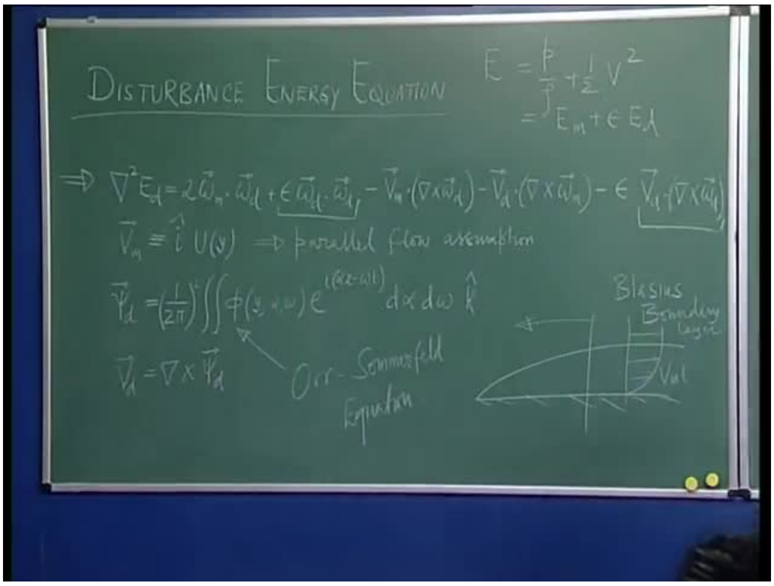
So, this was the starting point of the equation. There is nothing wrong up to here, but subsequently, what people did, they did study this with the assumption that, if you are looking at any disturbance field, it is either very localized or spatially periodic, and then, what happens is, then, those gradient transport term drops out. So, the gradient transport term drops out, leaving you with this equation. And, these are those dissipation, sort of balancing your interaction of the Reynolds' stress with the mean shear. But you realize that, this is, in a sense, if you are looking at the energy equation for the disturbance, then, this quantity, the mean quantity, mean shear, is already known.

(Refer Slide Time: 15:35)



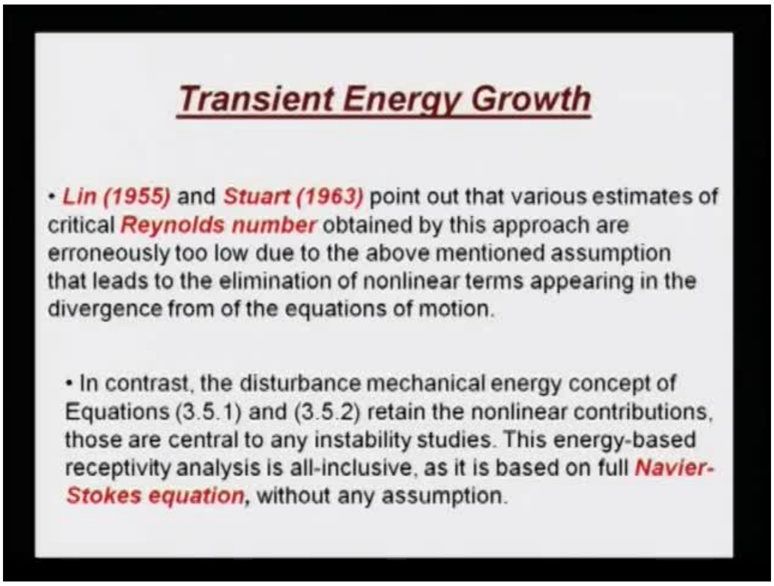
So, what you are looking at is a kind of an equivalent linearized version, because, this is a second order tensor; so is e v; that is also a tensor. So, in terms of that second order moment, this is a linear equation and what has happened, in making that assumption that the disturbance field is localized or spatially periodic, we have actually thrown the non-linear term, the initial term. And, this was pointed out by Lin and Stuart and they said that, of course, the critical Reynolds number obtained for many of the flows were too low and of course, that was tracked to elimination of non-linear terms, that we lost, when we made that localized or periodic assumption.

(Refer Slide Time: 16:04)



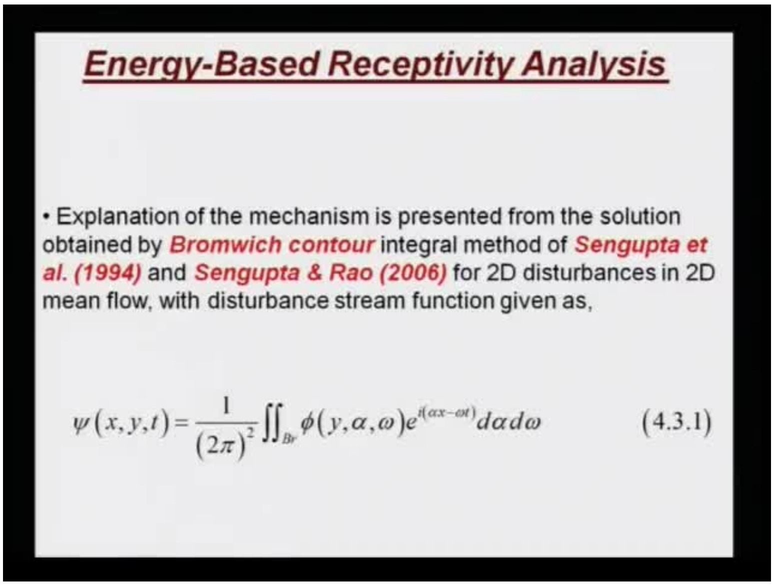
However, in contrast, if we look at the disturbance mechanical energy concept, that we have written there, the non-linear terms are very much there. You can see the non-linear terms can be kept, if we want to; however, even this terms also originate from the non-linear convection terms. So, they do not simply go away.

(Refer Slide Time: 16:23)

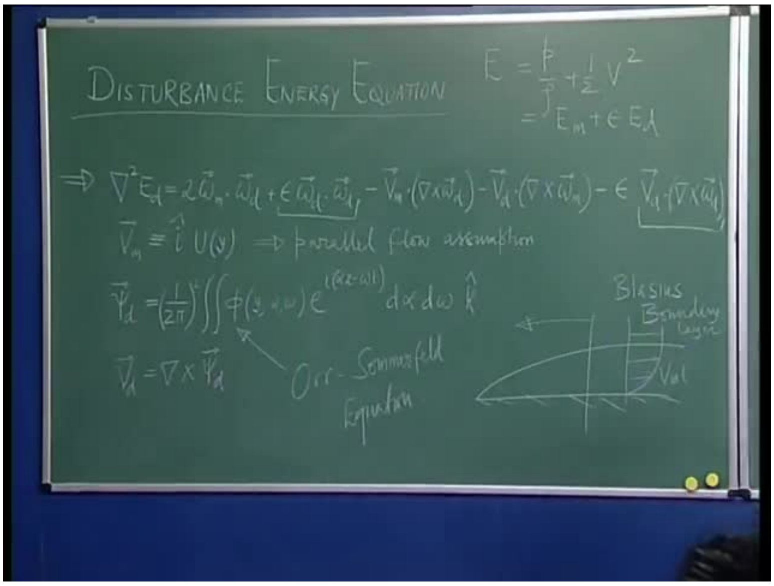


So, the non-linear contributions are kept intact in this instability study and this energy based receptivity analysis is all-inclusive, because it is based on full Navier-Stokes equation.

(Refer Slide Time: 16:39)

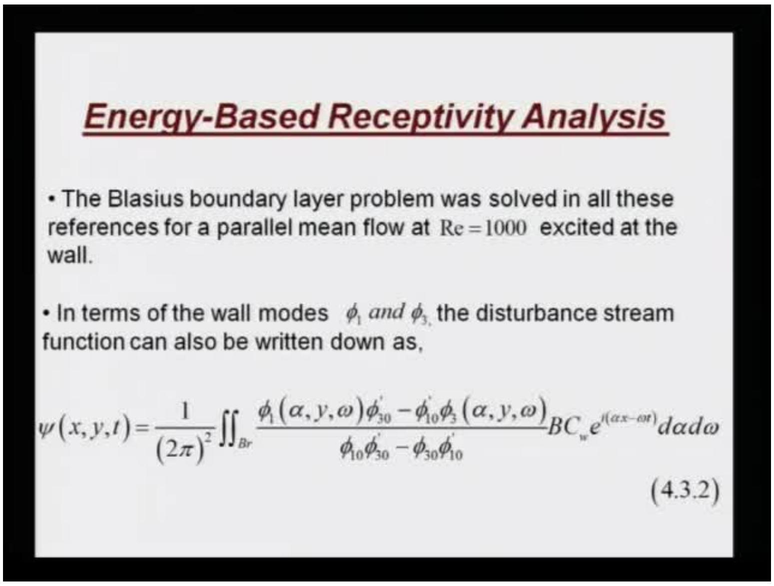


(Refer Slide Time: 16:46)



