Instability and Transition of Fluid Flows

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Module No. # 01

Lecture No. # 25

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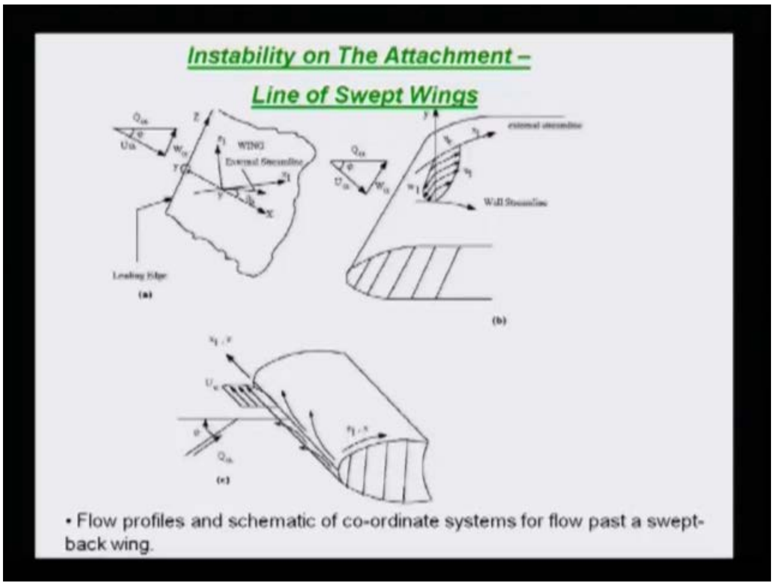

We were talking about instability that is noted on the attachment-line of a swept wing and this is the schematic. If you look at a top view it would look like this. The oncoming free-stream Q infinity subtends an angle alpha, drawn with the normal along which we have the U infinity. The projection of Q infinity along the normal to the leading edge is U infinity and there is a span-wise component W infinity going there. So, this is a plan view. In this plan view also, we show, how the streamline which is outside the shear layer looks like. So, external streamline refers to a streamline which is outside the shear layer. So, what you notice that, the component, the U component that we call as the stream-wise component, and the W that we will call as the cross-flow component have different relationship at different height.

So, what happens is, as a consequence, you have streamlines which may be aligned in this direction at the wall and as you go outside, you can see the cross-flow component keeps changing and that actually skews or twists the streamline. And, the external streamline is the one, where the W component has come to 0 and U has reached its free-stream value. So, from that point onward, your streamline will be the same, but inside the shear layer, it is all twisted. That is what to we are trying to show you here, by showing a asymmetric projection of the velocity profile over the wing. Now, what happens along the leading edge, that is what we call as the attachment-line. So, this is your attachment-line and you draw a plane normal to that, along the attachment-line, is called the attachment-line plane. And in that attachment-line plane, what happens, you have a flow in the span-wise direction.

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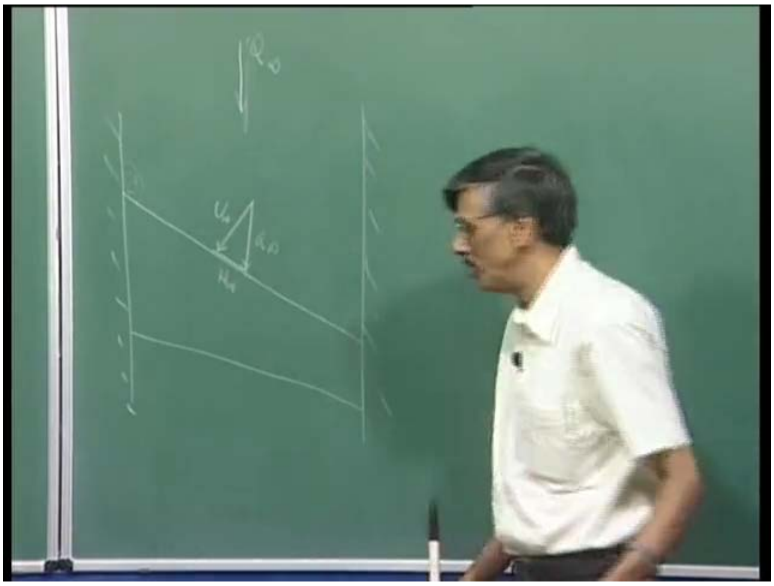

So, since that is the flow direction, so, we call that itself as U and it has no other component. And, this problem that I was telling you about also, when performed experimentally, what you actually end up doing is, you have, let us say a wind-tunnel, wall like this and you fit in a kind of a shuttering, which will connect from wall to wall. The idea is, not to have any of these tip vortices and which will affect the flow in the near vicinity, but you do it in a wall-to-wall fashion. And then, of course, you will have these vertical structures forming there; that sort of migrates along the span-wise direction.

You would also have similar such vertical structure created here, corner vortices here, but because the oncoming flow is the... So, this is what we are drawing there as Q infinity. And that, we split it into U infinity and W infinity, that is what we are talking about. So, essentially, Q infinity is like this. So, that is composed of these two components. So, this component is what we are calling as U infinity and this, we call it as W infinity and this is your k infinity. So, what happens, this is far away from the wind, but in the attachment-line plane, what happens is, you do not have this normal component; the flow is in the span-wise direction.

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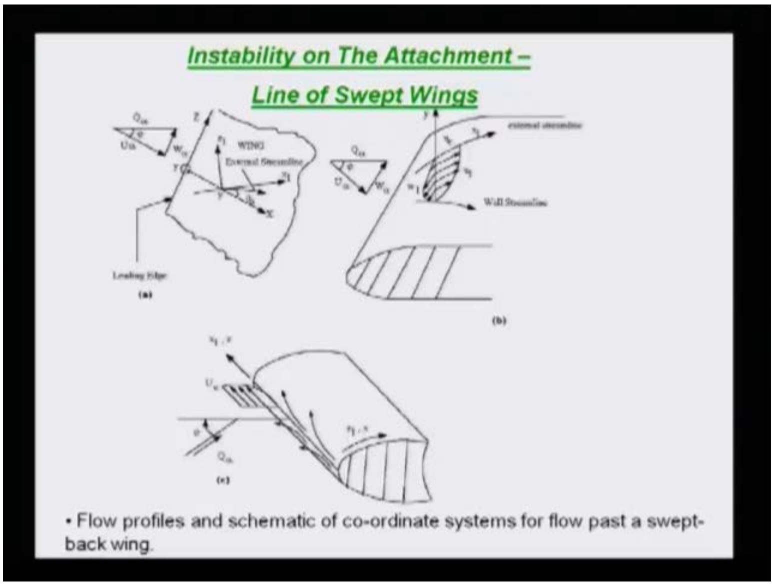

So, that is what we are calling, that itself as the stream-wise component. So, the flow that is in the span-wise direction is the flow direction. So, that is why, there is a kind of an exchange of access system; that is why, you would notice that, we have identified that axis as x 1 and z. So, why x 1, x 1 is the external streamline fixed coordinate system. So, viewed from external streamline, that would be the x 1 direction and from the wing fix direction, that would be your z direction; because, that is the span direction. So, basically, it is a kind of a description that we are talking about, that viewed from a geometric perspective, that is the span-wise direction z and viewed from the flow direction, that is actually stream-wise direction.

So, that is why, you notice that, we have called it simultaneously as x 1 and z. Now, we have to be very clear about this and you can understand, then, from the flow perspective, this is the, because the flow is in this direction, so, this will be your span-wise direction. So, this we are writing as z 1, but from the geometric perspective, it is the x direction.

So, that is what I thought, I will spend a little extra time and explain to you that, the subtle aspect of it. Now that we understand this, it is pretty much clear what we are talking about. So, we have a plane, attachment-line plane. That is where the flow is from here to here, in this direction. And, over that, the flow wraps up on top and below that attachment-line, flow wraps on the bottom. So, that is your visual picture. Now, let us see, what happens to this.

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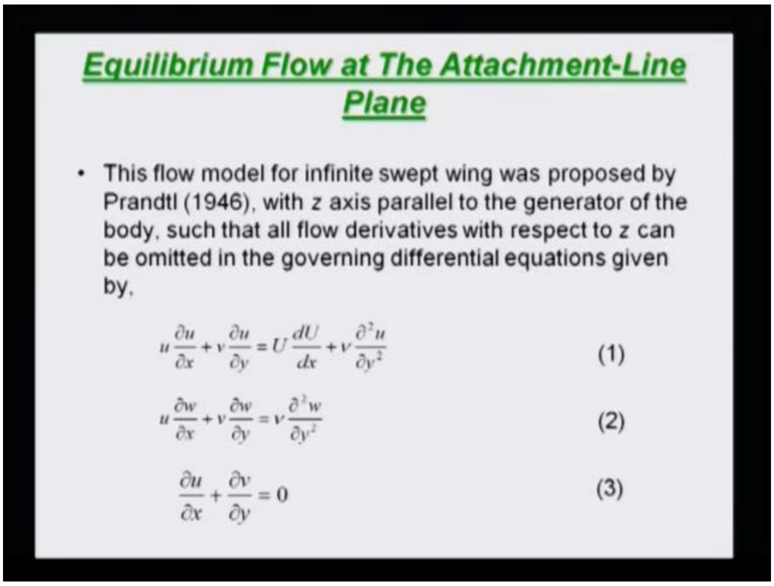

I told you that, it was once again Prandtl, who always does things before everybody thinks. He suggested that, we can look at the governing Navier-Stokes equation and then, consider the flow to be occurring over a swept wing, but whose span is infinite. So, that means what? You have still, a flow in the geometric perspective in the z direction, but the variation with z is 0; that is your meaning of infinite swept wing. So, you will have a z component of the velocity, but the derivative with respect to z will all go to 0. So, that is your infinite swept wing assumption. So, Prandtl said, let us consider the flow, idealize like this, and then see, what happens to our equilibrium flow itself. So, if you look at it this way… So, z axis is parallel to the generator of the body, because that is what it is, like a body of revolution kind of a thing, such that, all flow derivatives with respect to z can be omitted in the governing differential equation. So, this is your x momentum equation.

So, this is, we are writing in wing fixed coordinate system. So, we are writing it in the wing fixed coordinate system. u is the velocity component in the x direction; v is the velocity in the y direction and then, what is this term; you can recognize it; this is the pressure gradient term in the x direction; and this is your viscous diffusion term. What have we done here, of course, we have taken a boundary layer approximation so that, the stream-wise gradient is, diffusion is negligible compared to wall normal diffusion. And what happens to the z direction? Of course, because of infinite swept assumption, that term is 0. If del u del z is 0, all its derivatives also be 0.

So, second derivative automatically falls off. So, that is your x momentum equation. Look at, now, your, you do not have to do anything about the y momentum equation, from the boundary layer equation, we know that. And, look at, now, the span-wise component, the z component. So, you have u del W del x plus v del W del y equal to nu times the wall normal diffusion of W. So, this is ok. So, there is no applied pressure gradient in that direction. Whatever it is there, it is in the stream-wise direction. And, this is your continuity and you can very easily see that, del W del z is 0 because of infinite swept wing assumption. So, this is the equation, and now, let us goes ahead, and try to simplify it.

