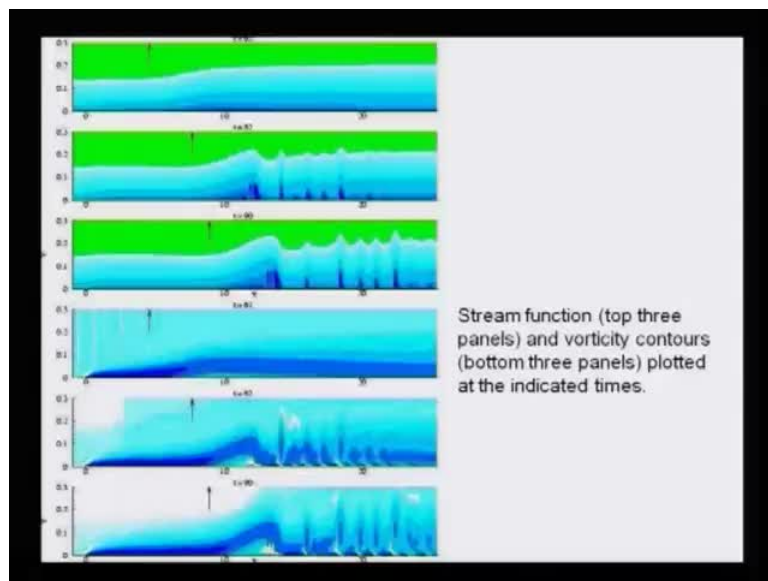


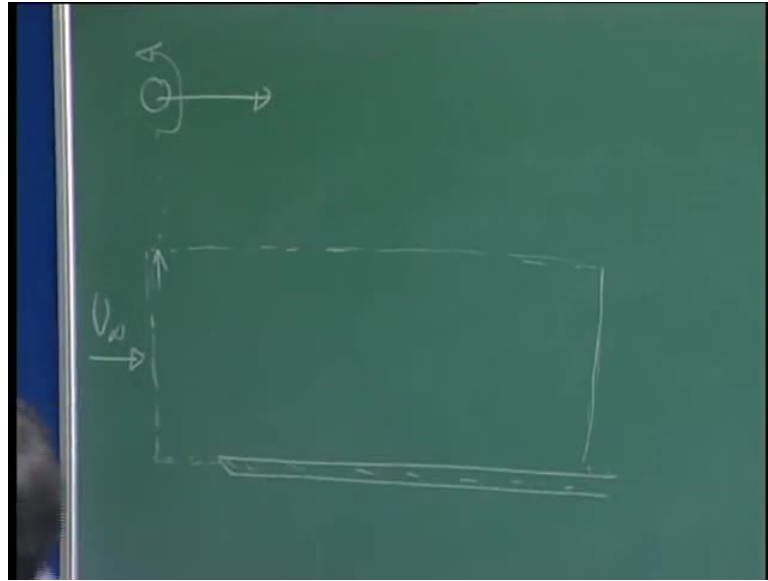
Instability and Transition of Fluid Flows
Prof.Tapan. K.Sengupta
Department of Aerospace Engineering
Indian Institute of Technology, Kanpur

Module No.# 01
Lecture No.# 23

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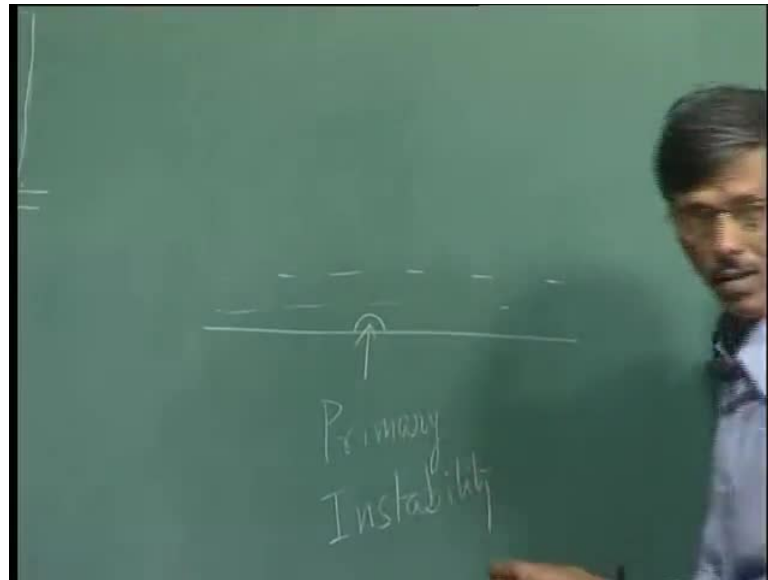
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We started talking about vortex individual instability as computed, and this was kind of a typical set of results shown for a computation, done in a domain like this. What we have basically, is a sharp leading edge flat plate. The flow is coming at 0 angle of a **diac**, and then, over and above in this computational domain, you have a discrete vortex that could be discuss or inviscid, it does not matter, that is conducting very far up. The instantaneous location of this vortex is indicated by an arrow like this; as you can notice there is an arrow here and so on.