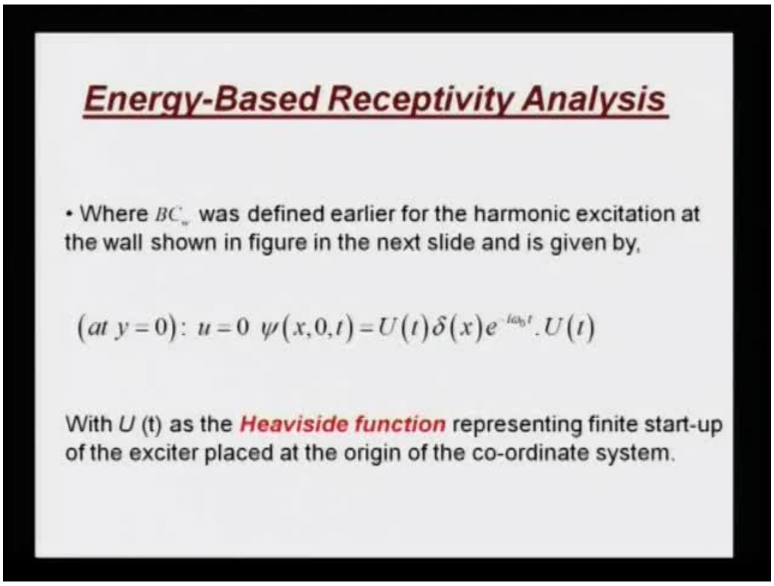
Now, let us spend a little time in understanding how we go about it. Well, how we go about it is the following; we define the disturbance field in terms of a disturbance stream function with a Fourier-Laplace amplitude and please understand that, psi d, I have written it as a vector, because if I have flow in the x y plane, psi is in the k plane. So, that is what we have done, because, with the help of such a vector notation of psi d, I could obtain the disturbance velocity. Disturbance velocity is nothing, but the curl of psi d.

(Refer Slide Time: 17:23)



So, that is what we do. And then, having described the mean flow and the disturbance flow, we can take a curl of this velocity fields and get the vorticity components. So, we can, perhaps, get everything in there. Now, if I am looking at the Blasius boundary layer problem with a parallel mean flow assumption, then, in terms of those wall modes, phi 1 and phi 3 are the wall modes, because they are the ones that will decay with the height; that means, if I excite the flow inside the shear layer, this disturbance stream function can also be written down like this; and you can identify very clearly, the dispersion relation appearing in the denominator. And, this is the type of input, the excitation field that we give.

(Refer Slide Time: 18:36)



We have talked about various kinds of possibilities and we can talk about, a sort of a impulsive excitation, as we have done for impulse response; or we could take a strip excitation, we can take Gaussian excitation; all kinds of things have been discussed. And, these equations have to be solved, subject to the various boundary conditions and in this case, please do understand that, we are interested in space-time dependence. So, we cannot just simply take the signal flow route, signal problem route, where we just simply say that, time dependence is as given by the imposed harmonic excitation, but we do need to start the problem at a fixed time. So, basically, that is where this Heaviside function U of t comes into picture; that tells you that, this problem was started at a finite time.

(Refer Slide Time: 19:20)

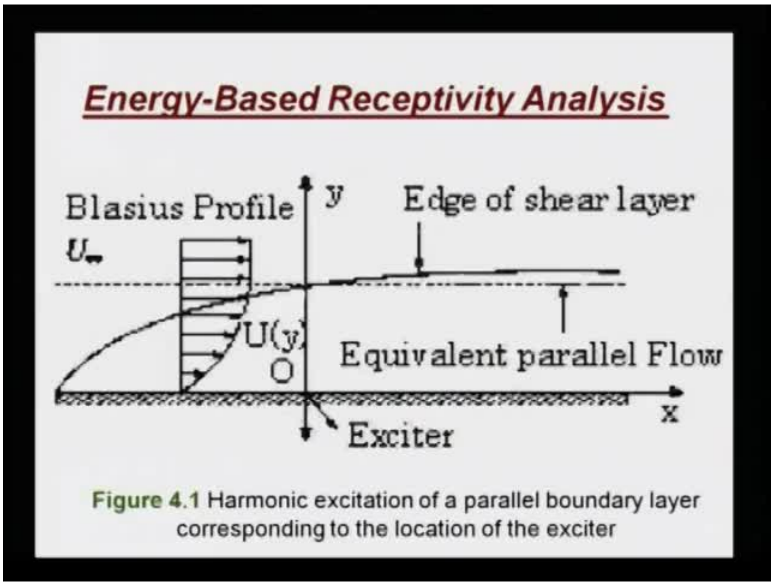
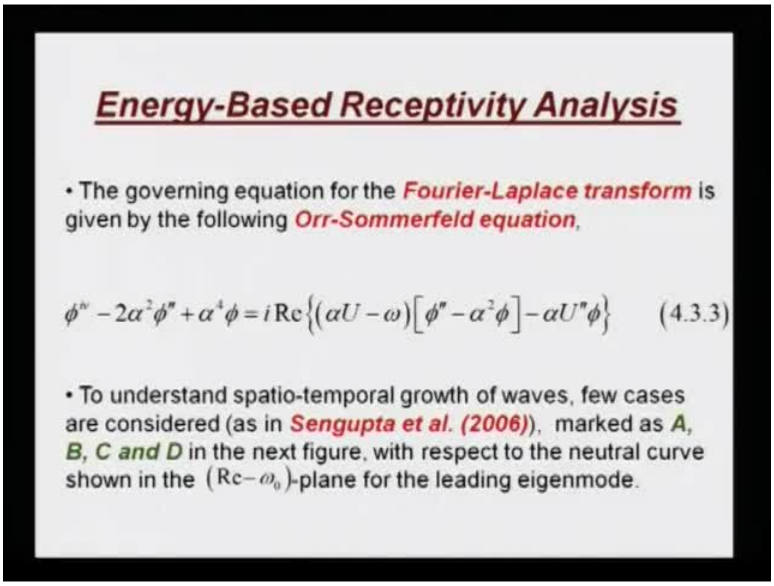


Figure 4.1 Harmonic excitation of a parallel boundary layer corresponding to the location of the exciter

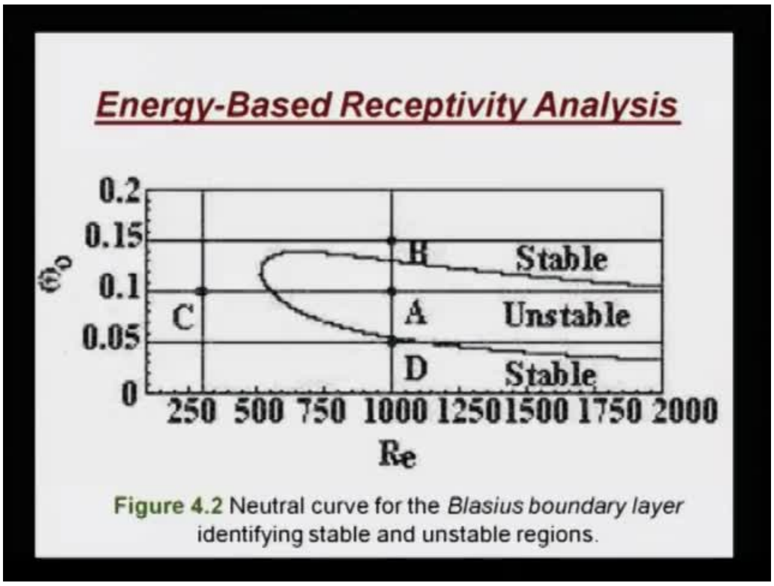
So, we basically solve this problem. We have this Blasius profile on the mean flow like this, and let us say, we position the exciter at a location like this, and so, we consider as if, this actual shear layer is replaced by an equivalent parallel shear layer with the same thickness, at the location of the exciter; and then, we study. This is interesting because, without this frame work for a parallel flow, we do not have wherewithal to prescribe the origin. Now, we can even prescribe x origin for the flow, at the location of the exciter. We can also start off to give a meaning of what t equal to 0 means; t equal to 0 means, when we start the experiment, when we start the excitation.

(Refer Slide Time: 20:20)



That is how we got that Heaviside function coming through the boundary condition. And then, the governing equation can be simplified. This is quite well known.