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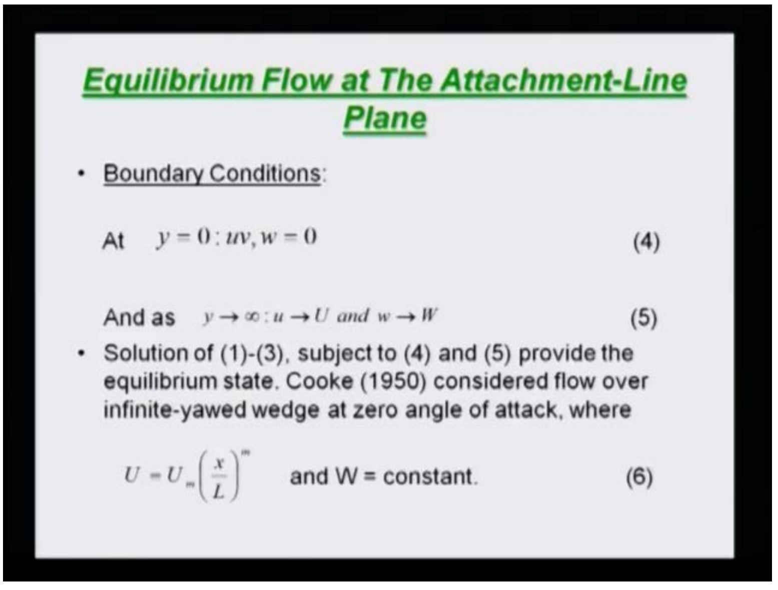

Now, before you do that, note down the boundary condition at the wall, all the three velocity components are 0, and when you go far away from the wall, outside the shear layer, u would approach capital U and w will approach capital W. And, now, if I now try to solve those equations, those two momentum and the continuity equations, subject to these two boundary conditions, to get the equilibrium state, Cooke actually thought of extending what Falkner and Skan had already done for flow with pressure gradient. We have talked about Falkner-Skan profile. So, Cooke basically, extended that same idea, that here, we are talking about an infinite-yawed wedge.

So, if the flow is like this, so, you have a wedge here, but that is infinite in this direction; so that, if you are looking at 0 angle of attack here. So, basically, the wing itself, that attachment-line plane itself, I am considering there is a wedge there; and the flow is coming right at the leading edge of the wedge. And then, we can prescribe the variation of U like what you do in Falkner-Skan profile, it goes as x to the power m. And, what (( )), the quantity, because, we are talking about infinite sweep. So, we have a W component, but it does not vary with z. So, that is why, you have the other vary there, W was equal to constant. So, from now on, what we would be talking about, that we will be talking about extension of Falkner-Skan weight solution by this idea of Cooke, as Falkner-Skan-Cooke profile.

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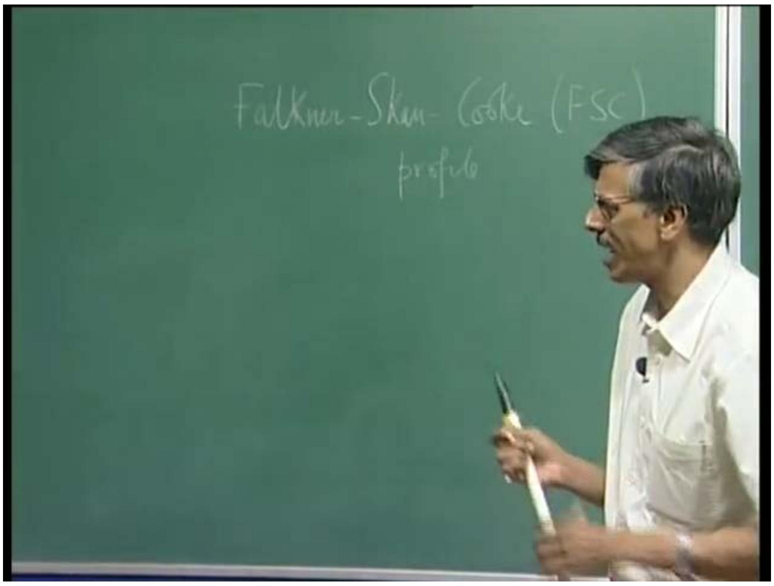

So, we will be talking about Falkner-Skan-Cooke profile. So, we are basically looking for the description of this Falkner-Skan-Cooke profile.

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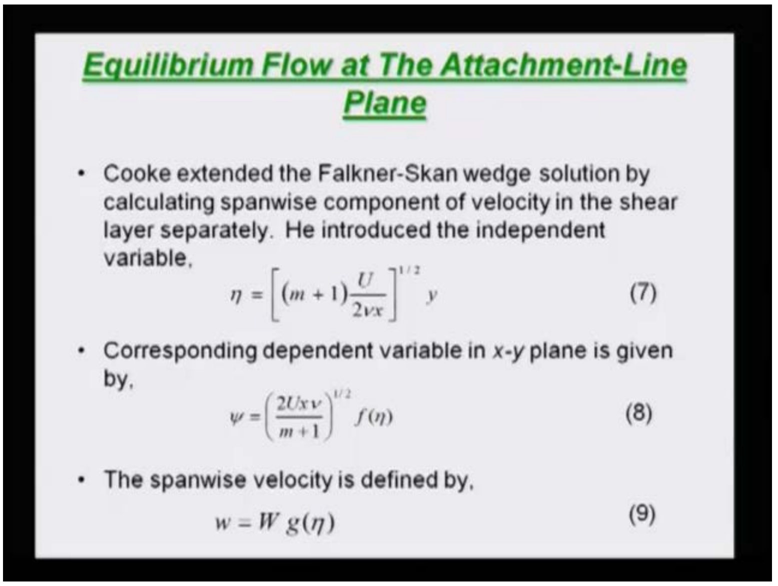

So, what we do is, we go through that same exercise; we define, stretch the wall normal direction by introducing eta; m is some kind of a pressure gradient parameter, like what we have done, remember; and would be something like x by U e d U e d x; that is your pressure gradient parameter. So, that is what it comes here, and you scale it by this kind of r e by square root of x kind of a thing and it is the y coordinate that is stretched; so, you are stretching that way. You also define a stream function. What is the idea here, of course, when you introduce a stream function, then, in the x y plane, you do not need to worry about mass conservation; it is automatically satisfied; existence of psi will give you that. And, we are talking about a span-wise component of the velocity, because, outside the shear layer, W is a constant, but inside what happens, it will vary; that is why, we saw the streamline skews. So, we introduce another non-dimensional dependent variable g of eta and this psi, we define it with respect to a non-dimensional stream function f of eta. So, basically, we are trying to depict the flow in terms of two quantities, a stream function, which is given in terms of its non-dimensional value f and a non-dimensional span-wise velocity component, which is given in terms of g eta.

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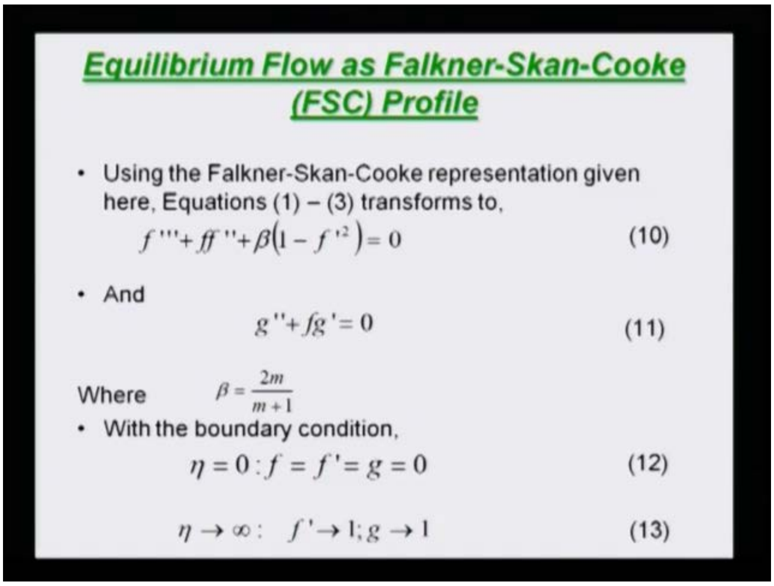

So, this is how we could go about describing the flow in terms of two new dependant variable. We will substitute those things, those two definitions of psi, f and g and plug it in the governing equation that we have just now seen. What you would find that, the momentum equation will yield this one, the x momentum equation will yield this first equation and this is what something you have already seen. This is exactly like what we have obtained for Falkner-Skan profile also. And, beta is the combination of this term, 2 m by m plus 1; that appears there. It is a non-linear equation, but in the leading term, it is a linear and the z momentum equation gives you this equation.

So, what you notice, of course, directly from here that, f can be solved independent of g. So, first, you will be solving for f, by solving 10. Once you have the f, we can plug it in in 11 to get g. So, this should possibly help you in defining the flow field. So, what happens is, you need to solve these two equations. It is our ODEs. So, it is a third order ODE. So, you require three conditions for f and second order ODE in g will require two conditions for g. This five conditions are obtained through this. At the wall, you have a psi equal to 0, psi prime equal to 0 with respect to x and y; they are 0s. So, that translates into f and f prime equal to 0. And similarly, the W component is 0. That gives you directly, g equal to 0. And then, when you go to the edge of shear layer, what happens is, the stream-wise component reaches the value of 1; the span-wise component reaches a constant value. That is it. So, both of them, f prime and g, approaches 1.

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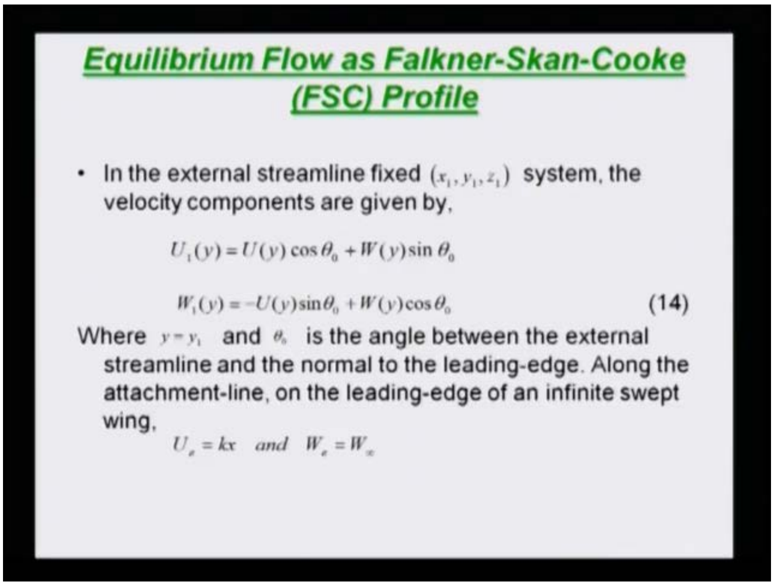

So, now, we do allow little bit of jugglery, because what have we gotten so far, we have gotten the mean flow, before we solve those two equations; that will define our capital U and capital W. Now, if you go back, go back and see, what is this theta naught; theta naught, we defined it as the angle between the external streamline and normal to the leading edge. So, let us go back and take a look.

