Instability and Transition of Fluid Flows

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Module No.# 01

Lecture No. # 26

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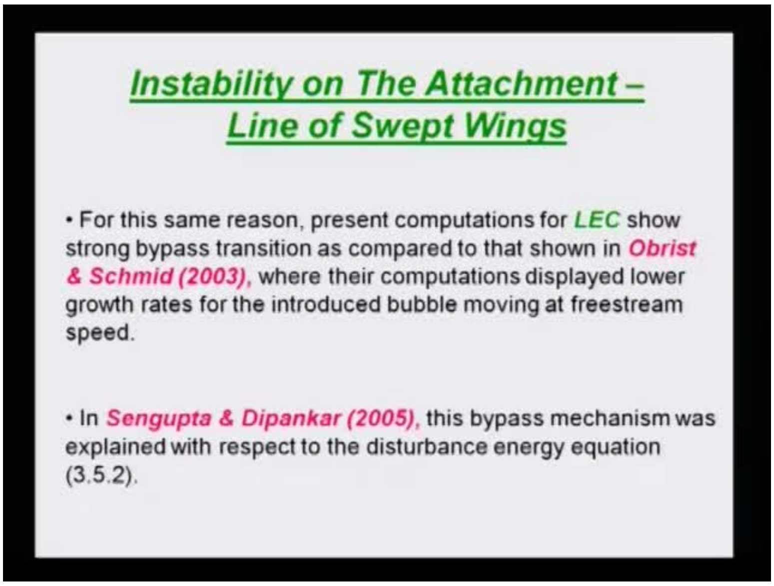

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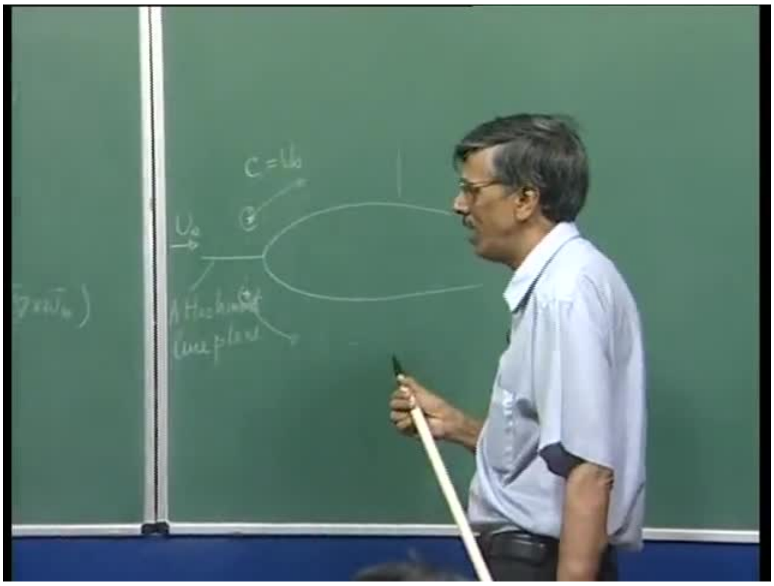

Let us conclude our discussion on instability on the attachment-line of swept wing. We were talking about what other groups of people may have done. One specific instance was thus worked by Obrist and Schmid, in a couple of back to back paper in J F M. They investigated the flow field of this kind, that, if you have, let us say, a flow past your swept wing and this is your attachment line plane; then, what they did was, probably, let us say, this is the edge of the shear layer. So, they took symmetrically positioned vortex on either side, outside the shear layer and then, they allowed this vortex to move at a free stream speed. So, it is like this. So, this will perhaps, go this way; that will go this way; this vortices, that, we may have, may be counter clock wise pair, but they would be moving at C equal to U infinity, where the oncoming flow here is given by U infinity. And, what they reported was that, there was not much of a growth rate inside the flow.

So, this was kind of a different problem that, what we discussed. We were discussing about a periodic vortex moving in this plane itself and I, kind of suggested that, one could perhaps, investigate what happens to a flow domain. If we think of it as some kind natural barrier and then, we can construct a flow domain like this and due to this inflow unsteadiness, what will happen for the three dimensional flow field; that would give us some fairly a decent idea, what happens as a consequence of attachment line instability for the flow over the wing. And, over the wing, then, you are basically talking about mechanism being present. So, we are talking about, if there is a stream-wise instability, if there is a cross flow instability, or the combination of the two, everything should be picked up by a three dimensional direct simulation of such a flow.

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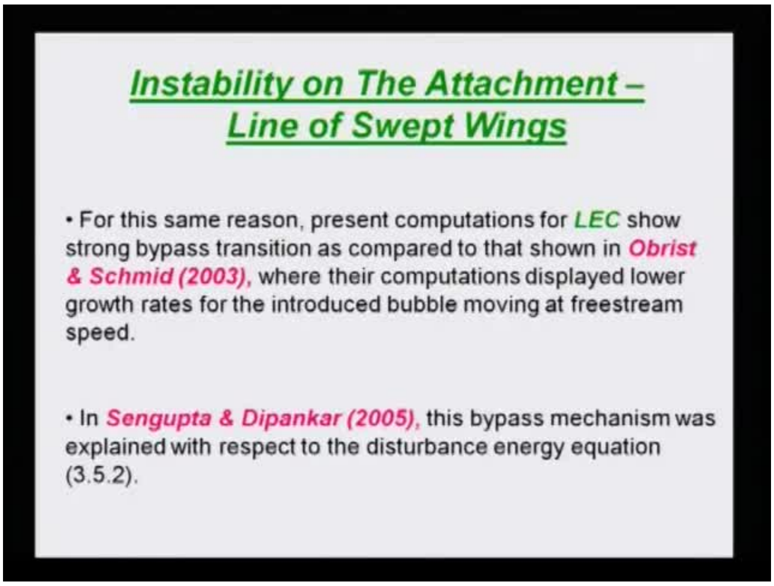

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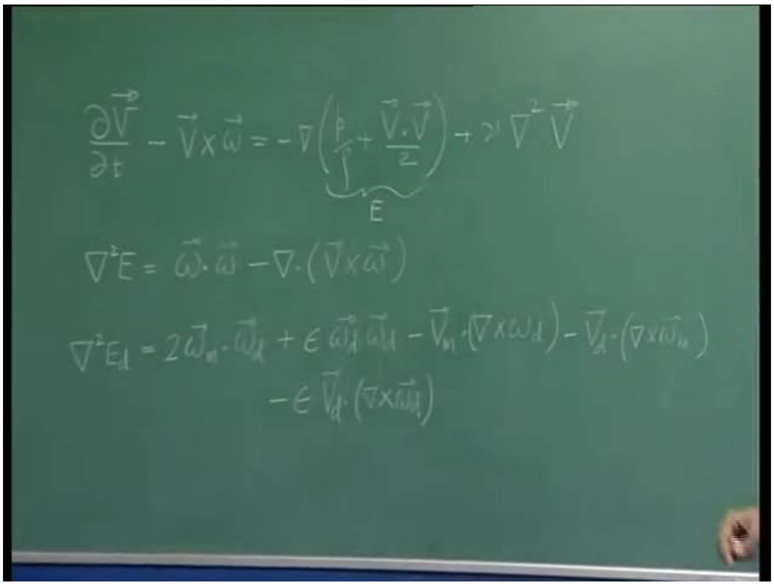

So, that was, was what was perhaps attempted in Obrist and Schmid; but they did not see any effect of it; probably, they did not also study what happens in the attachment line. Maybe, their attachment line flow was completely quiet; there was no unsteadiness in there. In contrast, what we have talked about, we talked about a bypass mechanism, that showed how the flow became unsteady right on the attachment line plane itself. And, that bypass mechanism was once again explained in terms of a disturbance energy equation. Just to recall what we have done about the disturbance energy equation, we start off with rotational form of the Navier-Stokes equation; then, take a divergence of it; that gives us the spatial distribution of this total mechanical energy E, which is given by this pressure plus the kinetic energy ahead.

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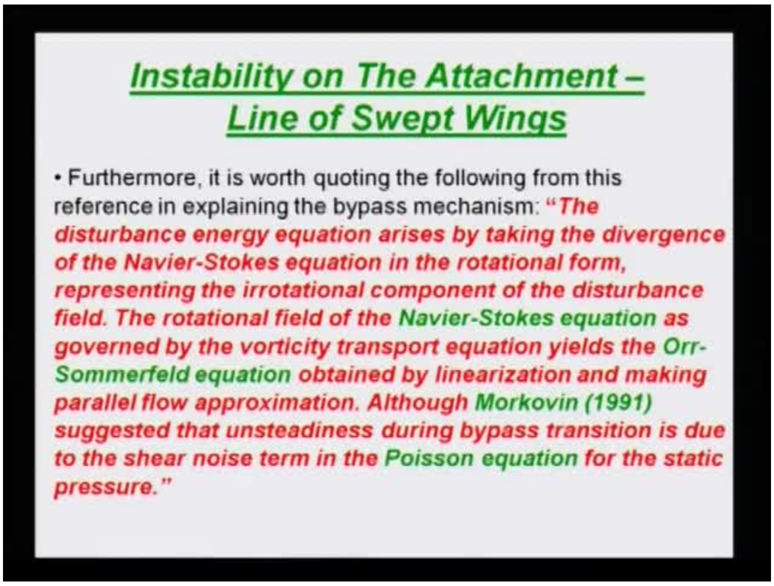

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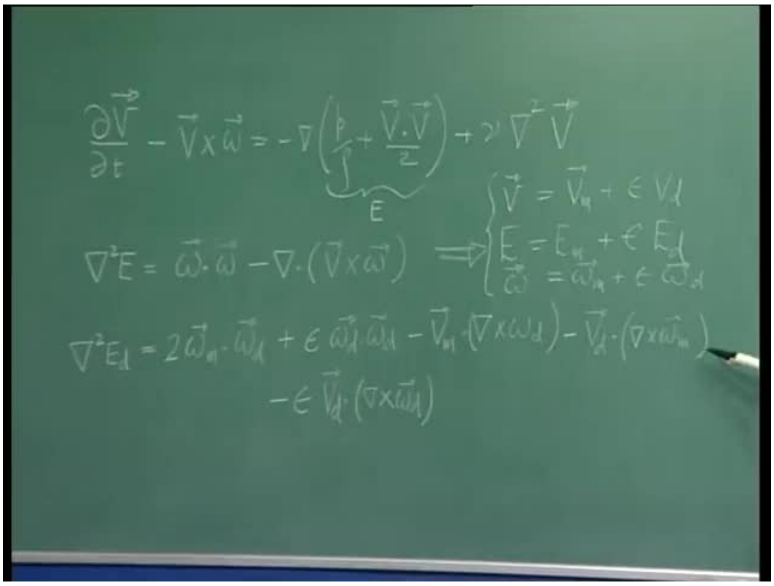

How (()) space is given by this. And, this is what we are talking about; quoting from that paper by a Sengupta and Dipankar, we noted in that paper that, the disturbance energy equation arises by taking that divergence of the Navier-Stokes equation in rotational form. So, this is the total mechanical energy and then, what we did, from here, we split the total energy, mechanical energy in to a mean part and a disturbance part. So, same way, you can do the splitting for the velocity field also, into a mean part plus a disturbance part and the same way, you could do the vorticity field also, split into a mean part and a disturbance part. If we do this, then, this equation represents this.

