Instability and Transition of Fluid Flows

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Module No. # 01

Lecture No. # 24

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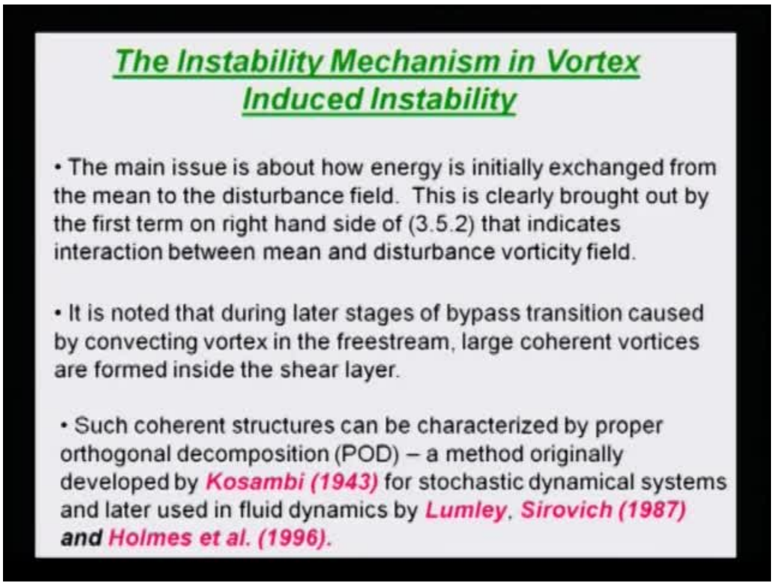

Last class, we had been commenting further, about vortex induced instability. Today, I will just bring one more aspect of, description of this instability, by using, what is called as the proper orthogonal decomposition. This is a specific technique, which was brought into dynamic consistent studies pretty early, but which was only exploited much later. So, we will talk about some of these issues. What we understand by vortex induced instability is, essentially, written out here that, how initially the energy is transferred from the equilibrium flow to the disturbance field; that is the main issue. If we can predict that, we have predicted the primary instability and we know, once primary instability is triggered, that immediately brings in its wake, secondary and tertiary instabilities, that leads to eventual bypass transition. This is what we called as a bypass transition. So, during the later stages of bypass transition, you do actually notice, large coherent vortices form, which also are, something similar to what you, one sees in fully developed turbulent flow.

In this context of vortex induced instability and creation of coherent vortices inside the shear layer, we should recognize the early seminal contribution by Kosambi in 1943. He wrote a path breaking paper, which remained largely ignored, for a long time. The idea was basically, to look at a dynamical system, but whose dynamics is determined by stochasticity. Stochasticity means, it is a time dependent, probabilistic event. So, if such a thing happens, how do you understand the dynamics of it? This is what Kosambi thought about, and he said that, let us try to look at the dynamics of the system, and try to project it, in a deterministic basis. So, this is rather important. So, it is basically, a projection of a stochastic system, onto some deterministic basis functions. And, this deterministic basis functions are orthogonal, and they are basically, denumerably infinite. So, if you are looking at a truly, a continued system, you will have a very large number of them.

But then, Kosambi suggested that, you basically look at such a system, but focus upon the energy content of each of this modes; and you decide upon yourself that, you want to describe, let us say, the first 80 percentage of the energy, or first 99 percentage of the energy, and then, you would see, this number of modes required to describe that quantum of energy, is not necessarily infinite; they are finite in number. So, that was the catch and this was seized upon by professor Lumley, who had been investigating on application of this idea into fluid mechanics, for quite some time. There are the early efforts, given in this monograph by Holmes, Lumley and Berkooz, titled dynamical systems; you can take a look. Then came, another breakthrough, which was a practical method of computing this proper orthogonal decomposition. This was also done by Professor Sirovich of Cornell, and it is called the method of snapshots.

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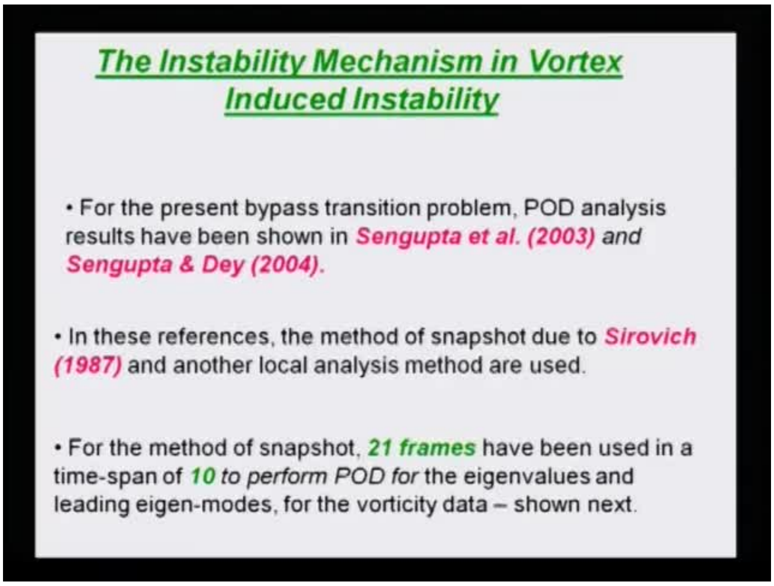

This was a major breakthrough, you can say, which was proposed by Professor Sirovich, in a three part paper, which actually revolutionized fluid dynamics. Even today, we keep using POD, for a great many of our investigations; and there are lots of new insights, that have been, really been, in introduced by us, in terms of explaining fluid dynamics. For example, for vortex induced instability, we have shown it in this fast track paper in JFM 2003. And, there was this conference cum a journal paper in 2004, where we showed, how we can use POD for bypass transition and tried to capture those coherent vortices and characterized them. And, as I told you, in these, we have not only used the method of snapshot by Sirovich, we also used something, akin to singular value decomposition.

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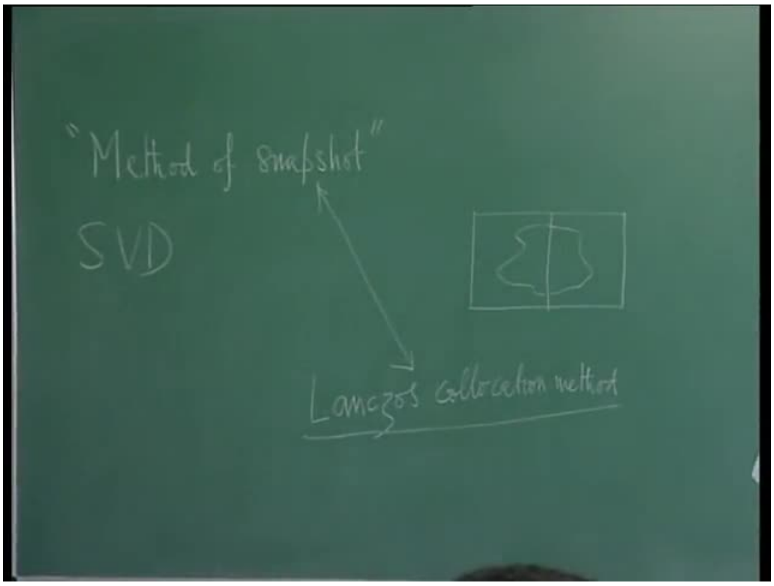

In linear algebra, you may have heard of Singular Value Decomposition techniques. So, this is essential used. The issue about this local analysis method, it is very robust. Suppose, if I take a domain here, and then, split it into two and then, I do POD of this part, and POD of this part, by using this orthogonal collocation method, Lanczos collocation method, what we found that, this method, as compared to method of snapshot, this is quite powerful too. And, what we could do is, we could get some picture here, let us say, a structure like this and we did the analysis separately of this part of the domain and then, we saw, there is virtually, seamless matching of the two. This is a very powerful method; unfortunately though, it requires tremendous amount of work; whereas, this method of snapshots require very fewer number of snapshots, and you can do those calculations better. But nonetheless, they produce different quality of results and you can understand that, this kind of thing, that we are talking about, it is a technique of signal processing.

So, whether you want to talk about visual signal or audio signal, you can do it. In fact, in a commercial entertainment industry, this kind of, POD kind of technique, is routinely exploited in compacting data. You can understand that, if I take the full dynamical system, the signal may be given by, let us say, 100 megabytes of data, and if I see that, 99 percentage of the energy is pretty well predicted by, let us say, 1 percent of this data, then, so be it; that is a huge saving. So, these are some of the techniques, also used as data mining technique. People use it, but none, nonetheless, we will not talk about that in great detail. Let us look at, what we can do in studying vortex induced instability.

