Instability and Transition of Fluid Flows

Prof. Tapan K. Sengupta
Department of Aerospace Engineering
Indian Institute of Technology, Kanpur

Module No. # 01

Lecture No. # 27

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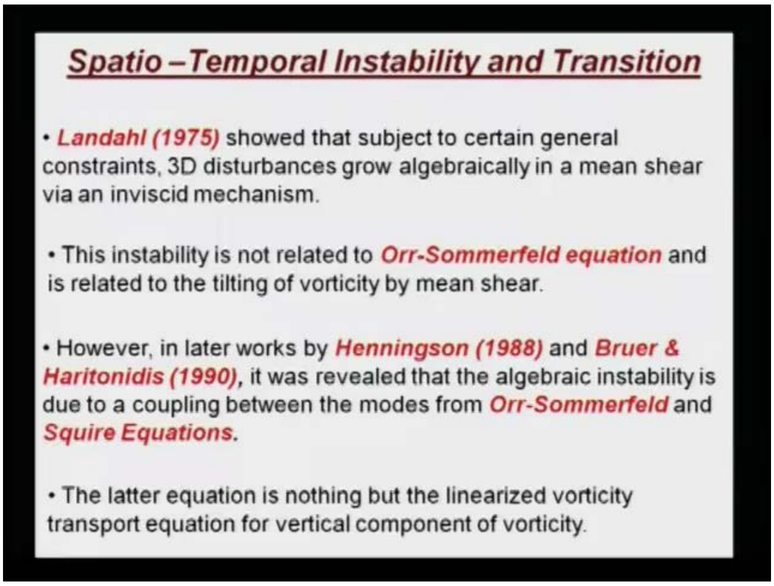

So, having discussed spatial and temporal instabilities in isolation, we have started discussing about spatio-temporal instabilities, where you would see, the disturbance will grow both in space and time simultaneously. And, in this context, lot of work has gone on. One of the earlier works, where Landahl actually discussed inviscid mechanism, where you found that 3D disturbances can grow algebraically in time, also in space. And, this mechanism of algebraic growth is distinctly different from the viscous instability given by Orr-Sommerfeld equation. Subsequently, Henningson and Bruer and Haritonidis revealed also, algebraic instability that is due to a viscous mechanism, a coupling arising from Orr-Sommerfeld equation and the Squire equations. Squire equation is nothing but linearized vorticity transport equations.

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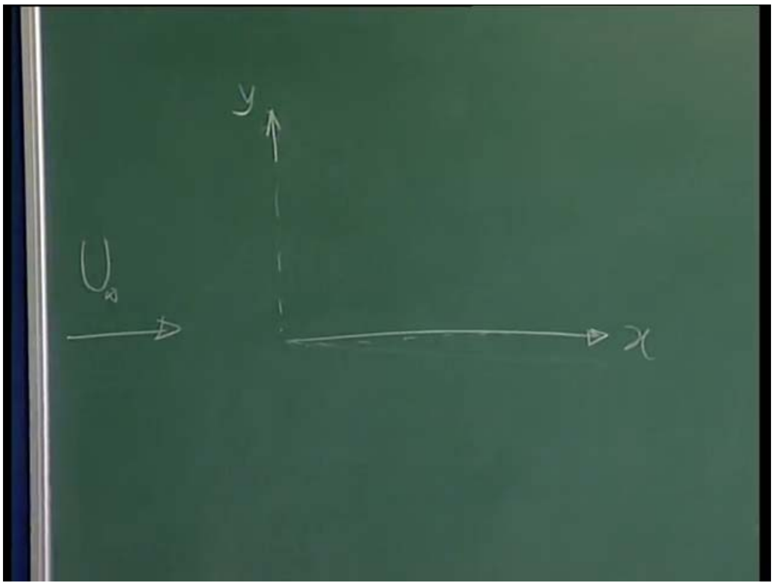

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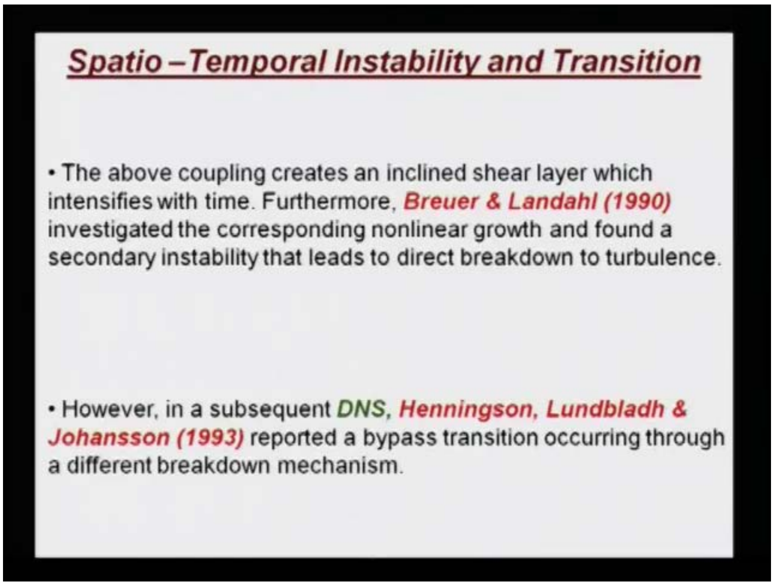

So, suppose this is the plate that you have. Let us say, in the x direction, the flow is. So, this Squire equation is nothing but the linearized vorticity transport equation for omega y. Now, what happens here is that, there has been sporadic efforts going on and off, of course, all of it had started from the original observation of Markovin that, apart from the classical viscous or inviscid road, you could also have bypass transition. So, there were all this efforts, which were really looking for the bypass mechanism through which you can get spatio-temporal instability and in the context of the work that we talked about, coupling between the linear modes, given by Orr-Sommerfeld equation and Squire equation, Breuer and Landahl did find out that the if you go to the corresponding non-linear stage, you could find secondary instability, that could really lead to a spectacular growth of disturbance into directly leading to turbulence.

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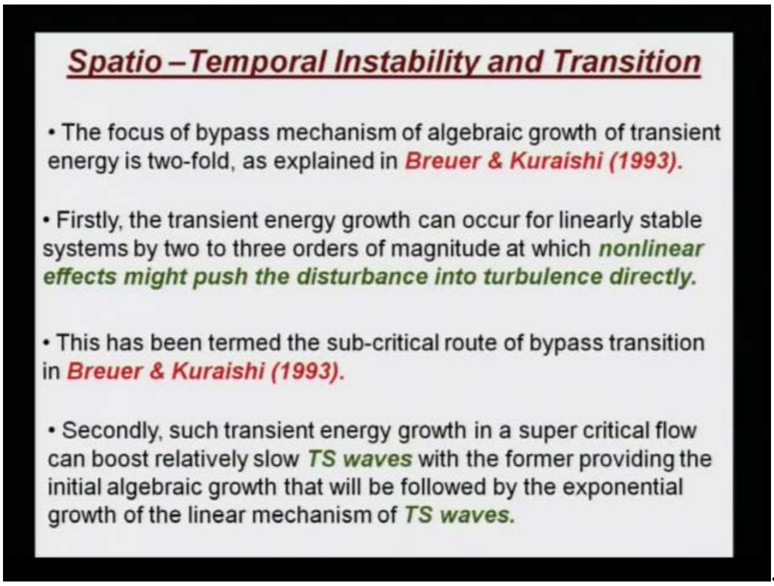

While this things are going on, there are lots of efforts that has also gone in, in solving the full Navier-Stokes equation without any assumption, that we called as a direct numerical simulation. In one such effort, Henningson and his co-authors talked about a bypass transition mechanism, which do not depend on any of this previous linear instability mechanism. So, basically, when you are looking for any bypass mechanism, people were looking for algebraic growth, it has been noted. And, the reason that this is of interest is that, it could lead to a transient energy growth itself as the primary event, which will lead to basically, growth of disturbances by order of 100 or 1000, where non-linear effects can directly come in and you would get a turbulence directly. If this happens, and it happens in those scenarios, where viscous instabilities are not predicted, then, that will be the sub-critical route as pointed out by Breuer and Kuraishi. The second route could be that, you are in the first critical stage, so, you already have weak Tollmien-Schlichting waves.

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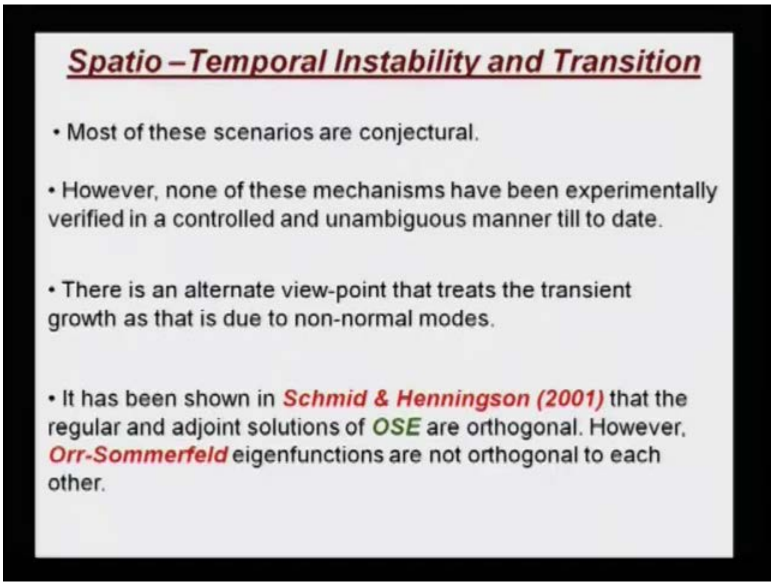

And now, over and above, if they have a layer of algebraic growth, so, you will have a compounded effect and that would lead to turbulence stoke. So, this two are the motivations that, people want to study algebraic growth. We have now basically, stated quite a few of this scenarios. They are conjectural, in the sense of theoretical projections. They have not been systematically studied, experimentally, in a controlled manner, so that, you can really say that, unambiguously that, this is exactly the mechanism by which things are happening. There is of course, the alternative view point that, this algebraic growth could occur due to the presence of modes, which are really not the normal modes that we obtain from the solution of Orr-Sommerfeld equation. They are basically, the non-normal modes. And, Schmid and Henningson are the proponents of this and they noted that, Orr-Sommerfeld equation modes constitute a set with respect to the adjoint of the Orr-Sommerfeld equation; however, each of the Eigen functions of the Orr-Sommerfeld equation themselves, they are not orthogonal to each other, anyway.

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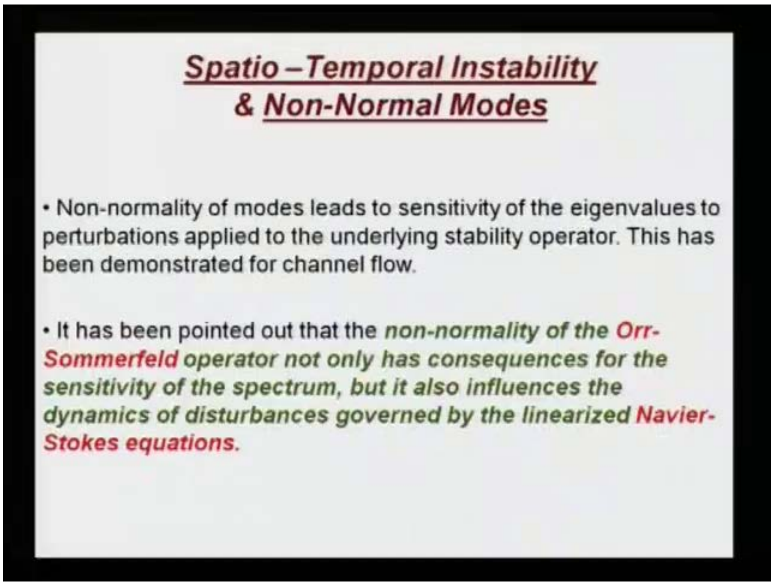

So, they are essentially non-normal. Despite that, it has been noted that, when you have non-number modes, they make the system very hypersensitive to background disturbances. This is due to the property of the stability operator of the governing equation. And, some calculations have been done for channel flow and it has been shown that, this non-normality of this, could buoy sensitivity of the spectrum, show a completely a different dynamics as given by the linearized Navier-Stokes equation.