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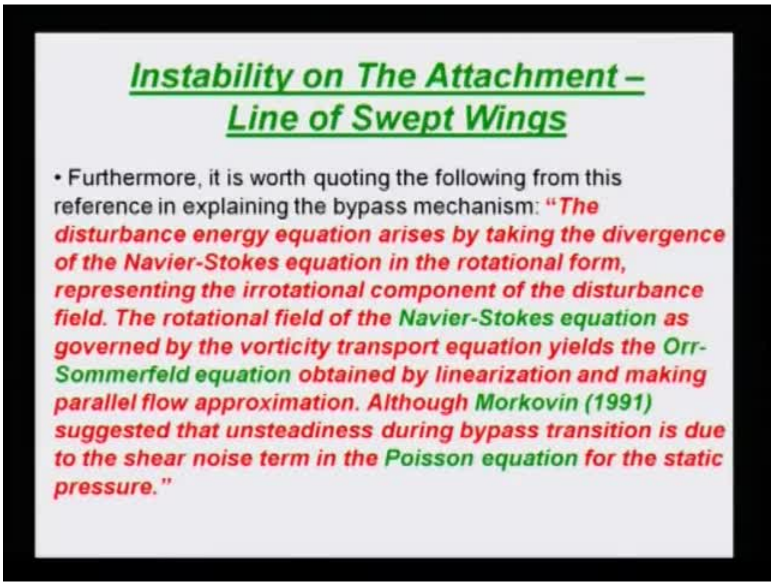

So, that is the first part. What we are talking about, that rotational field of Navier-Stokes equation as governed by the vorticity transport equation yields the Orr-Sommerfeld equation.

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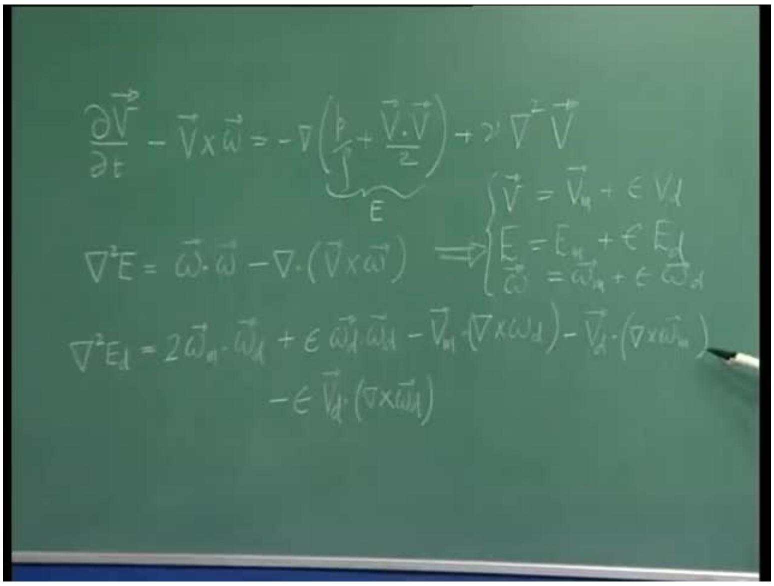

Essentially, this is the parent equation. This has everything in it. This is starting off from the full Navier-Stokes equation. So, if I talk about the velocity field there, like any vector, we should able to show it as a composite of rotational plus irrotational part. And, the vorticity transport equation is obtained by taking the curl of this Navier-Stokes equation.

So the way you can see that, progenitor of Orr-Sommerfeld equation is the rotational part of the Navier-Stokes equation; whereas, this equation has both, because this comes directly from Navier-Stokes equation. However, what we have done, when we take the divergence of this equation to get this, we do get the irrotational part that contributes to it. So, that is what we are saying that, this equation is interesting, because, on this side, maybe what resides is the irrotational part of the field and this side, we can very clearly see the presence of the vorticity vector as well as taking a, this cross product of velocity times the vorticity gives higher order rotational effects.

So, although we are looking at the irrotational part, the origin, the forcing is by the rotational field. So, this is something that we must understand. And, as this E is from the pressure head plus the kinetic energy head, so, one would be tempted to really hug back onto what Morkovin might have suggested.

Morkovin, sort of guessed that, if you write out a, similarly a Poisson equation for the pressure alone, then, you would also get similar such term. I could split it up, I could write it in terms of del square of p d by rho and then, I could put all the terms on this side; and then, one would talk about all this forcing etcetera as some kind of a shear noise term. However, Morkovin did not really go through this kind of steps to explain his point there.

So, we understand that, this Poisson equation that we have written for disturbance energy actually shows the coupling between rotational field of the flow field with the irrotational part. That is what I explained. This side shows you the irrotational part; this side shows you, the origin for the rotational part. And, this was also a very important point you note that, this mechanism of creation of disturbance energy does not depend on dimensionality. Whether we have a two dimensional flow or a three dimensional flow, this mechanism remains valid. This we will find out, when we come to the next module of the course, where we talk about non-normal modes spatio-temporal growth of disturbances. You will realize that, for the last couple of decades, people have been talking about newer mechanisms of instabilities. Unfortunately, those kind of mechanisms are very much biased in favor of three dimensional flow field and they do not work for two dimensional flow field.

In contrast, what we have shown in discussing bypass transition, in this part of the course, is that, we can see it in the flow visualization that, the disturbance field is two dimensional, because we saw the dye filament were lifted up in its own plane. So, it remained two dimensional, the mean flow, the disturbance field at onset was two dimensional; later on, there was a lateral spreading. So, three dimensionality would come in, but the beginning of the thing could be very much true for a two dimensional flow fields. That was also explained in terms of our numerical results, which was based on 2 d formulation.

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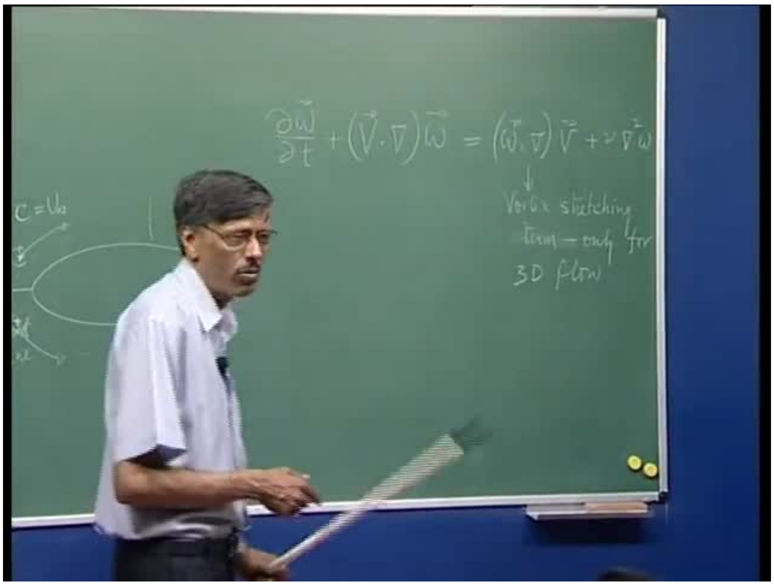

So, we can see, this is rather, a very more all-encompassing scenario, when we write down this disturbance energy equation. We noted that, a significant unsteadiness with large spectral bandwidth disturbance can be created without vortex stretching mechanism. See, this was also another thing, that we have noted, that for a long time, the discussion is fully developed ((over)) inflow has remained, with writing of the vorticity transport equation for general flow field and that vorticity transport equation, looks like this. We have this term plus v dot del, the vorticity and on this side, we have that vortex stretching term omega dot del. So, what happens is, if we are talking about 2 d, of course, this itself, this operator itself is 0 and it does not make sense. So, this vortex stretching term can only be present for three dimensional flow fields. And, this was very elegantly used, this observation, the splitting of vortices transport equation in this form, was very elegantly used by (( )) and his school, where they showed, how energy cascades due to this non-linear term; from small value of wave number to large value wave number, the energy cascades.

So, we will come back to this discussion in the second part of the course. However, if we are looking at two dimensional flow field, we do not have this term at all. So, basically, this over-reliance of talking about the reason behind large spectral bandwidth due to vortex stretching is probably over-emphasized; because, we have shown in this part of the course that, it is indeed, in 2 d flow we get it, through the bypass mechanism. we have shown, how quickly you get the spectrum full; from the calculation also, two d calculations. So, that should really be able to convince us that, we could get small scales in transitional and turbulent flows by our two d mechanism itself.

If we look back a little farther, we have shown that, there could be other mechanisms of creating any length scale by a dispersion. If you recall, we did talk about those periodically passing vortices. When we talk about this periodically passing train of vortices, they excite a very large, integral multiples of fundamental frequencies. For each frequency, we create a chosen k from the Eigen value analysis. And so, when I have infinite number of such circular frequencies, I have corresponding wave numbers. So, we are talking about this over-reliance of vortex stretching term is little bit of a historical legacy at this point in time. It is time for people working in the field to realize that, there are this two other potent mechanisms; one is the dispersion mechanism, that we showed in term for the case of periodically passing vortices; and, the other case is the bypass mechanism, where we obtained the similar picture, bandwidth filling case, for a periodic vortex passing by.