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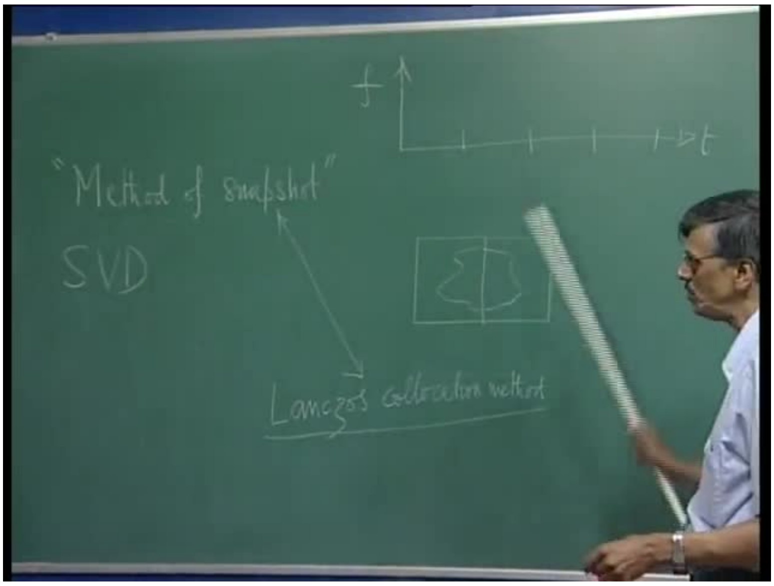

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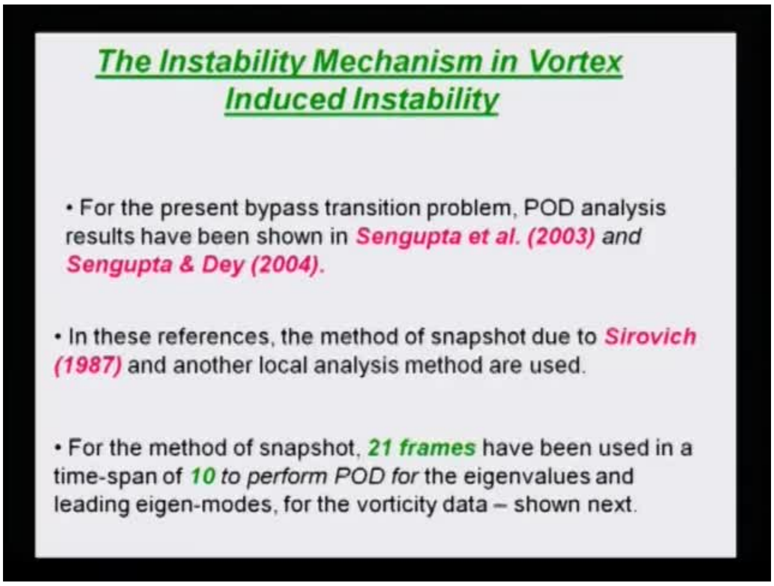

What we do is, basically, we take the time series. So, if this is the time, and we have some physical function, say f, and then, we look at intervals of 10. And, if we look at this time span of 10 and then, just simply, take 21 frames. So, these are those snapshots, we are talking about. So, each of the frames are constituent of the method of snapshots. So, we are basically taking 21 snapshots, over a time interval, non-dimensional time interval of 10, and then, performing a POD. So, that is what it is. So, while this SVD or this Lanczos collocation method, gets you the spatial Eigen values, and those require handling a very large number of, size of matrices; whereas, method of snapshots, basically takes a temporal series, that is what we are talking about. We are talking about 21 frames, in the time frame of 10, and then, perform the proper orthogonal decomposition technique.

Once you do that, you can get the Eigen values. If you are looking at the disturbance velocity field, then, those Eigen values represent kinetic energy of the disturbance field. If you are looking at the disturbance vorticity, then, you are looking at the disturbance enstrophy. So, we actually, can take both these routes; however, we realize that, vorticity dynamics is more central in understanding fluid dynamics. So, we basically, most of the time, look for POD analysis for the disturbance vorticity field. And, what we found is, the Eigen values and the Eigen functions are as given here, for this simple problem of vortex induced instability.

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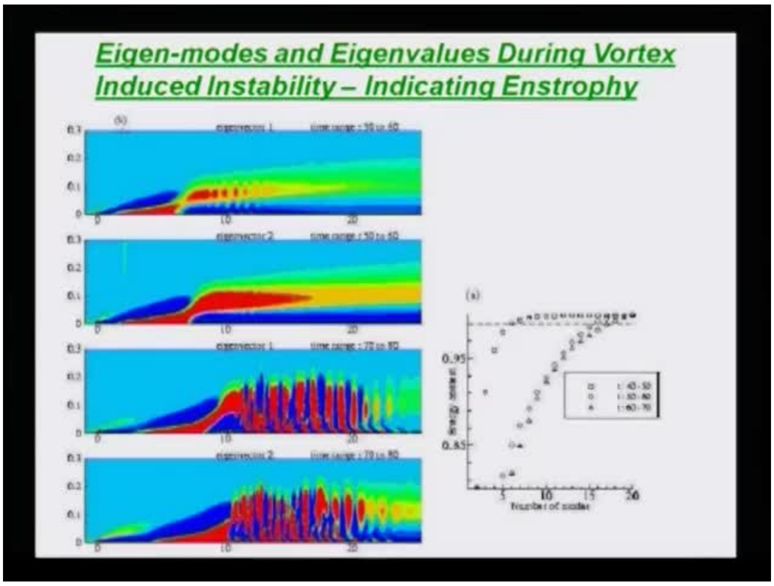

So, we have this domain, and this is where the vortices are, vortex is migrating. We can basically, just simply, look at how the flow is evolving. What you notice that, when the primary instability has not started vigorously, like the time interval between 40 and 50, then, we look at the way, these are the way, the energy cumulatively increases. So, this first mode may account for almost 80 percent; the second mode, accounts for, here; it could be about, say about, another, 5 and 6, about 0.11; 11percent of energy, second mode carries. So, first mode itself carries about 81 percent. Then, it is about 91, 92; then, the third one takes about 95.5; the fourth one takes you upto here, and this is a dotted line, which is indicative of 99 percentage of energy. So, although we are talking about a physical domain of having very large number of modes, method of snapshots reduces those thing, by converting it into, in terms of only 21 modes; the number of frames, determines the number of modes. So, you understand, why I emphasise on method of snapshots by Sirovich. This is that, that instead of looking at spatial Eigen modes, of modes of very large dimension, we are looking at very few number of snapshots, when we get this.

Well, later on, time permitting, we will talk about most recent efforts in our lab, where we use much more accurate POD analysis and the kind of instability scenarios that we have seen, in various external, internal flows; it is amazing. But nonetheless, you notice that, here, 99 percent of energy is given by first five modes, during 40 to 50. The moment your primary instability starts, and vortex instability occurs, then, you see what happens. Then, you see that, to reach that 99 percent, we virtually exhaust, all the 21 modes that we have; maybe 16 to 17, that kind of a difference. So, this are given at various time intervals of 40 to 50. The second data set is 50 to 60 and the third data set is 60 to 70. And, what we can do is, we can actually compare, let us say, the Eigen vectors at two time intervals. This is, these two are Eigen vectors 1 and 2, during time interval 50 to 60; and these are the same two Eigen vectors, but now, during the time interval of 70 to 80.

Well, the direct comparison will tell you, what is really happening. This is about the insipient stage; you can always see this multilayered disturbance of vorticity structure, but cellular structure started forming. And, once they develop, in time, you can see that, each one of them, actually creates this vertical eruptions that we are talking about. So, basically, not a bad deal; you take a real, true, time-dependent flow field; you do the numerical solution. Now, you do the POD, and you get a glimpse of what is happening. And, you understand that, this is a fairly a decent tool to use. Why, because, you are not making any assumption; you are actually getting the non-linear solutions and analyzing them. So, basically, accurate solution of Navier-Stokes equation, plus, it is a proper orthogonal decomposition hand us down, the very powerful technique of studying non-linear instabilities. See, in doing this, getting this results, we have not done any approximation, whatsoever. It is almost a concurrent development, where we develop that energy based receptivity theory. We have shown in the previous couple of lectures that…There also, we did not require any assumptions of parallelism or linearization, etcetera; directly using the Navier-Stokes solution, we could find out, how the forcing is created for the disturbance energy, and that was that. And today, we are giving a complementary picture, that you can look at POD and you can make use of this.

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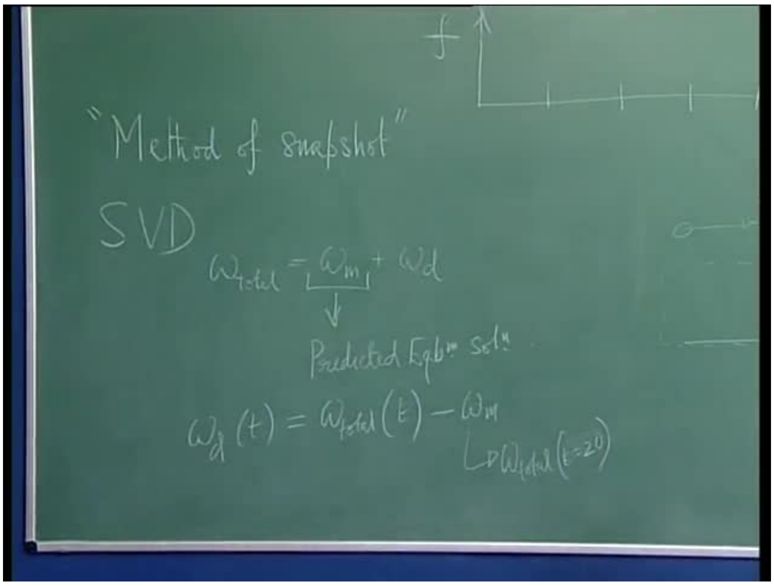

Well, there are much more such pictures available in our 2003 paper. So, you can take a look at them. Although, here, I have written on the y-axis as energy content, it is not truly so; it is enstrophy, actually, because, we are working with a disturbance vorticity. And, on this side, we have number of modes, that is given there. So, I think, this was that, I wanted you to, tell you about POD. So, how do we talk about this? So, basically, what we do is, we solve for the vorticity. So, if I call that as a total quantity, that is nothing, but omega mean, plus, say, omega disturbance. I am not writing (( )), but that is understood; that it may, or may not be a lower order quantity, but at the initial stage, it could be. Now, what you do, you get this and if you recall, while doing that energy based receptivity theory also, we talked about an equilibrium flow; that could be the flow, before the instability start occurring. So, this is kind of predicted.