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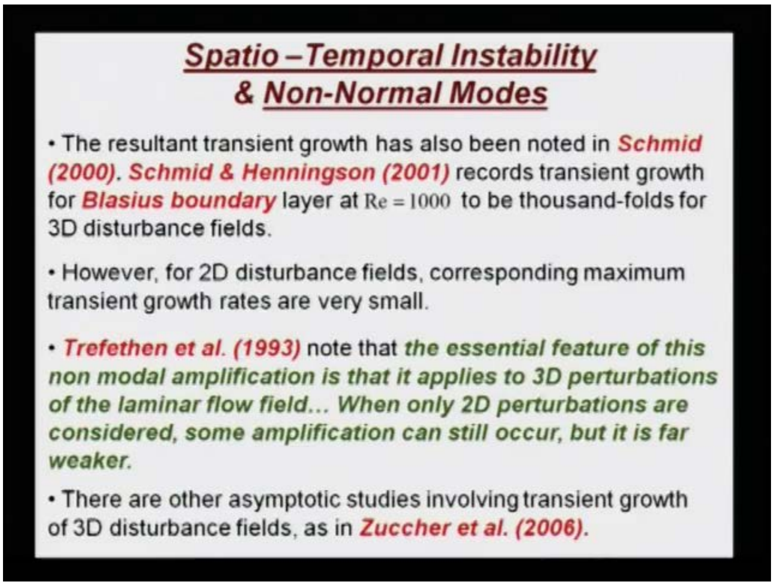

Now, when we take stock of this, we notice that, the transient growth that occurs as a resultant phenomena, that for, even for a Blasius boundary layer at Reynolds number of 1000, you can notice that, there is a 1000 fold increase in 3D disturbance field. When you look at the corresponding 2D disturbance field, this growth rates are far too smaller, that led Trefethen and his co-authors to comment that, the essential features of this non-modal amplification, that it applies only to 3D perturbations fields; and when you consider the same for 2D perturbations, this is a far weaker mechanism.

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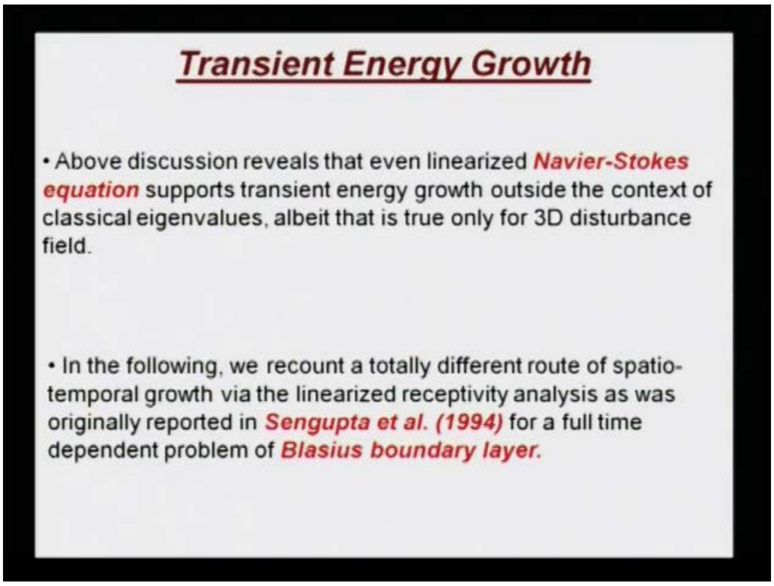

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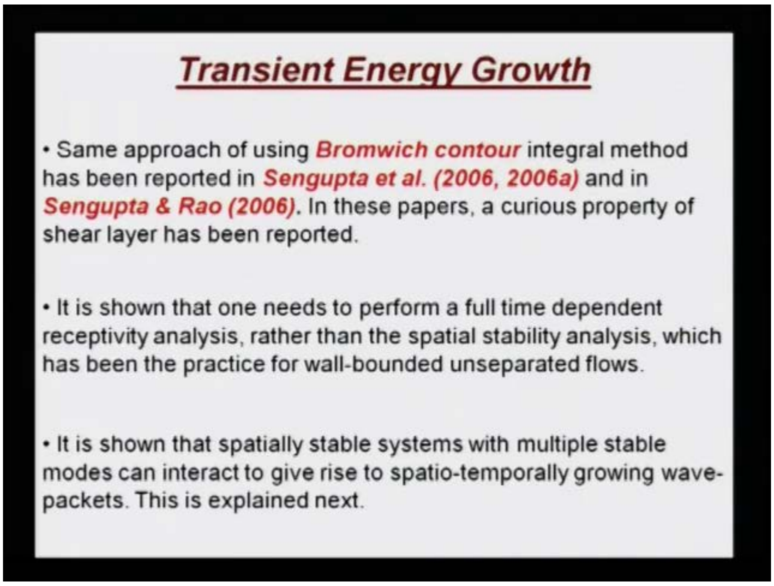

There are many other asymptotic studies involving this transient growth of 3D disturbance field, as late as this work by Zuccher in 2006 in j frame. However, what we are looking at that, in all of this cases, one has talked about, predominantly about 3D disturbance field. And, you know the legacy of 3D disturbance filed is attractive because, even for fully developed turbulent flow, we have a mechanism of vortex stretching which is present in 3D field. So, people have tried to sort of synthesize this two point of view and all looking for 3D disturbance field as alone. Of course, a different route that has been espoused by us, following our work on linearized receptivity analysis of Blasius boundary layer. And we find that, we do not need to talk about either spatial or temporal; we can perform Bromwich contour integral as stated here. They have been going on by various groups in our team and we found out a very curious feature of the shear layer instability that, when we perform a fully time dependent receptivity analysis, instead of performing spatial analysis for wall bounded un-separated flows, we found that, even though the flow, if studied in a spatially stability point of view, we find that, they are stable, multiple modes are present; they are all stable.

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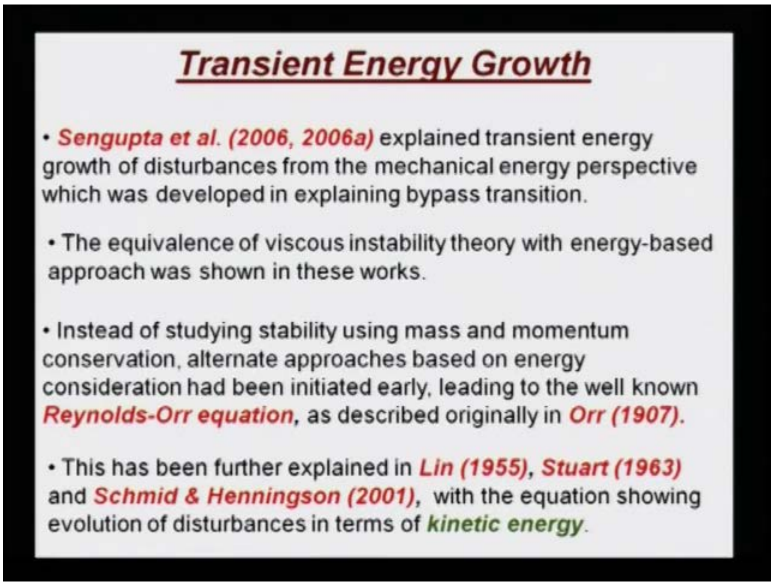

However, when they interact, they can lead to a kind of a spatio-temporal growth. This was something that was not known for more than last 20 years, but then, what is, in essence, happening here is that, this can also be explained from our mechanical energy equation that we have developed. This was done very recently, few years ago and we later on also showed that, this mechanical energy perspective is basically, all encompassing, because it comes from the Navier-Stokes equation and it can include, both the viscous as well as inviscid mechanism. The main term comes from the non-linear convection term. And, the equivalents of viscous instability and the energy-based theory were also established. That we have talked about in recent times, and we are going to talk about even more; however, we note historically that, people have tried to study stability of flow from energy consideration. This goes back all the way to the time of Orr and Reynolds who actually developed an equation called Reynolds-Orr equation. This was further studied in and explained in Lin, Stuart's work. You can also find it in Schmid and Henningson's monograph, where an equation for disturbance kinetic energy was developed.

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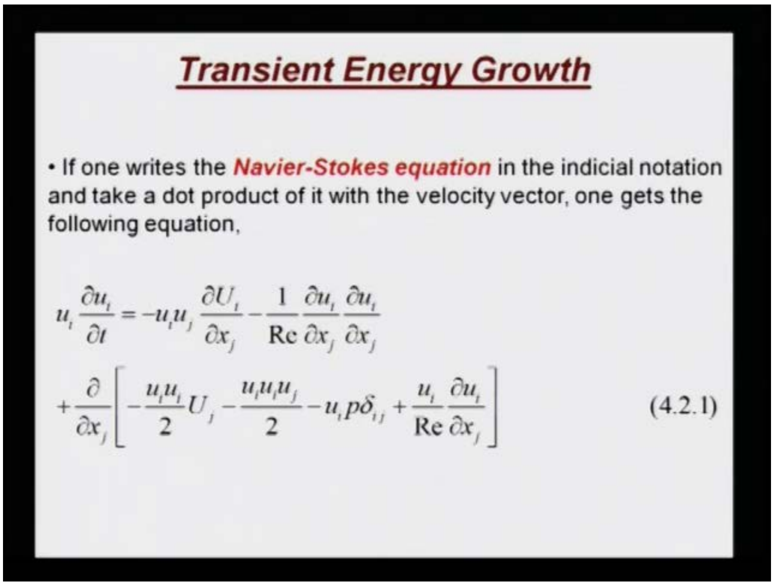

So, let us look at, what this could be. This is basically obtained, by looking at the Navier-Stokes equation, if you write it in indicial notation, and then, take a dot product of it with respect to the velocity itself. So, I will get this term, coming from the local acceleration term and then, the convection terms will come in two sets; one is due to the action of the disturbance stress on the mean shear and another is complementary term that comes as a gradient transfer term here, which also includes the pressure term; a triple correlation of the disturbance term as well as some viscous term coming here. And, this is the usual pure viscous diffusion term. The reason that we write this is, because, if I look at this, this is nothing, but a divergence term. So, if I take a volume integral over the whole domain and if we go on a very large domain, where some of these disturbance quantities goes to 0, then, we will notice that, this term will not contribute. And, what about this term? This term is something like a second moment term. This is also second moment term, but when you are looking for the second moment term evolution equation, then, this is already known this is due to the mean shear.

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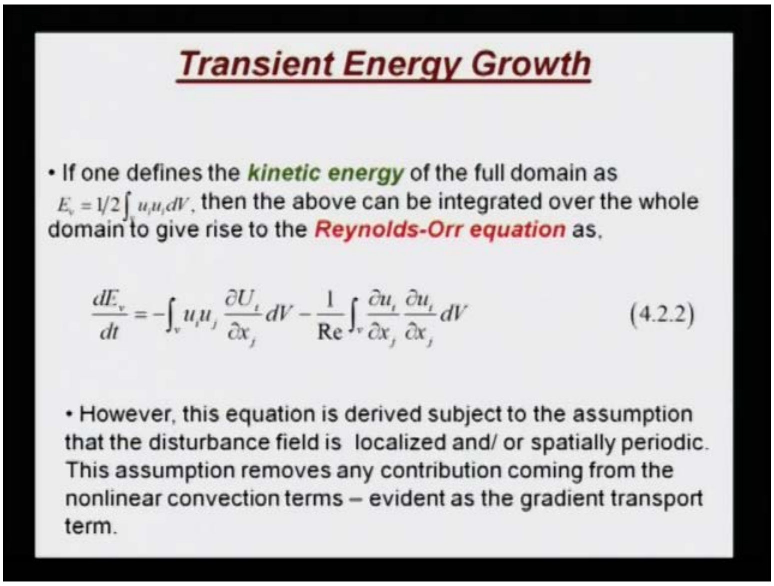

So, that is like a linear term, when you look at the evolution of the second model. In particular, you would be interested in talking about a particular quantity, is the kinetic energy which is also second moment, but which is nothing, but half of u i square. So, if I write down the second moment matrix, so, these are the some of the trace of the matrix; that half E square plus v square plus W square, integrate over the whole domain; call that as E subscript v. So, that is your kinetic energy of the full domain. And, the previous equation that I wrote, we saw that, if we integrate over the whole volume, then, the left hand side will give us this time rate of change of Ev and the right hand side, you get this term, that comes from the effect of mean shear on the second moment term and this is a viscous term. Now, this equation is derived, subject to an assumption that, the disturbance field that we are talking about is localized so that, if we go very far field, it is gone, going to go to 0, or it could, at the most, be spatially periodic, so, it will cancel out.