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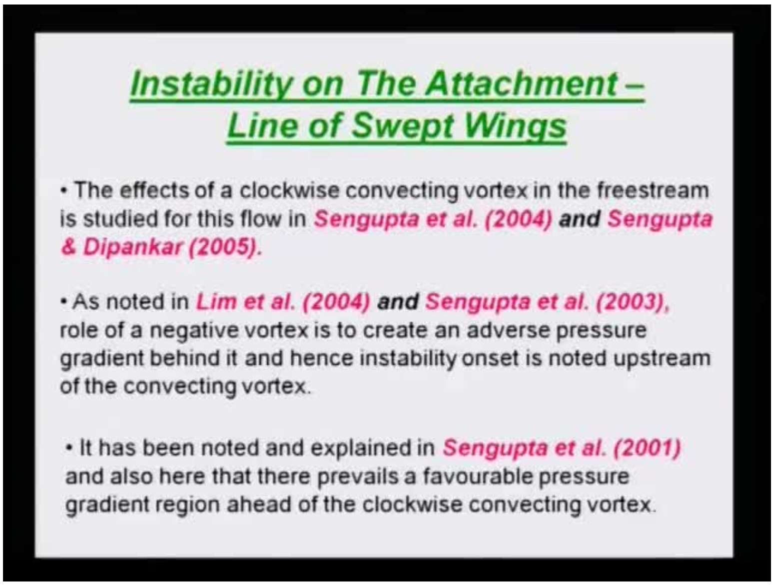

So, we should be able to understand, what we have been talking about. If I look at the attachment line instability, we have seen in the last class, the effect of counter clockwise rotating vortex. What happens when we have a clockwise rotating, clockwise convecting vortex in free stream; these were also studied. You see, I will, almost the same thing that we studied for Blasius boundary layer. Whatever we saw there, it happens here too. And, we also note that, if we look at clockwise rotating vortex, ahead of it, you are going to see favorable pressure gradient. So, we should see that, if we do the calculations.

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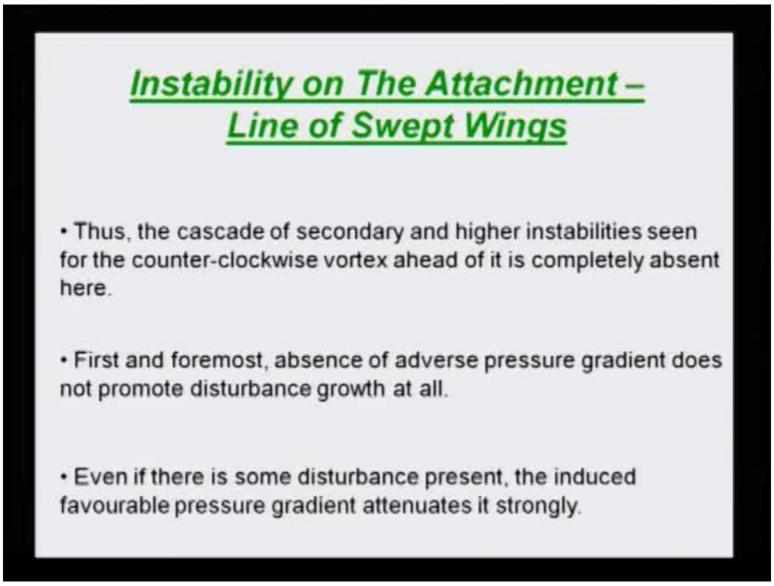

When we look at clockwise rotating vortex case, we are going to see none of this secondary or tertiary instabilities, that we saw for counter clockwise case; because, there everything was triggered by adverse pressure gradient and the cascade was accelerated because of this primary to give rise to secondary and tertiary. But here, what happens, this scenario is completely different. Ahead of the vortex, for a clockwise rotating vortex, we are going to get a favorable pressure gradient. Even though you can create some kind of large disturbance, it is going to be attenuated due to forward base and there is no question of secondary or higher instabilities showing up there.

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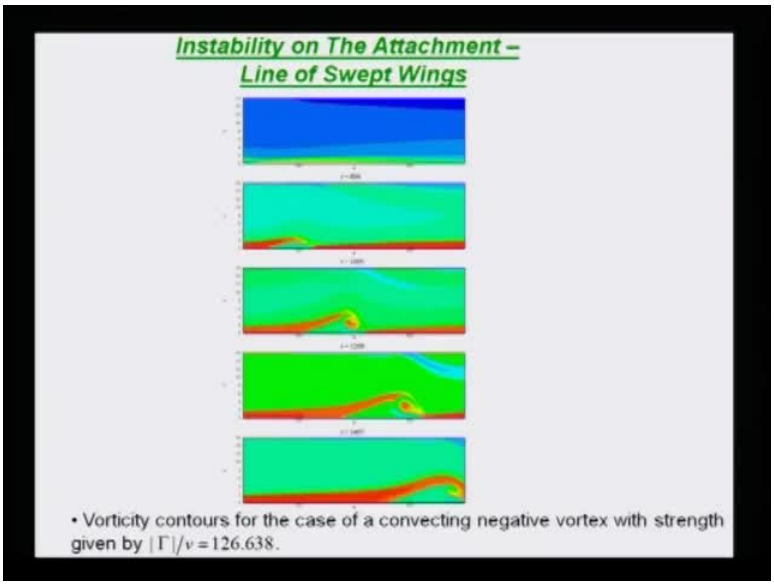

So, this is something that, we would like to see that, the disturbance growth is not promoted and even if there is some, they would be attenuated very strongly ahead of it. And, if there is something going to take place, that should be behind the vortex. So, that is what we see in this picture, rather very clearly. Now, you can see what is happening. We are seeing the vortex. This is the outside the range. Here, the vortex is somewhere here and this lifting up is happening behind. So, you can reason out that, the vortex must be here and then, you can also see, just ahead of the vortex, the boundary layer thins down. So, you can see the vortex must be exactly on top of this.

So, you do not need to even to provide the arrow. And, as it goes along, we have noted from that sketch, recall that, flow past, the rotating and spinning, in translating cylinder, we saw how the stream line converges and diverges. For this case, stream line diverges just behind the vortex. But again, if you go little farther down, again stream lines are narrowed down. So, this adverse pressure gradient is very localized; very localized and that is what causes a single vortex to be created. You are not going to see a large sequence of vortices. So, that is what you are seeing, that with time, there is a simply passby and nothing much happens; flow regards this back to its undisturbed condition. And, that is what a good receptivity calculation should reveal. And, all this things happen…The top frame is from t equal to 804; the bottom frame is around t equal to 1400. So, in that period of time, we see all this things happening.

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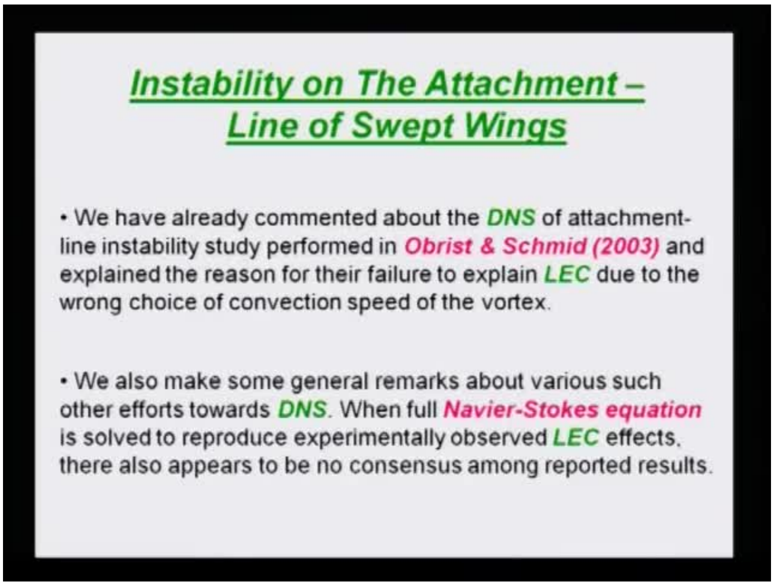

So, I have talked about the DNS of attachment line instability as they thought they were doing; but they were actually doing something else. Now, you understand…Well, the hope was that, if I take this two pairs, then, if there is something that is happening on the attachment line, that will be automatically picked up; because your computational domain included both the top surface as well as the bottom surface. But unfortunately, they did not pick it up; that is due to the wrong choice of the convection speed of the vortex. So, that is one thing we can say.

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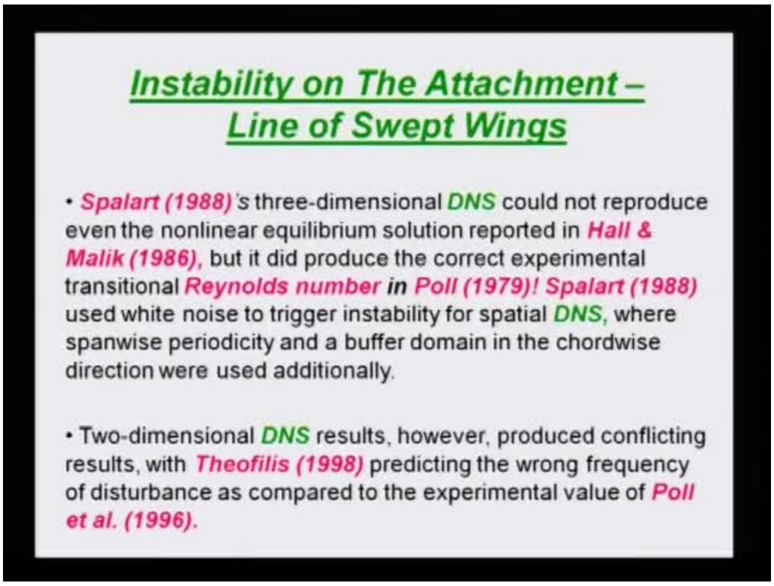

Various other people have performed direct numerical simulation. When full Navier-Stokes equation is solved to reproduce, examine, experimentally observed leading edge contamination effects, there were many researchers providing some different versions of their own results. That is something that we just simply briefly discussed, because it is important that, not only we should have a powerful computational tool, you should also have the physical understanding of what you are computing. For example, Spalart did a 3d DNS. He could not even produce the equilibrium flow itself. And, that was experimentally reported by Hall and Malik in 86. So, what was wrong?