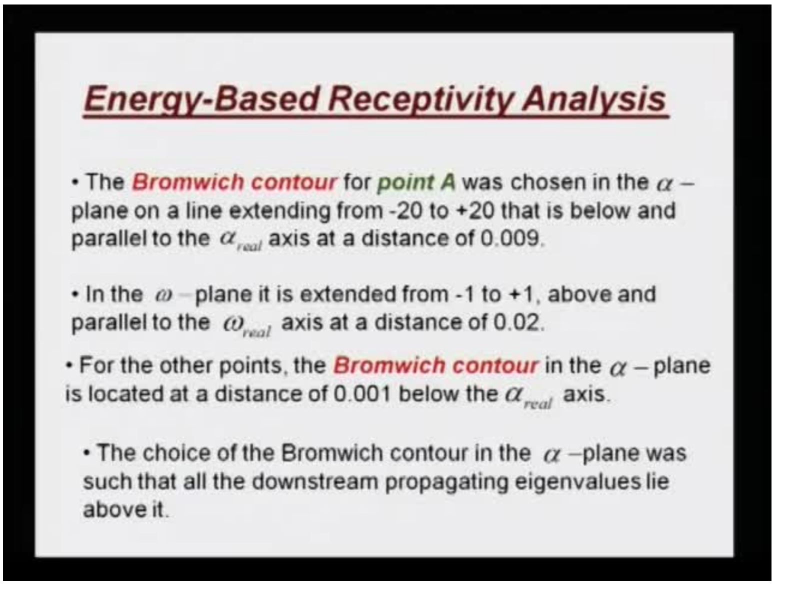
(Refer Slide Time: 20:36)



Now, what we do is, try to understand this problem, the dynamics, by this Bromwich contour integral, by taking various points and this is what we did. We took four points. For various reasons, we will see why we have chosen these four points. Well, we, first of all, we wanted to keep the Reynolds number constant and along this Re equal to 1000 points, we have chosen three points. The point A corresponds to where we expect the leading mode, the Tollmien-Schlichting mode to be unstable; that is what the point A represents. In addition to this Tollmien-Schlichting mode, we will have additional stable modes. The point B actually, corresponds to one such case, where we do not have an unstable quantity.

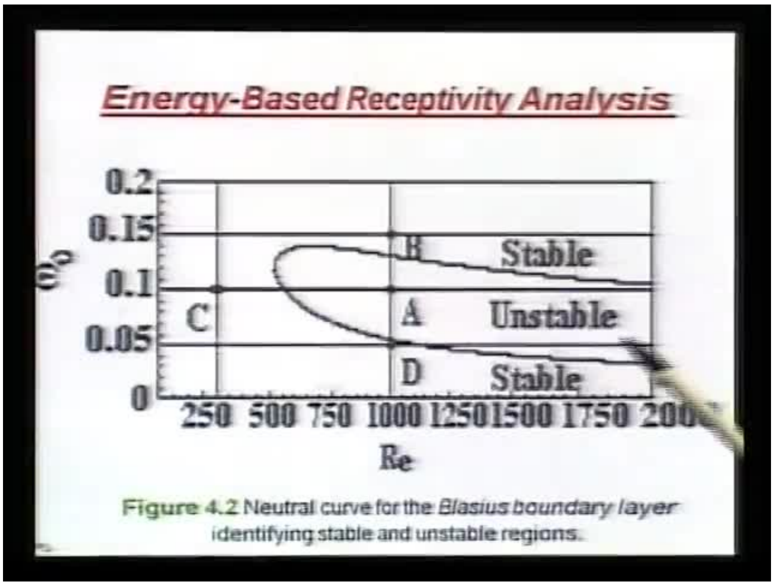
So, this is on the higher frequency side. If this was, A was for 0.1, this is 0.15 and here, if we do a, sort of a stability analysis by grid search method, we will find out the various modes to be stable. In contrast, we are looking at the point D which is also spatially stable, but which is very close to the neutral curve. See, here, if I create a excitation corresponding to this parameter, I am going to see a disturbance field, which is hardly decaying. Finally, this point C, which is far upstream of the neutral load, it corresponds to a case, where the Orr-Sommerfeld equation does not predict an instability at all, for any frequency.

(Refer Slide Time: 22:10)



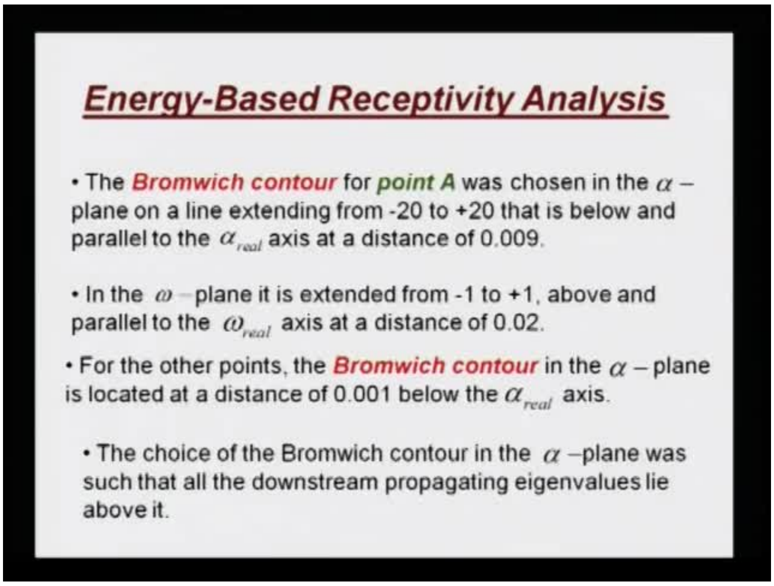
So, these are the four points that we had studied and the Bromwich contour, I explained to you in the last class also that, theoretically speaking, in the alpha plane, you would like to take it from minus infinity to plus infinity; and because, we are looking at the asymptotic part of the solution, for which alpha could be very small.

(Refer Slide Time: 22:37)



So, this values, like plus minus 20 along the real axis… So, we take four representative points; A is the unstable TS wave point. So, one of the mode is decidedly unstable; whereas, B represents a point, where all the modes are stable and they are away from neutral curve. So, the decay rate will be significant. In contrast, point D is also stable point, but it stays very close to the neutral curve.

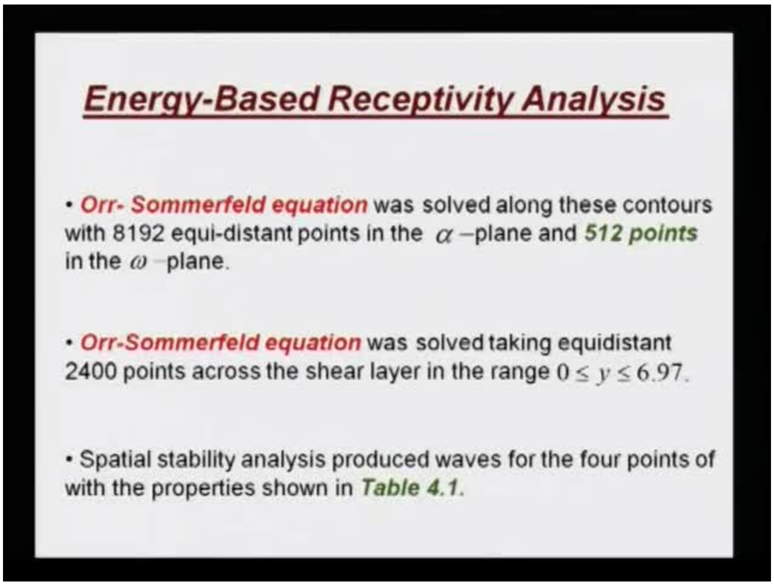
(Refer Slide Time: 23:27)



So, you may see the resultant disturbance field, may have a sustained a signature of disturbance. Finally, point C corresponds to the case, where it is significantly ahead of the neutral loop, implying that, here, all the frequencies are stable. And, we did talk about, in the last class also that, we have to choose the Bromwich contour, for any point; will have to choose the point in such a way that, all the downstream propagating modes should be above this Bromwich contour in alpha plane. So, that is what we did. We chose a Bromwich contour, which is 0.009 below the real axis and it extended from minus 20 to plus 20, in the direction parallel to the alpha r axis. So, that was your alpha Bromwich contour. Similarly, the omega Bromwich contour, we have to take it, it should be placed so far above, so that, any mode that are there, they must be below; because, you know that, this is coming with a minus sign here. So, all the modes have to be causal. So, we cannot have causality violated; that is why, we do indeed, place the Bromwich contour in the omega plane, at a height significantly higher, compared to what we took in the alpha plane.

So, this is at 0.02 and here, we have once again, taken a range of minus 1 to plus 1; whereas, the, for other part, the Bromwich contour, the alpha plane, remains more or less same. We could actually bring it closer to the alpha r axis, because we know that, unstable mode is not there. So, we can bring it closer to the alpha r axis and the omega axis will remain the same.

(Refer Slide Time: 25:12)



So, there is no such change for the omega contour. Now, this will perhaps, give you an idea of what kind of effort is needed, even in solving a linearized Navier-Stokes equation with adequate accuracy. For example, in the alpha plane, we have taken 2 to the power 14 terms. So, that is, sorry, 2 to the power 13, 8192 equidistant point in the alpha plane. And, we had taken 512 points in the omega plane. And, this was what we did in a 2006 work and you understand that, in the shear layer itself, we have gone only about 7 delta star; we have gone a distance from the wall to 7 delta star and we had taken 2400 points for the Orr-Sommerfeld equation. And because we are used a Runge-Kutta method, so, we had to take the mean flow, which should be having twice the number of points. So, a Blasius boundary layer, with 4800 points has been obtained.