This kind of assumption actually removes any contribution coming from the nonlinearity. And, what we are noticing so far that, most of the time, the non-linear convection terms are important. Even if you start thinking of a Rayleigh’s equation, that was essentially the role of the non-linear convection term. Then, Orr-Sommerfeld equation also, we looked at the non-linear convection term, but how it exchanges energy with the viscous diffusion term, that thing came about. But in this equation, Reynolds-Orr equation, you lose the non-linear term altogether.

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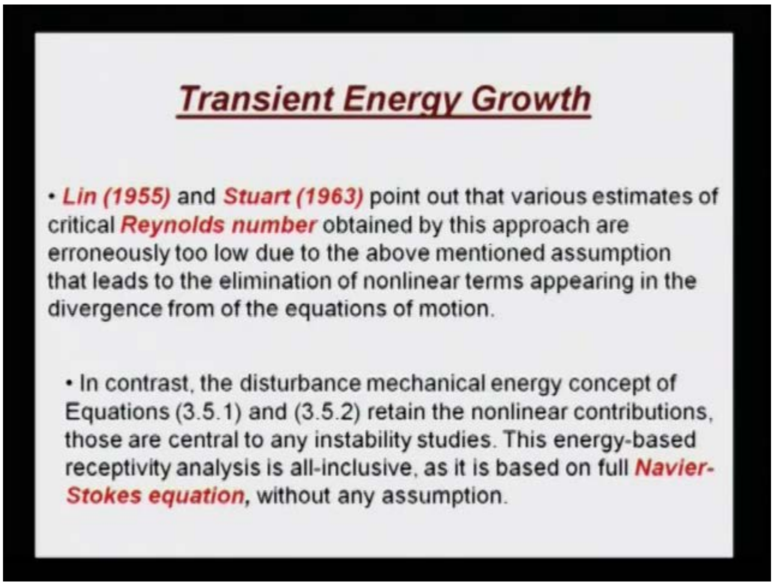

So, no contribution comes from the non-linear convection term and then, what might happen, as a consequences, you will get a critical parameter like critical Reynolds number, which is abyssmally low and this was noted quite early by Lin and Stuart. And, they said, of course, this gives you a kind of totally unphysical results. For example, for Blasius boundary layer, it could give a critical Reynolds number of less than 10. So, you can understand that, there is something, that is totally wrong and that is due to the elimination of the non-linear terms.

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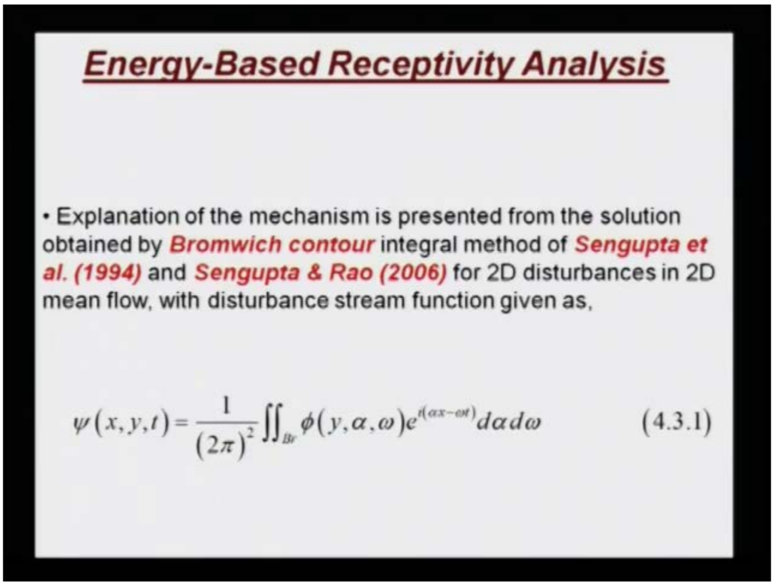

If you contrast that kinetic energy equation with respect to the mechanical energy equation that we have developed, we have talked about in great detail, we find that, the non-linear contributions are very much there. We have estimated their various effects and this energy based receptivity analysis is all-inclusive, based on full Navier-Stokes equation without making any assumption. We can study it in its linear form as well as non-linear form. Now, we would like to relate that energy equation approach with what we are talking about here, in terms spatio-temporal instability, the viscous instability. However, we are going to study it in the context of Bromwich contour integral. One interesting difference between this Bromwich contour integral method with the classical Eigen value analysis based on Orr-Sommerfeld equation is that, this is certainly, is not based on normal mode analysis. So, what happens is, you can look at the effect of all the modes simultaneously together; this is something unique and this is very important and secondly, we are not making spatial or temporal approach; we are looking at it together.

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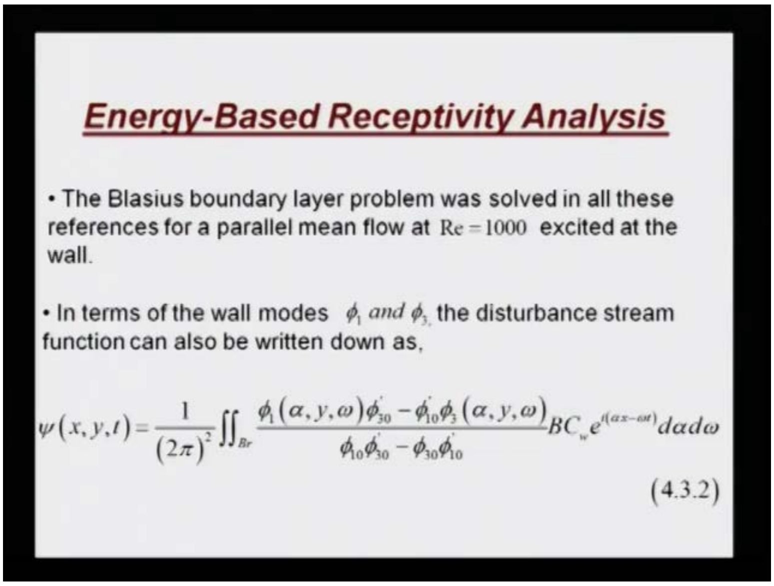

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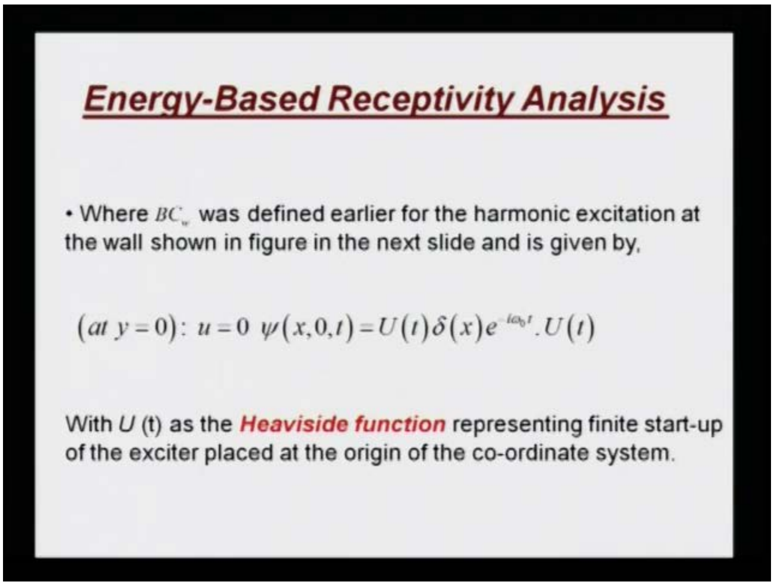

So, this two are the major reasons and that should recommend for itself that, this is a good way of looking at it. So, suppose we look at a Blasius boundary layer problem with reference to a (()) parallel mean flow, go back to our standard solutions that we have studied, then, if we write in terms of the wall modes phi 1 and phi 3, the disturbance stream function would be written like this. And, you can very clearly note that, the dispersion relation is in the denominator. This is how we coupled the receptivity and the stability equation. In addition, this condition BC subscript w tells you, how exactly the shear layer has been destabilized from the wall. So, that is essentially a boundary condition coming from the wall. So, we can have various formalism of this quantity; that we have already seen.

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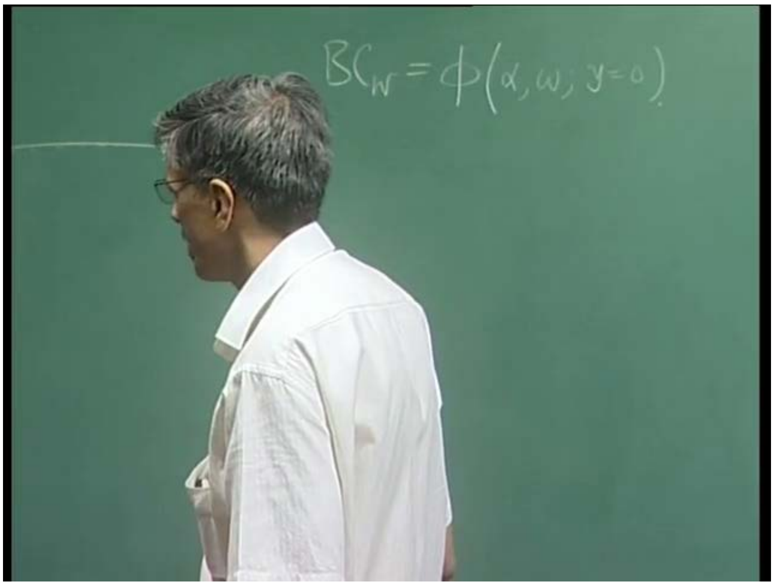

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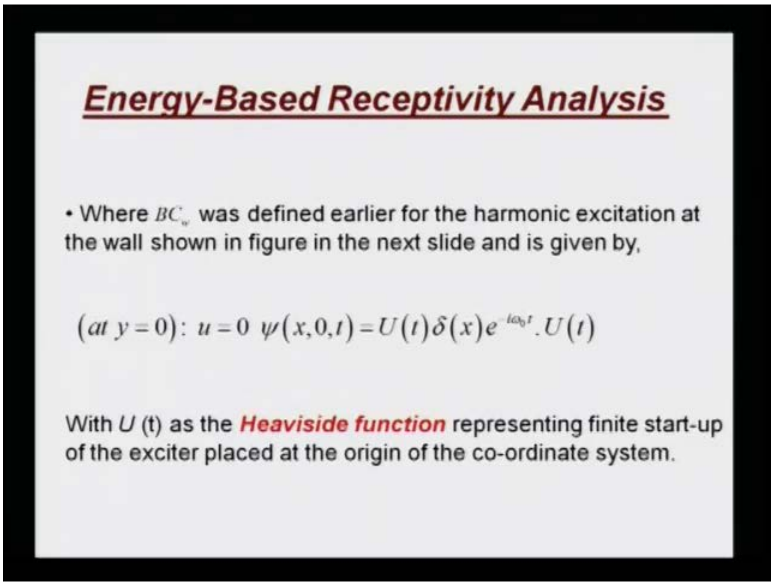

So, this BC w was nothing, but your, something like the Fourier Laplace transformer of the disturbance stream function phi. So, that could be for any alpha, any omega, evaluated at y equal to 0; that is your BC w. And, you can notice that, there are the possibilities at the wall, you will have low slip condition and we can have a kind of a localized disturbance source, which is indicated by this delta function. That is one thing in terms of spatial localization and we also wanted to know about, it is that finite startup time would come through this kind of Heaviside function.