But it did produce the so called transitional Reynolds number from the experiment of Poll. So, this is, really comes with a exclamation mark, because, if you cannot even get the equilibrium flow, but you do get the disturbance flow, it could be a coincidence. Why we should not repeat what Spalart did was, Spalart actually used white noise to trigger instability. And, we have seen, our explanation here shows that, it is a vortex induced instability. So, we have to create a vortical disturbance. If that coherent vortical disturbance is never going to be represented by a white noise, that is one thing. And, then, this kind of artificial constraint dictated by the computational constraint, took a finite span-wise domain and they said, flow is periodic in that direction. Although, I do not think, this is a major issue as such, because our calculations were done for two dimensions. So, you do not really need to worry about the three dimensionality of the flow. In addition, there is a buffer domain that was used in the, in chordwise direction.

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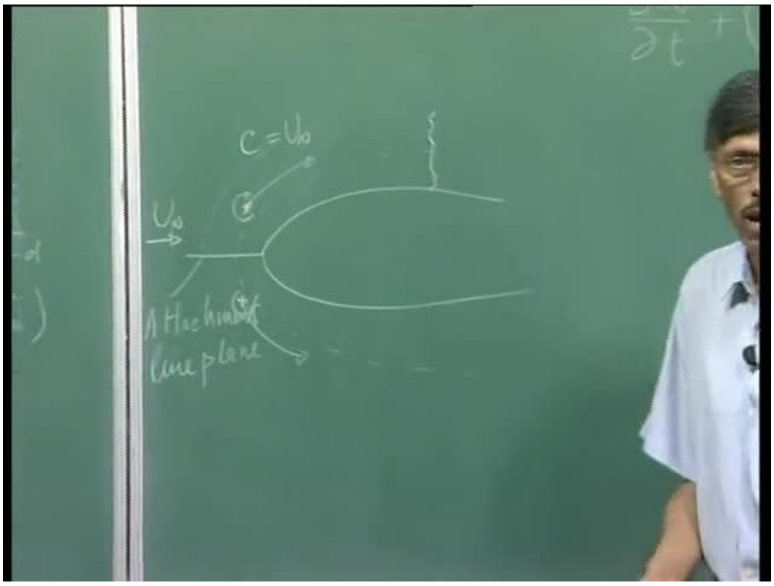

So, basically, if one is solving like this, so, there is buffer layer at the outflow; that was, what was done by Spalart. But unfortunately, I mean it was not too successful.

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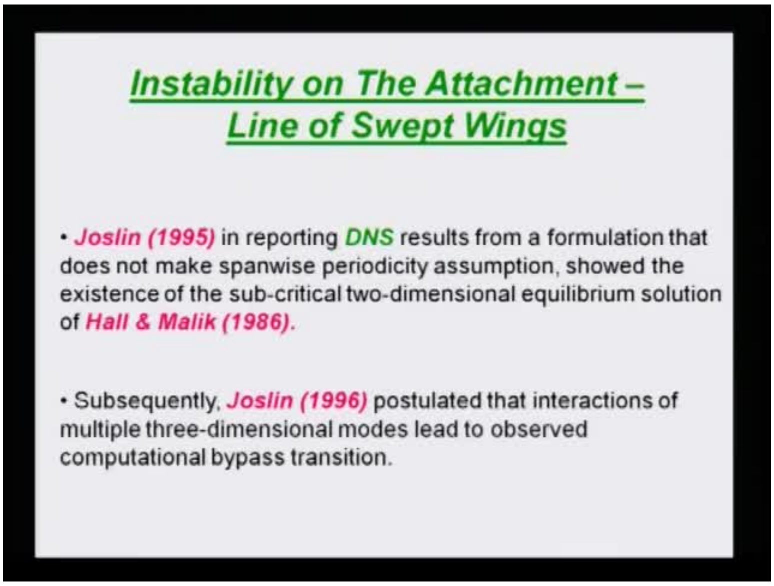

Two dimensional DNS were performed by Theofilis, who predicted wrong frequency of disturbances as compared to the experimental value of Poll et al. While Spalart's physical model was somewhat flawed, my understanding is, Theofilis's failure was totally due to wrong numerals, and not posing the right problem. In contrast, Joslin did some work in NASA-Langley; he did not use any span-wise periodicity assumption and could get to show the existence of sub-critical 2 d equilibrium solution of Malik and Hall. So, that was shown. Subsequently, Joslin went ahead and computed 3 d flows as well and was quite eminently successful in showing this bypass transition in a computational frame work.

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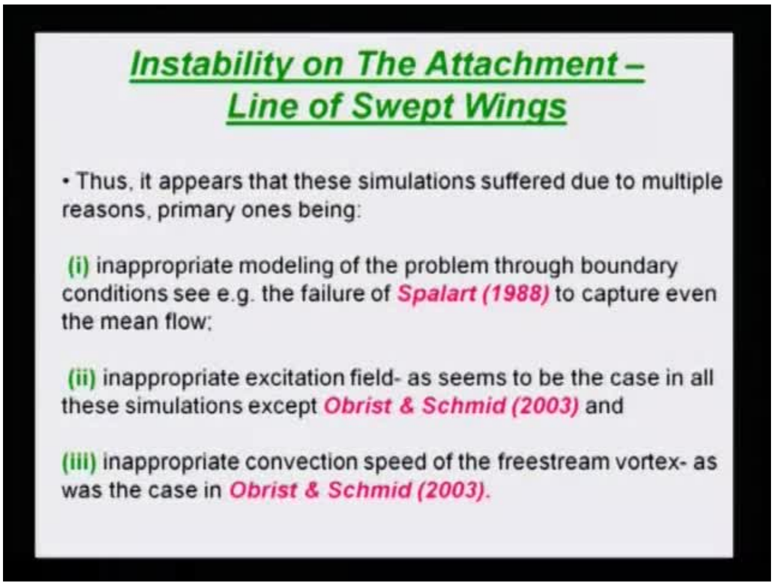

So, thus, it appears that, all these simulations suffered due to various reasons. The primary ones, of course, is the inappropriate modeling of the problem through the boundary conditions. For example, look at the failure of Spalart, who could not even capture the mean flow. Inappropriate excitation field, as it was the case in Obrist and Schmid; they did not take the correct propagation speed. So, the boundary conditions or the excitation field was not appropriate; it was going at the wrong speed. These are the two reasons that, we should be attributing to the failure of these authors.

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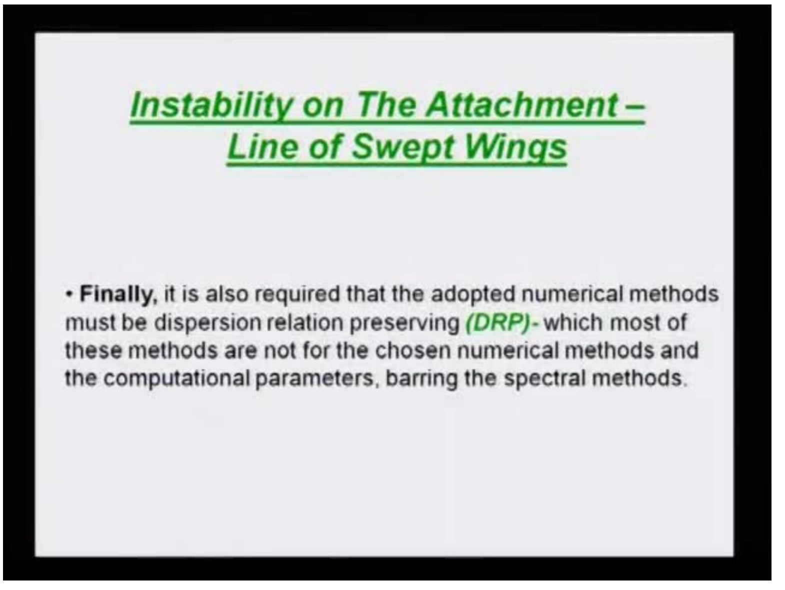

So, finally, I think, we would like to comment up on it. I just made a comment that, Theofilis' results were wrong; that is probably due to this reason that, we have to understand that, when you are computing transitional flows, we have to have a method which really satisfies the dispersion relation; because, that is everything. If I take a numerical method, the numerical method dispersion relation is totally different from physical dispersion relation; that will not make any sense. And, unfortunately, this concept itself, was not existing in late 80s very much.

Computationally, that you have to develop dispersion relation preserving numerical methods came in to existence with those famous paper by Tom and Webb, 93 and 94. So, later on, lots of people became aware and we also developed a series of DRP schemes. We specifically zoomed into the parameter space, where we should reside to attain this DRP scheme. This is something that we must do. Spectral method is probably, the gold standard of computing, because, it just provides you the exact physical dispersion relation provided you do, let us say, Fourier spectral method. Unfortunately, though, with such spectral methods, what you have to be worried about is the following that, you have to take some kind of periodicity. And, the spacing of the grid points either are uniform or given due to some various polynomials, like (( )) polynomial, dictating where the point is going to be.