(Refer Slide Time: 26:42)

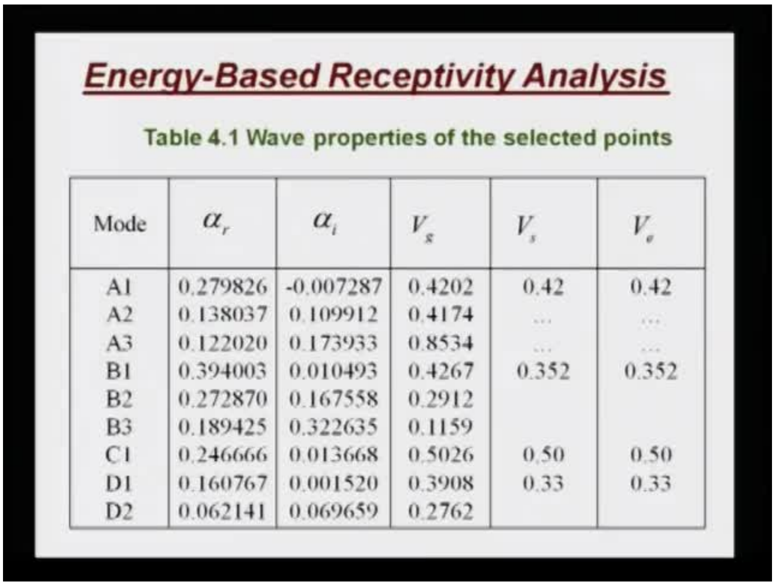

Energy-Based Receptivity Analysis

Table 4.1 Wave properties of the selected points

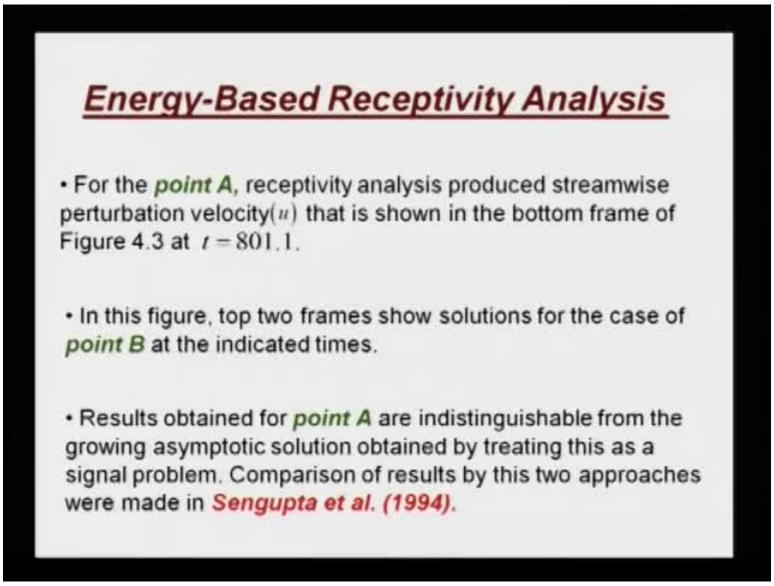
Mode	α_r	α_i	V_g	V_s	V_e
A1	0.279826	-0.007287	0.4202	0.42	0.42
A2	0.138037	0.109912	0.4174	...	...
A3	0.122020	0.173933	0.8534	...	...
B1	0.394003	0.010493	0.4267	0.352	0.352
B2	0.272870	0.167558	0.2912		
B3	0.189425	0.322635	0.1159		
C1	0.246666	0.013668	0.5026	0.50	0.50
D1	0.160767	0.001520	0.3908	0.33	0.33
D2	0.062141	0.069659	0.2762		

So, this is the kind of effort that is needed and if we perform a stability analysis, purely spatial stability analysis, then, we get the properties of this four points as indicated here. As I mention to you, the point A was inside the neutral loop. So, it has a unstable mode; you can see it from this sign, minus 0.007289 and that would also tell you, why we chose the Bromwich contour in the alpha plane; that was at minus 0.009. We wanted to keep this mode above the Bromwich contour; that is what we had done. The corresponding alpha r is given here. It is about 0.279. So, as these are given in terms of delta star, you can see what kind of wavelength we are talking about; it is of the order of tens of delta star.

Now, the other two modes that you get from the spatial analysis, A2 and A3, they are stable and you can see the growth or decay exponent is significantly high. So, if this is minus 0.007, it is plus 0.1. So, it is almost like 14 times, 15 times higher and this is even higher. The wavelengths are almost like, half of it; wave numbers are half of it; so, wavelengths are double. So, these are much more longer waves and if you calculate the group velocity, this is what you get. The group velocities is of the order of above 40 percent of the mean flow. Please be careful about this; this is what is called as signal speed; this was what was defined by Sommerfeld. He wanted to identify the wave propagating disturbance in terms of a signal and that speed is indicated in this column. And, this is the energy propagation speed that would come along, when we formally apply the disturbance energy equation in terms of e d.

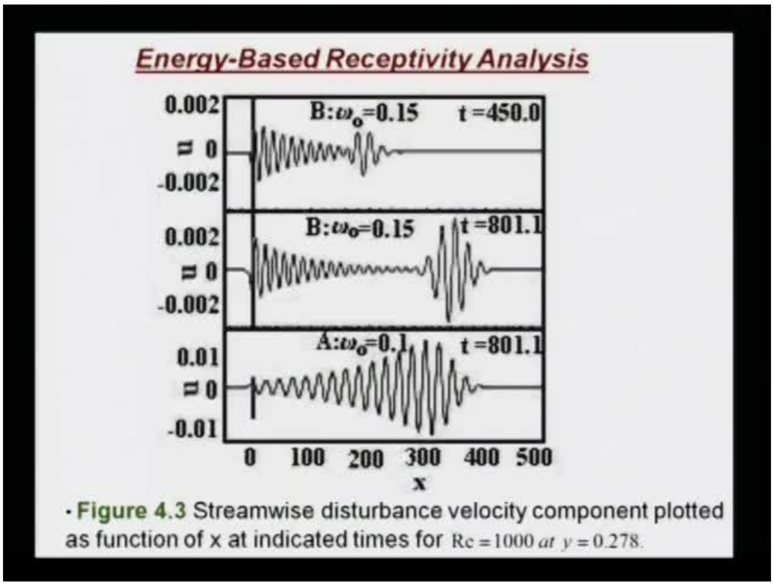
So, this two columns, let us not worry about; as we go along, we will signify what they are. For the point B, which was at, corresponding to omega equal to 0.15, we still have three modes; but all three of them are stable. And the least stable is 0.01, so, that is, you can see, it is a quite significantly damped solution. The group velocity ranges are different, but the leading mode has almost similar group velocity, like the unstable modes. The point C, corresponds to the one that is far ahead of the neutral loop; and you can see the, here, the decay rate is, decay exponent is even larger compared to the point B. And, point D is the one, that we chose at 0.05, omega equal to 0.05, which was sitting very close to the neutral curve and that is what you indicate; you see that, the decay exponent for the leading mode is quite small 0.001; whereas, the other mode corresponds to this.

(Refer Slide Time: 30:36)



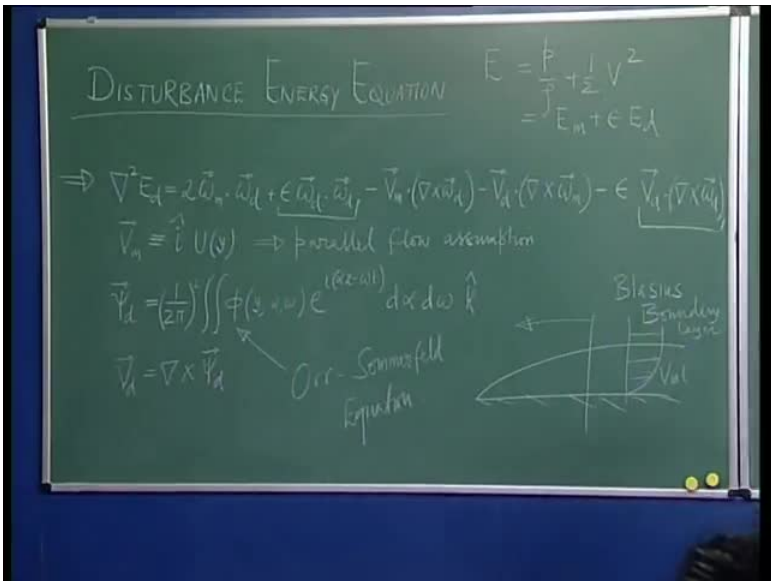
Now, if we keep this in mind and try to perform the analysis, we can figure out the various kind of solutions that we get.