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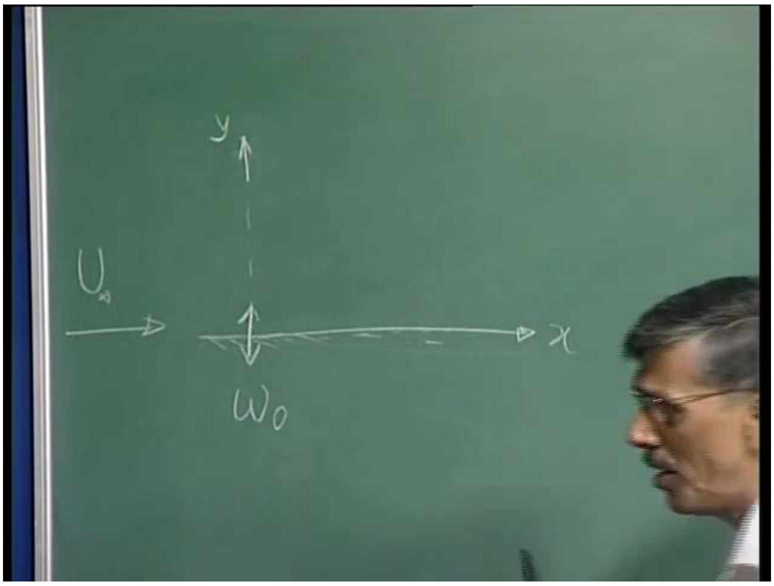

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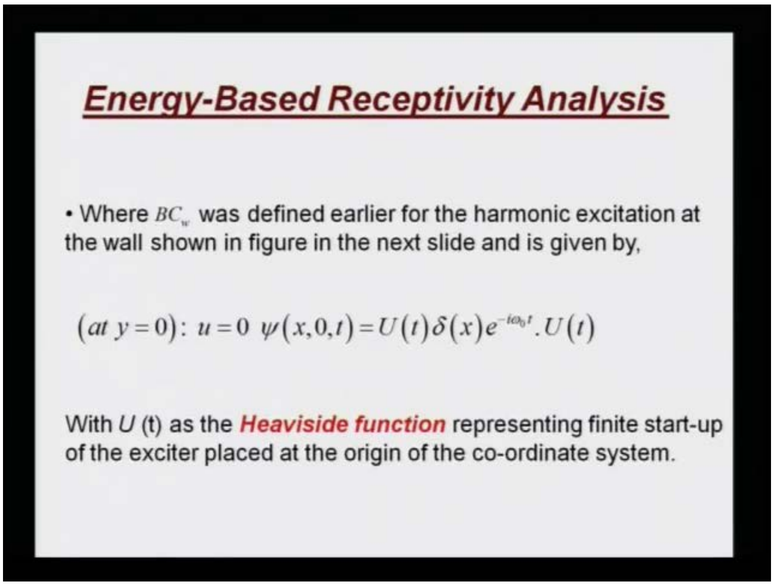

So, we are basically talking about some kind of a, let us say, if we talk about a flat plate and we position a exciter at the origin, that is what we are doing. So, we have a exciter, which is excited at a frequency omega naught and it is harmonic. So, that is why we are given E to the power minus i omega naught t and its localized nature gives us delta x and it is started at equal to 0.

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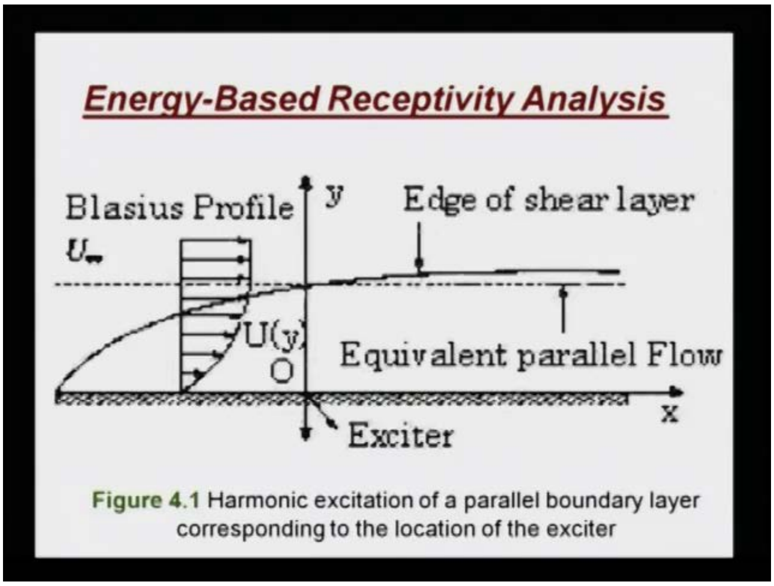

So, this is something we have seen already for a unstable system and this is the picture. We can look at the exciter sitting in a Blasius profile. A typical profile is plotted here. And, what parallel flow assumption implies is that, at the location of the exciter, you find out what is the shear layer thickness and you consider the flow actually consists of a flow, that does not change with x, having the same shear layer thickness at the location of the exciter. This is what the parallel flow approximation means.

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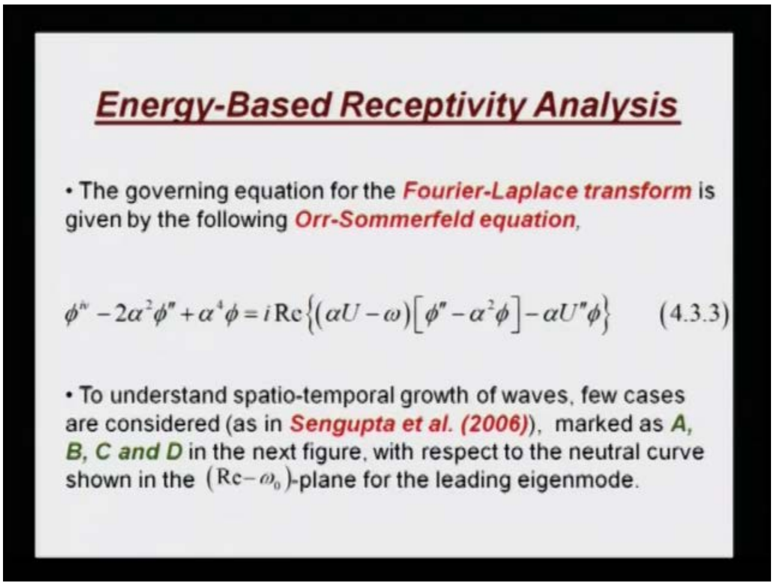

So, let us see, what we get out of this. We have already recounted what happens, when we look at the spatially unstable system, where we looked at the Fourier Laplace transform, again given by this Orr-Sommerfeld equation; and, we now try to study this problem again, but now, what we want to do is, not only study spatially unstable system, but let us also try to study, what spatially stable systems do.

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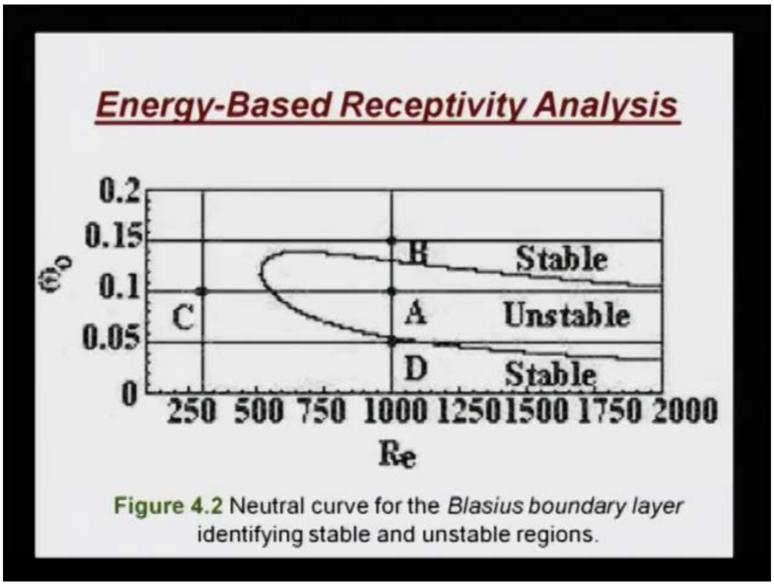

And, this is what is shown here, with respect to receptivity studies, where we had positioned exciters corresponding to the various locations here, marked as A, B, C and

D. A is what, we have already studied; that is very much inside the unstable core of this neutral loop; B is a stable, but it is above the neutral stable curve and D is also stable, but it is below and C is a subcritical point. So, these are the four points that we are going to study next and see, what do we get. Considering the fact that we already have seen what was the solution for A, we made the comment that, what we get from the full spatio-temporal analysis looks essentially the same thing that, we would have gotten from this spatial analysis itself.

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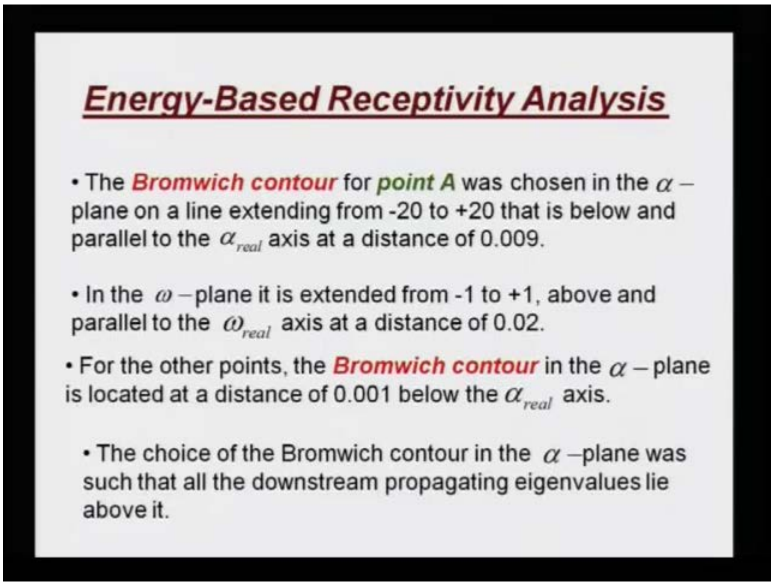

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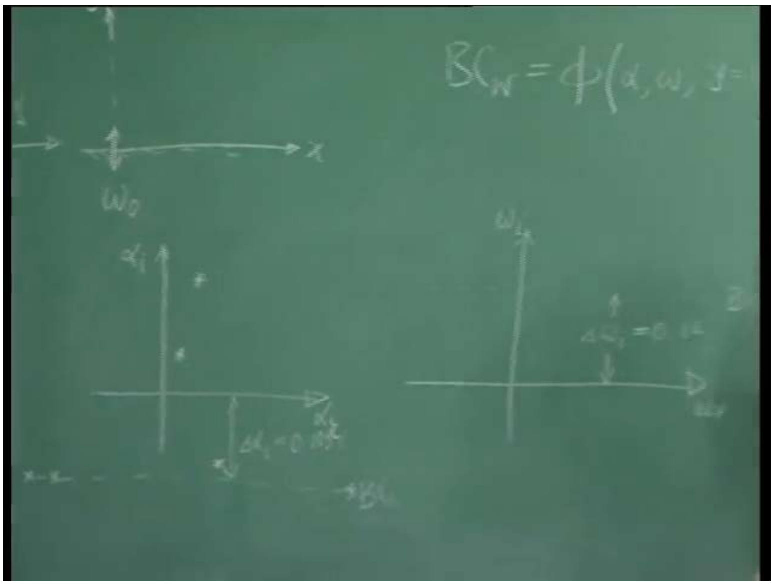

And for the point A, what we had done, we had basically lift at the Bromwich contour in the alpha plane. So, we have alpha r and alpha i plane and we have similarly a omega r and omega i plane. And, when it came to choosing the Bromwich contour integral, we chose a contour which is parallel to the alpha r axis and we did take it at a distance of something like 0.009. Why we are doing it, for obvious reason, for Reynolds number of 1000, we see a maximum growth rate is of the order of minus 0.008. So, we wanted to keep it as close to the… So, there would be a maximum unstable mode somewhere here and so, we want to keep it like this. Now, what is the reason for it? You want to do it for the reason that, there would be many many modes. So, in this case, maybe there are two modes, in addition to this unstable mode. Then, we want the effect of all these three modes to be visited upon along the Bromwich contour. And when we take it as close to the unstable point as possible, then, we do get its maximal effect, without losing numerically anything.