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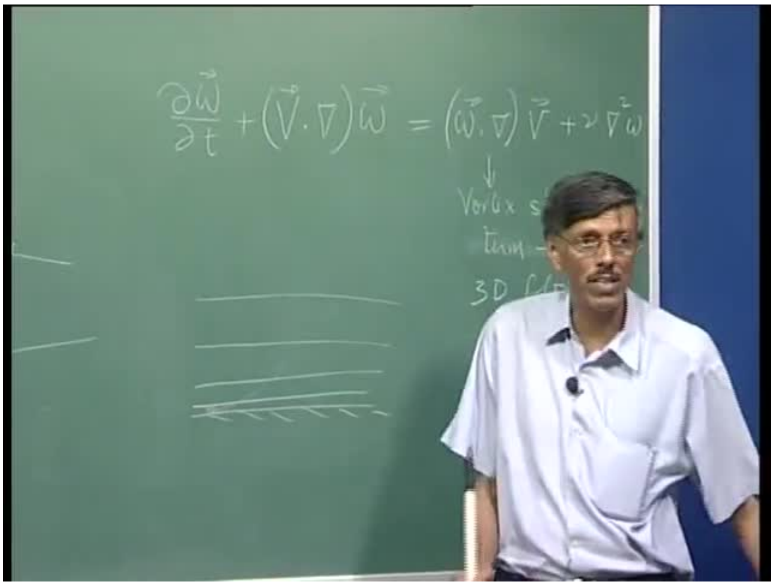

So, you could probably use that kind of information to stretching of points in defining the equilibrium flow. But once the disturbance are created, you are going to create some of this coherent vortices, I could have a, sort of a stretching like this, let us say, in the wall normal direction, we may have this kind of spacing; as we are going away, we are stretching. So, when I am creating a small vortex here, it may be this thing is quite adequate. So, I have this vortex, but then, later on, it is going to move like this; I may not have a complete control in computing such an event, where the grid spacing is very sparse, and this does happen, because, this point distributions are apriory imposed by the rules of the (( )) polynomial; whereas, if you work with non-spectral method, there is a possibility that, you could choose more general grid point distributions and that is something that, we should be aware of.

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Now, this is where we come to an end to this discussion. We are going to now, start off with a new topic. We are going to talk about various other complimentary efforts. Let us take a stock of the situation. Initially, we started off with Rayleigh's criteria of inviscid mechanism. Then, came the viscous mechanism through Orr-Sommerfeld equation and we found TS waves; that was created by viscous instability mechanism for external flows. Now, the classical approach that one follows is basically, you first identify an equilibrium state and then, you study its stability with the help of a linearized governing equation. Now, if I do some very well designed controlled lab experiment, I can see, they actually match quite well and this is especially true, when you see, when this forcing is due to thermal forcing, like a Rayleigh-Bernard problem or some kind of a centrifugal instability, that we could have in a , let us say, (( )) flow type of scenario.

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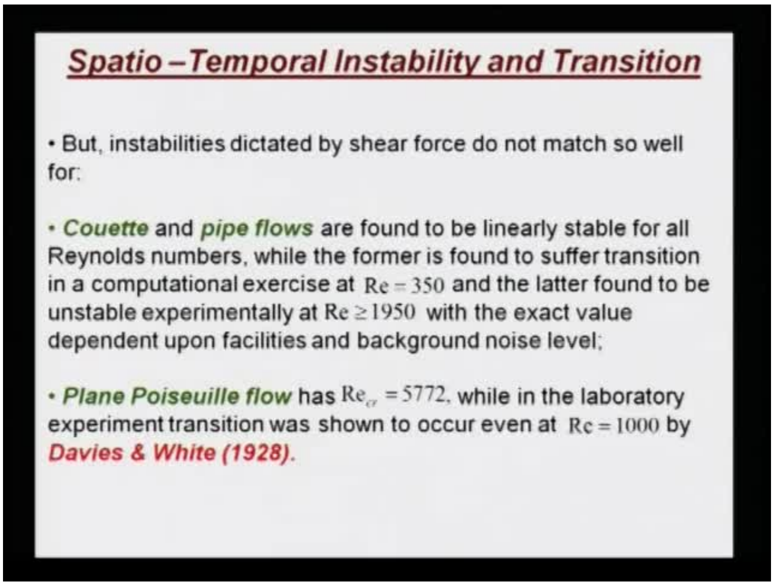

Those results match the lab experiments, theoretical and computational, theoretical and experimental results match very satisfactorily. But if you look at the general scenario, where instability is predominantly dictated by shear force, there we feel, by a great deal. The classic example is the Couette flow or Poiseuille flow,the pipe flow and we find that, both these flows are unconditionally stable; but we know such flows do become unstable. So, there is something wrong in that approach, that linear stability fails to pick up. For example, Couette flow has been shown in computational exercise to suffer transition at a Reynolds number of 350, but linear theory says, it would not be unstable at all; whereas, pipe flow we know that, it is the threshold limit is around 2000 and if you have it, the Reynolds number above of this, we can get this. Exact value depends on the facilities, the back ground disturbances, all kinds of things. If you look at the flow inside a plane channel, like what is called a plane Poiseuille flow, the linear stability theory gives a critical value based on displacement thickness of the order of 5700; whereas, this lab experiment reported by Davies and White in this Royal Society paper in 1928, they clearly established that, you could actually have your transition as low as1000.

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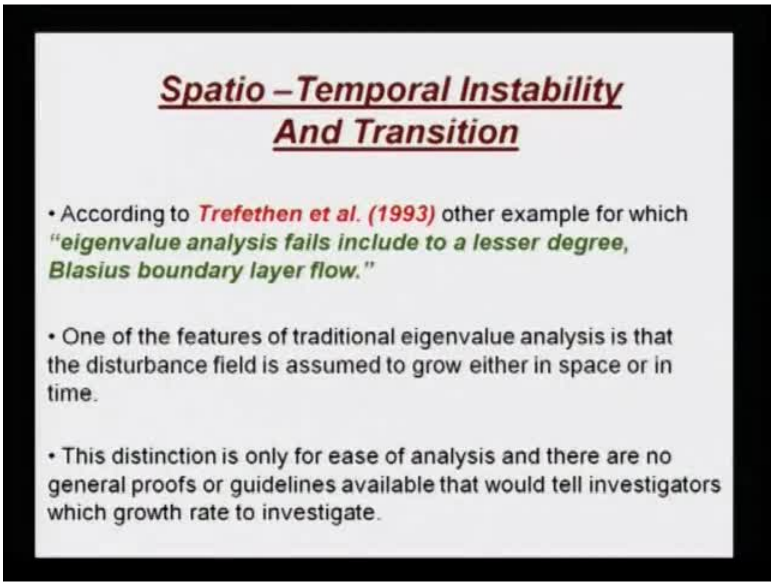

So, if this is something, please note that, we have been talking about the Orr-Sommerfeld equation solution and its ability to explain the instability in external flow was validated by Schuhbauer and Skramstad experiment. But according to Trefethen and his colleague, even this Eigen value analysis is not always successful; I mean, provided we do a very tailor-made theoretical calculation and the corresponding experiments, like in Schuhbauer and Skramstad experiment, we can show one to one correspondence. But for the same wall bounded flow, for many other situations, the linear theory does not work. You recall we discussed about (( )) mode; you recall that, at low frequency, just simply do not have anything. So, there are this thing. So, that is why, the Trefethen and colleagues do write that Eigen value analysis actually fails, even for the Blasius boundary layer flow.

So, we should be humble enough to know, where it works and where it does not. One of the features of traditional Eigen value analysis is that, the disturbance field is assumed to grow either in space or in time. This is entirely a failure of the method, because, physically how do we know upfront, whether the disturbances are growing in space or in time or who knows, it can grow both in space and time; and that is the title of this topic is, that we are trying to look for cases, where we are going to see simultaneous growth in space and time. Those will be called as spatio-temporal instability and they can also lead to transition. We note that, this distinction between spatial and temporal instability is only for the ease of analysis, and there are no really general proofs or guidelines those are available that would tell, which method to really follow.

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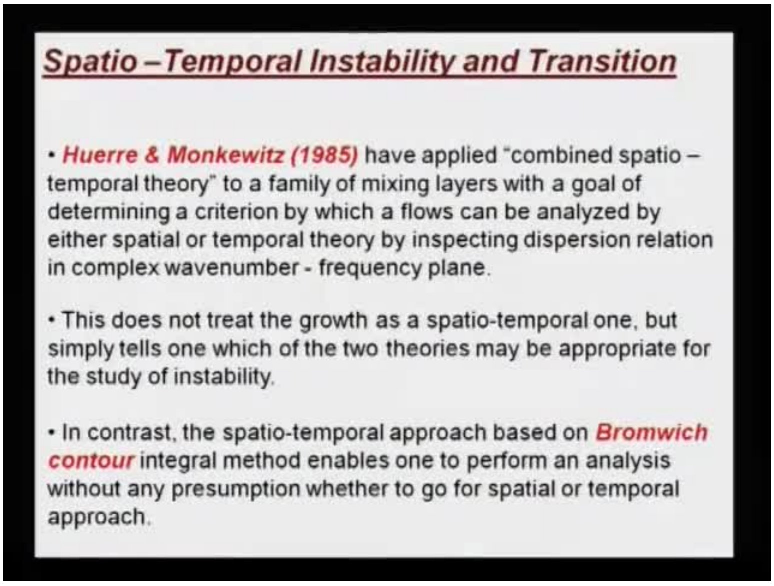

Now, some work has been done trying to identify which method to choose; whether one should take a spatial method or a temporal method. A prime example is this review paper by Huerre and Monkewitz. They actually, claim to have applied combined spatio-temporal theory to flows, a mixing layer. They were interested in finding out a criteria by which a flow can be analyzed, using either the spatial or the temporal theory. And, what they did was, they inspected the dispersion relation in complex wave number frequency plane.