The top three frames show the stream line contours for the flow field, for which the calculation is started when this is far upstream in the computational domain, and then, as we go along, for example, this result is shown as, t equal to 62; the vortex is now in our field of view, inside the range of the computational domain given by this arrow. What you see as an effect of this is, there is a kind of a thickening of the shear layer. This is exactly what we said that you get - vortex is a contour clockwise one, then it will tilt to scar it out. That scaring effect, you can basically see, leads to thickening of the shear layer. If you look at it a little later time, this was t equal to 62; these are t equal to 82. You can see actually, what has happened; this scaring leads to a bubble formation, and since the cause is moving, the result in bubble also moves.

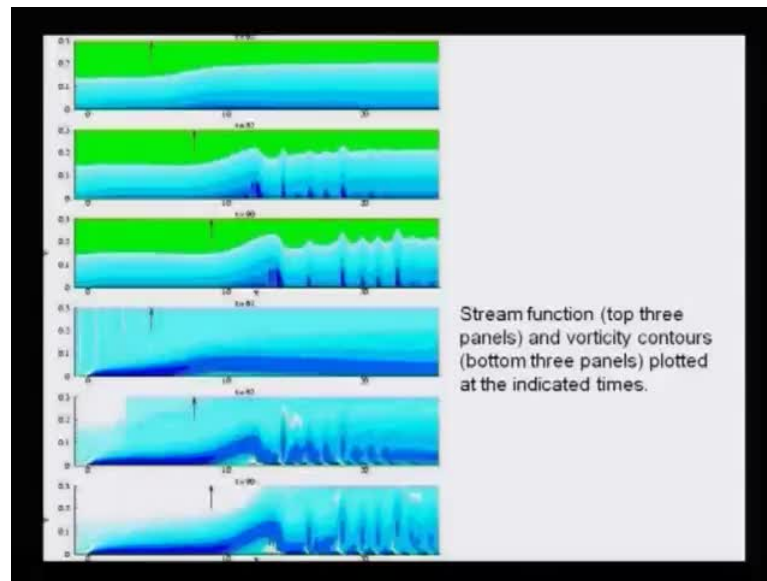
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That unsteady separation bubble, as it moves what it does? It creates an altered pressure gradient around itself, does it not? So, think of the following, if you are having a flow like this **quiescent** flow, well, the stream lines are somewhat like this. It just simply diverges because of the thickening of the boundary layer.

Now, if I create a separation bubble like this, then what happens? Locally, on this side I am going to create a strong adverse pressure gradient; and that adverse pressure gradient will farther destabilize.

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So, if this is a result due to a primary instability, then, the primary instability itself will cause space pressure gradient ahead of it, and that would lead to the secondary and tertiary and so on and so forth, and you can say what really has happened. As we saw in our analysis in the last class that a single convecting vortex creates an adverse pressure gradient ahead of it that leads to a major primary bubble forming, and you can see the distance of separation is quite ahead; but then, that itself creates a secondary and tertiary instability ahead of it, and this movement, actually, is very pronounced. So, what happens is once the primary bubble starts, you actually switch on a cascade process. This leads to a sequence of instabilities, and this is what you see, in this kind of dark blue shade, indicating locations of secondary and tertiary bubbles forming.