But if I keep the Bromwich contour further down, I will still have those effects coming in from all the three points, but you may lose out in terms of numerics. So, that is why, choosing this contours are important; you try to find out from the grid search method and then, polish it with the Newton-Raphson search; and then, you find out which is the most damaging one; and try to locate the Bromwich contour in the alpha plane as close to this as possible, but below this; why, because this Eigen value corresponds to downstream propagation. So, all the downstream propagating modes should be above the Bromwich contour. If I choose a Bromwich contour like this, the answer will be meaningless; because then, we are giving into a row that, as if the disturbance is going upstream; and that would be funny, because if I do it like this, with respect to this contour, this will be upstream propagation and what happens, this will then become a stable mode also; because it has a sign alpha in negative and if it is below Bromwich contour, it will be a stable upstream propagating mode.

But from our Eigen value analysis, we have found out that, it is a basically downstream propagating mode and this is also unstable mode; that is why, we will have to choose this. So, basically, we understand every bid of whatever we have learnt, it is not gone to waste. We have to do the stability analysis; we have to do the grid search; we have to do the Newton-Raphson polishing; we have to identify the location of individual modes and then, what we could do is, we can do this Bromwich contour integral. So, Bromwich contour integral, that is what I kept telling you all the time, it is apriori not given to us; we will have to do some extra work. And, in contrast to this Bromwich contour here, below the real axis, here, what will you do, here we would do in such a way that, it would be above all possible Eigen value. Why are we doing this, because you see, this multiplied by E to the power i alpha x minus i omega t. So, basically, then, we need all the Eigen values have to be below this, otherwise, we will get to a non-causal situation. So, we want to satisfy the causality condition. So, we try to put it above.

And in this case, in the omega plane, we choose it at a fairly a high value. So, this value, we have taken something like 0.02. That is what I promised to you that, we will discuss in detail. So, today we are telling you, how to choose this Bromwich contours. The Bromwich contours have to be chosen in alpha and omega i plane like this and then, once the Bromwich contours have been identified, we can solve Orr-Sommerfeld equation, starting all the way from one end to the other, theoretically speaking, you should like to go from minus infinity to plus infinity.

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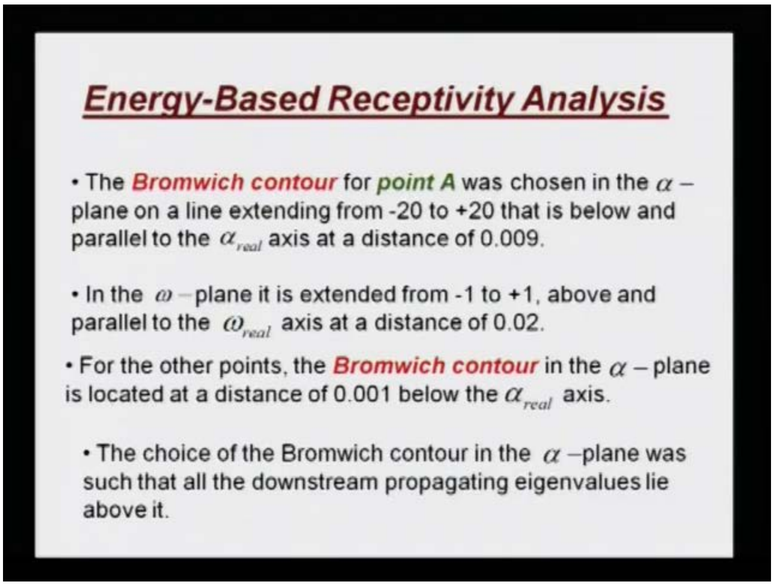

So, however, we do not want to do this for one reason that, I mean, we have a finite resolution; we have some delta alpha r possible, and what we have done, we have treated minus 20 itself as something like minus infinity. In the context of Eigen value analysis, what kind of alpha are we get; it is always less than about 1 or so. So, in such a case, compared to that, these are the distant edge. So, that is what we have figured out that, we can take it from minus 20 to plus 20 and in that interval, we take the number of points which are 2 to the power something; because you want to do a very good quality, a 50, so, we want to take a 50 with radix 2.

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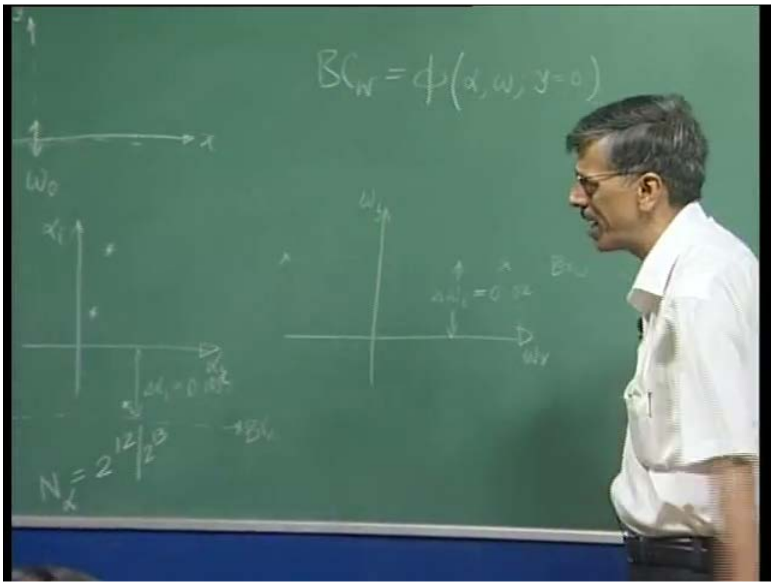

And, we take actually here, 2 to the power 12 points. So, let me call this as number of points in the alpha plane. So, that is something like 4096 points. This 4096 points because, 2 to the power 10 is 1024. So, it is 4096 points. We have actually also done it even higher, 8192 points. So, that is the kind of number of points that we take. We have also to take a large number of points in the omega plane and some of the calculations we have done were again taken from some omega range.

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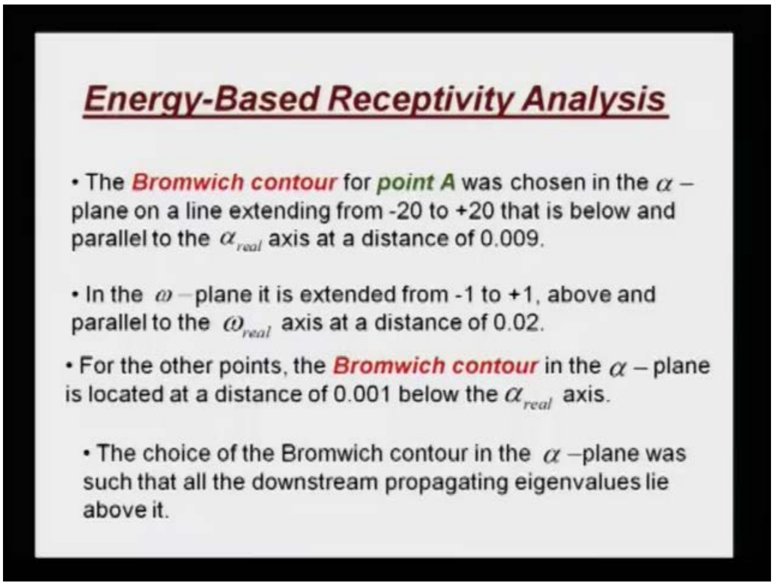

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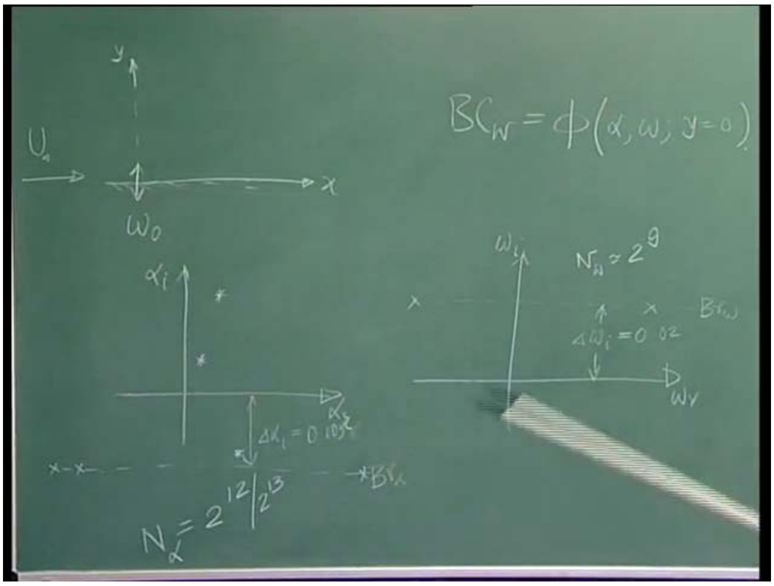

This omega range is, interestingly we have taken far too smaller, minus 1 to plus 1 and we have taken something like, number of points here is 2 the power 9 (( )). Well, this is something what we did about 5 years ago; if we were to do it today, we would be a little bit more, in a better situation; we may take more number of points. What happens here is, when I take the number of points here and I take omega max, that means what? Delta omega is getting fixed and the moment I fix delta omega, that fixes my t max. So, I am doing actually a simulation over a finite time range, that is dictated upon by 2 pi by delta omega.

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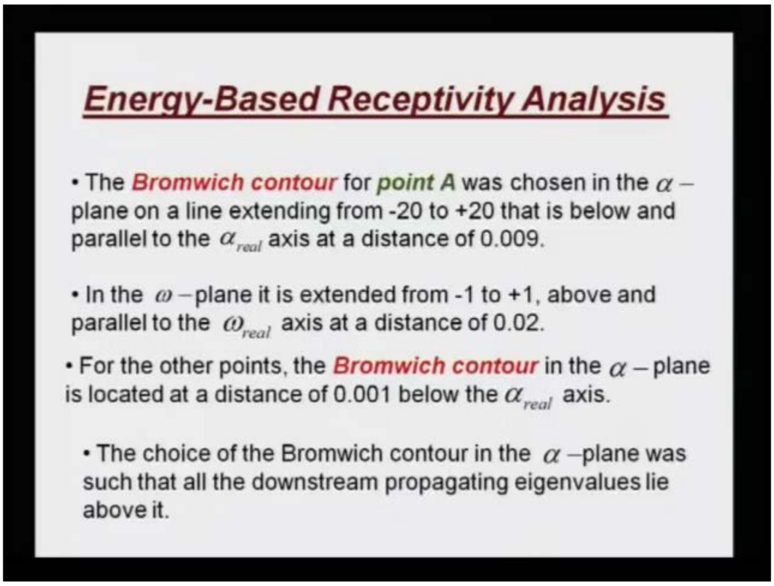

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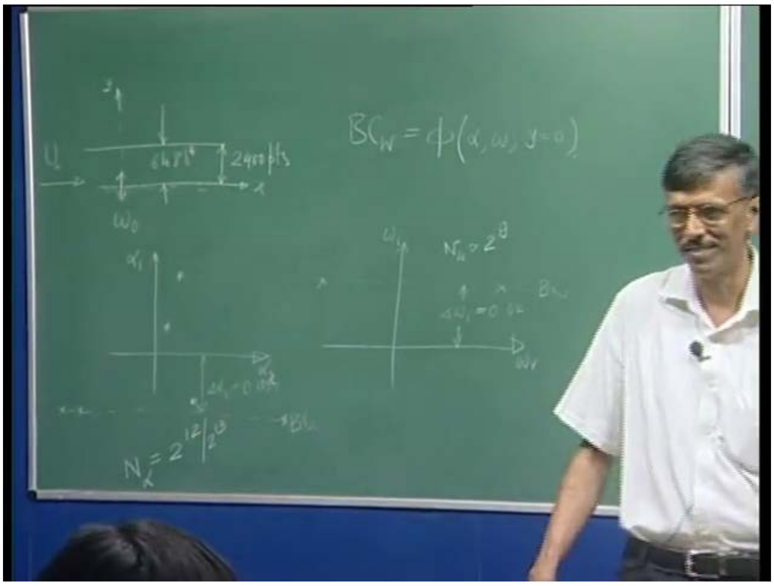

So, what happens is… That means that, simulation is valid only for a short time and we wanted accuracy; that is why we have been forced to take it from minus 1 to plus 1. It is not that, we will like to take minus 1 to plus 1; one would like to take it, may be from say minus 20 to plus 20 in the omega plane; we have to take corresponding points. See, basically, you will be solving all this Orr-Sommerfeld equation that many number of times. Every combination of alpha and omega, you will be solving that Orr-Sommerfeld equation. And, we are talking about the combinations here. You can think of, this is 500 and this is about, say 4000. So, this gives you about 2 million points. So, you have to be solving Orr-Sommerfeld equation 2 million times. And, we are solving the problem, in a range which is not too high; this is about, may be 6 to 8 delta star. 6 to 8 delta star and how many points do you take in there? Well, I think, those of us, who actually only do CFD and do not do this kind of calculations, they will be quite surprised; we take about, well, the results, I am going to show, where we have taken some 2400 points; we have also taken 4000 points.