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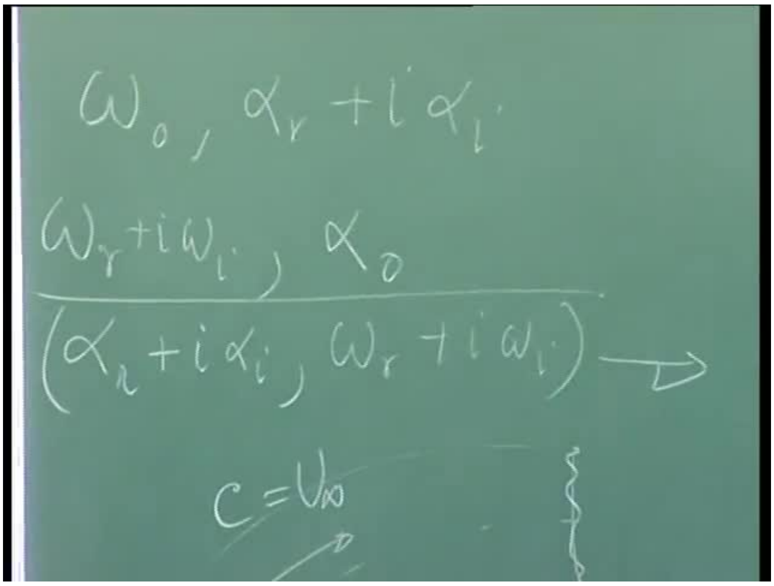

What we do in spatial theory, we take omega naught and complex alpha; that is your spatial theory. And, temporal theory, you will take complex omega and real wave number. So, what this people did, Huerre and Monkewitz, they just simply looked at the complex wave number and complex frequency.

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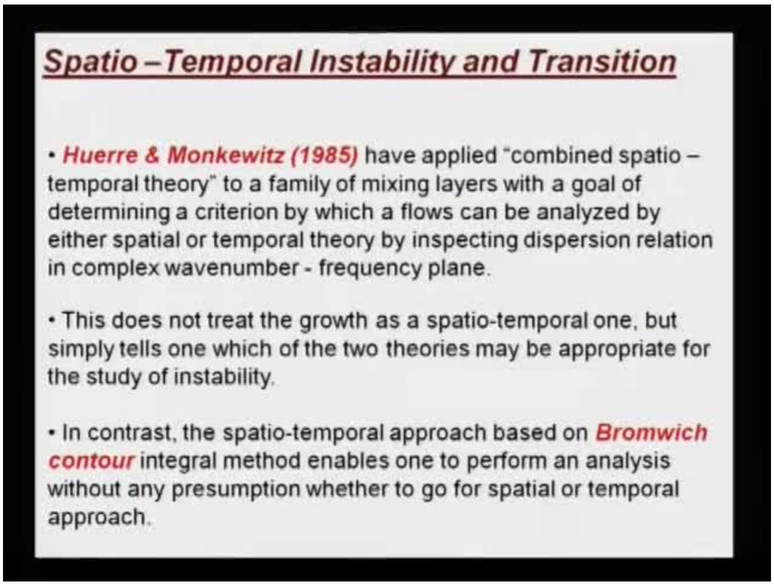

If you look at in this plane, and then, try to dissolve yourself, which method would be more appropriate? So, we need to understand that, this methodology is trying to identify whether to take either spatial or temporal. This method does not talk about doing spatio-temporal analysis per se. So, we must understand that, that is somewhat limited in its application; but if you recall, what we talked about, the Bromwich contour integral method, where we did perform the Bromwich contour integral going from, along the complex alpha and complex omega plane, parallel to the real axis, we did see that, we can work out the spatio-temporal behavior. What we discussed so far, for that part of the discussion, we found out that, if we are looking at unstable cases, the results that we get by this complete spatio-temporal approach and the results that we get from spatial approach coincide; that was the result that we obtained in 1994, and we kind of provided a certificate that, for that class of flow spatial theory is adequate; but in this part of the course, we are going to revisit it once again and see what happens.

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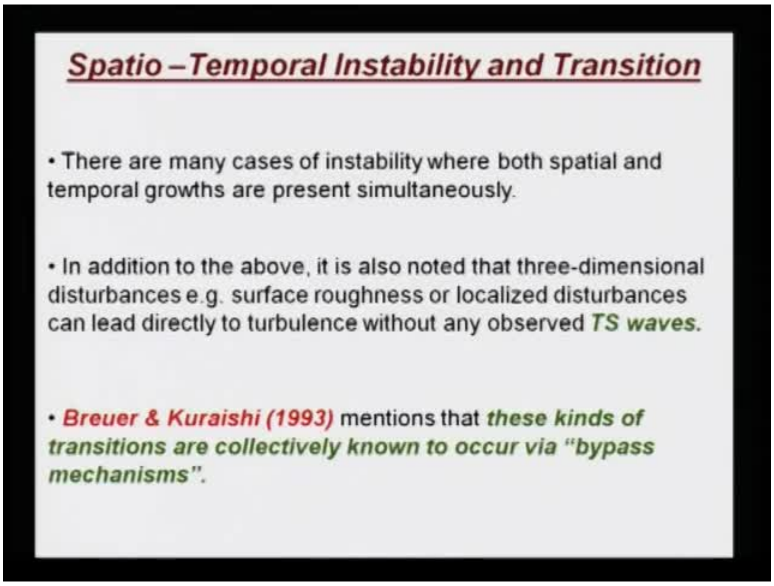

So, basically, in fluid flow, we will come across instances, where you would see the disturbances are growing both in space and time. It is quite natural that, we will have to abandon, those strictly spatial or strictly temporal analysis. And, in addition to that, we have also noticed that, if we create some kind of three dimensional disturbances, like localized roughness, a dimple kind of a thing, which need not necessary move like in (( )) experiment; you can just simply have a physically a roughness, a rivet on a aircraft wing. That itself, can lead to it, a flow in the wake of that roughness, a turbulent wedge will be created. You do not see any Tollmien-Schlichting wave in there; Tollmien-Schlichting wave, seeing is what? if you recall the Schuhbauer and Skramstad experiment, they did excite the system with a single monochromatic frequency, single frequency. So, a monochromatic disturbance field.

But if I put a sort of rivet on a aircraft wing, then, what is the frequency? I am having a free stream turbulence coming, that has embedded in it, a kind of a frequency spectrum, not a single frequency. So, as a result, for each frequency I am going to get a wave number, if my dispersion relation is correct; however, the total response field will be sum of all these individual components. And, that is where, in a really a practical case, in a natural setting, expecting to observe TS wave, is a bit of a over-expectations. We are not going to see a scenario, where in natural case, you will get a just single frequency excitation; that must be very specially designed. For example, if I talk about the flow inside turbo machine, you have the stator, rotor, etcetera. So, the stator will sort of eject vortices in its wake and that will be impinging on the rotor; and we saw the Kendall's experiment, where he rotated those two cylinders at the end of a pivot and then, rotated it and then, those are ejecting vortices at a fixed frequency; and there, you could see something, a signature of a TS wave-like scenario. So, you saw there, a TS wave packets; but in general, you do not expect that.

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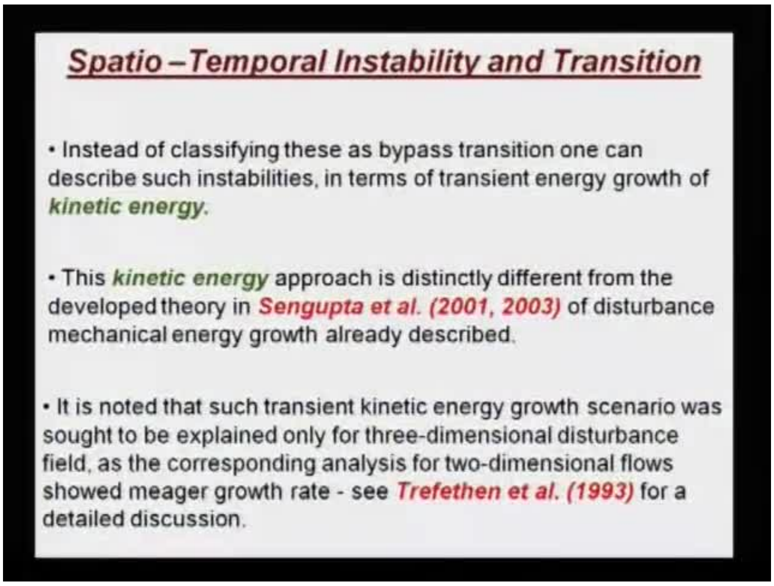

Now, there is a lot of activity that goes on in some of the Scandinavian countries and a large number of them, they have actually focused upon, what was called by Markovin as bypass mechanism. So, there are a large group of people who try to do. So, Breuer and

Kuraishi, in their 1993 paper, say that, whenever you see such things, where you do not see a TS wave, let us call them as a bypass mechanisms. So, essentially, even this spatio-temporal event, people have tried to talk about as a bypass transition; what people have tried to basically follow in those approaches was, basically, they looked at the kinetic energy of the flow field and see, how this kinetic energy changes with time.

And, if you see a large transient initial growth, that is what prompted them to call those kind of phenomena as bypass transition. So, this is not a sustained phenomenon; this is just a transient energy growth of kinetic energy. And, I must point out that, this kinetic energy is completely different than what we are looking here; this is the total mechanical energy, whereas, those studies involves studying this kinetic energy. We will see what each of it leads to. It is noted that, if you have such transient kinetic energy growth, it somehow works for three dimensional flow field; because, if you do the corresponding analysis for two dimensional flow, the growth rates, even at those early times, are very small. These are orders of magnitude smaller. This has been very clearly pointed out by Trefethen et al. In that1993 paper in Science, they just do talk about it that, this so called transient kinetic energy growth, is probably a problem mechanism for three dimensional flow field, but not necessarily two dimensional one.