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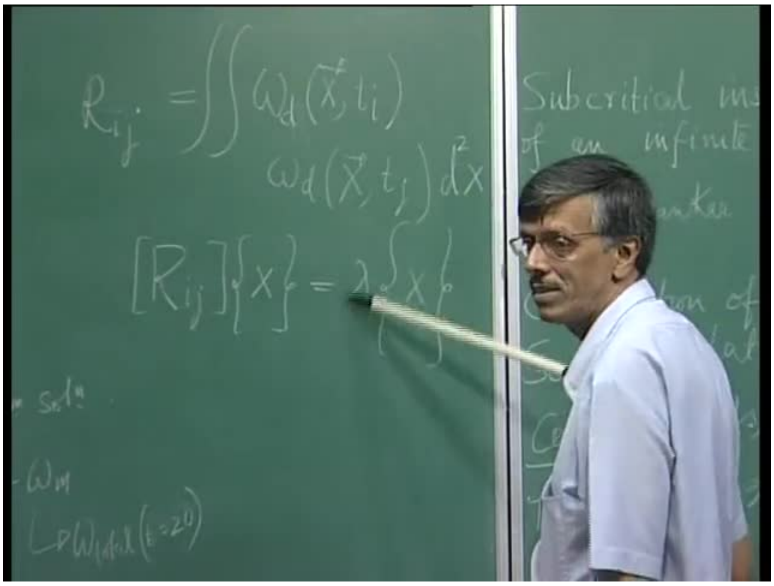

So, maybe, in this case, we would have taken at t equal to 20 as equilibrium solution. So, if I have this, then, it is rather easy. So, omega d, at any time, I could write it as, omega total at that time, minus the equilibrium solution. And, that equilibrium solution, as I was saying, it could be the omega total itself, at a earlier time; maybe at t equal to 20. For this particular case, that we have shown here, it is like this. So, what happens is, at each and every time, I am getting this disturbance vorticity. Then, what I could do is, I could create a co-relation matrix, which I, whose element I could call as a R i j. So, i and j index refers to different time instants and that, I will be talking about say omega i at… So, this is omega disturbance quantity at ith time frame, for some x vector; I will rather, write it as t i. So, that is what it is, and then, multiply by omega d, this at t j; and then, I perform the integral over all the collocation points. So, in this case, we are doing a two dimensional calculation. So, we will be integrating over the whole domain. So, this is this. So, once I have this R i j matrix, then, I can calculate R i j times, say, some Eigenvector, is nothing, but lambda times x.

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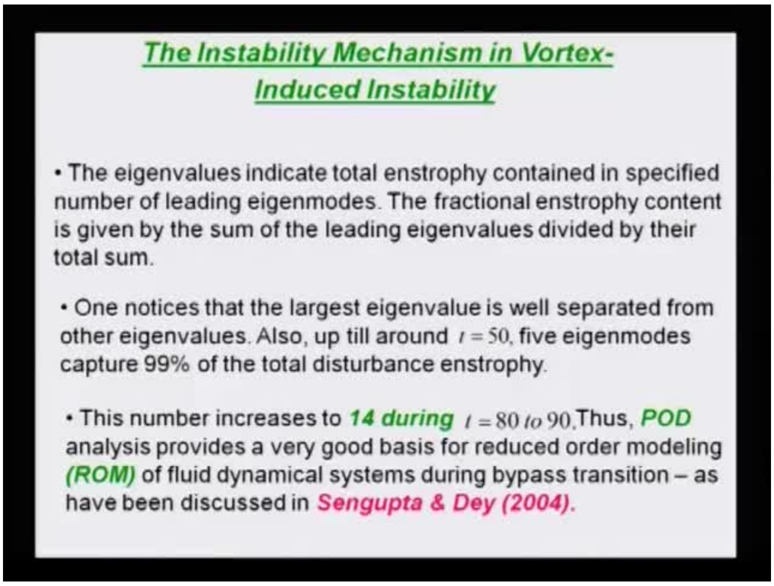

So, once you actually compose this R i j matrix, you just, simply have to calculate its Eigen values and Eigenvector, and you know that, there are many ways of doing it. And, that is what we are talking about here, as the Eigen values. So, those Eigen values indicate the total enstrophy, are contained in specified number of reading Eigen modes. And, if I look at a fractional enstrophy content, basically, I add up lambda i, divided by sum of lambda i; that gives me a fractional content. That is what we are talking about. We take the partial sum over leading Eigen values, divided by the total sum.

So, that is how we define. And, one notices that, largest Eigen values is well separated from other Eigen values; also upto, until about t equal to 50, 5 Eigen modes more or less captured 99 percent of the energy, total disturbance enstrophy. And, the number increases to 14, during t equal to 80 to 90. So, that is this. This POD analysis is also given alternative names. People talk about reduced order modeling. Reduced order modeling, because, instead of looking at an infinite dimensional system, you are basically talking about, say, in this case, say, 14 modes. That, the idea is the following; that, if I could reduce the number of basis, which defines this dynamical system, quite adequately, then, there is a possibility that, I could change the governing partial differential equation into a set of coupled, ordinary differential equation.

The number of those ordinary differential equations happens to be low. So, we may be talking about evolution equation of POD amplitude. So, basically, what we are talking about, well, I think, we will comeback to more detail of POD later, but let us just keep our discussion today, in a qualitative plane. Basically, we show that, this Eigen modes consist of a time dependent amplitude and a space dependent part. So, this R i js are obtained from the space dependent part and this amplitude or the time dependent part, those evolution equation I am talking about, in reduce ordered models, are those dynamical equations for those amplitudes. You can just take this representation of space-time separation, put it in the Navier-Stokes equation, make appropriate modifications or manipulations; you hope to get those dynamical system model for those time dependent amplitudes. And, this amplitudes that we are talking about here, I have only 14 of them. So, this is what was originally proposed. We have taken it to a different level, because, we consider that, we have the solution correctly available with us.

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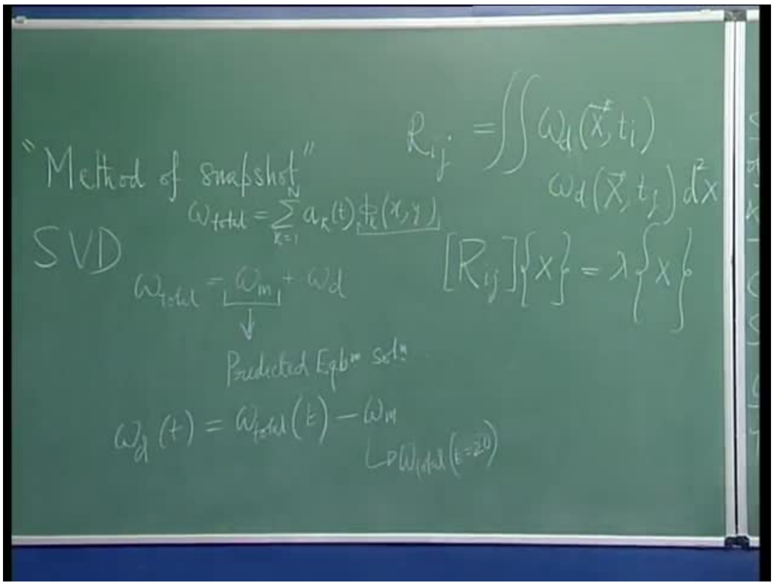

We have this available all the time. So, if I now get the spatial modes, I can actually, since omega total is written as summation of, let us say, a time dependent function, and the Eigen function here, which is a function of, let us say, x and y, and k goes from 1 to n, then, what we are talking about, since this POD analysis helps us getting this spatial dependent functions, like what we plotted, in the previous transparency…

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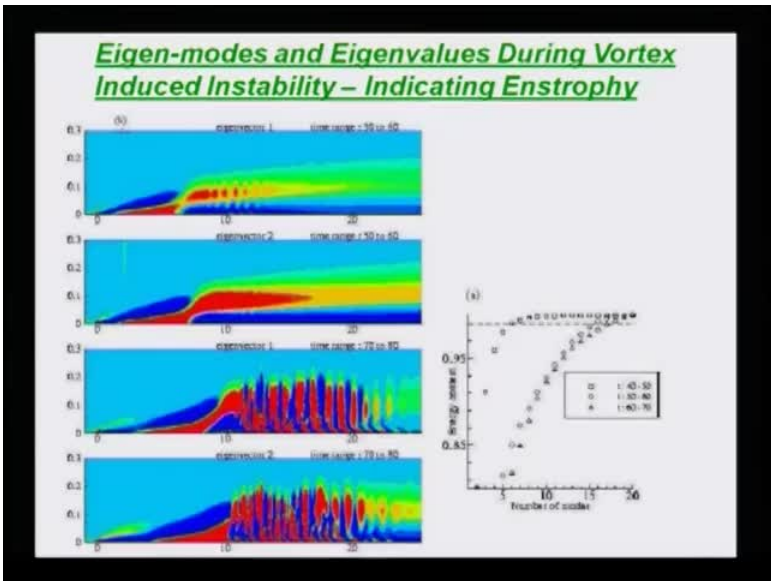

In the previous slide, we did show, these are those basically, phi of x, y. Here, we are showing phi at different time intervals. So, this is the phi 1; this is phi 2 during t equal to 50 to 60. And, this is basically, a phi 1 and phi 2 during t equal to 70 to 80. So, what exactly we are doing, we are actually, taking the data over a finite time interval, and make, saying that, during that time interval, the first mode, on an average, looks like this. So, this is a statistical depiction; these are not qualitative, quantitative, deterministic model of the function, because, the system is varying. So, we are projecting a, kind of a, average value. Now, once we do that, so, we have this; we have this; we can easily obtain it. Why and how, because, these modes are orthogonal. So, we can use the orthogonal property and we can actually, from the accurate simulation, we can calculate, how this a varies with t. This is the unique approach that we have chosen. We have faith in our direct simulation result. So, we do this and in doing so, we have come out, over the last 2, 3 years, with some fascinating pictures of how this things are obtained as a function of time, directly from D N S results.