If you look at it at a little later time, at t equal to 90, you can see the primary bubble has not moved very much; but look at the speed at which the secondary and tertiary events have occurred. You can also notice, the edge of the shear layer, which is given by the edge of this blue line, you can see that these are sort of lifted up; we call this as vertical eruptions.

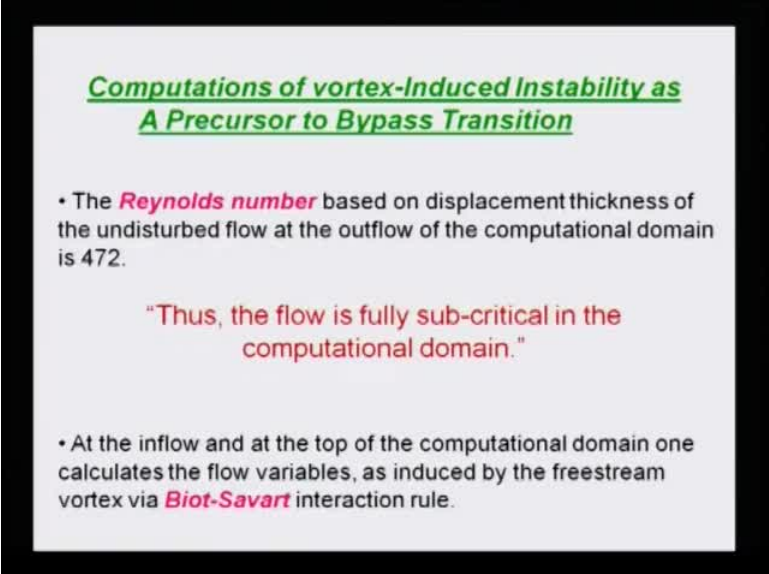
These individual bubbles indicate lump vortices. These lump vortices, as they all move down stream, are also picked up. These vertical eruptions are a trade mark signature of even, fully developed turbulent flow.

We will talk about turbulent as you go along. You will see that even there, at each cycle of turbulence, you have various stages of flow development. One of the stages is called, the ejection stage, and during the ejection stage you do see such vertical eruptions.

Basically, through this control numerical experiment, you can pick up those basic prototypical modules, which you expect to see even for fully developed turbulent flow. So that is what it is, and if this is your stream line contours, which shows little subdued effect, you can see much more magnified effect if you plot the vorticity contours, and that is what you are seeing here.

Once you notice these vorticity contour plots, you can see this thickening of the shear layer at, t equal to 62; and here you can start seeing that effect of this primary bubble growing, leading to secondary and tertiary instabilities. Here, you could very clearly see these vertical streaks - the vertical eruptions.

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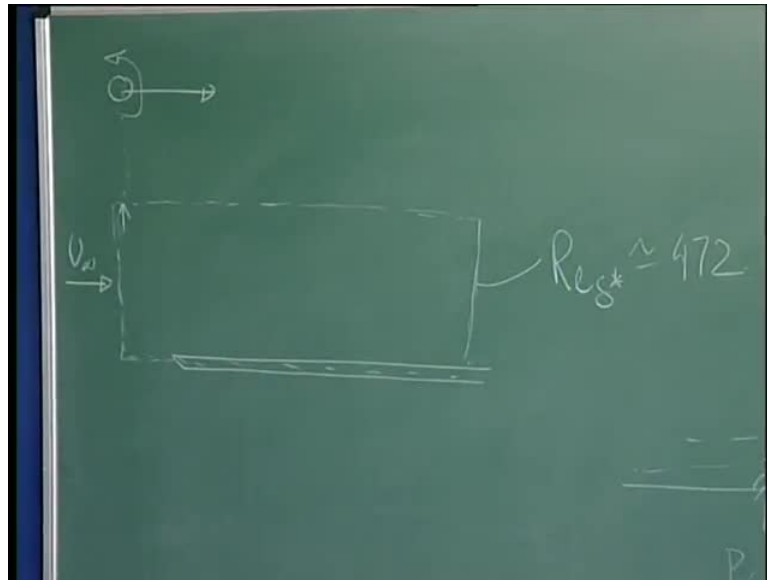
**Computations of vortex-Induced Instability as
A Precursor to Bypass Transition**

- The **Reynolds number** based on displacement thickness of the undisturbed flow at the outflow of the computational domain is 472.