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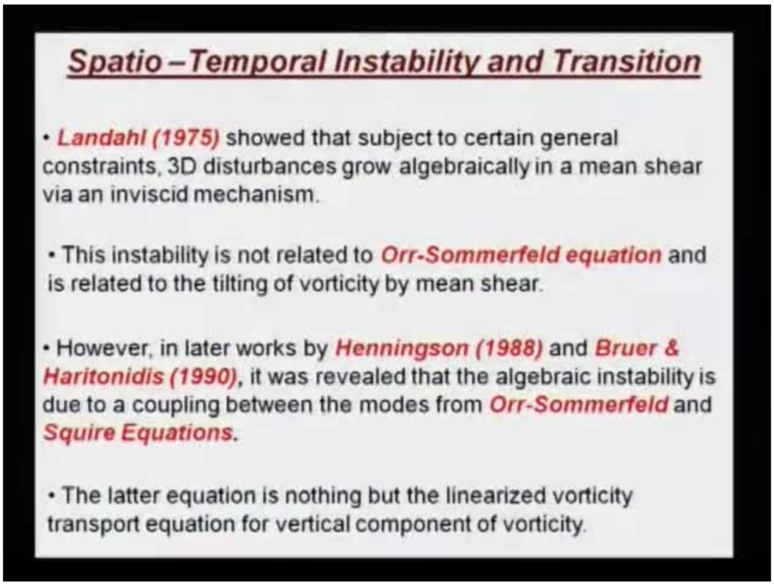

In the context, actually, this work of this so called bypass transition, following original idea of Markovin, was by Landahl. Landahl did lots of studies, starting from this 1975 paper; what he did show that, you could see a 3 d disturbance, that grows algebraically. See, now, if we look at the growth via Orr-Sommerfeld equation, the growth there was exponential, e to the power minus alpha i x or e to the power omega I t. So, those are exponential growth; but Landahl did show that, if you, sort of subject the equilibrium flow to some constraint, then, you could get a three dimensional growth, algebraically.

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Now, you understand that, there is a distinction between algebraic and exponential growth; because, if I, let us say, plot time along this, then, what happens? See, initially, let us say, the background disturbances are rather small and this is growing exponentially. We will not see like this; it will slowly pick up; once it picks up some threshold amplitude, then, it grows like this; whereas, if you have a algebraic growth, then, what happens? Then, it could just simply go like this. Say, suppose, it is like a linear growth. I am not even talking about quadratic or anything; that could be significantly faster than exponential growth.

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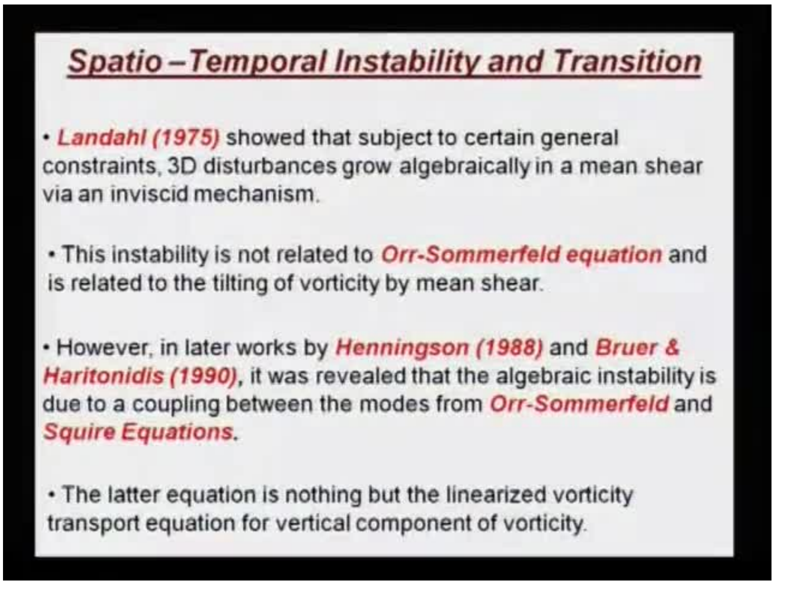

So, Landahl was basically probing with the idea, that you could have in the initial phase you could have a algebraic growth and that could take your disturbance to a significantly higher level. For example, here, the distinction between the two could be thousand-fold or ten thousand folds. Whereas, at a later time, if this (( )) and this goes, this oversteps. So, this is something that was proposed by Landahl and what he said, this is not related to Orr-Sommerfeld equation; this is related to tilting of the vorticity vector by the mean flow itself. So, this is some sort of a lifting mechanism of the vorticity field. Now, does it not ring a bell? What we did with vortex (( )) stability; we are trying to lift the shear layer. So, we did see, some such thing already in action. In later work, Henningson, probably from the same school, as well as Bruer and Haritonidis, they reveal that, the algebraic instability is possible, if you conceive of a coupling of modes between the solutions obtained via, solved solution of Orr-Sommerfeld equation and a Squire equation.

Now, this Squire equation is nothing, but the linearized vorticity transport equation for the vertical component of the vorticity. You know, Orr-Sommerfeld equation itself is a, is an equation for the vorticity, linearized vorticity transport equation. But if you look at the vertical velocity component, then, vertical component of vorticity, then, the corresponding equation is called the Squire equation. So, you may have a Orr-Sommerfeld mode in the x y plane and then, you can look at the vorticity in the y direction; because, if I have a flow in the x y plane, the vorticity is already in the z plane.

So, that is your Orr-Sommerfeld mode. So, Orr-Sommerfeld mode will be in the z plane, but we are talking about omega y. So, the wall is, a flow is in the x direction, wall normal direction I will have. So, that is your governing equation given by Squire equation. We have written down the linearized vorticity transport equation; we can simply write the omega y component.

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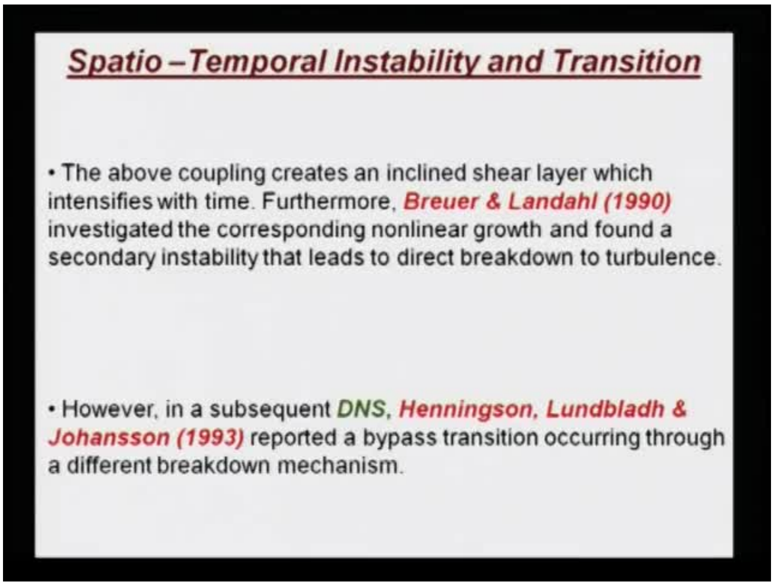

This coupling of Orr-Sommerfeld and Squire mode creates an inclined shear layer which intensifies with time. Later on, Breuer and Landahl collaborated and investigated the corresponding non-linear growth; all those Orr-Sommerfeld mode as well as Squire mode, we are talking about the linearized equation. So, they extended the study to include non-linear growth and that indicated that, there is a possibility that, you could get a secondary instability, which can directly take the flow to turbulence state.

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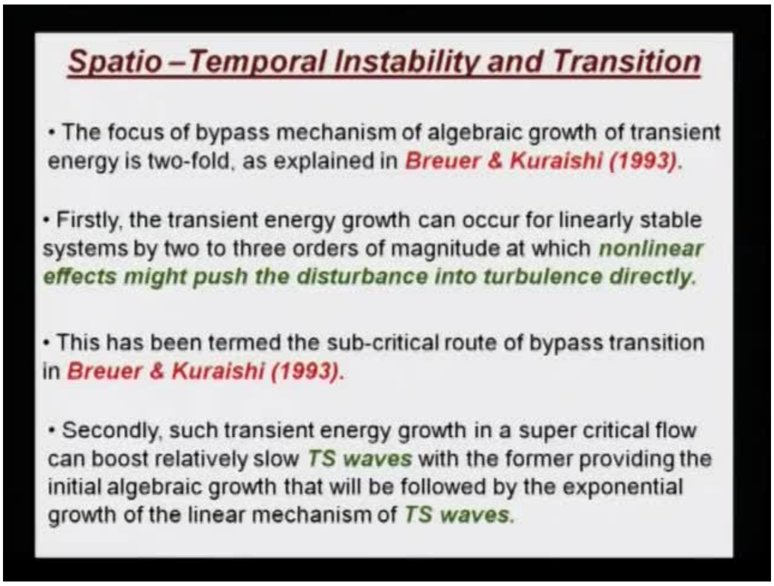

In a subsequent direct numerical simulation by Henningson, Lundbladh and Johansson, they are again reported some bypass transition through a completely different breakdown mechanism. What do we conclude from this, either there are too many computing mechanisms or multiplicity of theories may show limitation on, of each of them. So, we need to really focus our attention. Why people are interested in algebraic growth, that is what I was trying to show to you. This was explained in Breuer and Kuraishi’s paper. Firstly, the transient energy growth can occur for linearly stable systems by two to three orders of magnitude; after which, directly non-linear effect can take over.

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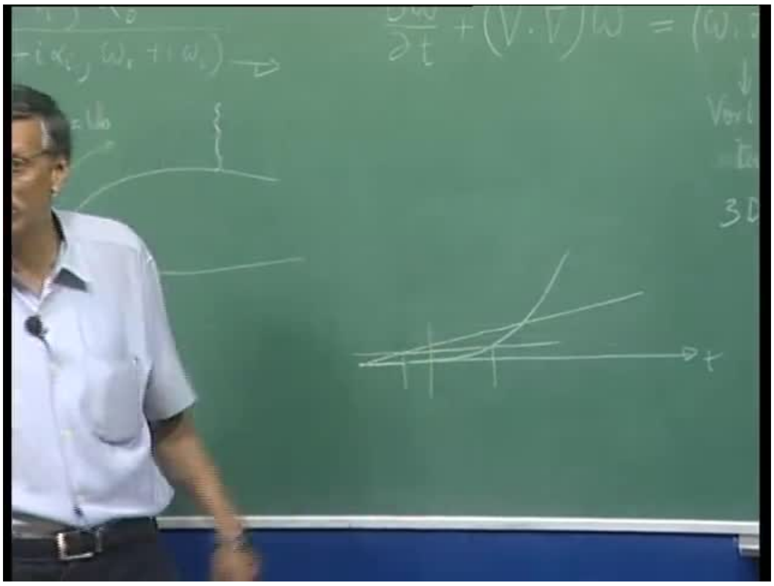