You need to have that kind of resolution to pick up this waves, as accurately as possible. So, basically, you can think of the resolution, 2400 times, about 4000 points, in the xy plane; that is about 8 million points, within only 8 delta star. This is interesting because, those of you who do CFD and you may have seen some publications, people say, we have done a very well resolved calculation; we have taken 20 points inside the boundary layer or 100 points inside the boundary layer and here we are talking of thousands and thousands of points. Because, boundary layer is about 3 delta star. So, here, you are looking at 1000 points within the boundary layer.

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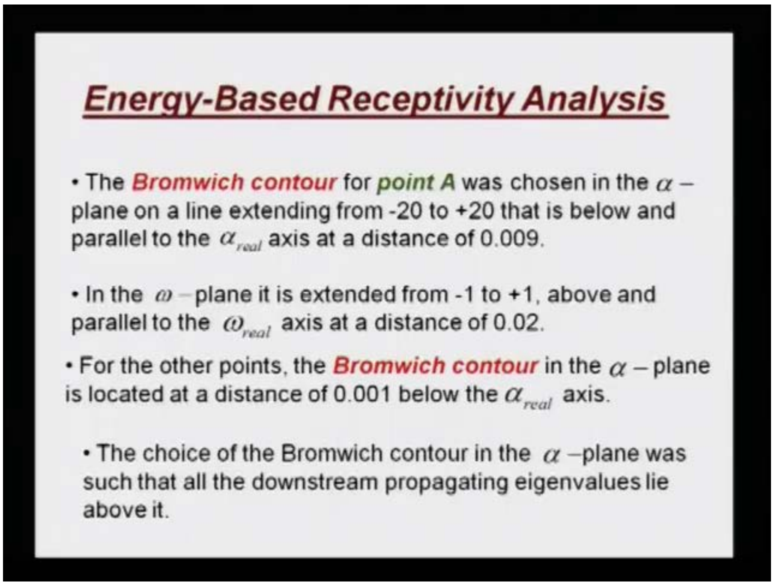

So, computing equilibrium flow is one type of activity; computing a disturbance flow is activity for the grownups, who have come up matured and have the confidence in solving the real disturbance quantity. It takes that much more effort and to get it, get your linearized Navier-Stokes equation, to give you a quality results, which picks up all those Eigen values of the associated Eigen modes correctly, you have to go through that kind of effort.

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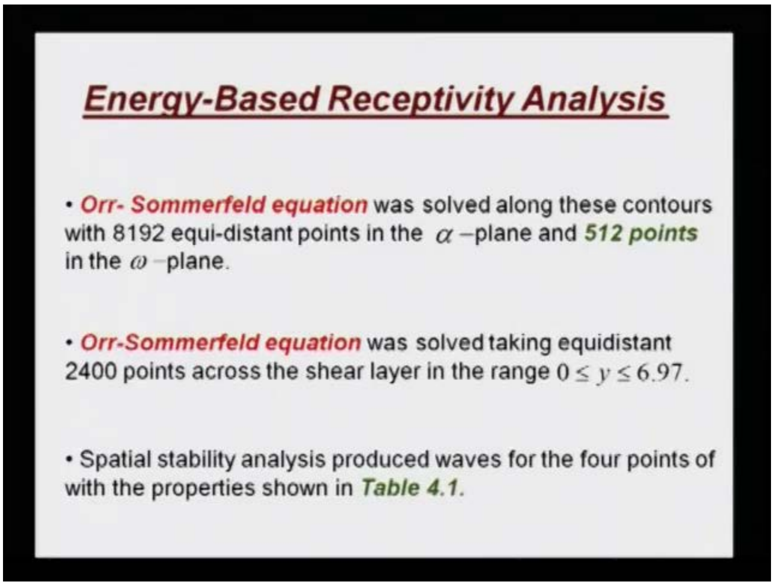

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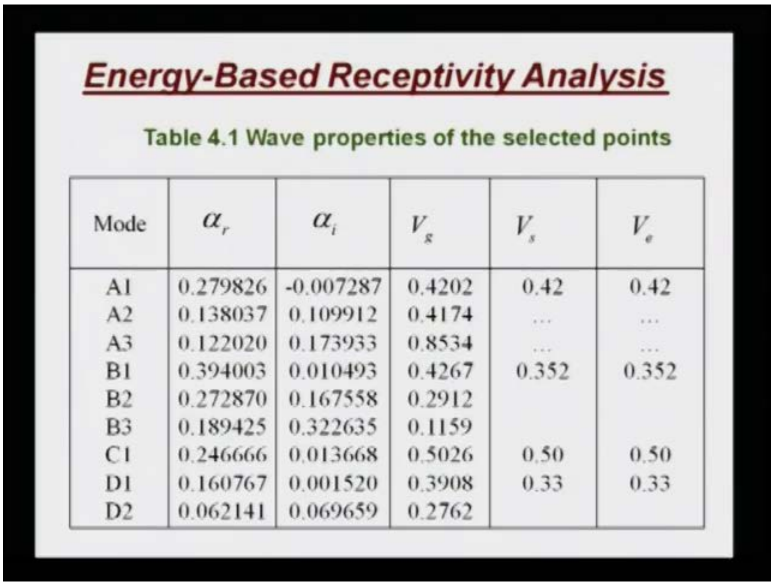

Mode	α_r	α_i	V_g	V_s	V_e
A1	0.279826	-0.007287	0.4202	0.42	0.42
A2	0.138037	0.109912	0.4174	…	…
A3	0.122020	0.173933	0.8534	…	…
B1	0.394003	0.010493	0.4267	0.352	0.352
B2	0.272870	0.167558	0.2912		
B3	0.189425	0.322635	0.1159		
C1	0.246666	0.013668	0.5026	0.50	0.50
D1	0.160767	0.001520	0.3908	0.33	0.33
D2	0.062141	0.069659	0.2762		

So, this is something, I just thought I will sometime explain. So, today was that day. We did now figure out, and as I told you that, in the results that I am going to show, these are basically, 2 to the power 13 points in the alpha plane and 2 to the power 9 in omega plane and y range is from 0 to 7. I have taken about 2400 points. And, you will see, once we look at the grid search method along with the Newton-Raphson search, for all this 4 points, we get this kinds of modes or the point A, we found out, it was inside the Newton loop and that is indicated here with a negative value of alpha i. And, you can also see, it is a downstream propagating mode, because the ((group)) velocity is 0.42. The other two modes that you find, they are stable; wave number is the half of this unstable wave number. This is also same thing. They are decay rates, they are huge and the group velocities are of also the same kind; this is almost half u infinity, this is, almost close to 1 infinity. All these are, what we call as the signal speed and the energy propagation speed, we will talk about it; we will come back to how we obtain it, but we know, how to do it; we have done it.

So, the point A1 is inside the neutral loop and then, all this 3, point A is inside the neutral loop and has only three modes. And, we now know, because of that essential singularity, we can represent any arbitrary disturbance in terms of this three modes plus the point at infinity. We have already done that. The point B, point B was where? It was above the neutral loop. So, there, we find that, we again get three modes and all the three modes are essentially stable; this is the least stable, but still it is plus 0.01. And, the group velocities indicate all those three modes to be downstream propagating. And, in contrast, the C1 which was a subcritical point, there we find, you have only one mode, with alpha as 0.25; alpha i is also indicative of a stable mode and the group velocity is half of u infinity. The point D, which was again at the same Re equal to 1000, but little below the neutral curve, there we have only two modes and those two modes have alpha i given by this, and the vg is indicated by this. So, essentially, all the things that you are seeing in this table, corresponds to downstream propagating modes.

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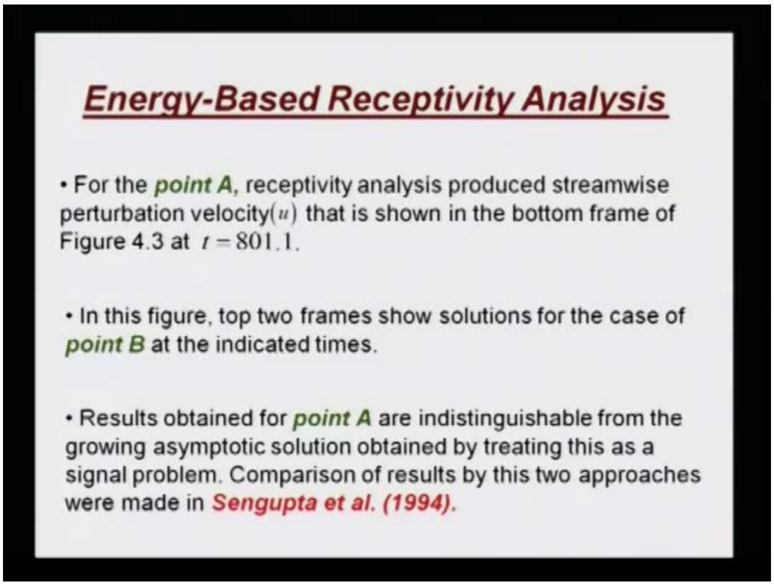

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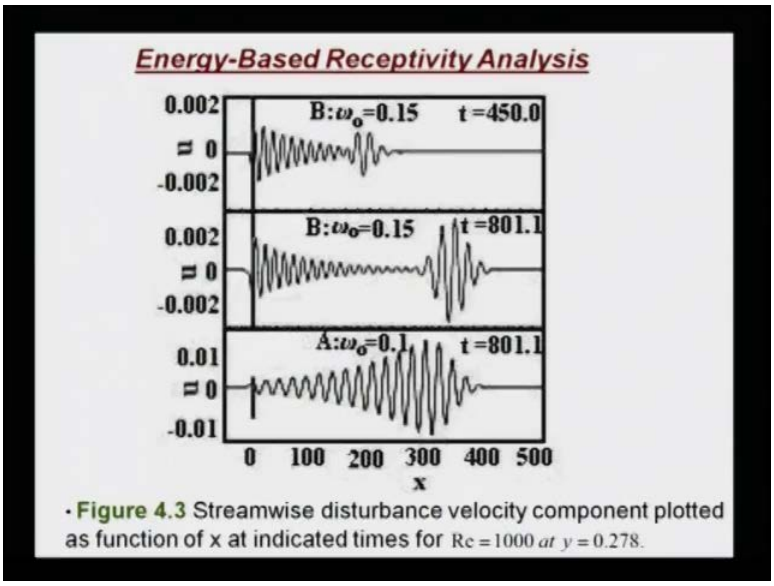

We will come back to this table again, when we talk about the results. Now, we have seen that, the point A, we do get the solution to be unstable and that is what is shown in the next slide. Let us take a look at the slide at the bottom, the bottom frame, that corresponds to it and at a time like 801, we see the solution like this. This is your location of the exciter and you see that, this wave is growing. This is bounded by this decaying front. So, as time progresses, you get this initial growth followed by this decaying front, this is what we notice. And for point B, we are considering a case, which is above the neutral curves. So, that corresponds to about 0.15. This is the solution at t equal to 450 and this is the solution at t equal to 801. And, what we note that, the spatial theory says, it is a stable mode. So, that is what we have seen; it is a damped wave, that is coming up. And, but that is preceded by this wave front, and that wave front, actually, initially grows and then, it decays. At a later time, we see this decaying front is, decaying wave, asymptotic wave is very much there, but this is always preceded by this.