"Thus, the flow is fully sub-critical in the computational domain."

- At the inflow and at the top of the computational domain one calculates the flow variables, as induced by the freestream vortex via **Biot-Savart** interaction rule.

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This is exactly what one would see in a computation. Let me also tell you that, the Reynolds number, that was based on the displacement thickness, at this so if I look at this out flow- Re_{δ^*} , that is roughly about 472.

So, what does that mean? Inside the whole domain that we are computing, the flow is sub-critical with respect to Tollmien-Schlichting wave; because we know that Tollmien-Schlichting waves are generated at 520. So, the whole domain flow is supposed to be sub-critical; but because of this steady forcing we can create a sequence of unsteady events, and that is what we call as bypass transition. We would note this very emphatically, that the flow is fully sub-critical in the computational domain, despite which, we are seeing all this.

What you do basically is, effect of this convecting vortex is fed through by calculating the individual velocity of the inflow and at the top of the domain. You do not try to calculate what is happening in the out flow because, we do not know (()) if there are some instabilities. If we say that the velocity induced here is due to some above interaction, then we are basically precluding all the events. So you never do that. Most of the time at the outflow, what you do is, you give a boundary condition, and you allow things to happen.

We do such a thing while you fix the input disturbances at the inflow, at the top of the competition domain; and that you do by using 8.36(()) of interaction. That is fairly straight forward.

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**Computations of vortex-Induced Instability as
A Precursor to Bypass Transition**

- At the outflow, fully developed condition is applied for the wall-normal component of the velocity (and using the same in **SFE**, one can obtain the vorticity boundary condition at the outflow from Equation (3.4.2).
- At the top frame, one notices incipient unsteady separation on the wall: The primary instability.
- In subsequent frames, one notes secondary and tertiary events induced by the primary instability.
- In these computed cases, one does not notice **TS waves** and the vortices formed on the wall are essentially due to unsteady separation that is initiated by the freestream convecting vortex.

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$$\left. \begin{aligned} \frac{\partial \psi}{\partial \xi} &= 0 \\ v &\sim \frac{\partial \psi}{\partial \xi} \end{aligned} \right\} \frac{\partial^2 \psi}{\partial \xi^2} = 0$$

Primary Instability

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Now, I told you that, at the outflow, you do not use induced condition due to the vortex; but what you do is, give a softer boundary condition, which is called here as, the fully

developed condition, in the sense, we give the wall normal component of the velocities stream wise gradient, so x_i is along the stream direction.

So, V itself here is like your $\text{del psi del } z_i$, am not writing the scale factor. There is a scale factor also involved. So, this is basically nothing but your H to V . Then when we are doing this, this is equivalent to something like this.

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Computations of vortex-Induced Instability as
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$$\left. \begin{aligned} \frac{\partial v}{\partial \xi} = 0 \\ v \sim \frac{\partial \psi}{\partial \xi} \end{aligned} \right\} \frac{\partial^2 \psi}{\partial \xi^2} = 0$$

Primary Instability

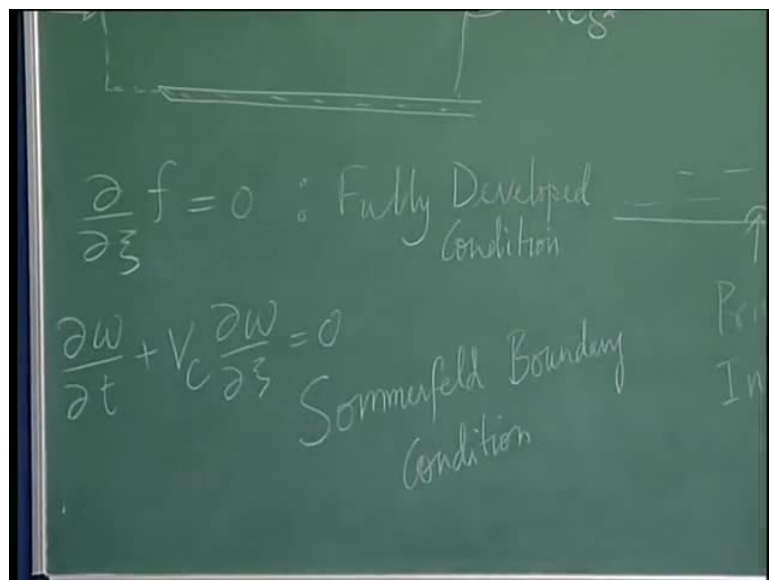
$$\frac{\partial}{\partial \xi} \left(\frac{h_2}{h_1} \frac{\partial v}{\partial \xi} \right) + \frac{\partial}{\partial \eta} \left(\frac{h_1}{h_2} \frac{\partial v}{\partial \eta} \right) = -\omega h_1 h_2$$