(Refer Slide Time: 30:49)



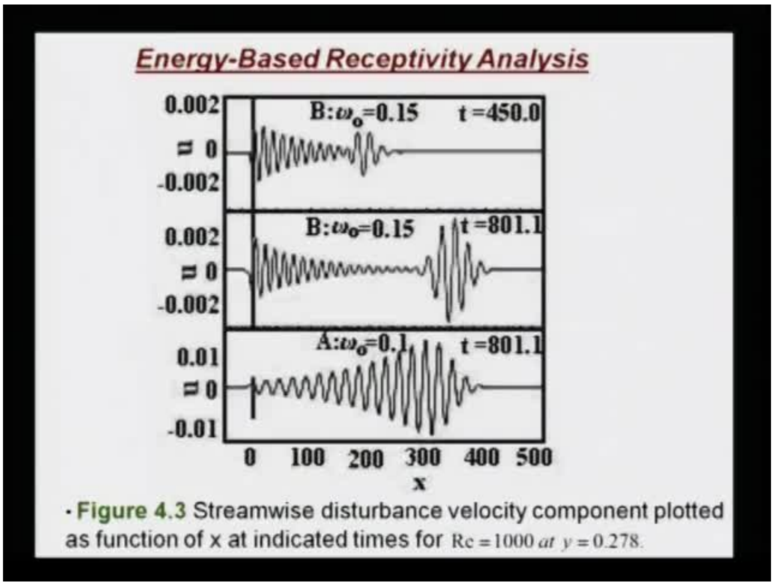
This is what we are seeing here, if we do that Bromwich contour integral; obtain the phi and from the phi, we do double inverse transform to get psi d; and from psi d, I can obtain the V d. So, the V d has a component, which is a stream-wise component u and a wall normal component v.

(Refer Slide Time: 31:50)



So, this u is plotted here. So, for the point A, for which the circular frequency was 0.1, the solution is shown here, at a significantly large time, t is equal to 801 and what you see, is basically this; that you have a local solutions and so, if we do the Bromwich contour integral method study, then, we saw the Orr-Sommerfeld equation along alpha and omega plane; and we do get the value of phi for those values of alpha and omega. Then, we perform this double inverse transform here, to get psi of d. Then, we would take a curl of psi of d to get the disturbance velocity.

(Refer Slide Time: 32:05)



• Figure 4.3 Streamwise disturbance velocity component plotted as function of x at indicated times for $Re = 1000$ at $y = 0.278$.

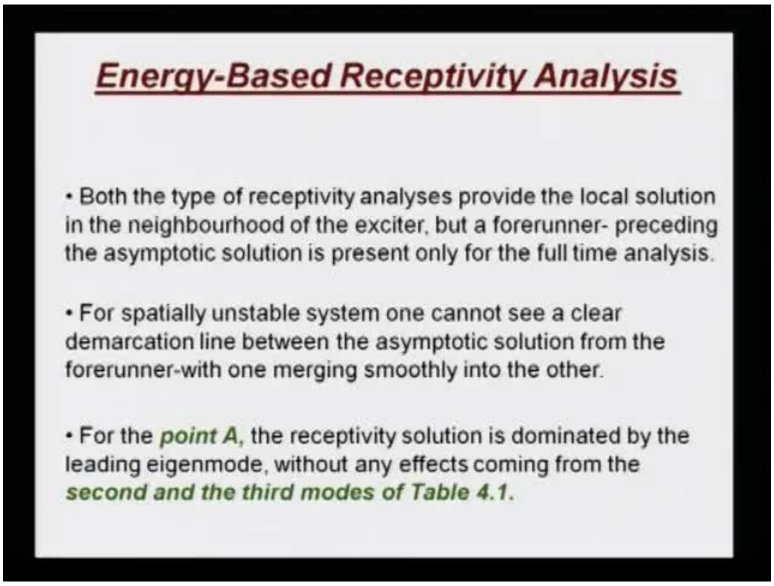
And, the stream-wise component is u that is shown here, point A. For point A, which corresponds to omega naught equal to 0.1, shown at a significantly larger time, t equal to 801. Please do understand that, in the omega plane, we have taken 512 points. So, if I define my omega range from minus 1 to plus 1, then, what have we got? We have gotten the delta omega; we have the range; we have the number of points; we got delta omega. And, from (( )) limit, we can convert it into the corresponding t max; what is the maximum time we can get and that is what you are seeing here. So, if I take only 512 points, I can go only up to 801. And, if I have to take more number of points, for a longer time, then, I have to take more number of points.

So, this is the trade off that you will have to understand. At those times, when we were working, this was the limitation of our computer. What we notice is, of course, is a local solution, followed by a asymptotic solution here; and the asymptotic solution is followed by a front, which decays. And, this is what we would expect, because this is nothing, but a wave packet. We are studying it as a spatio-temporal entity and we see that, the disturbance at a finite time, can only reach up to a finite disturbance, a distance. If I were to take more number of points, so that, I can go to much larger t, I would perhaps, see it would fill up the whole domain.

So, that is what we do, see. So, this is what you see for point A; but however, when you do it for point B, this is how the flow really works; and according to our spatial stability analysis, this was a perfectly spatially stable system and that is what we are seeing. The asymptotic part is strictly decaying; however, at t equal to 450, and at t equal to 801, we do sees this wave-front and this wave-front, actually increases with time. And, this is something that was not understood before. Now, you can note that, for both this points A and B, we have done the simulation for Reynolds number of 1000 and we do get, at the identical time, almost a similar wave-front.

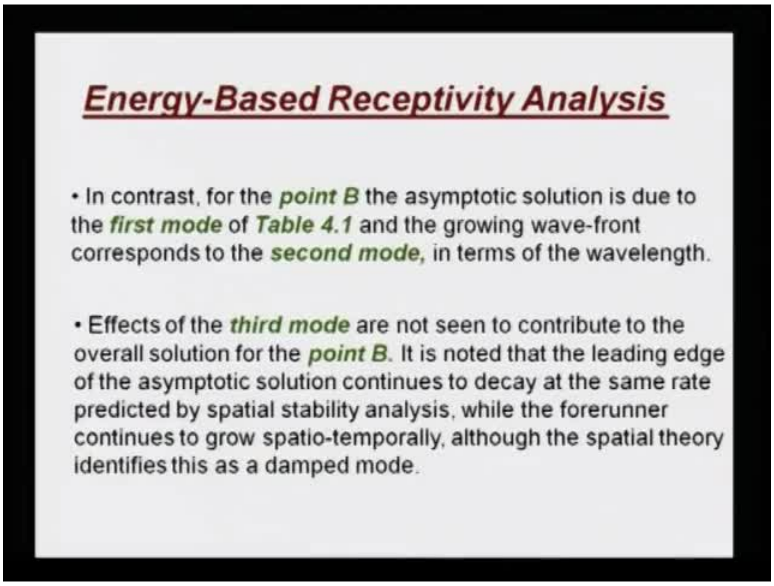
So, the question is, whether the spatio-temporal wave-front that we are seeing, is a function of Reynolds number or it is due to interaction of multiple modes, is something, that is an open question at this point in time even. And, this solution that we are showing you here, has been obtained for the point at the inner maximum. Inner maximum is about quarter of a delta star; that we have seen for this Reynolds number case.

(Refer Slide Time: 35:22)



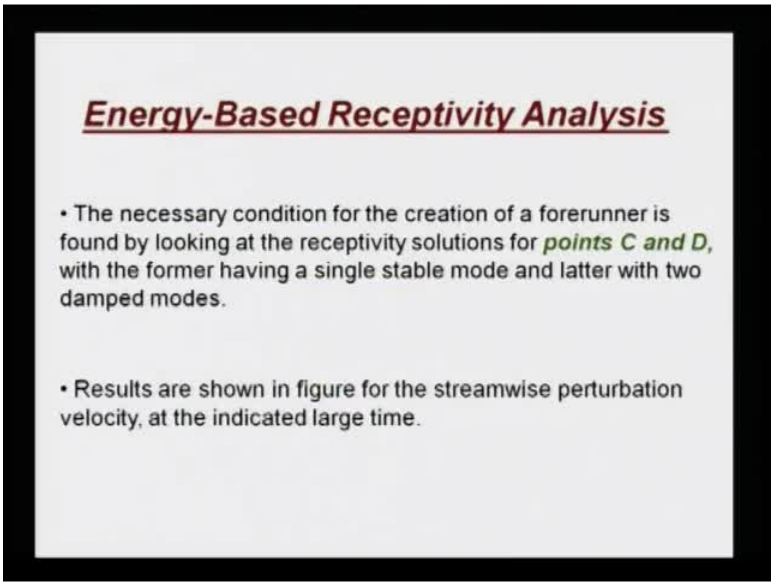
Now, that is what we discussed it, for the point A; receptivity solution is dominated by the leading unstable mode, without any effect coming from the damped part on this solution. We did not see any moderation, because, we have the solution. So, we could calculate its growth rate.