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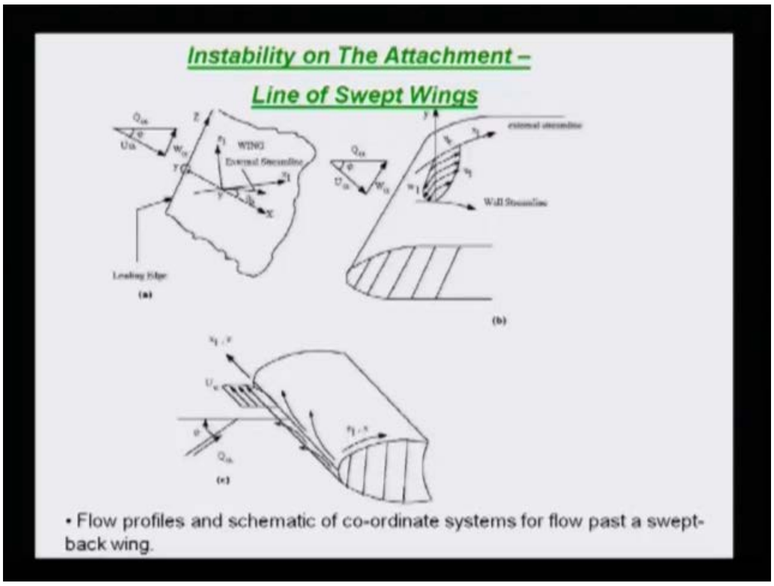

So, that is what it is. If you look at the plan view, if you look at the plan view, so, this is your x axis, wing fixed x axis and this is the projection of the external streamline on the plan view. So, this theta naught is the angle between the two. So, if I do that… So, I have a component of, x component, that I am calling it as capital U of y; then I have a component that is capital W of y; and I am trying to find out the component along x 1, y 1 and z 1. So, this is essentially the whole idea. So, I want to get it in the wing fixed coordinate system. So, that is what we are doing here.

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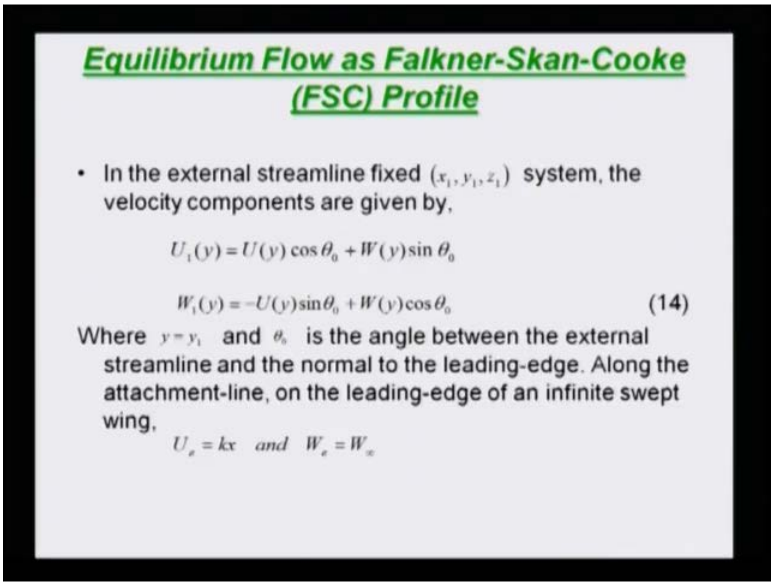

So, we have obtained, u and capital U and capital W. We know what this angle theta naught is, then, we can obtain this component in the external streamline fixed coordinate system. This is what we get. And, you can realize that y and y 1 are the same, because they are perpendicular to a wall. So, you do not have to distinguish between the two. And, you have obtained the velocity profile. And, what happens, along the attachment-line, on the leading edge of an infinite swept wing, we know U e, the edge velocity happens to be proportional to x itself. This is, this is exactly like what we did for stagnation profile, if you recall. Well, you may have done it in some earlier courses. I did not, did I do it? I think we did not do it in this course, the stagnation point profile, the Hyman's flow; we have not done it.

But if we look at Hyman's flow, that is what you would notice that, there would be, U e would be equal to k x and v U would be some k y with a minus sign. But in this case, what happens, you do not have a y component there. So, you have a, this, and this is directly proportional to the x component; whereas, W e is equal to W infinity itself. So, because, once you are outside, the flow does not change any more; that is the definition of infinite swept condition; that W e will remain same. There is no additional span-wise pressure gradient; that is, that is the whole idea.

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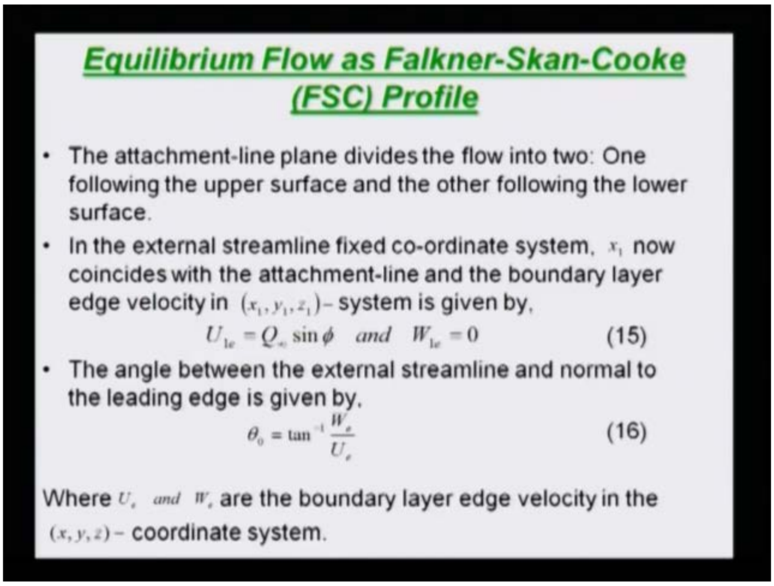

So, once we have this edge condition provided for U e and W e, we could now think of the attachment-line prime, that it divides the flow into two; one part follows the upper surface; the other follows the lower surface, and if I now focus my attention on external streamline coordinate system, x 1 now coincides with the attachment-line; that is what we have discussed; that x 1 and z are same. That is what we are saying. And, the boundary layer edge velocity, in this external streamline coordinate system would be given by this. U 1 e should be equal to Q infinity sine phi; what is phi? Phi is sweep angle; phi is the sweep angle. What happens to this? This has to be equal to 0, because, that, you are in the streamline direction.

So, you cannot have any other component; flow is along the streamline. So, that is what you have there. And, there is no cross component; that is it. And, the angle between the external streamline and normal to the leading edge is given by tan inverse of W e by U e. So, this is again, going back to your wing fixed coordinates system. So, the wing fixed coordinate system component defines your theta naught. Now, so, you understand U 1 e means the edge velocity in the external streamline fixed coordinate system. W 1 e is external streamline fixed coordinate system and that is by definition has to be equal to 0.

But that is not necessarily the case, when you look at, in the wing fixed coordinate system. There, you would have a stream-wise component and you would have a span-wise component. So, this is how we are going to define the flow field.

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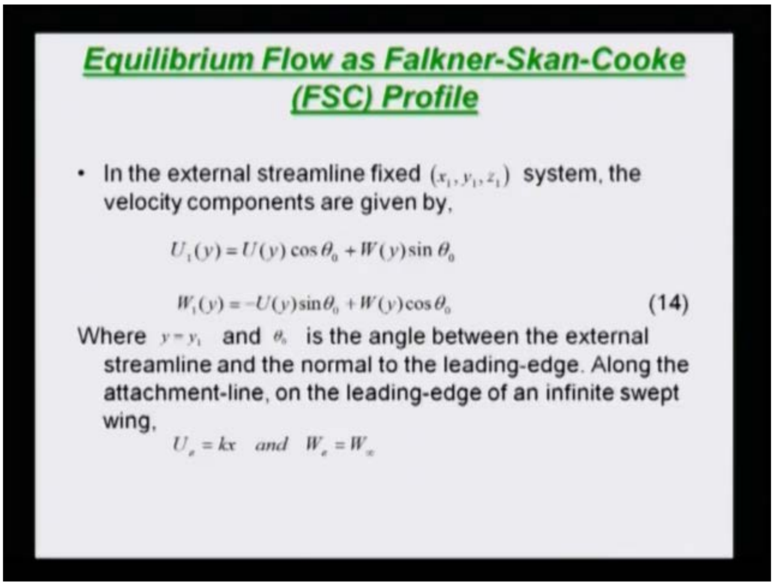

So, what happens? We have written down that expression for U 1. If you recall, let us go back there. Here. So, if I know, these two quantity, cos theta naught and sin theta naught, then you are done.

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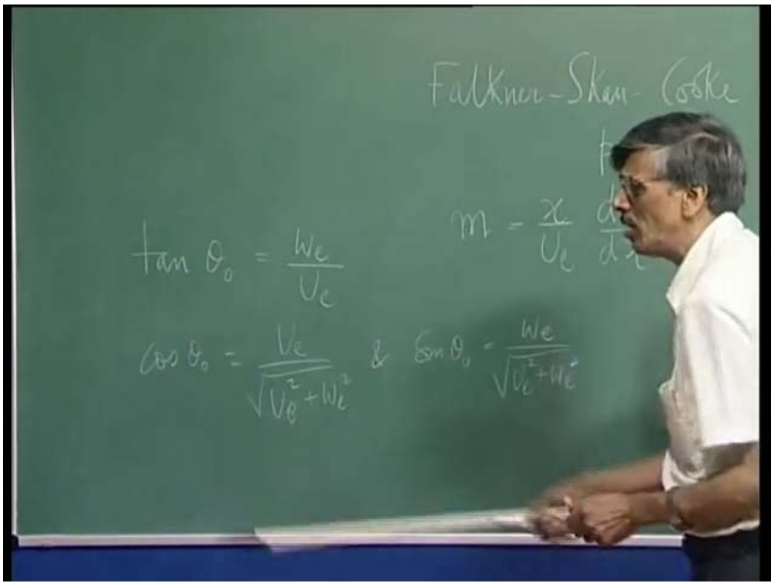

What is it, we have already seen? Tan theta naught is W e by U e. So, what will be cos theta naught?

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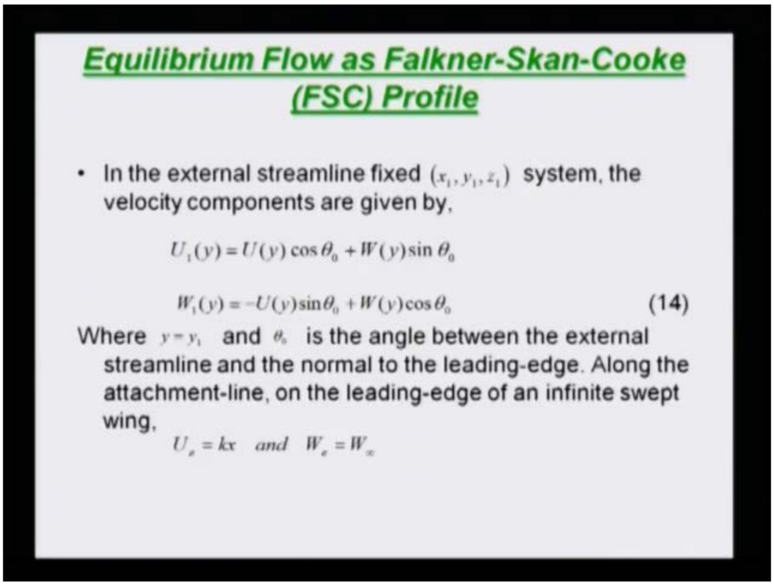

So, we will substitute it that, but what about U? That also we can define in terms of f. We have the definition of psi. So, we can differentiate it with respect to y to get U. So, this will be substituted in terms of f prime cos theta naught; we will use this expression, sin theta naught we have it, and this also, we can define it in terms of g.