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The secondary derivative of ψ with respect to ψ equal to 0 that is what we are applying there. We can also use the same thing in our stream function equation. What is the stream function equation? If we recall, we have this $\frac{\partial^2 \psi}{\partial x^2} + \frac{\partial^2 \psi}{\partial y^2} = \omega$, that is the definition of the vorticity times $\frac{1}{\rho \nu}$.

So, in this equation also if you substitute this condition at the outflow that gives you the condition on ψ derivatives with respect to y ; that is what you apply there. So what we are saying is that, we apply the vorticity boundary condition at the outflow. If I know the ψ , I can calculate this from there; I can calculate the value of the vorticity at the outflow that satisfies the fully developed condition.

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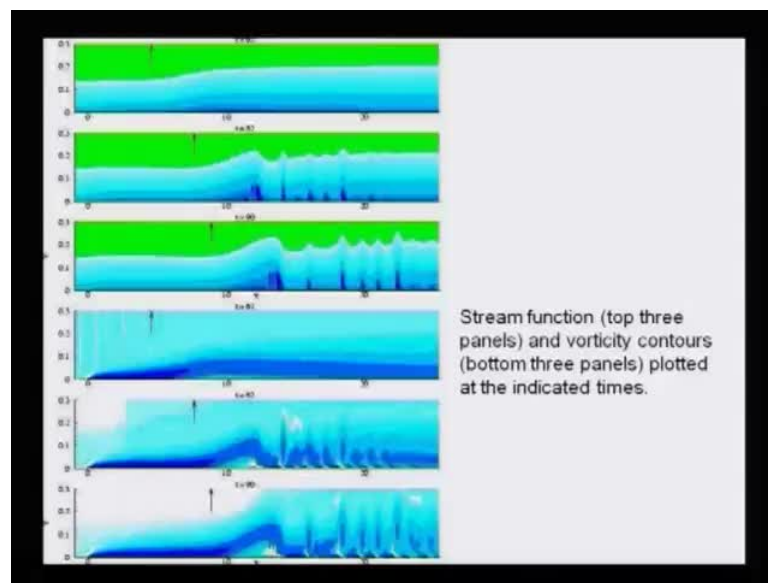


This is one way, the other way to do is, the Sommerfeld boundary condition. In the Sommerfeld boundary condition what we usually do is that, it is more aesthetically satisfying and is also useful. Any condition given like this, well, any variable equal to 0 is what we call as fully developed boundary condition. What the name, the terminologies signifies- that the quantity under the investigation is fully developed; if you move any further it does not change any more, that is what fully developed condition means. Whereas, there is alternative way of looking at it; that if, as we saw, the vertical structure were moving out of the computational domain, it is not as if it is fully developed there.

It is sort of a propagation phenomenon; and to address that propagation phenomena, if I look at some quantity, let us see the vorticity, then I will say that it is going as a one dimensional signal, and I could give some kind of a conductive velocity of that signal- V_C , and that is going like this. This is what we call as, the Sommerfeld boundary condition.

Once again, Sommerfeld proposed this; and it is used quite extensively in computation. This is called Sommerfeld boundary condition, and some people also, well, call it as Radiative boundary condition, as if something is radiated outside. This is what you find in many commercial softwares also; they basically do this. What is the problem with this case? The problem is knowing what this V_C is going to be. It is not like 1D convection equation that is a scalar quantity that you prescribe it **upfront**.