(Refer Slide Time: 35:29)

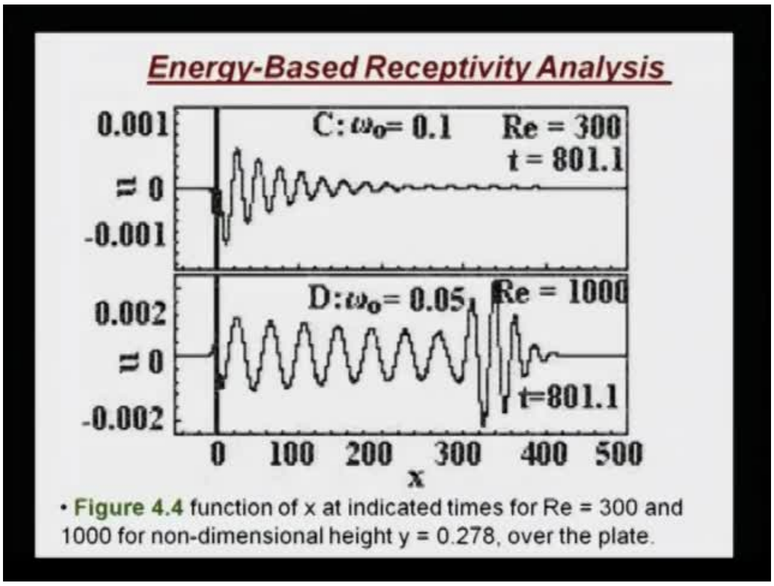


We can do it. And, we have done that; that helps us in making that last statement that, the growth rate matches exactly what is given by the first mode. So, the second and third mode does not give as much, in terms of the disturbance flow quantity. If you look for the point B, the asymptotic solution is once again due to the first mode and the growing wave-front, interestingly enough, the alpha r, the wave number of the wave-front, matches with that of the second mode. Effect of the third mode is not at all seen for the point D; not in terms of alpha i or alpha r.

(Refer Slide Time: 36:51)

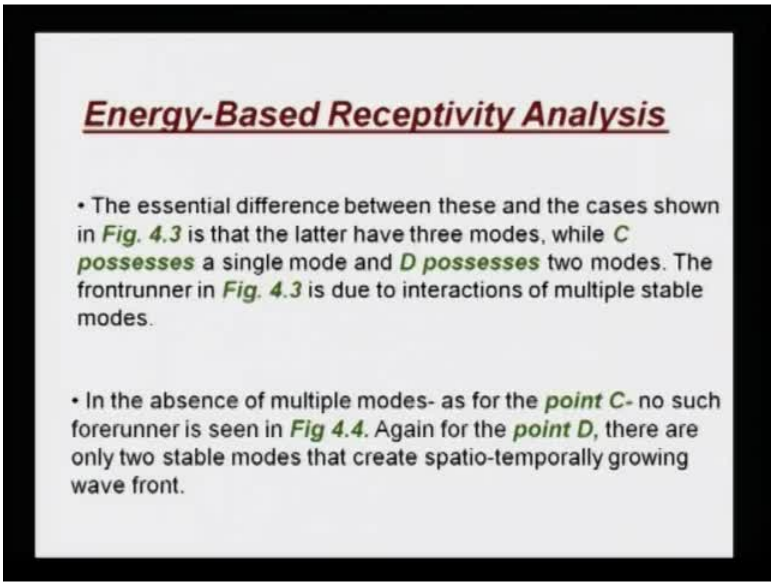


(Refer Slide Time: 37:30)

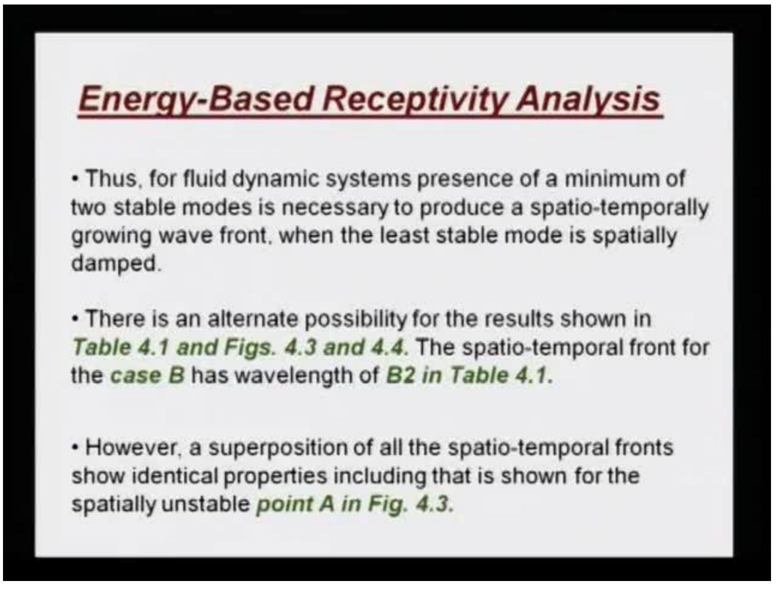


So, basically, this forerunner that we see, it continues to grow spatio-temporally, even though, the other part, asymptotic part of the solution decays. And, this is something interesting, that we need to understand. We do make an observation here that, the case that we see, that the necessary condition for the creation of forerunner, is found by looking at the receptivity solution for the points C and D. For point C, we have a single stable mode and later, point D, we have two damped modes. For point C and point D, the solutions look like this. For point C, we do not see any wave-front.

(Refer Slide Time: 38:04)

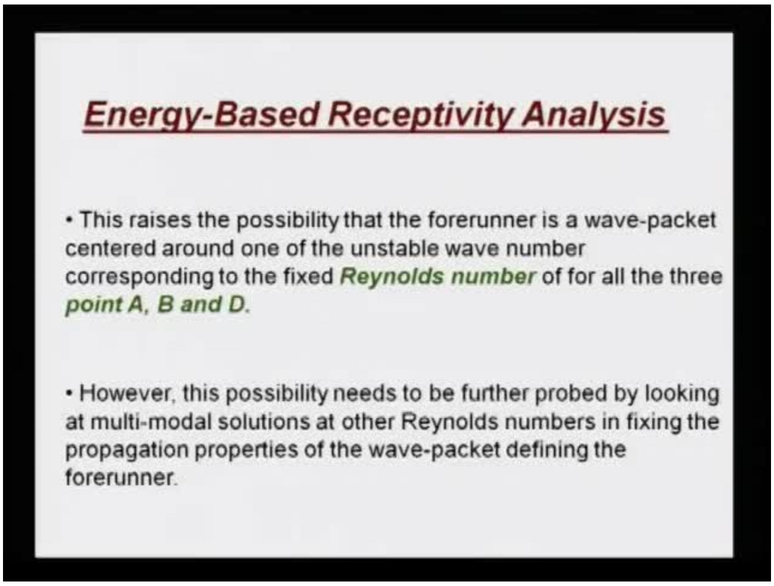


(Refer Slide Time: 38:31)



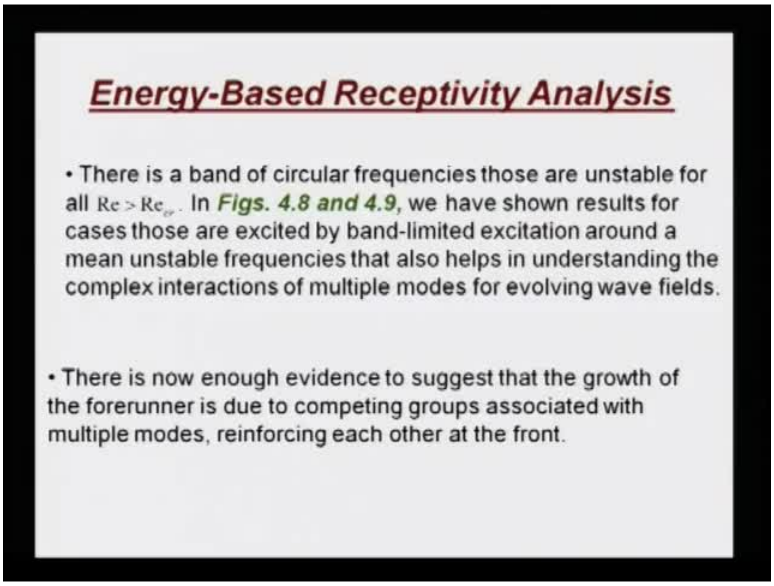
But please do understand, this is at a different Reynolds number; whereas, for point D, we have two modes; both are damped. But we do see a kind of a spatio-temporal growing wave-front. This is somewhat tempting; for 1, 2, now, conclude as if, that a necessary and sufficient condition for this spatio-temporal wave-front or the forerunner is that, you must have more than one mode; because, for C, they have a only single mode. At the same time, we must also keep the option open of future studies, where one should study a similar analysis, do a similar analysis, for different Reynolds number cases; because for point C, although we have a single mode, but this is also done at a very lower Reynolds number. So, this spatio-temporal growing wave-front, is this a strong function of Re, so that, when you reduce Re, it decays, or it is due to some kind of interaction of multiple modes; this is something that we need to understand. So, you know, this is what we observed that, the spatio-temporal front for the case B has a wavelength of B2. Whether this is just a pure coincidence or it is because that mode becomes important, is something that has to be further more studied in greater detail.