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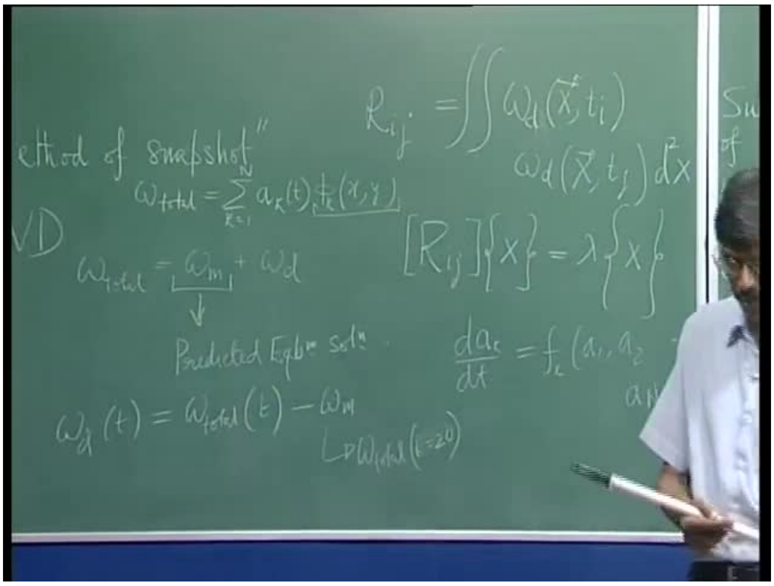

However, originally, Kosambi, Lumley and al, they were thinking of basically, getting some dynamical, system description of this modes. And, this could be some function, which could be functions of a 1, a 2, upto say, a n; we do not know, and that, they tried to obtain it from the Navier-Stokes equation. And, in trying to obtain it from the Navier-Stokes equation, lot of limiting assumptions have gone in, and in the process, people have lost track, of how good is, this evolution equation is. But, if you look at the way that we are talking about here, this is entirely different. We know this exactly. We have a very good description of i, then, we can get a k as a function of time, and we can plot it, and we can see a new classes of modes.

In fact, only last month, or maybe, just talked about this modes, this a ks, as some varying generic classes of modes. We have seen, there is a universality of flow, wherever you have vortex dominated flow. We compared two classes of flow; one is flow past a cylinder, a blub body; you know that, it sheds Bernard Karman vortices. So, you have coherent structures, and then, we also studied flow inside a cavity. And there, the lid motion creates a rotary, coherent structures, and what we found that, there is a universality of this classes of modes that we have obtained, in the external flow and the internal flow. This is the something that, we can say, with some amount of conviction, that a new portrait is emerging, by using POD in studying non-linear instabilities.

So, what we are now going to do is, we are going to talk about another example, where vortex induced instability comes into picture. Now, those of you, who may have had some exposure of doing some design course in Aerospace engineering, so, if you have done it, you already know. Those of you who do not know it, let me tell you, what is the scenario, that past second world war, the aircraft speed started increasing and with the increased speed, you know that, you cannot afford to keep the wing as straight. You have to sweep it and the moment you sweep it, you start getting a different qualitative picture of the flow. It is a very nice, fascinating tale, that, of course, during, and around second world war time, there was a resurgence in activities in developing jet engines. And, people realized, of course, that, you try to go at a very high speed, then, you will create normal shock waves, and that would lead to a huge drag penalty. And, it was so huge, that people kept talking about sonic barrier; that, as if human being will never be able to cross this speed of sound. And then, of course, engineers are, by nature, optimists. They do not give up so easily. They kept on studying it and one of the conference, I think, it was in Voltas, in 30s, when leading aero-dynamists came about and some of them even suggested that, one of the solution is, the speed that is normal to the leading edge of the wing.

So, if I sweep the wing, then, what happens? Although, my oncoming flowback number could be say 0.7, 0.8, but the normal to it, it can be kept, say, less than 0.4. And then, the resulting shocks, those will be created, they will not be normal shock; they would be oblique shock, and they will be weaker. And, there is also a possibility, which came about much later, that you make the shock absolutely weak, and those are called the super-critical wings. They have come about in 60s and 70s. In fact, these days, most of the aircraft that you fly, commercial, long distance airlines that you fly, all have this simple critical sections and wings. These are called natural laminar flow wings. We talked about it so casually, that it was bound to happen. But in the initial phase, it was very confusing, because, we have noted that, for two dimensional sections, for aerofoil sections, like what you would have in a straight wing, the flow becomes unstable, with an adverse pressure gradient.

But when the wing was swept back, then, people noted that, the flow transition is occurring over the wing, where actually, you have favorable pressure gradient, not adverse. And, this was a very confusing thing, and time permitting, we will come back and talk about stream-wise and cross-flow instability of three dimensional flows past a wing. People have now gotten, some understanding, of what causes three dimensional flows to be qualitatively different than two dimensional flows, in showing these instabilities. Concomitant with this story, there was also another interesting story. People found out, that sometimes, the flow becomes turbulent, right from the leading edge of the swept back wing; it, it was not so, for a straight wing. It was only for swept back wing, people noted that, the flow becomes turbulent, right from the leading edge. And, some of you may be doing a design course.

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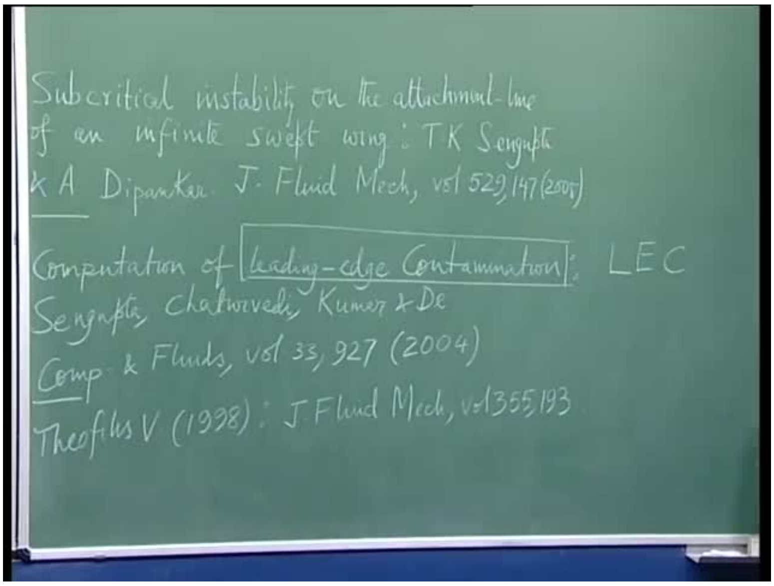

When you calculate the drag estimate for the wing, you assume the flow to be turbulent, right from the leading edge, for that reason; because, in actual wing design, if you are not very careful, if you do not take care, flow actually becomes turbulent, right from the leading edge. And, this problem is, what is called as Leading Edge Contamination, or we call it as LEC. As you can note that, even in 2004, we have been computing and trying to explain, why a leading edge contamination takes place.

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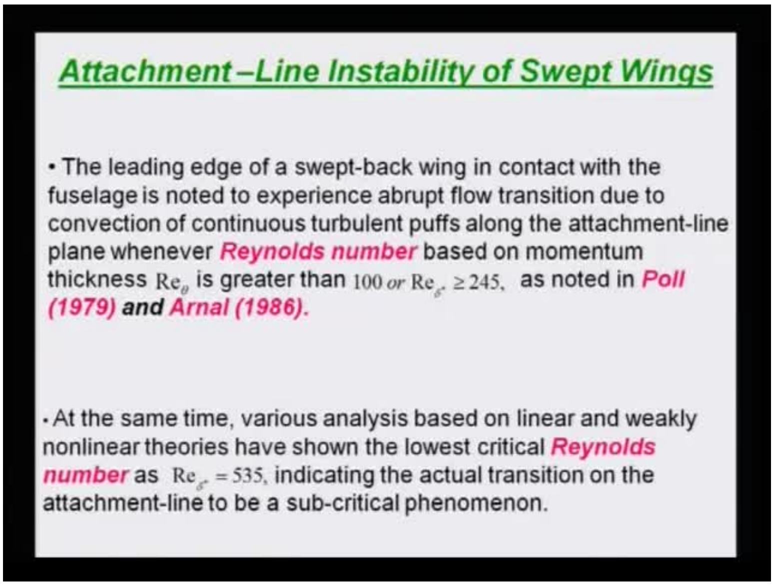

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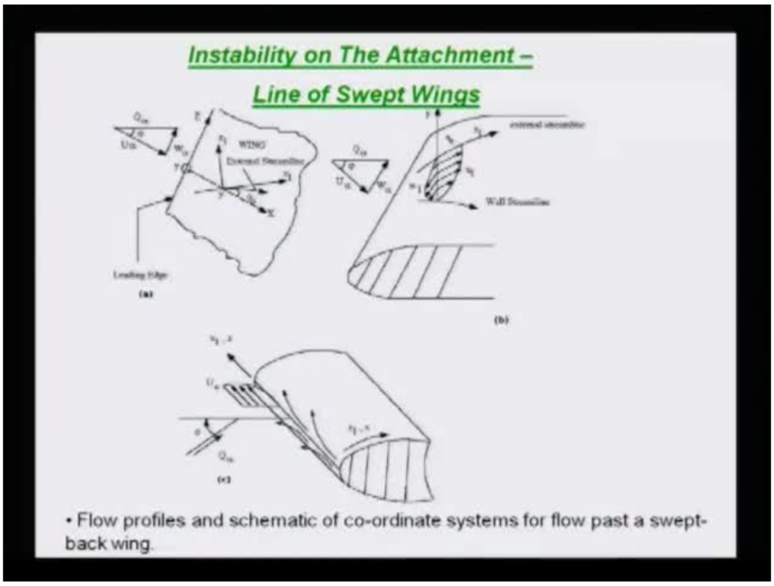

So, it was a kind of a puzzle, jigsaw puzzle to aerodynamicists for a long time. What is a attachment line? Let me first show you, with the help of a figure; then, we can come back. There, it is. Suppose, you have a swept wing. So, basically, we have drawn it in such a way, that actual q infinity, is the total velocity vector, which has a normal component, which is u infinity; and, this panelized component, along the leading edge, which I call as w infinity. Now, what happens, when I look at the flow field around the leading edge, you actually see that, when you have a swept back wing, you have a plane here, which is almost like your stagnation point plane. So, that is the attachment line plane. However, flow here is not stagnant. What is happening here? The flow actually goes, along the leading edge. So, if I am looking at basically, let me just explain to you, what we are seeing. We are seeing the following; it is mostly related to aeronautical domains. So, let us talk about it.