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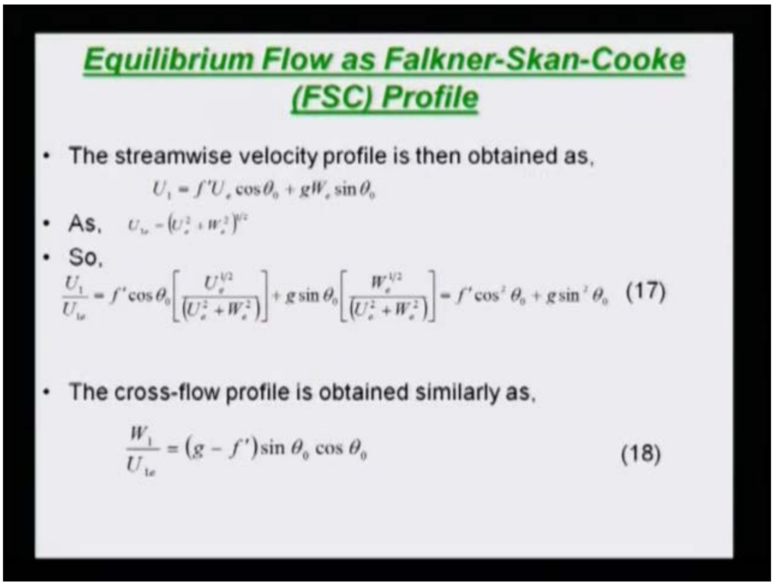

So, if we know this, then, there is no problem, because then, we can go ahead and we are going to get this. You can very clearly see that, U of y is nothing, but f prime times U e, because it is, U by U e is equal to f prime. Then, I have this cos theta naught and W is nothing, but g times W. And, if I define U 1 e as this, that is that; because, once you have the wing fixed component… And now, you want to go to the external streamline fix component. That only has U 1 e component. W 1 e by definition is 0; so it will be equal to this. So, let us substitute all of that here; f prime, cos theta naught, U 1 e; I have just written it like this. And, this will give me this component f prime cos square theta naught plus g sin square theta naught. At the same way, you have the other expression that is given for W 1 component. If I non-dimensionalize with respect to U 1 e, then, I get this. So, you can realize that, we will solve those two ODEs. So, we will get f and g distribution and from those information plus U e and W e information, we will have this expression for theta naught and we will get this.

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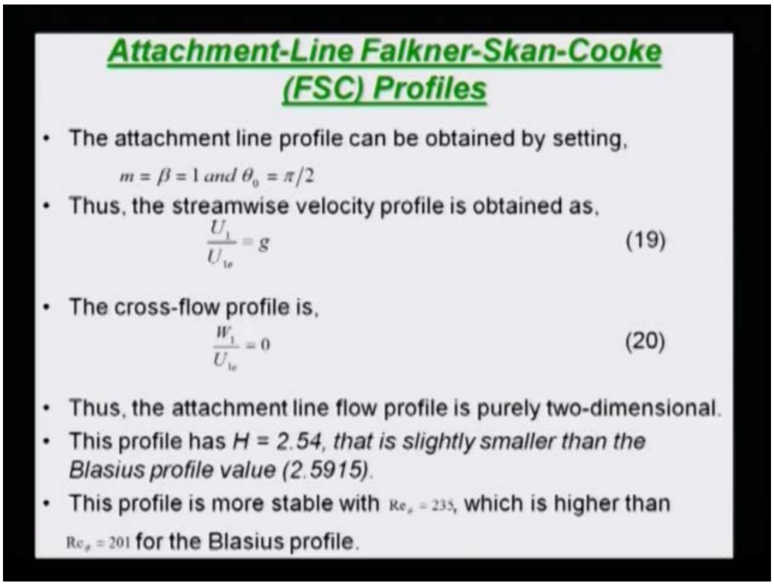

So, once we have the flow direction components, then, we are ready to define the equilibrium flow. That is what we want to do. Now, look at the attachment-line itself. In the attachment-line itself what have you seen?

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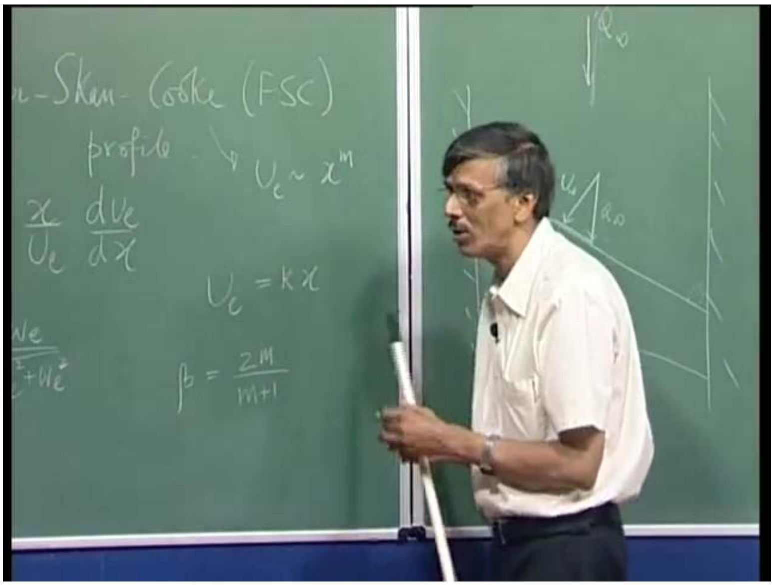

We saw U e is k x, whereas, here we said that, U e goes as x to the power m; that was the Falkner-Skan idea and that was extended by Cooke like this. So, what you find here, m is equal to 1. If m is equal to 1, what was beta? Beta was 2 m divided by m plus 1.

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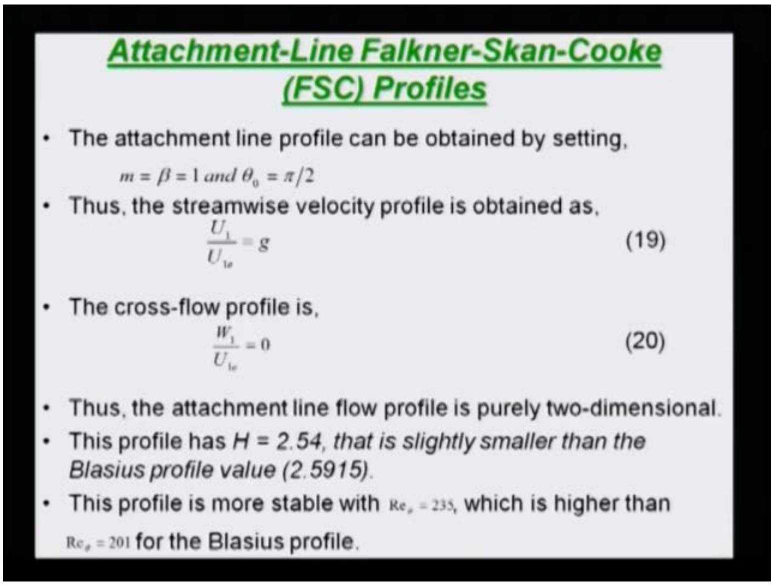

So, if m is equal to 1, beta is also equal to 1, that is what we are saying. And, what about the angle? What is the angle between external streamline and the normal to the leading edge? It is 90 degrees, because a flow has turned. You see, the one part is going up and another part is going down and in this plane, it is going the span-wise direction; that is why x 1 and z are synonymous; that implies theta naught equal to pi by 2. So, what happens is, we can use this information in the previous slide, what we have seen the expression, what will we get? The stream-wise velocity is just simply given by g. And, the cross-flow profile is equal to 0. So, what happens? This is the magic of Prandtl. He has shown that, the flow is two dimensional, because you have only one component of velocity; the other component is identically equal to 0. So, that, that is what it is. Of course, you see it, because the W 1 by U 1 e is that f prime minus g into sin theta naught cos theta naught.

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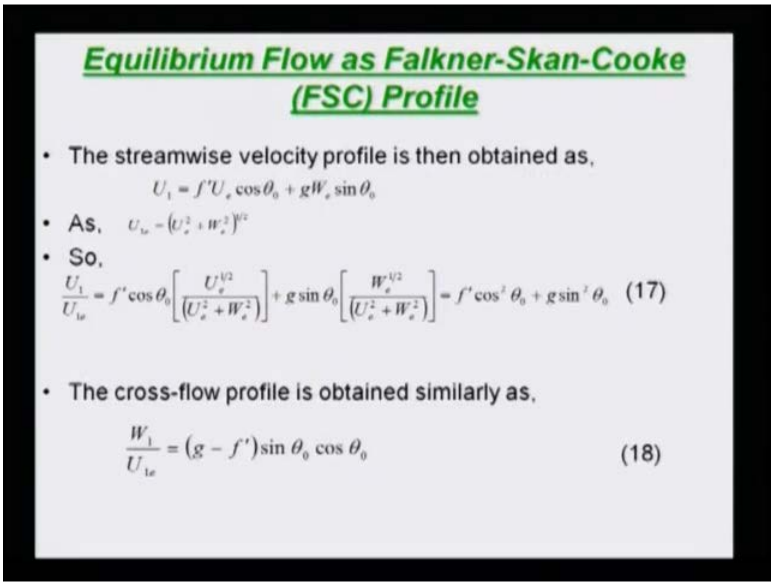

So, what happens? For 90 degree of course, this goes to 0, that is what we have established. So, W 1 is 0. What about U 1? You will get this part go away, only this part will be there. So, what happens is, you will have a two dimensional flow on the attachment-line plane, but you will be solving those two ODEs that we have seen already. First, we will be solving for f and f prime, from that Falkner-Skan-Cooke equation. And, the from that, we will go to the second equation, that was g double prime plus f prime g equal to 0; that we will be solving, so, we will get g. And then, from there, we have gotten U 1 and W 1. Well, if you see that, this was what was established by Prandtl himself.

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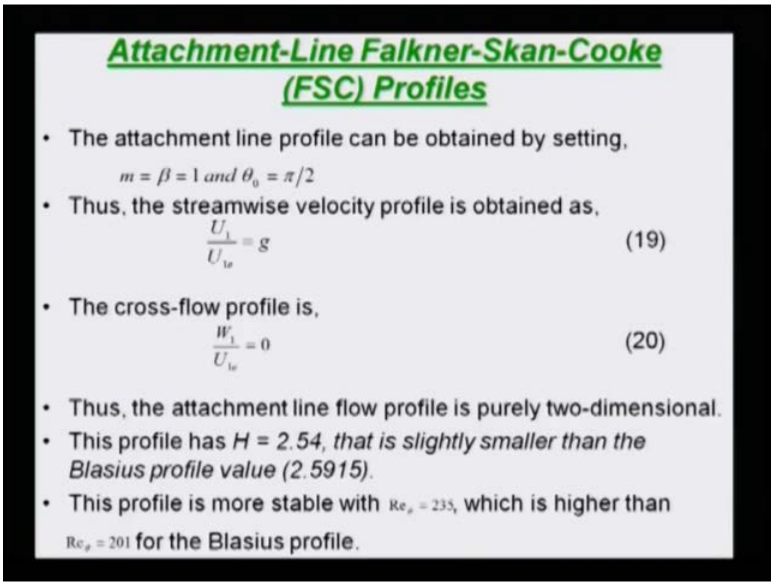

So, we notice that, this profile is purely two dimensional. And, you can do a little bit of computations and then, calculate the shear factor. The shear factor is 2.54, which is slightly smaller than the Blasius profile. So, that means what? Of course, it will be more stable. Smaller the value of H, more stable it is; you saw, the flow with adverse pressure gradient H keeps rising. So, you would see that, and this is what is obtained; that, if I calculate R e theta critical, based on momentum thickness, this is about 235, which is higher than R e zeta equal to 201 for the Blasius profile. So, this is slightly more stable, but you understand now, this is the attachment-line plane; it is not a stagnation point flow. It is, it is different, because the stagnation point flow for 2 d section, we have seen that, R e delta star, critical value reaches a value, which is greater than 10,000 as compared to Blasius profile of what, 520. So, that 520 actually corresponds to R e zeta equal to 201. And so, you can see that, this attachment-line plane is not as stable stagnation point flow; it is slightly more stable than the Blasius profile. This was what we pointed out.