And what you notice, this spatio temporal front that you are noticing here, is continuously growing and it is not really like, what one would call as a transient; you know, this is, we calculated up to 800 and it continues to grow. However, you notice that, there is some kind of a similarity between this leading part of this and leading part of this; in both the cases, it just ((say)). So, this is bound to happen; this is bound to happen because we are looking at dispersion. So, signal takes a finite time. So, at that time, energy has propagated up to here, the disturbance energy. So, you see, this two points A and B, A corresponds to a spatially unstable point and B corresponds to a spatially stable point; both have a wave front like feature; for A, you may not see it, because it has terminated into the wave front, but for B, you can certainly, clearly distinguish between the decaying part and the spatio-temporal wave front part.

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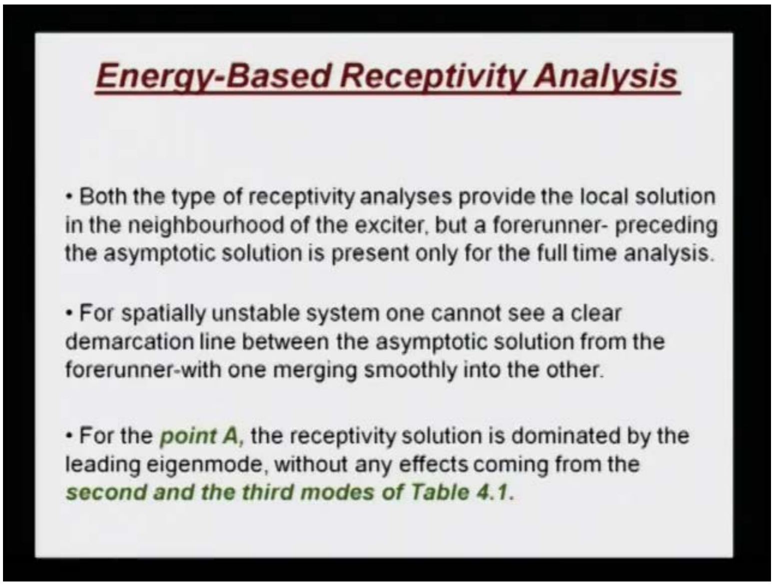

So, if I now, look at those other points, then, I could see something different, but before we do that, we just sum up what we have just now seen that, performing those, that receptivity analysis, we of course, noticed the local solution in the immediate neighborhood of the exciter; and additionally, we have a forerunner; this is the one, that precedes even the asymptotic part of the solution; and this, you can only get, when you do a full time analysis. If you just simply do a spatial stability analysis, or even spatial receptivity analysis, we have done it, remember, we called it as a signal problem. So, if we would have assumed that, omega only goes as omega naught, then also, we would not get this. So, it is not only that, will have to do the receptivity analysis, we will also have to do a full time dependent receptivity analysis.

And, for spatially unstable system, one cannot see clear demarcation line between the asymptotic solution from the forerunner, with one merging smoothly with the other. For the point A, the receptivity solution is dominated by the leading Eigen modes, say we had three Eigen modes and the one that is growing, is the one that dictates the fate of the packet; not much of a effect coming from the second and third mode. Why and how I say that, we have the solution. We have the solution, the asymptotic part; we can do an fifty analysis; we can calculate its alpha r and alpha i and we can make this observation like what we are saying; that, a leading Eigen mode dominates everything. The second and third, there are no such things, because, if I do the fifty of psi versus x, then, I will get corresponding phi versus alpha and I will see that, there is just a peak at A1; A2 and A3 there are no distinct dishonorable peaks.

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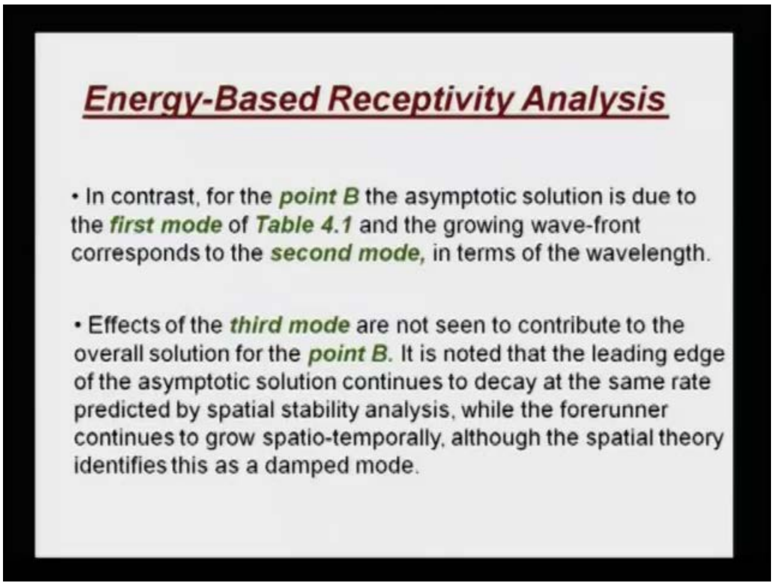

Now, in contrast, when you are looking at the point B, the asymptotic solution is due to the first mode. And, if you do the fifty of the forerunner part, then, you will see that, the wave number and the decay rate, etcetera, especially the wave number, corresponds to the second mode. But please do not think that, it would be always like, belong to one of the modes, because we have seen qualitatively, the forerunner of A and B looked similar. So, it is not necessary that, though that forerunner is associated with any particular Eigen modes per se; it could be something more.

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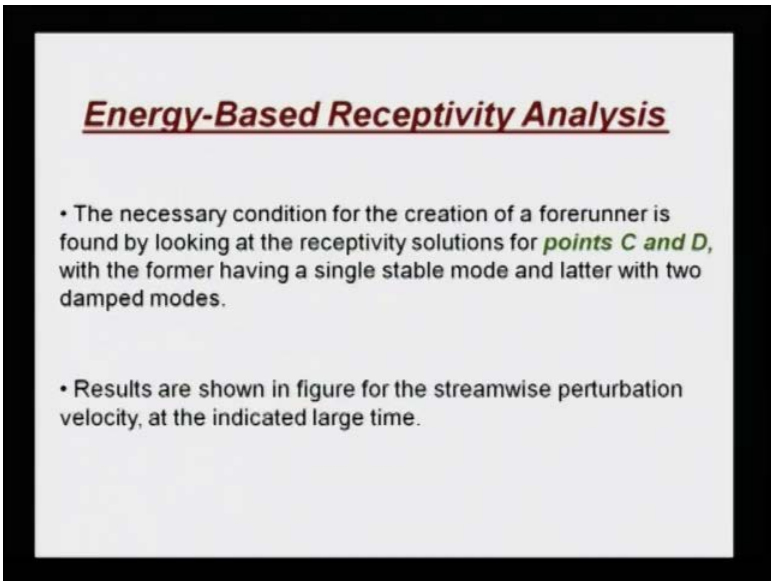

Effect of the third mode is not seen at all for the point B; that the leading edge of the asymptotic solution continues to decay at the same rate predicted by the spatial stability analysis; while the forerunner continues to grow spatio-temporally, although the spatial theory do not identify any growing mode at all. But we notice that, to get this spatio-temporal mode, we probably need to look at solutions for other points. See, D was a point which was below the neutral curve and C was a point to the left of the neutral curve.

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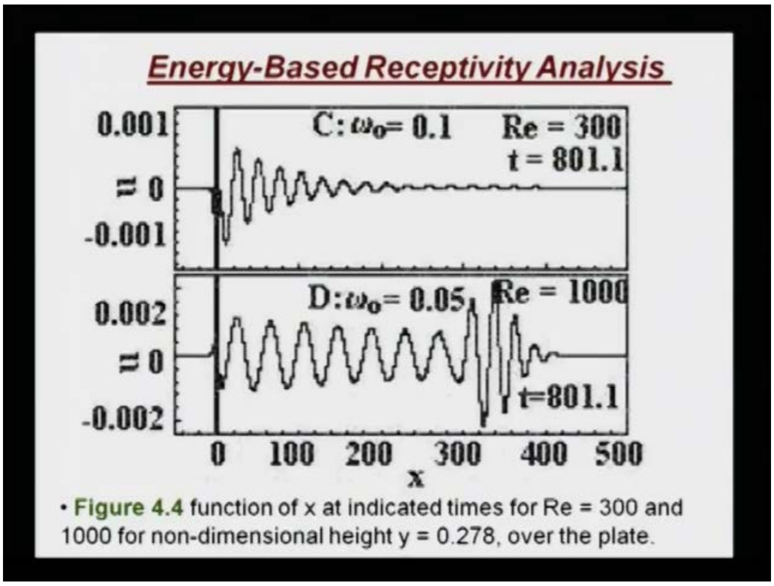

So, let us look at those two points and the corresponding receptivity analysis. And, this is how, we are going to see. For point C, there was only a single mode. And, that was a damped mode and that is what we are seeing. So, this is something. So, for C, the omega naught is kept the same, but Reynolds number is lowered. And, for point D, Reynolds number is kept same, but omega has been reduced to 0.05. See, A corresponded to 0.1, B corresponded to 0.15, D corresponds to 0.05 and here, what you could see that, this is very close to the neutral curve. So, the leading mode is damped, but it is almost near neutral, and that is what you are seeing; it is slowly decaying, but it has a sort of a leading mode here, a spatio-temporal mode. Let me just tell you about, little bit of historic fact. Sommerfeld student Brillouin, he was interested in investigating the effect of forerunner for electromagnetic wave propagation. So, if you look at the book written by Brillouin on waves, you will see that, he was more interested in looking at forerunners in electromagnetic waves and there he could not find.

And, in electromagnetic wave, you are looking at a non-dissipative system and what we are studying here, is a kind of a dissipative fluid dynamical system. So, what he was looking for in electromagnetic waves, we can see something of that kind, for a hydrodynamics. So, this was something, which was really out of the blue kind of thing, because people did not anticipate that, this would be there. So, when we figured it out, our motivation was trying to explain the dynamics of spatially stable system. I told you that, we were interested in investigating a case, where we would have the basic dynamics is given by a stable system. But yet, it supports very large spatio-temporal growths, like what you see in a tsunami kind of a scenario; because, if you look at the ocean boundary layer, it is a spatially stable system. But now, if the ocean boundary layer is excited by a delta function like earthquake at the ocean bed, then, what happens; that was the thing that, we were trying to investigate. And, our investigation did show this.

And you can very clearly see that, what people have narrated, their personal experience on the seashore during such events, that you see some small waves coming and then one or two large, big waves and then, everything quite (( )); exactly the kind of thing, that you are looking at here. Although, I would not stretch it any further, because this is Blasius boundary layer, one can do that; we leave it to our friend in oceanography to do all these kind of studies; if they can do such high fidelity calculations, like what we have been talking about here, there is a possibility that, this thing could be investigated.

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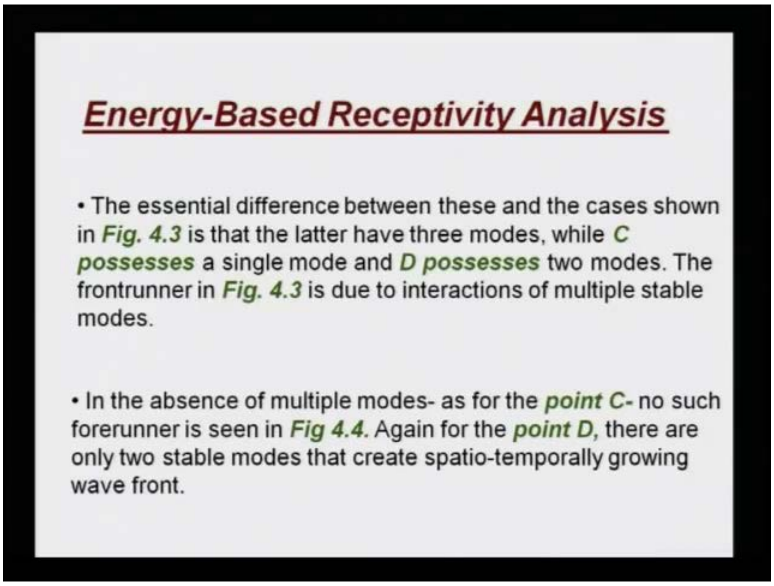

And, the main point that we are trying to make is, essentially the following that, not only we will be able to establish that spatio-temporal growth and decay of such packets, forerunner, which originally started in Brillouin's idea, looking for it in electromagnetic wave, which we have shown here for hydrodynamic waves, I think, you can also look at it from a directionality point of view; because this is all done in a 2D scenario. So, I am again, once again, giving an idea for further research. If anyone is interested, one can look at in a three dimensional field and then, you can also find out the group velocity and you will see, why a tsunami comes in direction X and does not hit this port B, this station B, because it would have directionality; from the group velocity, we can calculate vgx and vgz, and we can say, which direction it will go. So, there is a possibility of doing this. So, there are a lots of things.