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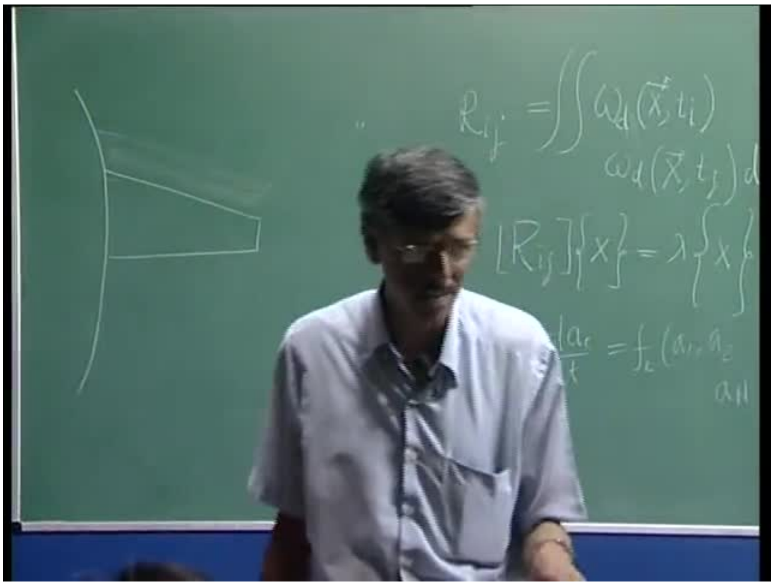

So, this is your, let us say, the fuselage, and this is your wing; and, what is happening here, is some kind of a vortical structures are getting created at this corner, and they keep moving along this plane, and in this plane. So, in this plane, it is like this, where I have shown you; if you project it out of the plane, then, that is your attachment line plane; on top of which, the flow actually wraps up like this. And, in the bottom, it rolls down like this. So, it is like, a kind of, a layer of separation. Attachment line plane is the layer of separation. So, along that line, what happens, why the flow becomes turbulent, that is a big mystery.

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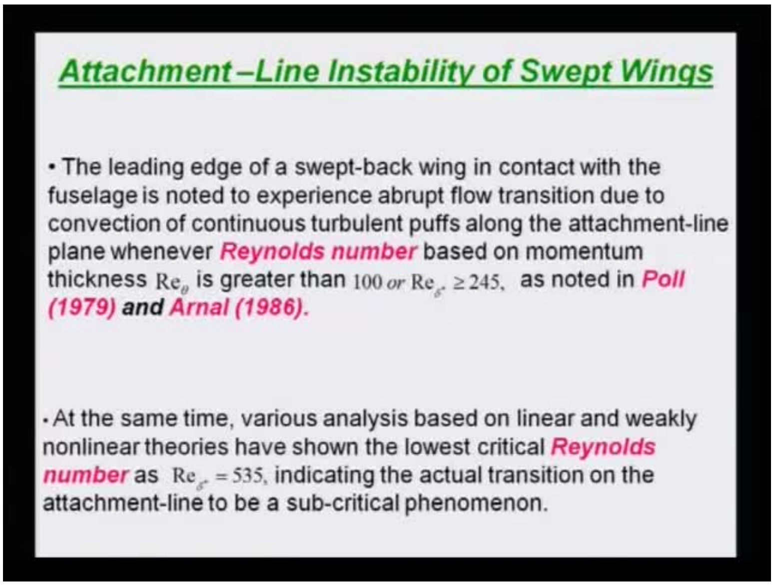

Now, let us go back and give you, a kind of, a running commentary, of what people were looking at that. When we look at the leading edge of a swept back wing, in contact with the fuselage, you notice that, there is abrupt flow transition. And, this was later on studied variously by Poll and Arnal, who noted that, this abrupt flow transition is due to convection of continuous turbulent puff along the attachment line plane, whenever Reynolds number based on momentum thickness, that is, we call it as Re theta, is greater than 100, which is equivalent to Re delta star of greater than 245. These are essentially experimental observations.

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So, whenever Re theta is greater than 100, or Re delta star is greater than 245, you see the flow becoming contaminated, right at there. So, how do they do this experiment in the tunnel? It is fairly simple. If you have a tunnel like this, so, these are your tunnel walls; then you fix a wing, from tunnel wall to tunnel wall. Then, what happens, flow is coming like this. Because of this junction, you will create eddies, corner eddies and because of the sweep, those eddies are not able to stay there. They are forced to convect this way. That is what they are talking about, that convection of continuous turbulent puffs. Well, you have vortices, but because of the inherently large Reynolds number, they could be quite significantly colorful, in terms of spectrum. That is why, they have been called as turbulent puffs.

See, while the experimental values of critical Reynolds number, given here as 100, based on momentum thickness, or 245, based on displacement thickness, people tried to study the stability of the flow. How do you study the stability of the flow? In this case also, once again Prandtl comes to our rescue. Way back in 1946, he has established, we will see it shortly, that the flow in the attachment line plane is essentially, two dimensional. So, here, on the attachment line plane, the mean flow, the equilibrium flow, is two dimensional. And, if you try to study its instability, then, the corresponding critical Reynolds number comes out as 535, if, so called weakly non-linear theories are invoked. If you do not use weakly non-linear theory, linear theory gives it a little, even higher value. That value, let me just quote to you, it is about 583; that was calculated by Hall, Malik and Poll in 1984.

So, we have a linear theory, which tells us a critical Reynolds number of 583.1 and if we invoke little bit of weak non-linearity, we can bring it down to 535. But still, it cannot explain the gap between 245 to 535. So, this was considered a kind of, a mystery; very much of mysterious state of affair. Now, let me just tell you that, this kind of sub-critical phenomenon, we thought that, we could explore it as a vortex induced instability, because, this is a prime candidate. You know, that is what we are talking about. Suppose, I have a leading edge of a wing, like this, and I have this attachment line plane, and in that plane, the vortices are convecting in this way, and this flow is two dimensional, then, it is quite natural for us to investigate this, as a vortex induced instability problem.

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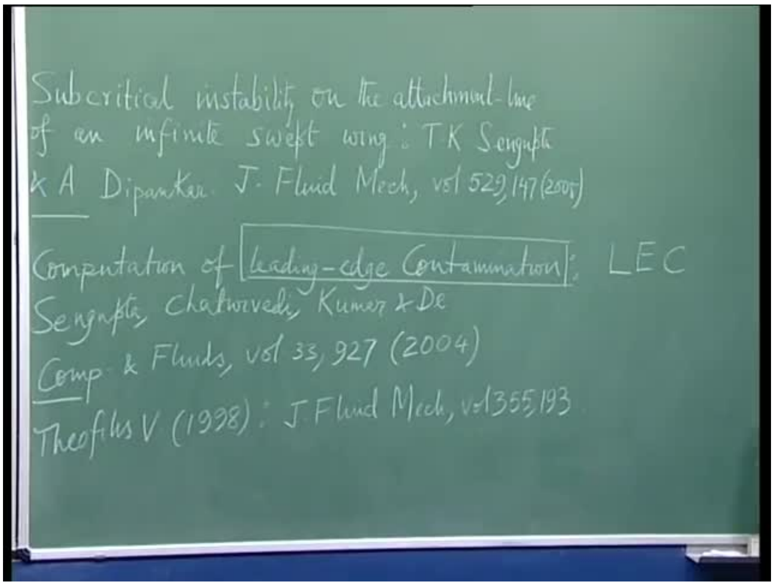

We have the source of vortex as the wing body junction and we have the flow to be two dimensional. Then, we have done this experiments, that we have seen, that, we could, depending on the sign, we could create instabilities. Why do not we explore this and that led us to this two papers. First, we did some computations in this paper in 2004, that was followed by a detailed study in that JFM paper, that I have. Of course, we have, once again, looking at sub-critical instability, because, experimental observation is 245; theoretical calculations is 583. So, it has to be a sub-critical excitation.

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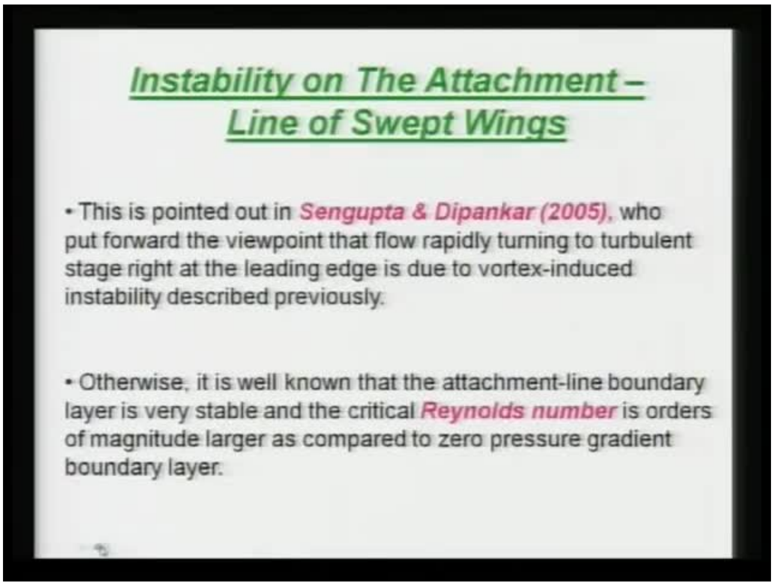

So, now, this is what we did, talk about in this JFM paper that, the flow that rapidly turns to turbulent stage right at the leading edge, might be due to a vortex induced instability; that we have already described. We know that, attachment line boundary layer is very stable and this observation that I make here, is very crucial. You see, there are two things. In two dimensional flow, we have a stagnation point flow, that the Hiemenz flow; for Hiemenz flow, the critical Reynolds number is something like, greater than 10000; whereas, zero pressure gradient boundary layer, it is 520. So, there is a factor of 20 involved in 2D flow; but in 3D flow also, we see that, that gap has come down, but still, what we can see that, that is slightly more stable, than your zero pressure gradient boundary layer, because, we saw the Hall et als' result indicated, a critical Reynolds number of 583.

So, it is slightly more stable. But here, what is being said, it relates to comparing this 2D stagnation point flow versus the zero pressure gradient boundary layer. So, 520 versus, greater than 10000; whereas, if I look at the stagnation point flow instability versus attachment line instability, it is not greatly different; it is also greatly different, because, for stagnation point flow, two dimensional flow, it is like more than 10000; for attachment line flow, it is 583. You see, this is one of the reason that, if you are talking about high speed flow, then, you will have to keep the wing, to have some kind of sweep back, and this kind of phenomena happens. Attachment line instability is due to the sweep back itself, and due to the corner vortices. This predominantly happens, if your sweep angle is more than 20 degrees. So, many a times, you will hear people talk about, if we do not have a sweep angle less than 20 degrees, then we have 3D instabilities.