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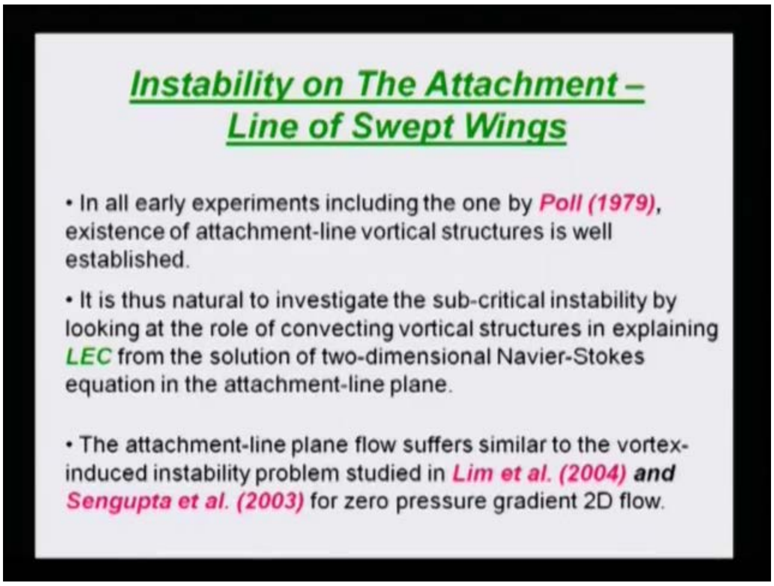

Now, having defined the equilibrium flow, we can start thinking of its stability. How do we do that?

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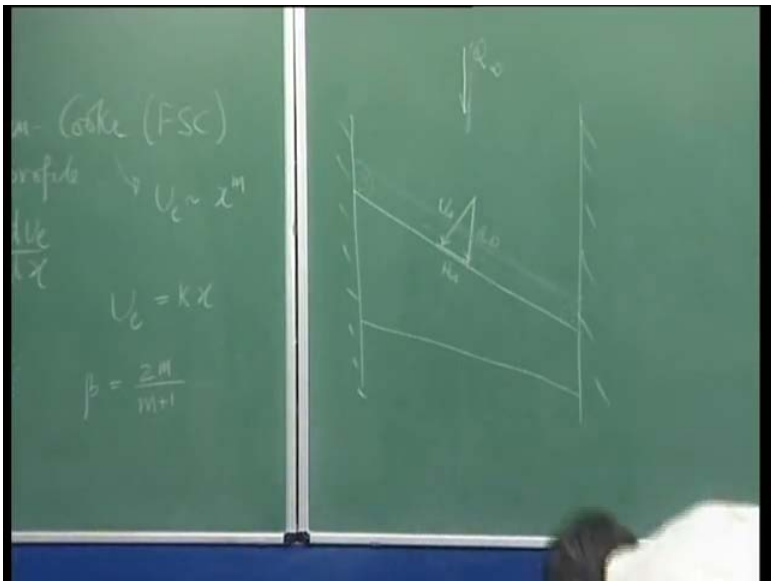

Before we do that, if we are looking at instability of the attachment-line flow of the swept wing, let us look at what people have noted experimentally. For example, Poll, D I Poll, did that experiment and instead of taking a wing, he took a long cylinder and he noted that, along this attachment-line in this plane, so, if I have this plane, in this plane, of course, there will be a lot of vortical structure, because of the, aided by the swept angle, they will propagate inside the shear layer or outside the shear layer.

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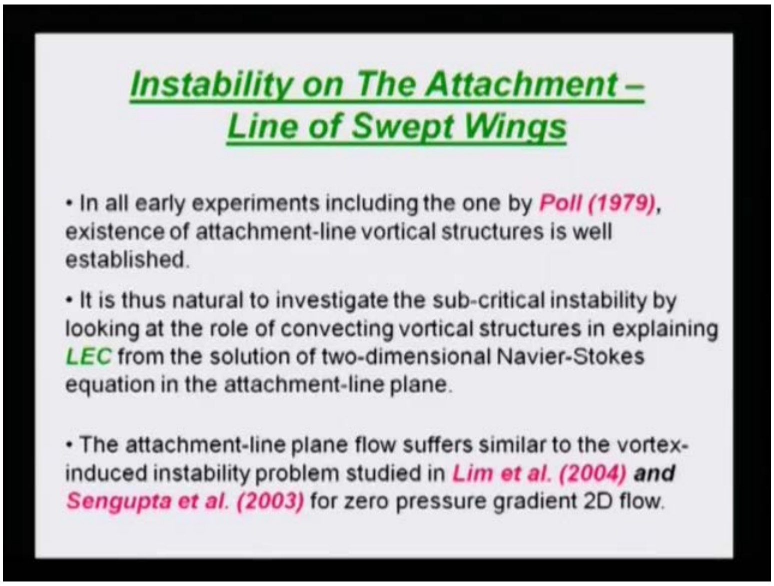

So, that is what is being said that, experimentally it was noted that, attachment-line vortical structures are very much there. So, we decided to investigate the subcritical instability by looking at the role of convecting vortical structure in explaining this leading edge contamination problem. And, what we would be able to do is, we have defined the equilibrium flow, but now, for defining the instability of the attachment-line flow, let us solve the 2 d Navier-Stokes equation. Now, that will be your, all inclusive; the flow itself is two dimensional and then, we will find out its instability by solving the 2 d Navier-Stokes equation. So, there is no assumption made, except the fact that, we are talking about the disturbance field also being two dimensional. We are not talking about 3d disturbance field. And, we, we are not talking about invoking Squares theorem or anything; it is just that, we are starting 2 d instability. Now, the situation is something similar.

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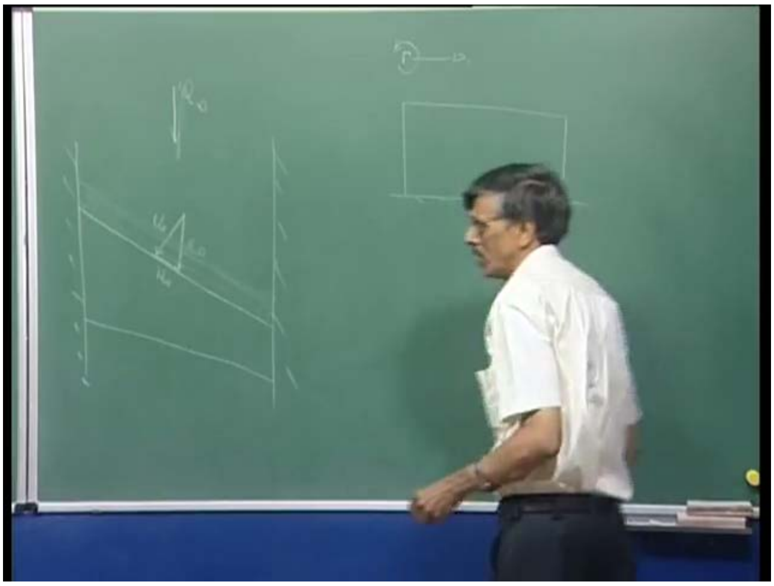

So, if I project this flow, I mean, if I look at it like this, the flow will exactly be the same thing that, we have done, studied before. We will be talking about a flow in a box and there is this vortices; they are going in this direction. That is what the experimental observation of Poll, finding… Everybody have looked at it, Anhal, they have reported the existence of convecting vortices in the attachment-line plane. So, we follow the same thing that, we have talked about just now. In fact, we are discussing this attachment-line instability as an example of that vortex individual instability. So, that is the whole idea.

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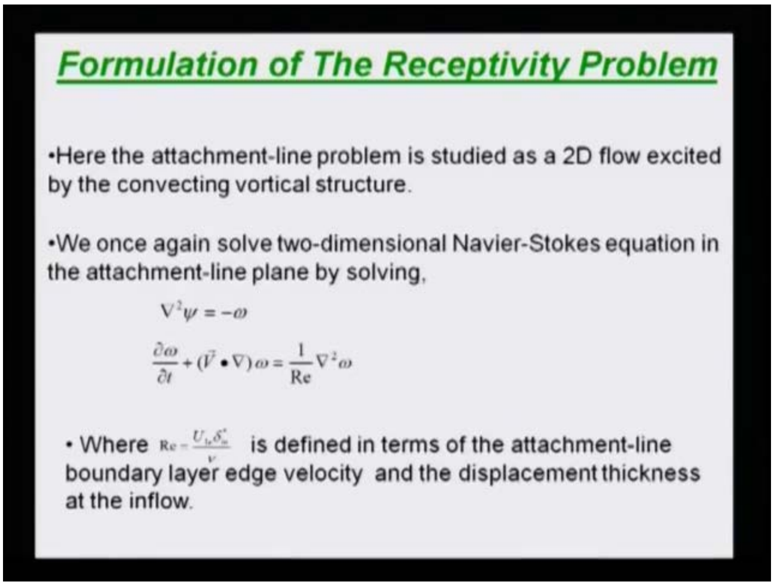

So, solve the Navier-Stokes equation. These are the two governing equation, stream function equation and the vorticity transport equation and here the R e that we are having here, is defined in terms of the velocity scale that is given by U 1 e. If you recall, that is the edge velocity in the streamline fixed coordinate at the edge. So, that we have calculated. And, the length scale we have purposely taken as some kind of a… See, your attachment-line boundary layer would be somewhat like this. So, you take this as your delta. So, this is your length scale. That is what has been used in defining this Reynolds number. And, he would completely agree with me that, for a two dimensional flow, stream function and vorticity formulation is the most accurate one. Because, you do not have any scope for spurious mass generation etcetera; existence of psi does that and the vorticity is a higher order quantity, which you are directly solving. It is much better than solving, let us say the primitive variable and then, converting those U and v components into vorticity; because there, if I get a solution of a differential equation as U and v, and then numerically differentiate to get omega, that accuracy will be compromised as compared to directly obtaining omega from a differential equation.

So, that is what we like to emphasize. There are other mathematical issues. That vortical field also happens to be divergence free, because it is a two dimensional flow, so, it becomes automatically divergence free. Why, because omega is in the z direction; divergence of omega in the x y plane is, of course, 0, because omega is not a function of x and y. So, that is a very significant plus point. While, if you solve for primitive variable formulation and although, by definition the vorticity itself, again in the, is in the z direction and again, it will be divergence free, but the magnitude of the vorticity calculated, will depend on numerical differentiation of U and v; that will be a shortcoming of that formulation.