Let us now try to summarize, what we have seen in this figures, that, while in the earlier two figures, for A and B we had three modes; here C possesses single mode and D possess two modes; the forerunner in figure 4.3 is due to interaction of multiple stable modes. Well, I write this with bated breath, because, this has to be qualified with further studies; however, if we do not have multiple modes, as was the case for point C, we did not see a spatio-temporal growing wave front. Then, can we say that, having more than one mode is a necessary and sufficient condition for spatio-temporal growth or was it that, in that Reynolds number of 300, corresponding to point C, this temporal dynamics is such, we do not see any spatio-temporal growth wave front. So, one has to really fill up the gap between 1000 and 300 and see, what is the history of spatio-temporal growth wave front.

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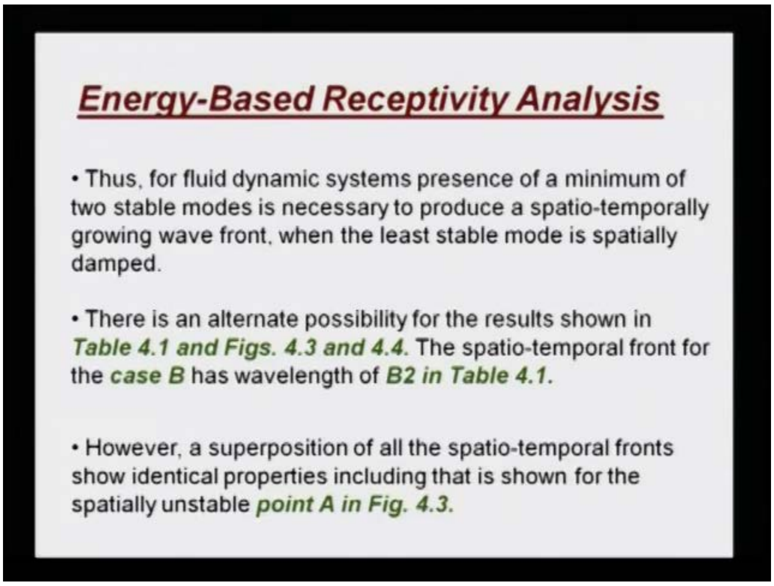

These are some of the things, that are looking straight at us and trying to encourage some of you into studying it and figure out what is going on here. So, I think, I have written this, but I will raise my hand up, if you object that, whether it is really a necessary and sufficient condition or not, I do not know, but at least for these four points that we have studied, that if we want to see spatio-temporal growing wave front, we must have more than one; that is what we saw. This is the alternative that we are talking about. The alternate possibilities is that, the spatio-temporal front for case B has wave length of B2, since one of the modes, but this is not necessary too, because if we superpose the spatio-temporal front for A, B and D, which are all for the same Reynolds number, they seem to follow each other. So, is it something, a function of Re all alone, that is what I am suggesting to all of you that, there is a possibility that, one could study the dependence of the spatio-temporal wave front on Reynolds number; that is the problem that needs to be solved.

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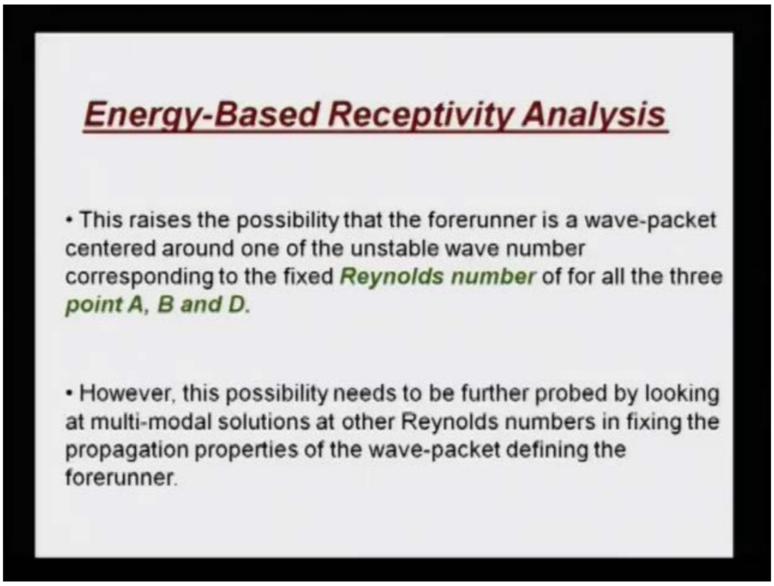

So, that is what I am making a conjecture here; I should be also allowed to make some conjecture. So, like, everybody does. So, we are saying that, this may raise a possibility that, the forerunner is wave packet centered around one of the unstable wave number corresponding to that fixed Reynolds number, for all the three points A, B and D. We do not know. This possibility needs to be probed by looking at multi modal solutions, at other Reynolds number and fix propagation properties.

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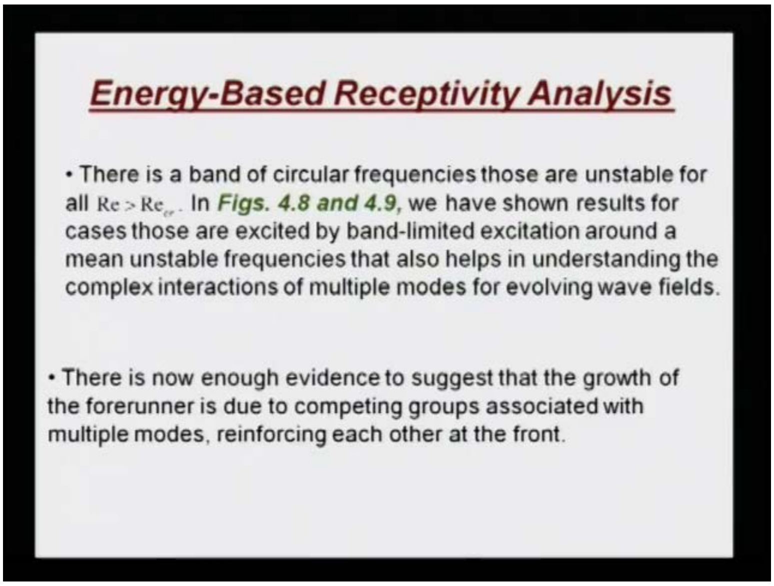

So, if anyone of you interested in doing a PhD, there is a topic straight ahead that could be looked at and solved. However, we will talk about what happens when I look at the dynamics of a Blasius boundary layer itself, where I excite the system in the band of frequency, where all the individual modes are unstable; but if I excite the system with that band, what happens to it; that is what we are going to study. That, even though we are looking at all the individual modes are unstable, we will see that, the overall dynamics may not show any growth at all. See, this is one of the thing that, we should be very very critical of this normal mode analysis. Studying things in isolation has actually impeded our understanding of fluid mechanics more than anything else. So, that is why there is so much of a need, to adopt Bromwich contour integral; however tough it may appear to begin with, I do not think we have an option at this point in time. There is no enough evidence to suggest that, the growth of forerunner is due to competing groups associated with multiple modes, which reinforces each other at the front.

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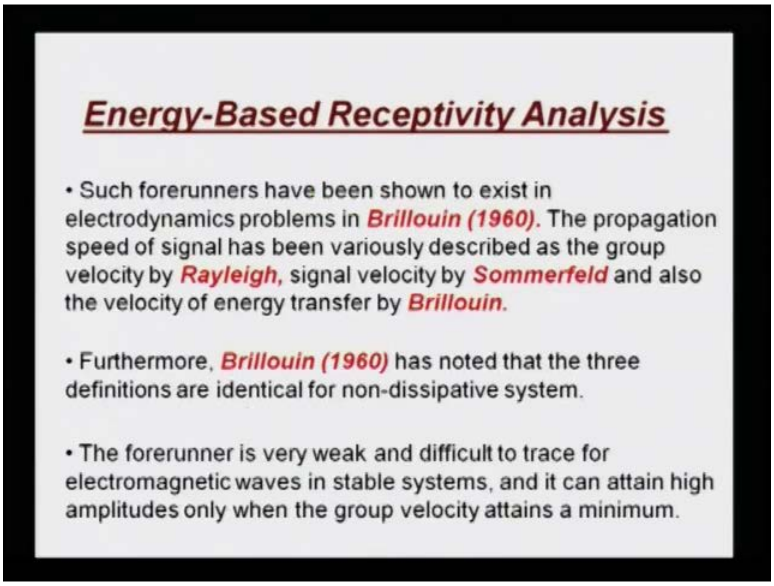

So, this is something that we need to be looking at and as I told you, just for a record, we look at the book by Brillouin. He was looking at problem of electrodynamics, in search of forerunners; the propagation speed of signals; this kind of disturbance propagation has been given in a different name. You know, we have been talking about group velocity, we are following Rayleigh. Rayleigh was the first person, who actually, originally followed Hamilton's idea. If you look at even older papers by Hamilton, Hamilton talked about interaction of multiple modes. But Rayleigh did put it in a firm foundation. Sommerfeld called it as some kind of a signal velocity and Brillouin was the one who used to talk about energy propagation speed; that is why, in the table you saw that, we had made three columns - group velocity, signal speed and energy propagation speed. So, we want to basically, figure out what this is. And, Brillouin made a sort of a prophetic statement that, if you are looking at non-dissipative system, all these three will be identical.

But if you are looking at a dissipative system, it may not be of… How therefore, problems of electrodynamics, this forerunner is very weak and it has been extremely been difficult to trace it for electromagnetic waves in stable system; however, below also noted that, if you are trying to look for it, you will only look for it, for the condition where group velocity actually attain some kind of a minima. So, you will have a bandwidth.

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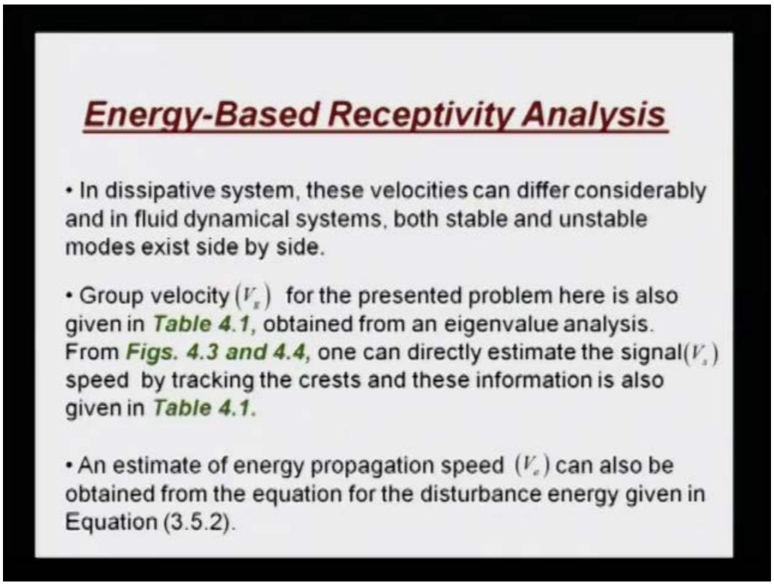

And if you plot the group velocity versus the, say wave number range, then, if you have a minimum of the group velocity in that range, it will be around there, you should get to see some kind of a high amplitude; we have not pursued it. So, it is just for your consumption, just lay out this fact. We look at dissipative system for fluid dynamics and we take the queue from Brillouin and say that, this velocity can differ considerably, because, we have both stable and unstable modes staying side by side. So, I think, I will stop here and we will continue our discussion of it in the next class, where we will define what is the difference between the group velocity and signal speed and the energy speed, how these things haven to be obtained, we will talk about that.