So, less than 20 degrees, you have a transition, going from 2D in stagnation point flow to attachment line flow. So, this is something that we must remember. Now, Theofilis and his co-authors, wrote in 2003 in a JFM paper that, till that date, there were two major issues, that remained unanswered. The first is, of course, related to the phenomena of sub-critical instability right in the attachment line plane itself. So, that is the first issue. This is what we tried to do it, in this couple of papers. There is also a second issue that, if the flow becomes turbulent at the leading edge, what happens to the flow subsequently, over the wing. So, you see, it could so happen that, it could have some kind of a turbulence set in, right here at the leading edge. But that need not go down. So, that is the second issue. Even if the flow becomes turbulent, as it goes along, and because it is a sweep back wing, so, the stream lines could go like this, and because of various, prevalence of favorable adverse pressure gradient, it, those flow may not be turbulent itself.

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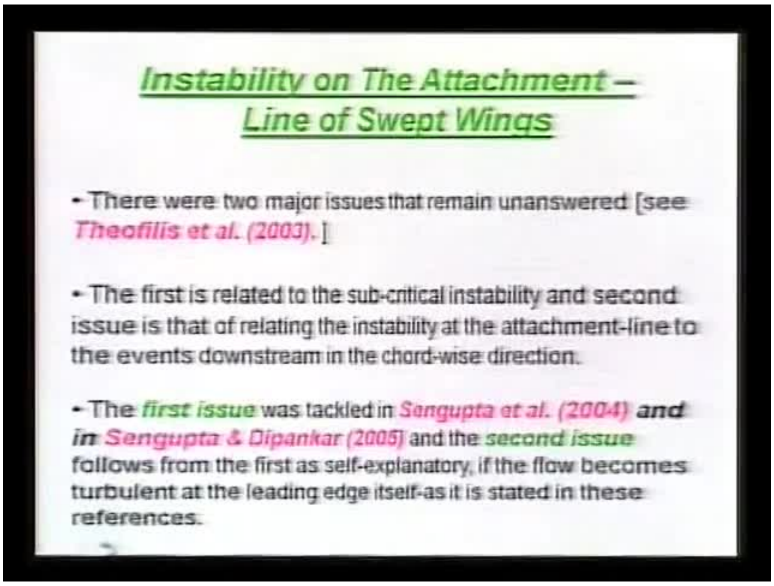

So, these are the two issues, that one did not know in 2003; and at least, the first issue of the primary instability, was explored by us, in this two papers, and that is what we are going to discuss now.

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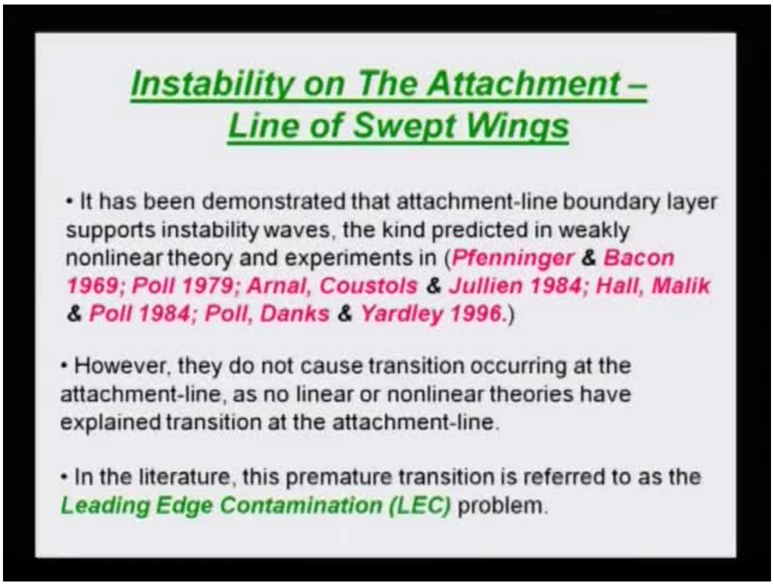

Now, people were of course, very much mystified for nearly 40 years. You can see that, Pfenninger and Bacon, in their 1969 paper, conducted experiments and what they were looking for, that an attachment line boundary layer, if we can see some kind of instability waves. And, they did experimentally noted this. This experiments were subsequently done by Poll, Arnal et al, then, Hall et al, and then, Poll et al again, and you can see, a continued interest, and it was remaining, sort of difficult to, sort of explain. In fact, I think, it was in one of the early paper by Theofilis in 1993, he made a remark, tried to compute 2D and 3D flows, and apparently, could not see any instability, occurring for the two dimensional flow. So, he made this claim that, two dimensional model equations considered, do not deliver sub-critical instability of this flow.

So, he was trying to write out evolution equation for 2D and 3D disturbance field, and this was in this paper in JFM 1998, he did that. Subsequently, of course, he teamed up with other people at Gottingen, one of the ((DL)) Labs, I think, it is Gottingen, and they tried to study this; and, one of his student also, did some computational work, to see basically, it was very interesting. They were trying to mimic our vortex induced instability scenario, and they were not successful. Now, the reason is easy to find out. We will talk about it, as we go along, when we talk about various efforts gone in there. But one thing, that you notice that, you do have instability waves, experimentally. But no one has established that, they will cause transition on the attachment line. We do not have any linear and non-linear theories, that explain this. So, in that context, this was something of a revelation. That is why, this premature transition is referred to as the leading edge contamination; so, as if, the flow gets contaminated by some unknown disease; and that was something, that remained unknown, for quite some time.

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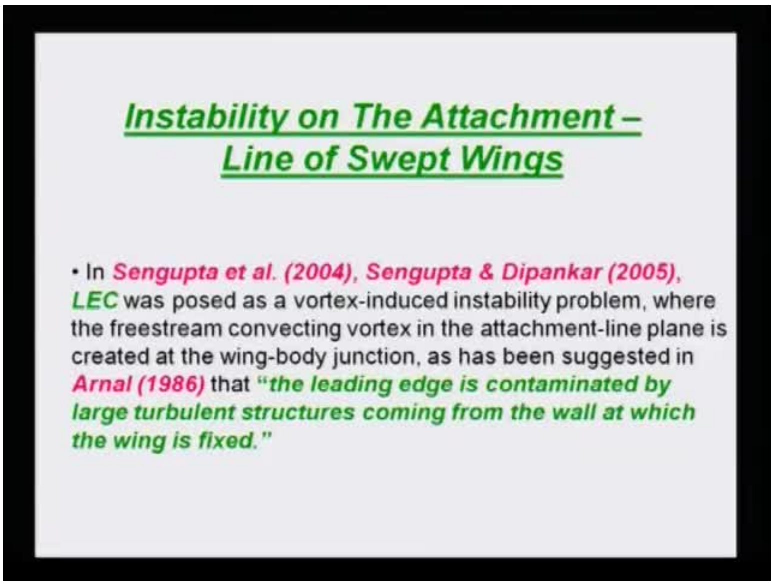

Now, what we did in this couple of papers, we just posed it as a vortex-induced instability and the freestream convecting vortex in the attachment line plane is being created at the wing body junction, because, this was very much apparent. Experienced researchers like Arnal, they have noted this that, the leading edge is contaminated by large turbulent structure coming from wall, at which the wing is fixed.

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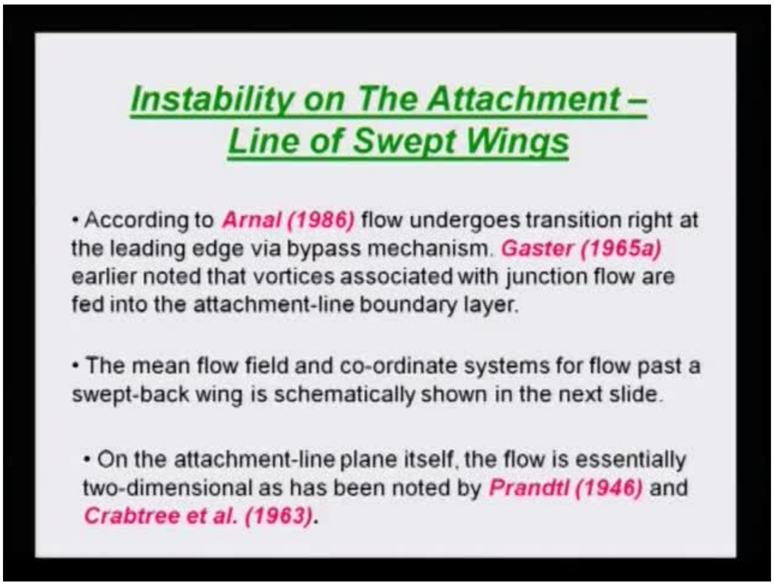

So, if that is so, then, it is very easy for one to go ahead, and do that. In fact, Arnal called this as a bypass mechanism. We would also call it the same. Gaster even noted very early that, if you can somehow make this vortices coming from wing body junction become weak, then, you would not have such a contamination. And, I think, he has a patent also; Gaster's device at the leading edge, how to do it. But, you know, like, these days, you can go to the tunnel and you can refine the wing body junction in a much clear way and you can do that; that is one of the story behind all the NLF wings. They spend, in designing that wing, in the last stages, they do this wing body junction design, in a very careful manner, so that, your section properties are good; your wing properties are good, but, you still should not suffer from leading edge contamination. And, that is done experimentally, so that, you do not create those large vortices at the wing body junction.