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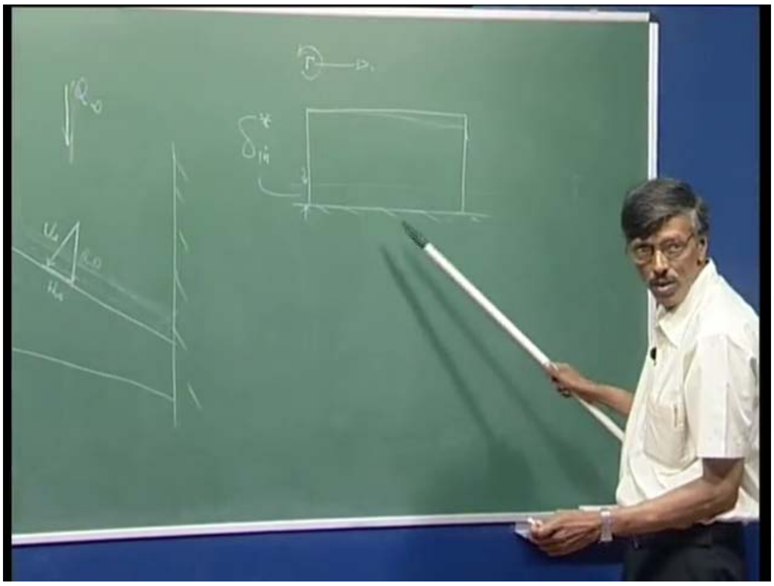

So, what we do here is a same thing. We are studying the flow in a wall. This is a, basically a straight line; we take 501 equidistant points in this direction. So, this is our stream-wise direction. So, this is your stream-wise direction and this is the wall normal direction that we are studying. So, this is essentially your leading edge of the wing.

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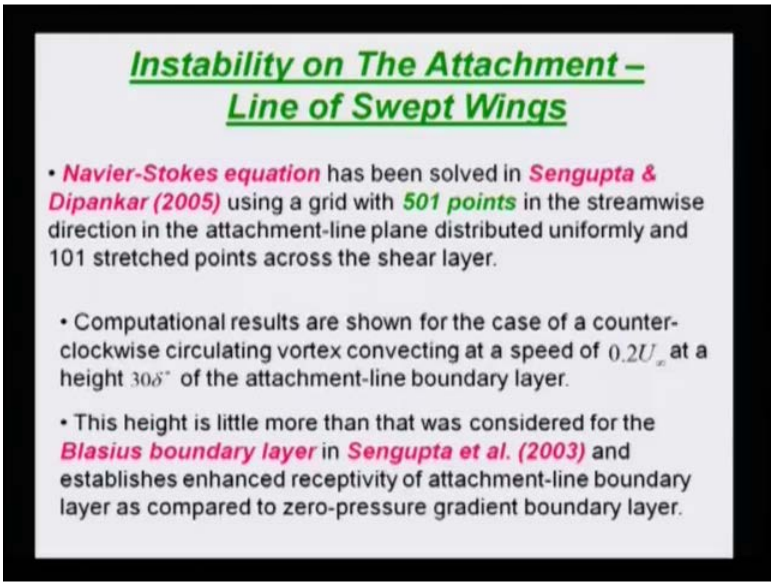

In this direction, we had taken about 101 stretched point in the boundary layer, in this g f m paper that we reported some results. Show you some result, one can do any number of calculation; there were those two papers that we talked about; one was in the computers and fluids, where we reported the computational result. And, this paper, this g f m paper, where we talked about the physical mechanism. In those publications, you would see there are many, many cases computed, but I will just show you one particular case of convection speed of 20 percent, with respect to the free-stream. And, think of the height of this vortex; it is about 30 delta star. This height is slightly more than what we have done in the experiment, where we have taken something like, what, 27.5 five delta star. So, will look at it and then, see, what do we get; how stable or unstable this is, with respect to the Blasius boundary layer.

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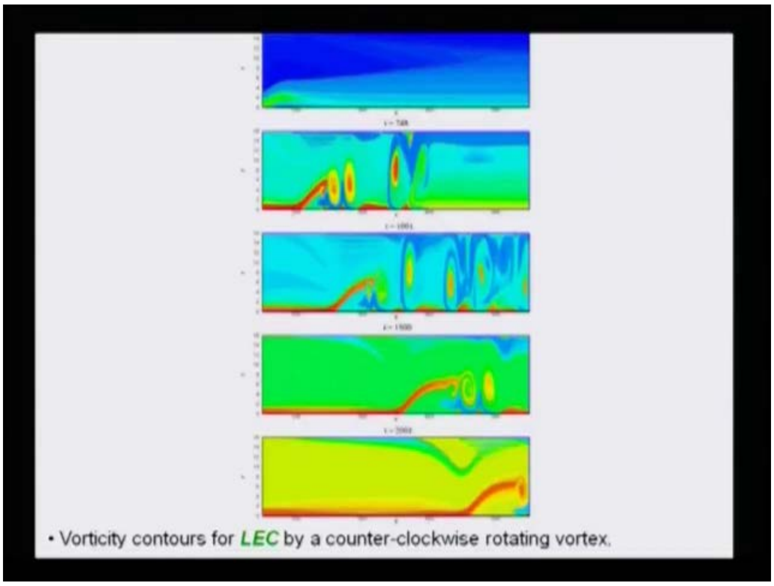

So, we look at some representative results. In the top frame is shown, for which the vortices, the vortex, one of the vortex, a periodic vortex we are studying. It is right outside. Please understand that, even this numerical experiment that we are reporting here, it is basically a unit process. We are trying to find out, if a single vortex passes by, then, what its effect is; that is what we are calling as vortex inducing stability. And, in this kind of experiments, what we are talking about, it is not necessary, just only a single vortex passes by; you get a stream of such vortices getting created here and swept away like this. So, each one of them will have its foot print.

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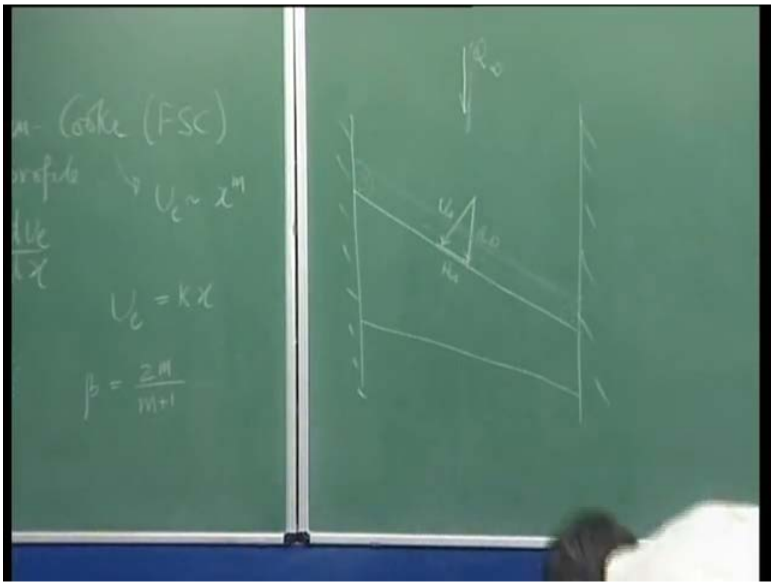

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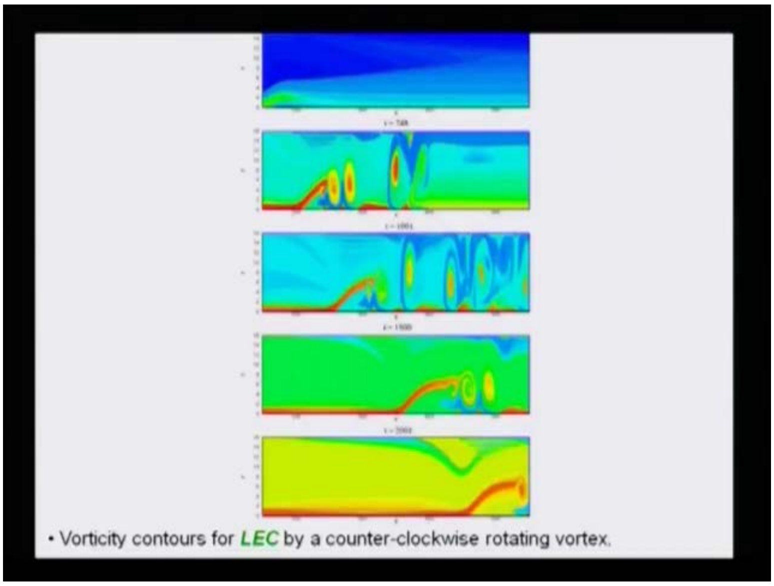

So, what we are noticing here is a typical output of such a foot print. So, in a actual flow, it will be a sort of a convolution of all this results taken together; but we are trying to establish by a numerical unit process, what this is like and in this case, what you notice that, this vortex is still outside the inflow. So, if I take it like this, this vortex could be somewhere still outside and it has started affecting the flow inside. How does it do it? Of course, it does do it; do it because, through this inflow boundary condition and the top of the computational boundary, you have some imposed condition; that shows the effect of the approaching vortex. And, you can already see, bulge there, indicating the flow has been affected. And you, you can, now, that answers your question, whether we have a constant shear layer thickness or not. It just simply starts affecting it somewhere here, unlike the Blasius boundary layer case. What did we see there? We saw two sites of instability; one was at the leading edge. Here, there is no question of leading edge; we are talking about infinite thing.

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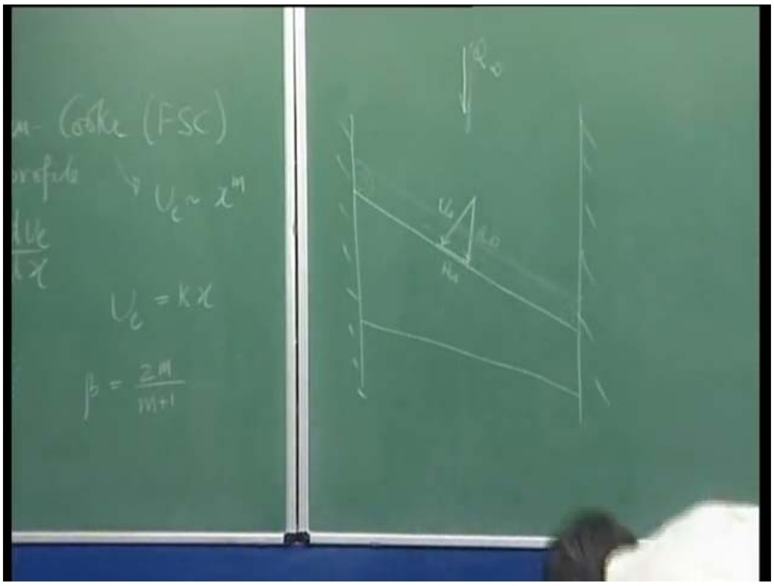

So, we have just simply taken a small slice of it. So, there is no leading edge site in consideration. And, what did we see there in the Blasius profile also? That it is selective; it does not happen anywhere, but at a fixed location. And, what was that fixed location? That is what we are doing through the assignment. We are finding out, at different xs the time history of pressure gradient; we will be able to find out why and where, and, it occurs and what value of c, it is the most potent. So, that is the essence of the assignment that you guys are doing.