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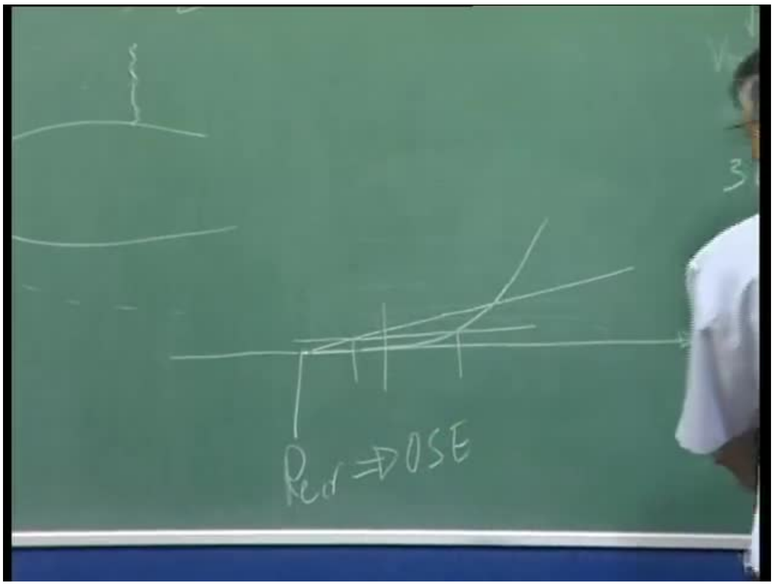

Suppose my non-linear threshold is here; so, then, you can see, for the initial algebraic growth, I have to go up to this time, whereas, if I had depended upon exponential growth, I had to depend upto that long a time. So, whether I wait for this time or I have to wait up to that time, depends on whether the algebraic growth is present or not. So, they are saying that, if you have algebraic growth, then, the non-linear threshold can be reached earlier. In the initial stages, by such a, such a mechanism and when that does happen, you talk about it as a sub-critical route, quite obviously; because, you know we have talked about the criticality with respect to the creation of unstable waves per se, but here, this is somewhat of a different case, that, I could have a flow, where this is my R e critical based on O S C. So, this corresponds to Orr-Sommerfeld equation and this, people are talking about, let me clean up this diagram and explain to you, what we are talking about.

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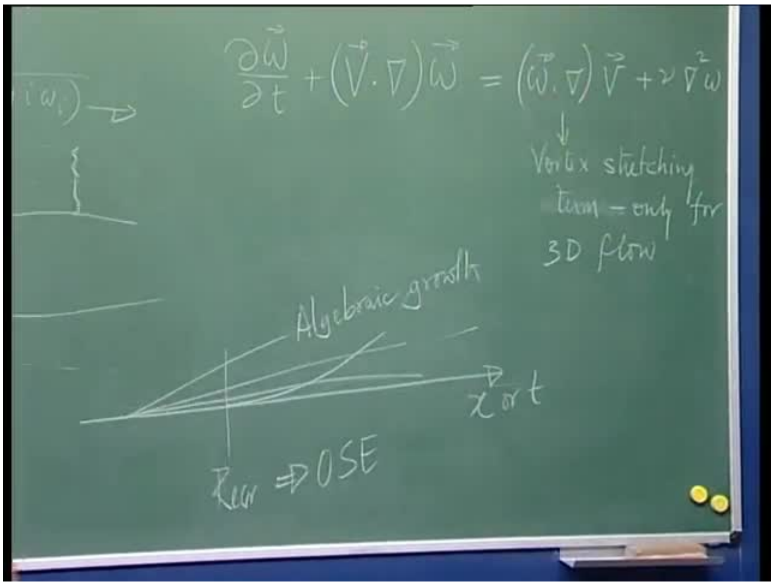

So, if I am talking about, say x or t, along this axis. So, if this is my R e critical, corresponding to Orr-Sommerfeld equation, then, if my algebraic growth starts taking off from early on, then, that could be a sub-critical thing. So, this is your algebraic growth. I could have different types of algebraic growth. I could have this kind of growth, I could have milder growth and so on and so forth. What happens with the O S mode? O S mode picks up from here, up to here it does not exists. So, it picks up from here, and then, it goes like this.

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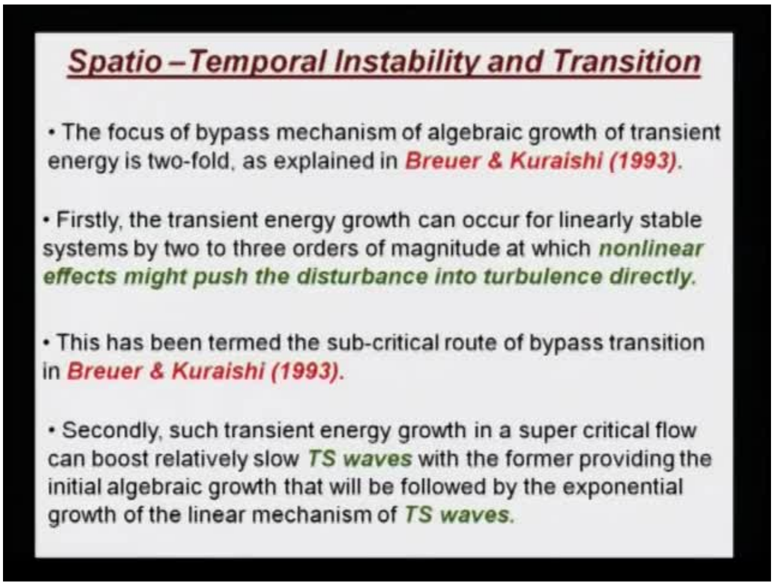

So, what happens is, that is what Breuer and Kuraishi is saying that, either it could be a sub-critical route or you could have a transient energy growth in a super-critical flow.

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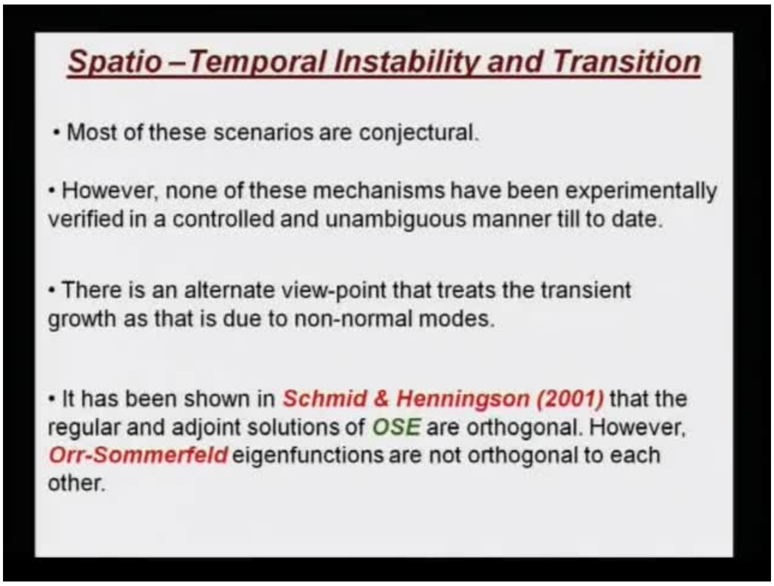

So, you come upto with the linear growth, then, it switches on to the O S mode and then, it suffers an exponential growth. So, that is the second route that, suggesting that, it could happen; that you could have a initially a algebraic growth, followed by exponential growth of, during the super critical stage. See, these are all, are very conjecture; they have not been verified experimentally by controlled experiment in a unambiguous manner. So, however, there is a alternative view point, that treats the transient growth as due to non-normal modes. Now, what kind of an animal is it, non-normal modes? This has been perceived by Schmid and Henningson, who wrote that, if you look at the solution of Orr-Sommerfeld equation or take the adjoint of the Orr-Sommerfeld equation, and the modes that you get, they are themselves orthogonal to each other; however, the Orr-Sommerfeld Eigen functions themselves were not orthogonal to each other; it is easily shown.

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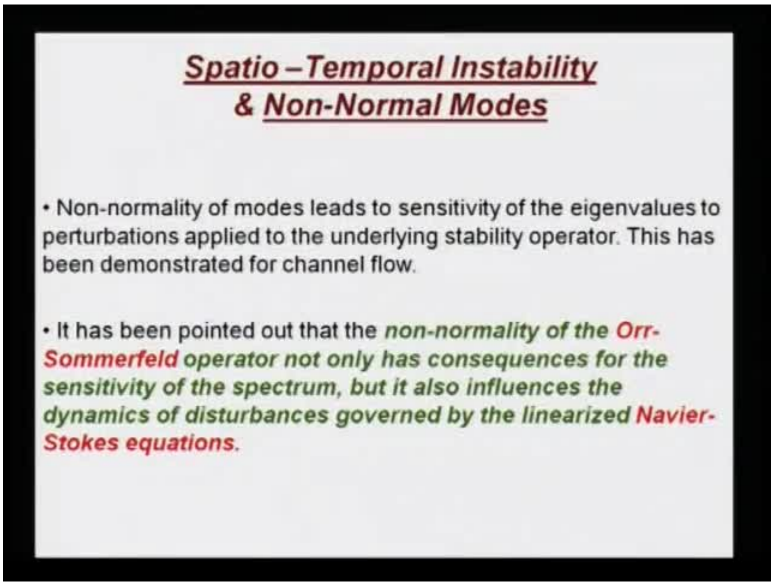

If that is indeed the case, non-normality of modes of a fluid dynamical system are very hypersensitive to background disturbances. So, theoretically, you are getting some kind of Eigen values etcetera; but they are so susceptible to background disturbances, that, you can get some kind of a spectacular growth in the presence of disturbances itself. And, this has been demonstrated for channel flow that, you have for channel flow, unlike that, the case of linear theory, where we saw that critical Reynolds number was 5772, but we know experimentally, it can trigger at much lower case. There it could be that, there is, the flow is not so called uniform flow; it is coming with disturbance. That, those are hypersensitive to disturbances. That kind of flow has been reported in Schmid and Henningson's monograph on this flow. I will stop here. We will pick up our discussion from this point onwards in the next class and we will see, what is the state of art in this particular area.