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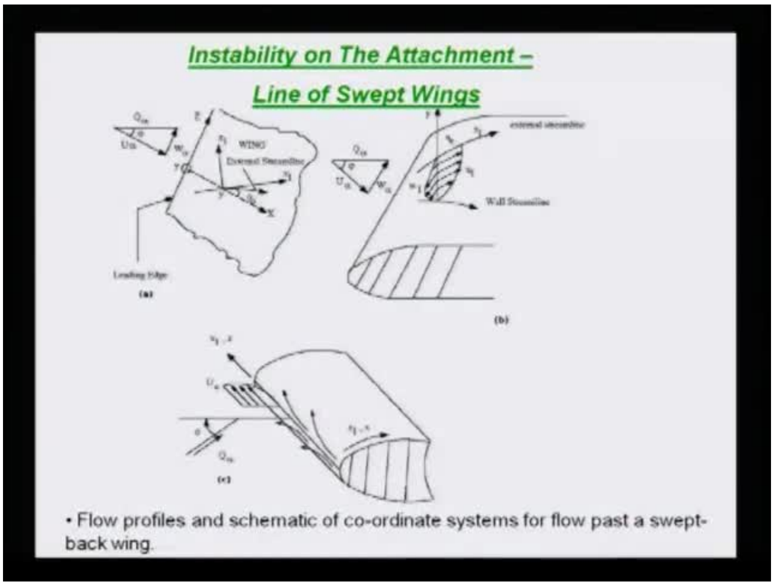

So, as I told you that, people had given us enough clue, how to go about it. We will show this flow field, etcetera, but we note that, this is a, another gift given to us by Prandtl, and also, in that book by Crabtree et al, that this flow in the attachment line plane is two dimensional. So, this might help us in studying this. I mentioned to you that, there were some computational work done by Schmidt's student, (( )) and Schmidt. They tried to compute the flow as a vortex induced instability, but they did not succeed, for various reasons; we will talk about that later. But, I think, one of the main culprit was that, they considered it as a three dimensional phenomena; it is not a two dimensional phenomena. That is why, they lost it; and then, second most important thing they did wrongly was, they thought, this vortices will move at freestream speed. And, now, you know, through your assignment problem also, that, you have a very selective basis for c, which is conducive in creating this. So, now, we have seen this. Now, let me familiarize you, with various coordinate systems that we use, in defining the flow. Like here, in the lower frame that you see, q infinity is the total oncoming flow field, which is at an angle alpha, to the normal, to the plate. So, you could get a u infinity and a w infinity.

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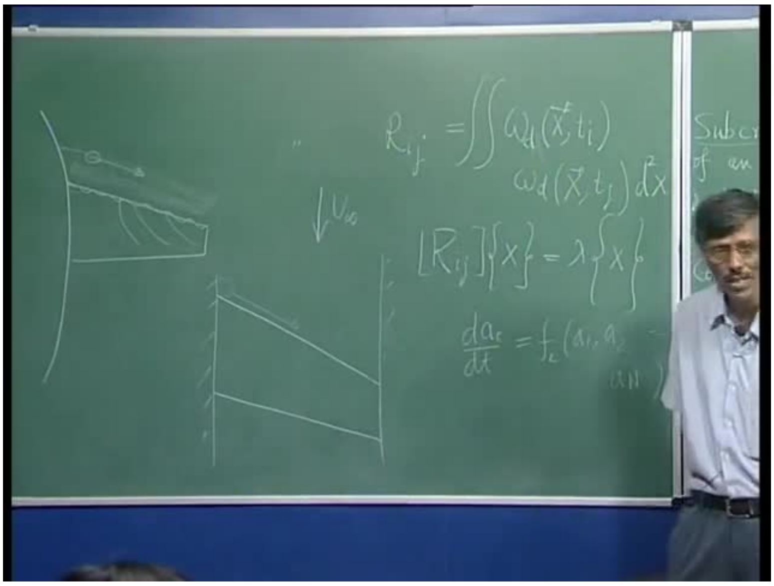

Now, in this analysis that we are going to talk about, we are going to talk about, as if this wing is infinitely swept; infinitely swept means, it is a never ending wing, on this side. So, there are no such wing tip effects involved, and that is not too bad, actually; you can see the equilibrium flow itself, creates a bias of the flow, going from this side to that side. And, even if you prematurely terminate this, you do, you are fixing this wall to wall; that is one of the way of doing a 2D experiment. But, in this case also, you will see a vortex getting trapped, created here, and that can wash out downstream, but it cannot propagate in this direction. So, that is why, if you look at this experiments, they are very trustworthy and reliable, simply for the reason that, the vortical structure that is getting created on the other wall, will not propagate against the sweep; instead, the vortical structure that is going to be created here, will naturally move along. So, that is your attachment line plane; the way we have shown here, it is extending towards you. And then, in that flow, what you notice that, there is this wall stream line; you can talk about a stream line, which is indicative of the flow direction on the wall. And then, once you move out of those shear layer also, you get another stream line, that we call as external stream line.

So, these two lines are not same. In addition, then, we have a wing fixed coordinate system. So, there is this wing fixed coordinate system; this capital X is normal to the leading edge, and capital Z is along the leading edge. So, that is that, and y is perpendicular. So, that is your wing fixed co-coordinate system. Then, we have here, external freestream, stream line based coordinate system. So, we show its projection on the wing itself. So, we project this on the wing itself, that is along this. So, there, we draw a tangent; that is our x 1 axis. Then, we have, perpendicular to it, is a z 1 axis, and y 1, or that is also, the y is fixed on the normal to the plane.

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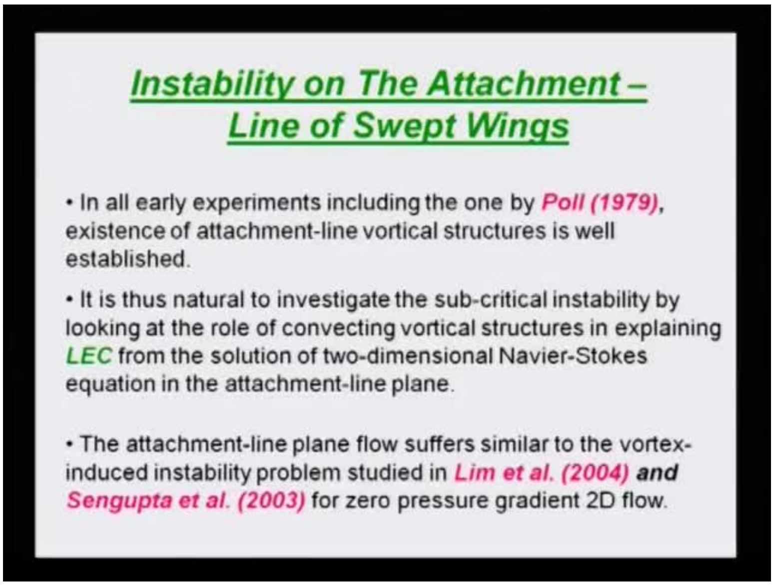

So, these are the various coordinate systems that we should be talking about. So, please keep this in mind that, we would be looking at this. I will pass on this slides to you, for you to take a look at it. So, basically, once again we note that, Young Poll has done this experiment and he showed, there is an existence of attachment line vortical structures. What he did of course, he did a interesting thing. He did not do it with the wing section; he did it initially, with a cylinder called ((log)) cylinder. Usually, when you do the experiment, you have a cylinder, which is perfectly normal to oncoming flow. So, he just swept it. So, he did those experiments, and that is why, we are going to study it at a sub-critical instability, by looking at the role of convecting vortical structure, in explaining leading edge contamination, from the solution of two dimensional Navier-Stokes equation in the attachment line plane.

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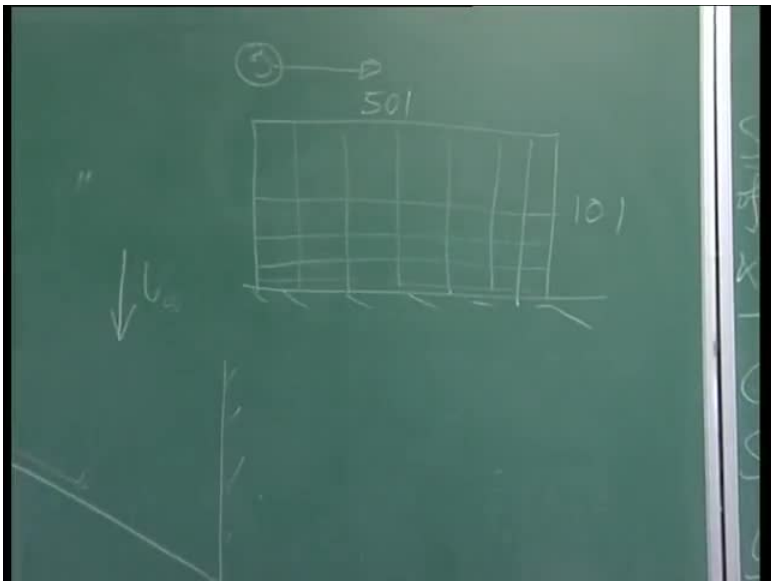

The attachment line plane will suffer similar to vortex instability problem. So, we have stated it time and again. What we do here is, basically, again, we take a two dimensional plane. So, the computational geometry is going to be exactly the same, that we have done it for the earlier cases. So, what we are going to basically study, is a domain like this. So, this is our wing now. Now, there is no question of taking leading edge, etcetera. So, we take a box like this; that is your attachment line plane. And then, you have vortices migrating outside the shear layer; and what we did is, basically, we did take about equi-spaced 501 points in the stream-wise direction. And, normal to this wall, we have taken a stretched grid, like this. So, this, this side we had 501 points; this side, they get 101 points. And, some results we will see shortly, will be for the case of a counter-clockwise circulating vortex, convecting at a speed of 0.20 and at a height of 30 delta star.