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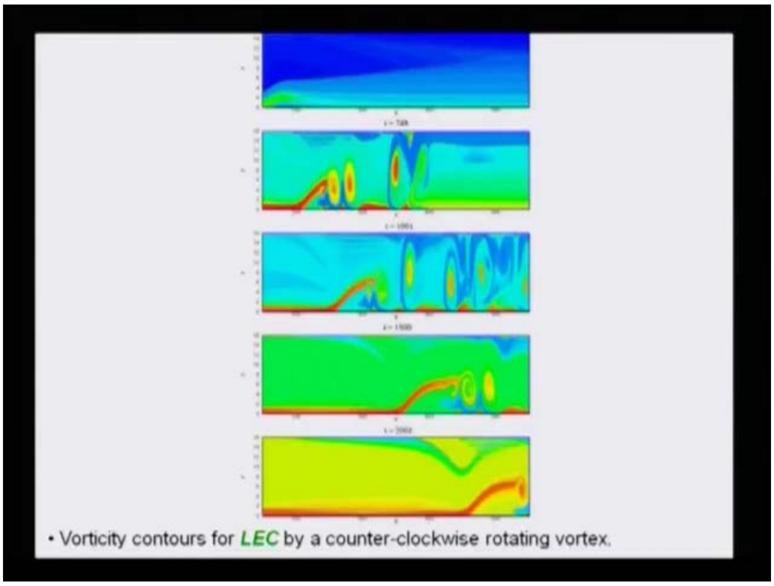

So, what you are going to report back to me, is a specific nature of the Blasius boundary layer. Quite unlike here, you will not have that. Here, things will have to essentially start from the leading edge itself, of the computational domain, right at the inflow. And, we are starting to see that. And then, as it approaches near, then, you can see that, it just breaks up into all those wall normal vortical eruptions. And, this is a solution at t equal to 6, 7, 8 and various times are given. I think, the third frame is at t equal to 1001; then, the last frame is at 2001. So, that is what you see, that, the vortices are the sweeping past and then, even in this case, what has happened? The vortex may be still somewhere here, but the disturbance is a swept away. This is due to one vortex, but there is no guarantee, or it would really be likely that, by the time this has gone away, another one is coming back in its trail. So, there would be repetition of this events and this is what we need to do.

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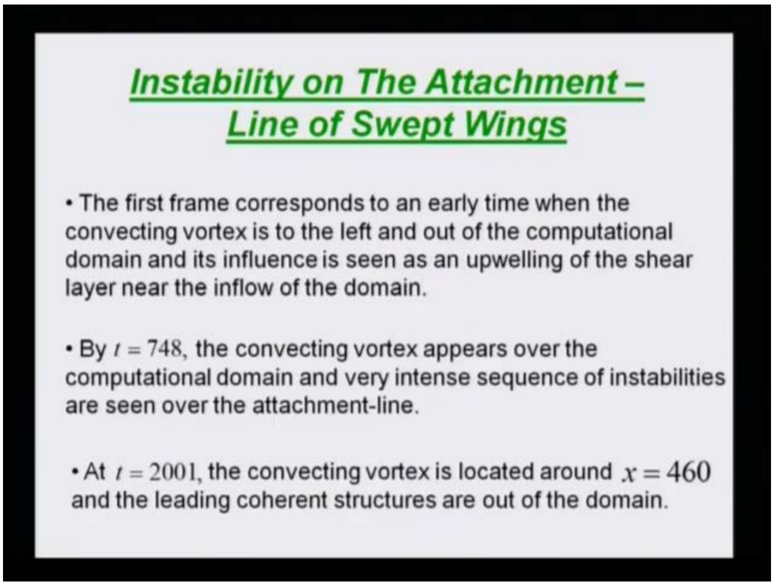

This also tells you, if you are an aerospace engineer that, how you would go about designing a trouble free wing. What would you like to do? Minimize the source of disturbance. We have seen, we have been talking about it all through, that there is no such thing as generic instability. We have seen for pipe flow, the flow can be unstable at 2000 Reynolds number based on diameter; it can be even at 100,000, depending on how you quench the background disturbances. So, here also, the boundary layer may be ready, but if I do not allow this convecting vortices to be created, I am done. So, basically, if I am trying to design a wing, I would pay particular attention in that design, where this convecting vortices, this corner vortices are not created at all, or if they are created, they go at a small, weaker strength and also, there is a selectivity of c, that you can actually take a look at that paper, that computers and fluids paper, where we have done some seven, eight cases; take a look at there and you will find out the receptivity of the boundary layer for different c.

So, what we just now saw, we can make this following comment. The first frame corresponds to an early time, when the convecting vortex is left to the, left out of the computational domain and its influence was seen as an upwelling of the shear layer near the inflow. By the next frame, we saw the convecting vortex has appeared over the computational domain, but we also see a very intense sequence of instabilities over the attachment-line. And, if you look at the last one, this is where the location of the vortex was; however, the leading edge of the, leading edge of the core end structures, those are getting created due to this vortex instability, they are out of the competition.

So, you can see that, it also goes at a very fast clip. So, that is the reason that, when you start designing a aircraft wing, you want to be, do, you want to do a, sort of a conservative estimate; you allows assume the flow to be turbulent right from the leading edge; because, if you do not stop the flow of this convecting vortices, it is very likely, the flow will become turbulent right from the leading edge, through this mechanism. We are not talking about how this leading edge vortices affect the flow over the wing surface later; but here, we are just keeping our attention focused slowly, on what is happening at the leading edge; it is on the attachment-line plane itself. Now, we also noticed that, such vortices created adverse pressure gradient, if it is counter clockwise, it creates an adverse pressure gradient ahead; the action of ((scarving)), that is what we explained. It tries to lift off the shear layer. At the same time, behind this vortex, what it does, it tries to push it down. So, what will happen? You will see a kind of a thinning down of the shear layer and that is what we saw in the last two frames. Wherever…So, looking at the last two frames itself, you can kind of tentatively guess, where the vortex is, where the vortex is.

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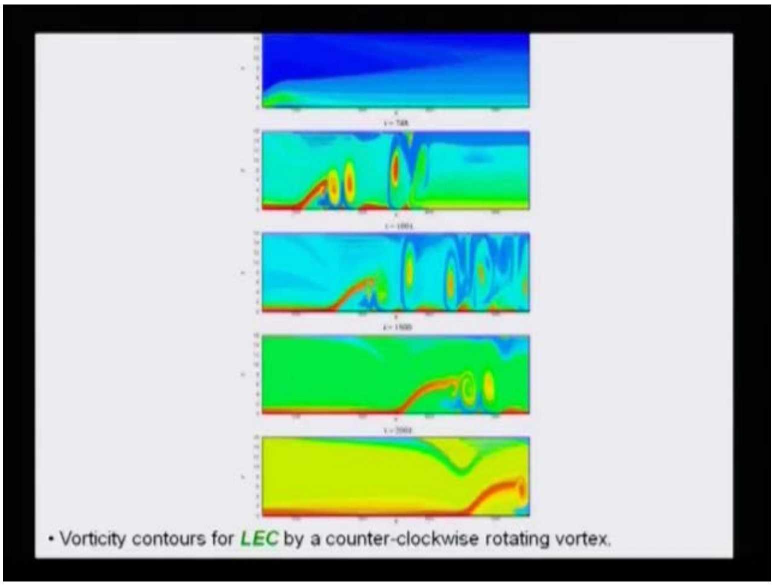

The vortex, like what we are seeing here, there is a thinning down on the, here. So, the vortex must be somewhere ahead of this. So, on this side, it is creating instability; on this side, it is thinning down the boundary layer. And, rest of the place where its effect is marginal, you can very clearly see that, it is almost like a constant thickness, a boundary layer. And here, you are also seeing the same thing, but now, here what has happened, that, it has created a significant thinning down, because, it has acted over a longer period. See, this is about 1500, this is 2000. So, over a longer period, the same favorable pressure gradient has acted upon. So, please do not try to view it like the usually, the Eigen value analysis always looked at; but what happens is, you have a time independent equilibrium flow and the instabilities warn of how the disturbance grow in space; however, in this solution of Navier-Stokes equation and the attendant description that we are talking about, we really do understand that, there is time element playing a big role.

That is what we are doing. We are calculating the pressure gradient as a function of time at a fixed location. And, we do see that; see that very clearly that, here, well it was getting affected here. Here also, you could see, the vortex is somewhere here, there is a little bit of thinning down. And here, it is, you can see, it has thinned down a little more; and here, the thinning down is marginal. But as time progresses, you can see, this is the trace of the free-stream vortex; that is why we did not even mark by arrow. If, I wanted you to understand that, but here, in this spot of the shear layer, it has acted over a long, longer period and that cumulative effect shows the significant thinning down of the boundary layer.

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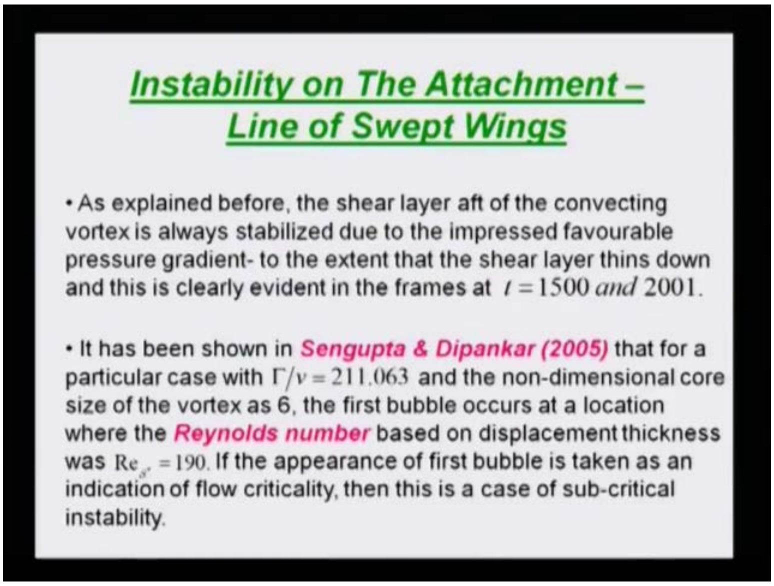

Now, this is what we commented upon. Now, we did talk about this, that we have looked at a very particular case of gamma v in terms of this; that corresponds to a core diameter of 6. The first bubble occurred at a location where the Reynolds number based on displacement thickness is about 190.