(Refer Slide Time: 39:36)



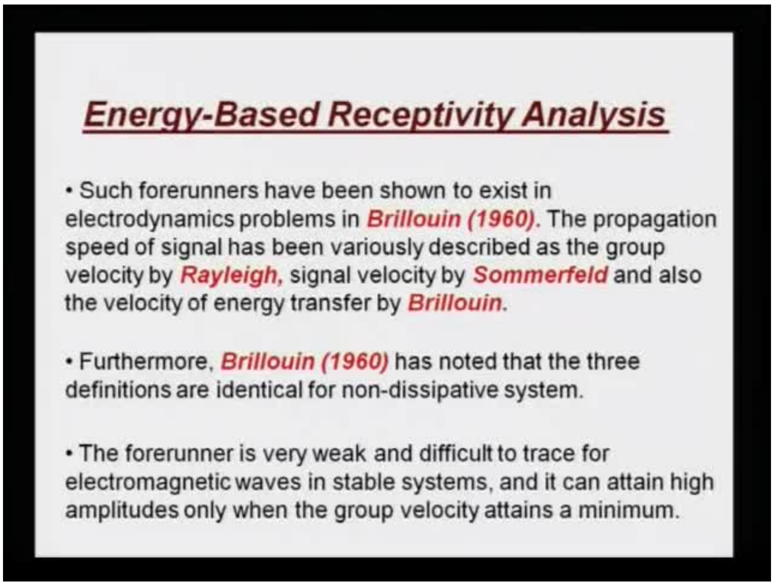
However, what we can say or do at this point in time is, to note that, if I depended solely upon this spatial stability analysis, then, of course, I would not be able to get the spatio-temporal front. The spatio-temporal front is obtained because, we performed a Bromwich contour integral method; because we treated the problem as spatio-temporal problem. So, that is something that, we must…And, we must further probe, at multi-modal solutions of Reynolds number, with Reynolds number and then, try to see, what is the propagation property of this spatio-temporal wave-front. This is a major, outstanding problem that needs to be really studied.

(Refer Slide Time: 40:30)



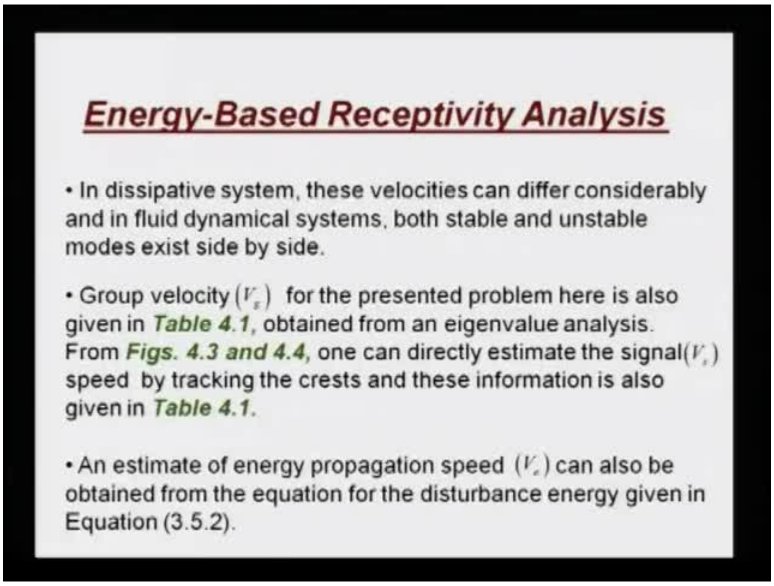
Now, next, what we want to study that, supposedly that spatio-temporal wave-front is an attribute of, a function of Re, and multiple modes are not so important; let us say, we grant that. but then, we want to study a case, where Re is, of course, greater than Re critical and then, we will consider the response of the system, where circular frequency has a bandwidth and in this bandwidth, all the frequencies are unstable. And then, what will happen? Because of the way we write it, they will mutually interfere; and now, this interference is both in alpha and omega plane. And, this is what we need to study. Let us try to find out what happens.

(Refer Slide Time: 41:59)



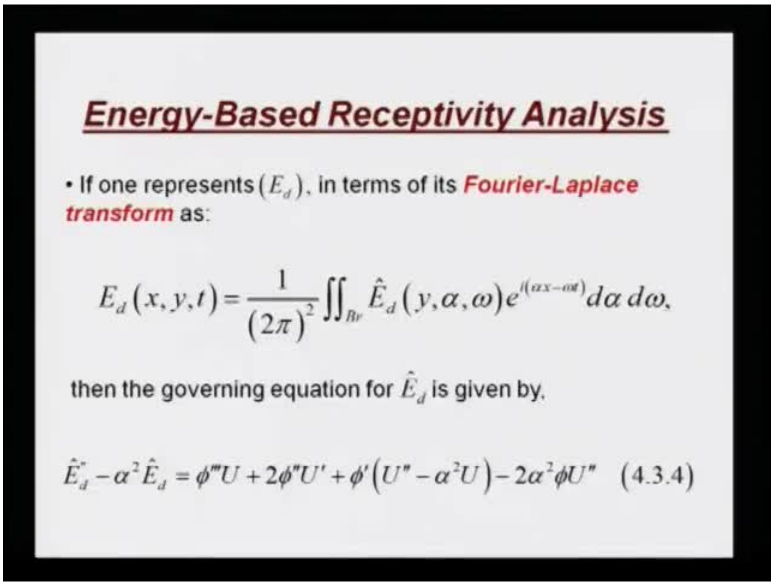
Now, although we make this observation, but we could keep our options open; however, we just simply mentioned that, the growth of the forerunner is, due to probably this multiple modes interacting with each other and they probably add to the phase. You know, this is what is the concept of groups, and this has been going on from the time of Hamilton and Rayleigh and so on and so forth, all the way up to the nice monogram by Brillouin. And, we talked about the properties of forerunner, which was studied by Brillouin for electrodynamics problem. The propagation speed of the signal has been variously defined by group velocity, which we adopt in our case; signal velocity by Sommerfeld and energy transfer velocity by Brillouin. Brillouin noted that, the three definitions are identical, if we are looking at non-dissipative system; but here, of course, our study involves dissipative system, because we are looking at viscous flow.

(Refer Slide Time: 43:04)



So, this does not hold. What was noted by Brillouin that, for electromagnetic waves, this forerunner is very weak, in case of stable system; only it can attain high values, when the group velocity is minimum. Something probably related to what people later on called as the absolute instability. Indissipative systems, this three velocities that we talked about, the group velocity, signal velocity, energy propagations, can be different, because we can have both stable and unstable modes, existing side by side.

(Refer Slide Time: 44:08)

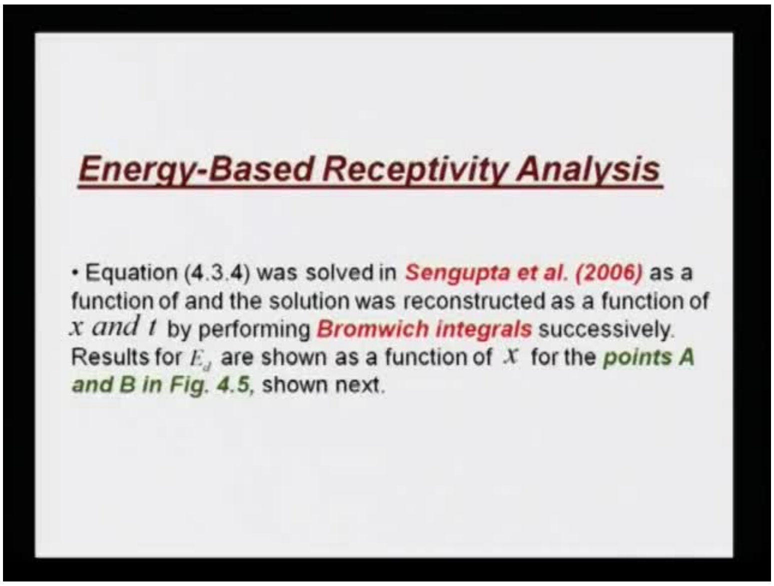


So, that is what one can do. Group velocity, we can obtain, as we do here, and shown in the table, obtain directly from Eigen value analysis. And, if I look at the signal itself, if I plot psi versus d or u versus x, then, I can calculate; I can calculate, let us say, the point of maximum, at what speed does it go. That is what we call as the signal speed. And, estimation of energy propagation speed, actually, would require that, we study this energy propagation equation itself; and this is what we study next. To do that, to develop an equation for E d, for this particular case, we can define the disturbance energy also in terms of its Fourier-Laplace transform. So, E of d, with a cap, is a strong function of y, height over the wall and also is a function of alpha and omega, and which we will perform the Bromwich contour integral, to get the quantity in the physical space.

What is the governing equation for this? Well, governing equation for this is here, given in the physical plane. So, now, we can substitute our representation for psi d, V m, E d like this and write it down here. Now, you can very clearly see, what is this quantity? This is your del square E d. If I take a y derivative twice, then, I will get this E d cap double prime; that is your second derivative with respect to y. and, this is your second derivative with respect to x, minus i alpha whole square will give you, plus i alpha whole square, will give you minus alpha square. And, these all this stops that we see on the right hand side, we can open them up and suppose, we have obtained those quantities from the Orr-Sommerfeld equation; that is what we have done. So, we can write it down here and you can notice that, the quantities that determine the energy propagation is dependent on U velocity, the shear, the curvature of the velocity profile as well as the third derivative.