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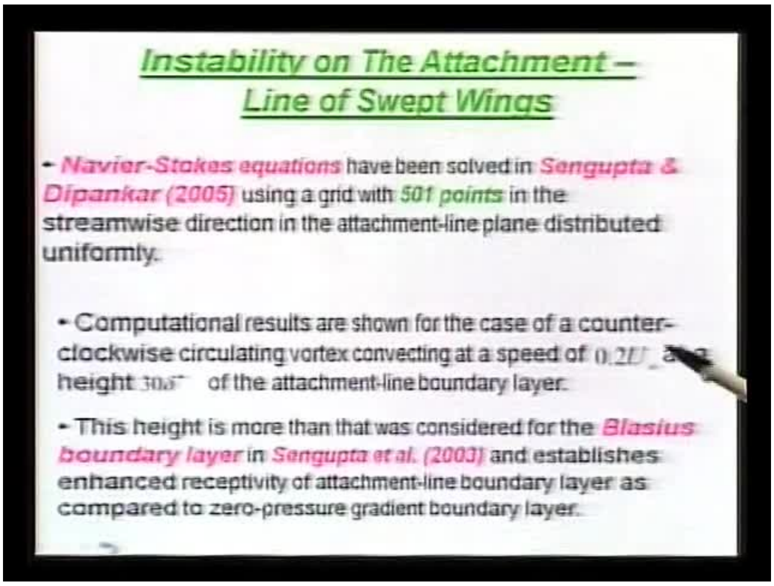

So, basically, we purposely took it little more than what we had done in the previous experimental case, to show that, even this attachment line profile, has a little higher receptivity, because we are keeping the vortex little farther away. So, as compared to zero pressure gradient boundary layer, we have purposely taken this height to be little more.

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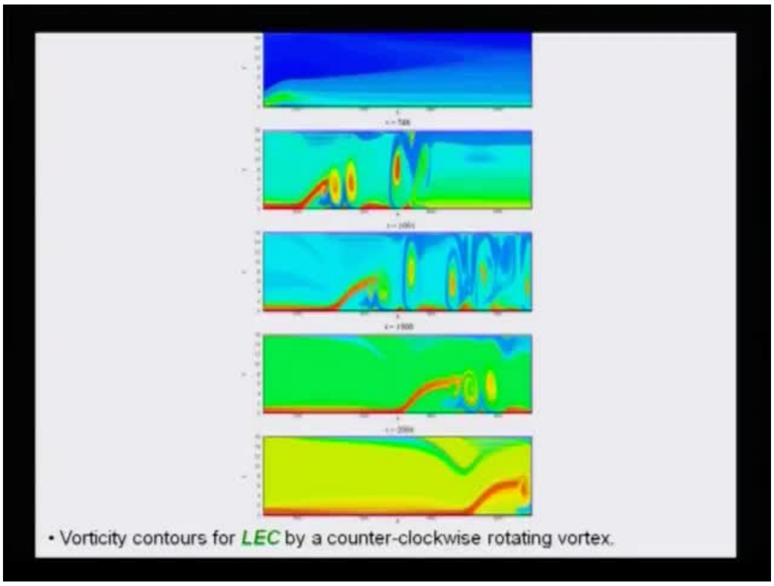

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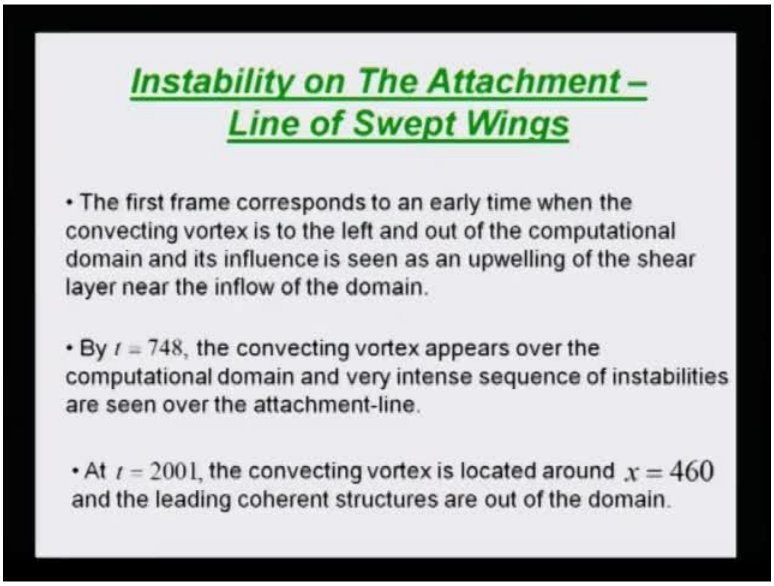

So, this is what you see, as the instability on the attachment line. At earlier times, you can see, again, this bulge in the shear layer, which actually degenerates into erupting vortical structure, at later time. And, this is what is noted, as the vortex, vorticity contours due to a counter-clockwise vortex. Now, in fact, if you look at the first frame, the top frame, that correspondence to a time, when the convecting vortex is outside the computational domain. So, even there, when it was out of the computational domain, its influence is clearly seen here. You can, as I said, that was that. The second frame, that is at t equal to 748, the convecting vortex appears over the computational domain, and the intense sequence of instabilities are seen. And, if we look at, at a large time, the convecting vortex is located at x equal to 460, while the leading edge of coherent structures, they have actually, moved down to the computational domain.

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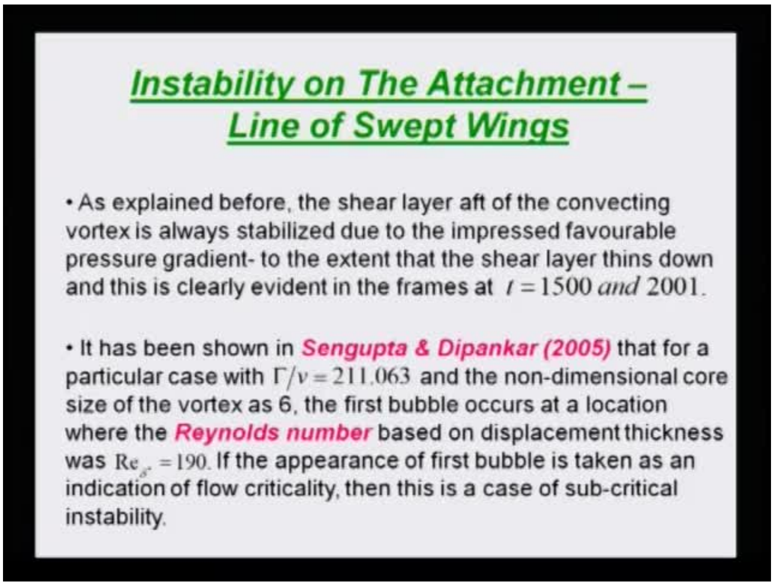

Now, the shear layer that is aft of the convecting vortex, is always stabilized, because, that introduces, a kind of, a favorable pressure gradient, that we have talked about. And, we can see, the shear layer would thin down there, in those region; that you can see actually, in those later frames. If you, if I show it again, you will notice that, in these cases, you can very clearly see that, vortical events are taking place here, but, the shear layer is thinning down, as you are going down; this side, you have a thicker boundary layer; this side, you have a thinner boundary layer.

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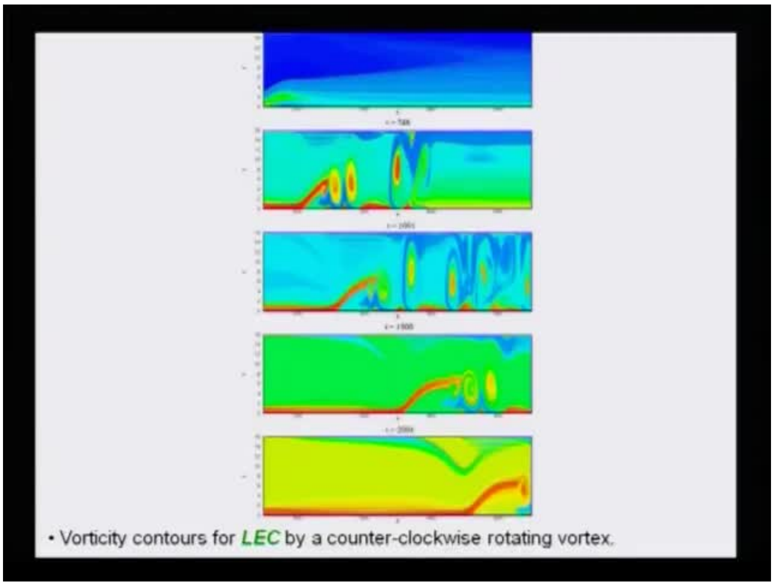

So, these are some of the things, that show one to one correspondence, between this equilibrium flow versus the Blasius flow, that we have studied.

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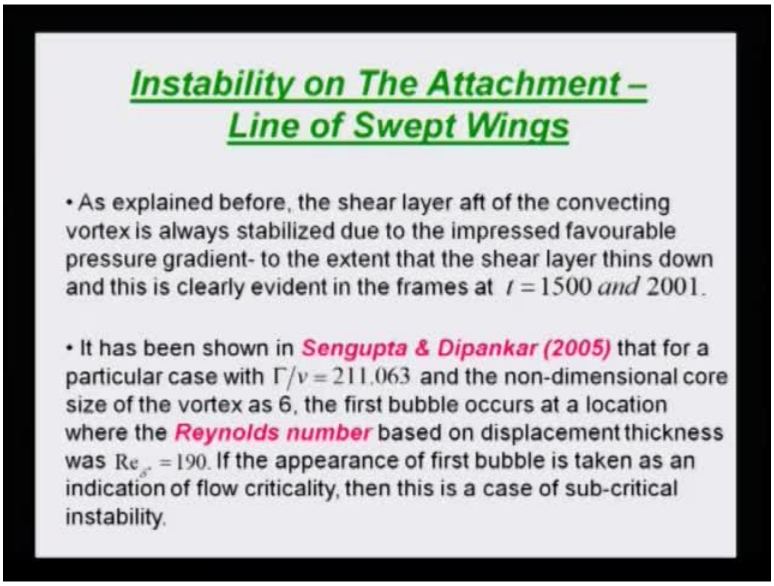

Now, we did show a case in this paper, gamma by v, about this kind of value. See, this is trying to make it, a kind of non-dimensional core size of vortex, as this. So, basically, what it is? You can see that, again, these are finite core vortices; these are viscous vortices, those are going. And, the non-dimensional core size is ok, and the first bubble occurs at a location, where the Reynolds number is basically, about 190, based on delta star. If the appearance of the first bubble is considered as indication of flow criticality, then, this is indeed a case of sub-critical instability that, we are trying to establish. I think I should stop here.