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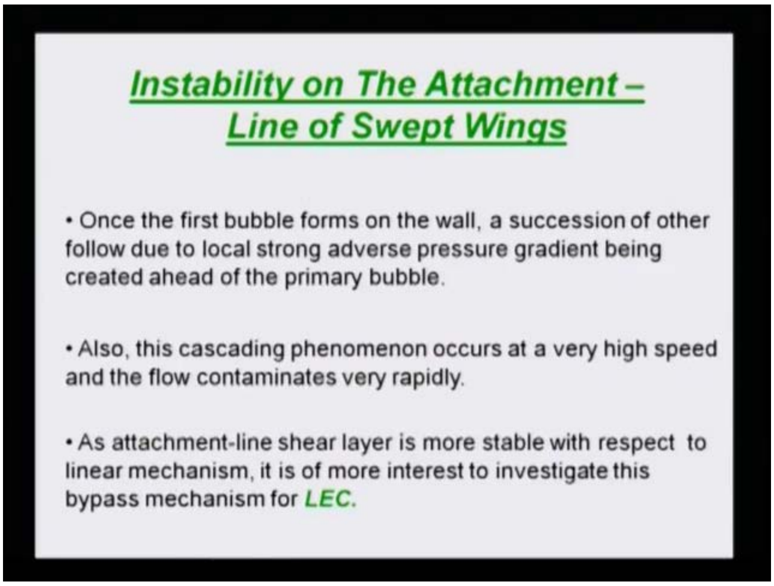

So, that, what was the R e critical? It was noted as 235. So, if I see instability event occurring at 190, that is obvious, that it is a sub-critical event. So, that is what we are saying, the flow criticality indicates a sub-critical instability. And, this is something we have all along seen in vortex individual stability. Once the primary instability makes its appearance, through the formation of a first bubble, a succession of many others will follow. They will be ahead of it, because the, locally if the shear layer bulges out, it creates a further adverse pressure gradient ahead of it. That is what you do, you do see a intensification of cascading effect of this adverse pressure gradient. And, this cascading phenomena occurs at a very phenomenally high speed. And, that is why the contamination just simply sweeps across. So, once you have an attachment-line shear layer, which is slightly more stable with respect to the linear mechanism, it is of more interest to investigate this bypass mechanism of LEC. We have, we can actually talk about this. This can actually provide a very nice basis for future investigation which we have not been able to do.

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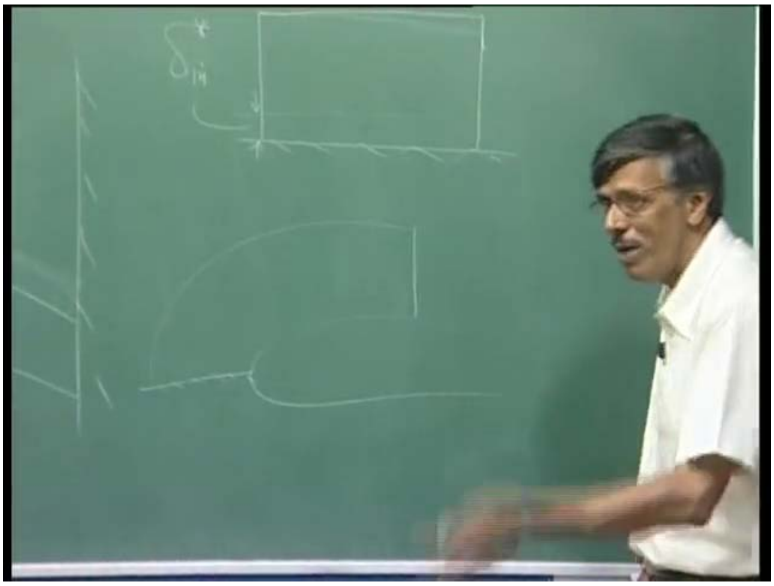

Suppose, I define this solution of Navier-Stokes equation in the attachment-line plane as the inflow of calculation where, I could do something like this. Let us say, this is the leading edge of the wing and this is my attachment-line plane. Now, I could do is, I could basically study the instability over the wing. This solution, as a function of time, can come from what we have done just now. So, it is like creating a inflow condition and then, you can study that. You recall, this is what we talked about; that we talked about the necessity, for first seeing what happens in the attachment-line and then, subsequently, what are its effects downstream, on the wing. So, that part of the investigation is overdue. Maybe somebody will like to do that, and it is going to be a very fantastic piece of work, because, most of the analysis or the research results, those are available in the literature, you basically split the flow into to two; either you look at the stream-wise instability or you look at the cross flow instability. But if you do a complete 3d analysis, you are going to see a combination of the two. So, you do not have to make any assumption. You are going to get a truly space time dependent three dimensional instability study. Maybe I am giving an idea, for doing some future research, for yourself.

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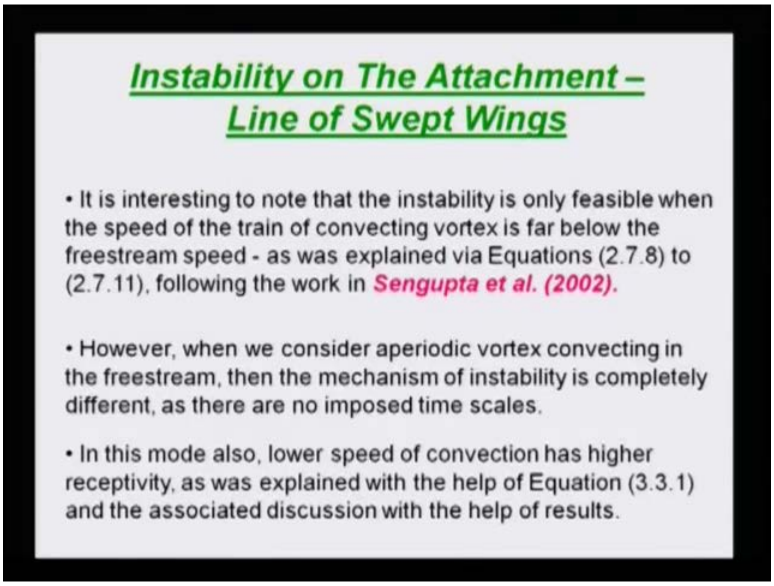

Now, we reiterate that, instability is only feasible, when the speed of this train of convecting vortex, remember, we did that case, where we have periodic vortices; that we established there. We also considered a periodic vortex. There also you found out that, you basically have to have a speed, which is lower. Why, because you recall, you are, you are now seeing it, in your disturbance stream function expression. What is important is U infinity minus c. So, that is why your c has to be less than equal to U infinity; that gives a large amplitude. So, that also, you will notice that, for a periodic vortex also, c has to be distinctly different from U infinity; that is what we are saying. But there are the differences; one of the main difference is, of course, in this case, periodic vortex case, you impose a space and timescale; here, you do not.

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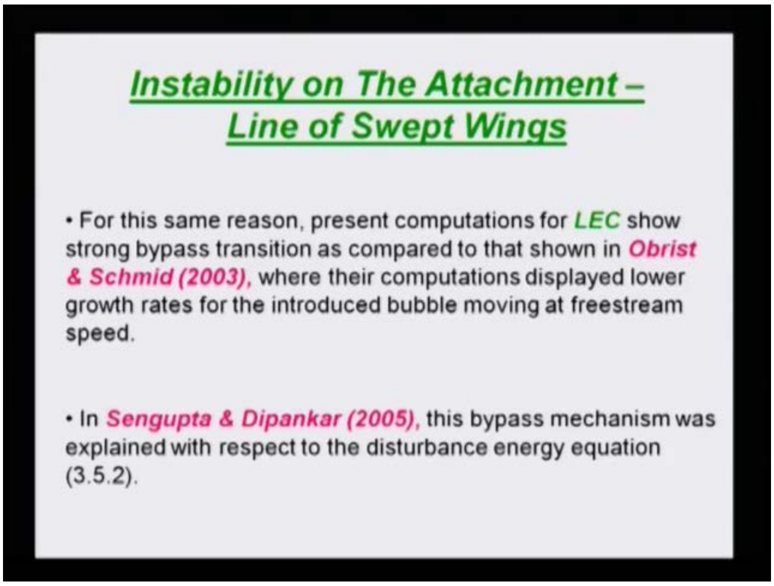

In this case also, we said that, lower speed of convection has higher resistivity; that we have seen with the help of those expressions for disturbance stream function. And, we have shown that, you can show the c of bypass transition. This was not possible. There was this case computation by Obrist and Schmid. They had some earlier inkling about what we have done in our fast track paper that, the vortex induced instabilities is important. So, Schmid was involved in studying in bypass transition. So, he asked his graduate student to look at it.

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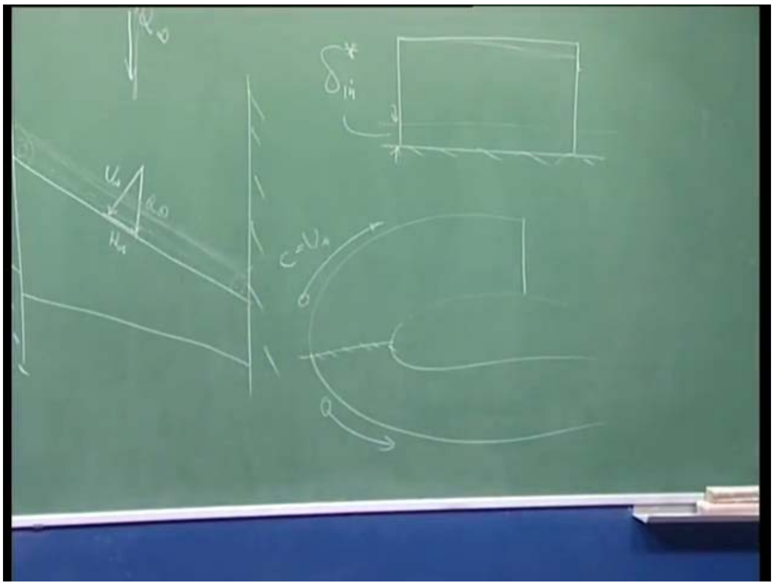

What he did was basically this. That you have this flow like this, and then, created two vortices, one going in this direction; another going in this direction; and both of them were set equal to c equal to infinity. See, look, I mean, there is a qualitative difference between what we did and what they tried. They tried to do it, study it as a three dimensional problem and they put this convecting vortex outside, but they, that was forced to go at freestream speed.

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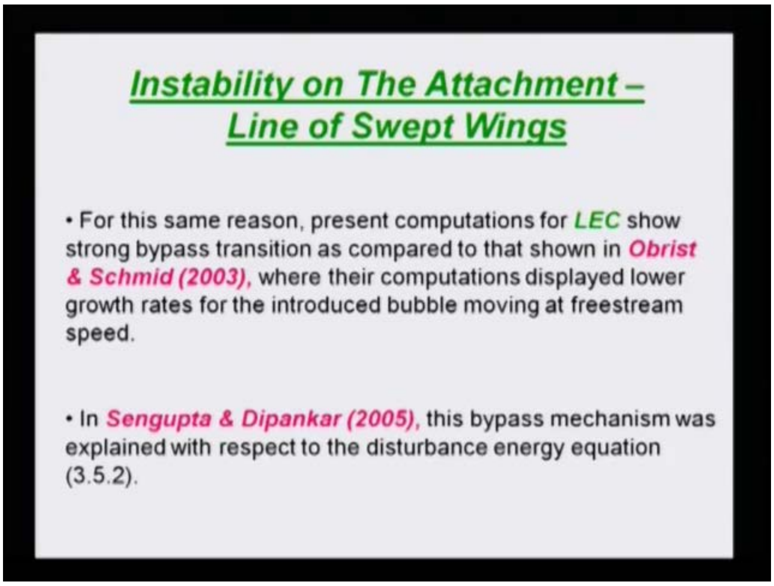

That is why, they did not see any effect at all, very very marginal effect. We can actually explain what we did; it was in the attachment-line plane itself. Then, we can show that, disturbance energy does get created significantly, if you do convect the vortex which is not going at this. So, I think we will stop here and will take up from this point in the next class.