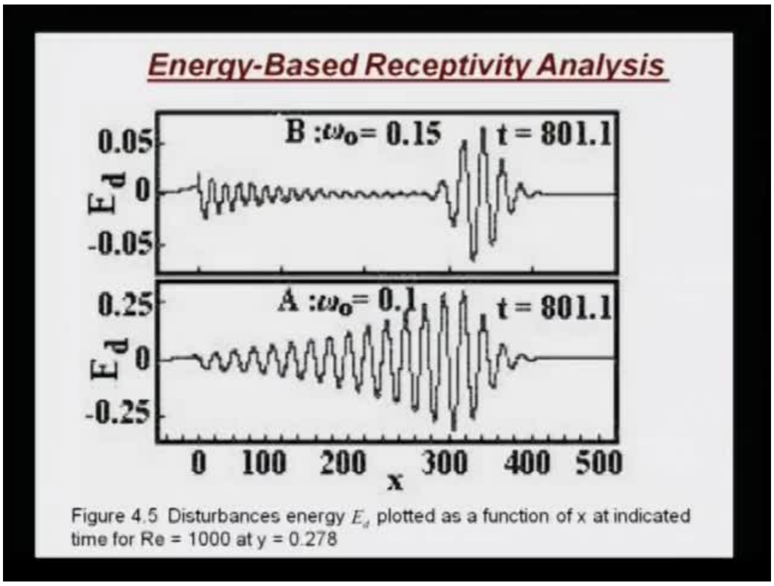
So, this is something interesting, because if you recall that, when we looked at Orr-Sommerfeld equation, we saw the Orr-Sommerfeld equation depended on the mean flow from the definition of U and U double prime. But here, when we are looking at the disturbance energy, it not only depends on the mean flow, but it also depends on the shear and the curvature and the third derivative of the mean flow. So, this is something interesting.

(Refer Slide Time: 46:35)



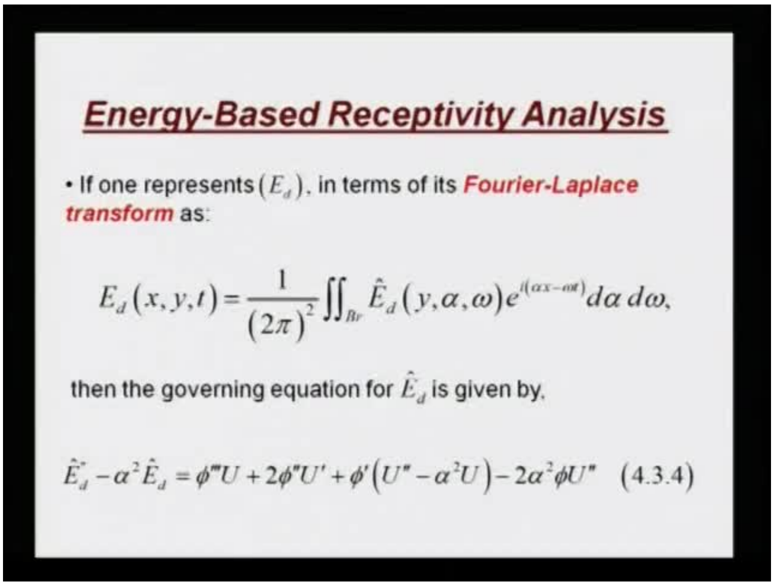
Now, this equation that we have just now seen, we can solve it again along Bromwich contour in alpha and omega plane.

(Refer Slide Time: 47:11)



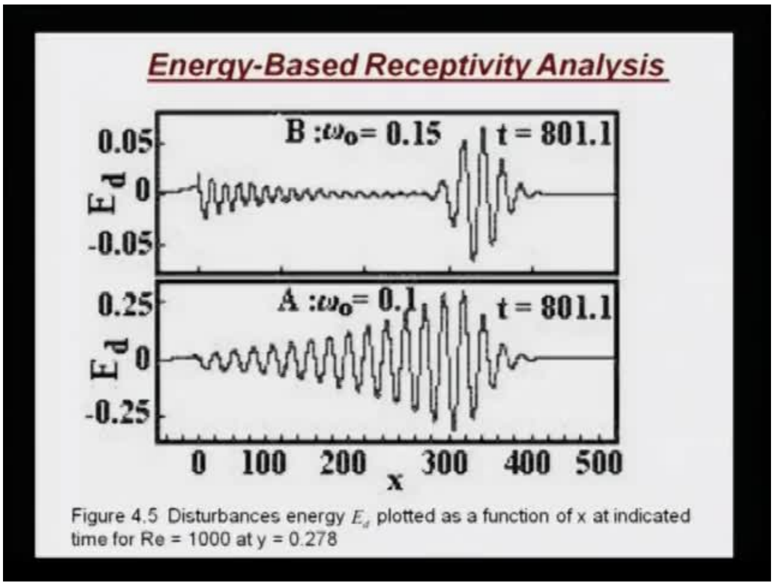
Now, once we have gotten that E hat of d, as a function of alpha and omega, for this Bromwich contour, we can do inverse transform and obtain this solution. And, I am showing you the solution for that point A and B, in that figure. A corresponds to, in the unstable point; B corresponds to the stable point. And, this is what you get. Well, of course, for the point B, you do have, from the Orr-Sommerfeld equation at spatially stable, that is what you are seeing here also. Please do understand that, when it comes to this equation, can you do a stability analysis?

(Refer Slide Time: 47:59)



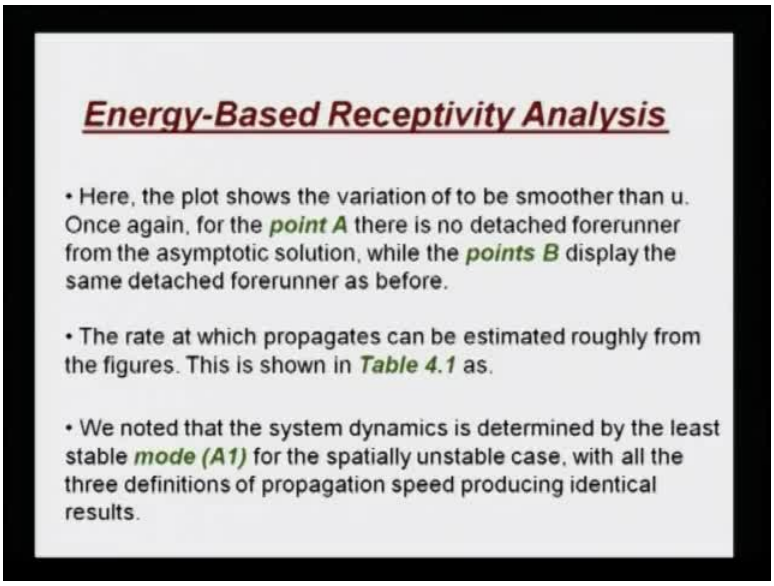
See, this comes strictly from the Laplacean and we cannot do the stability analysis like what we did for Orr-Sommerfeld equation. This is interesting because, there is, in this equation, if you look at, there is, this is the forcing term. So, this is not like your Eigen value problem. There is an explicit forcing coming in from here. And, if you look at the so called, transfer function here, transfer function does not involve any term related to the mean flow. So, stability of what we are studying? See, this is something you must understand that, this is a very interesting alternative viewpoint; whereas, in the Orr-Sommerfeld equation, the governing equation was homogenous; here, the governing equation is in-homogenous. In the Orr-Sommerfeld equation, we had a transfer of function, which was a function of the mean flow. Here, we have a transfer function, which does not depend on the mean flow. So, this is something that is interesting.

(Refer Slide Time: 49:18)



So, when we study E of d, by this Bromwich contour integral method, it is a very interesting thing, because what we are looking at here, is nothing, but a forced vibration problem, forced. What is the forcing, that is coming from, how the mean shear interacts with the disturbance shear. That gives rise to this kind of a thing. And, despite all that, of course, it is excited by the forcing; so, whatever may be the forcing property is, that is seen here also. For example, point B was stable; so, we do get a asymptotically stable energy. However, we do pick up the spatio-temporal growing wave-front. And, for the point A, which was inside the neutral curve, we do have a growing wave-front. And, there is this leading wave-front present in both of them. These are shown for Reynolds number of 1000 at a height of 0.278.

(Refer Slide Time: 50:05)



So, this is what we need to study. So, we can make the following observation that, the variation that we have seen, E of d versus x, this variation is much smoother than u versus x; or we would expect it, because it is squared, V squared term. So, if I have some discontinuity in V, when I square it, it would smooth out. Once again, for the point A, there is no detached forerunner, while the point B displayed the same detached forerunner as before. The rate at which this wave-front propagates, we can work it out from the figure itself. And, this was obtained in table one as V e; recall, we talked about V e; what was one Relova suggested that, we should also talk about energy propagation speed. So, since we have now got energy as a function of x and t, we can calculate this speed. And, that is how, it was obtained and it was noted in the table. Then, the system dynamics is determined by the least stable mode A1, for the spatially unstable case, with all the three definitions of propagation speed producing identical results. That is what we saw. So, I think I will stop here.

So, tomorrows' or next class, we will be talking about, what happens to this. Then, we will make a kind of a grand summary of whatever we have done so far. We have been focusing mostly about spatial growth. We will like to see what happens in flows, where we instead have temporal growth; that will be out next stop in our journey.