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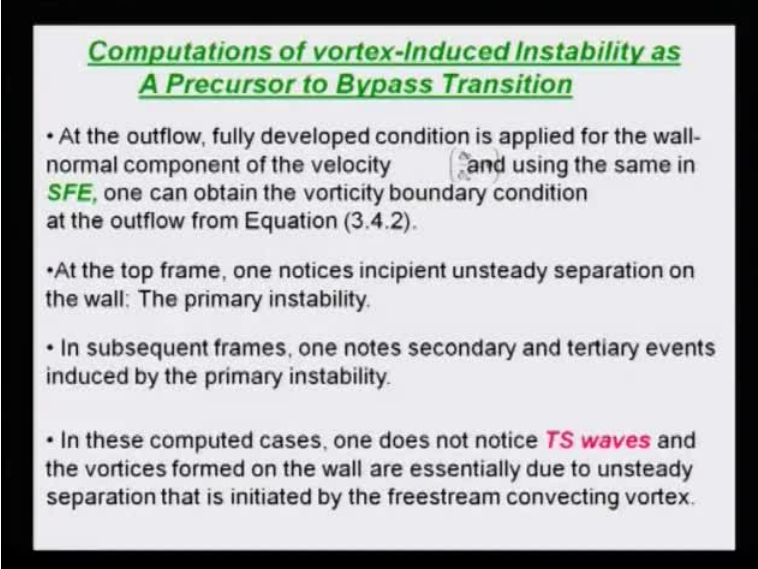


This is going to be something that is part of the unknown itself, because, as we saw in the, probably you can see the results, if we look at the last frame, if I look at it here, the vertical structure is going out; if it is going out then what is going to happen? What you find is that, at different heights you have different values of vorticity, and it is not necessary that it is going out.

Like a scalar vorticity, the vector, it can have different orientations at each and every point, and it can go at different speed. This is going to be a function of η which we do

not **an prime**. There are many ways of handling it, one of the way that we have tried to do is, calculate the velocity at the previous time stream, at the whole edge.

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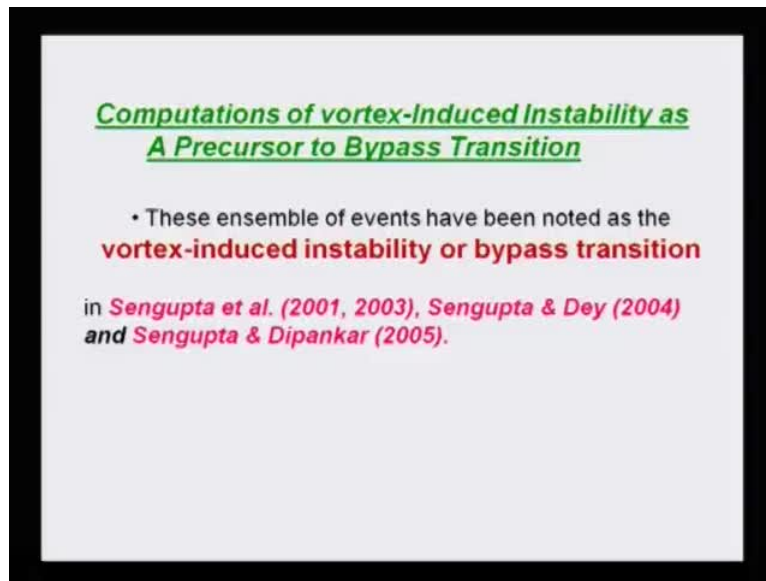
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At each point you calculate this, and then, you let it go out. This is what one does in terms of computation. These are important, these are not trivial issues. These are still part of research topics. We have seen is the incipient unsteady separation on the top frame, which is what we just now saw. That will indicate us the primary instability, and of course, in subsequent frames we could note secondary and tertiary events that is essentially induced by primary instability. This goes without saying that, we do not notice tollmien-schlichting waves on the vortices; those are formed in the wall essentially due to unsteady separation that is initiated by a steady convicting vortex in the free stream.

This was a quite interesting flow field. That is why, if you recall, the Doligalski wrote in a review paper in 1994 that, how a single convicting vortex affects a flow? That is a matter of intense research, and that was not known. We just carried it on, and we our self referred satisfied, because this confirms our experimental observation. We do get the theoretical and computational verification of this.

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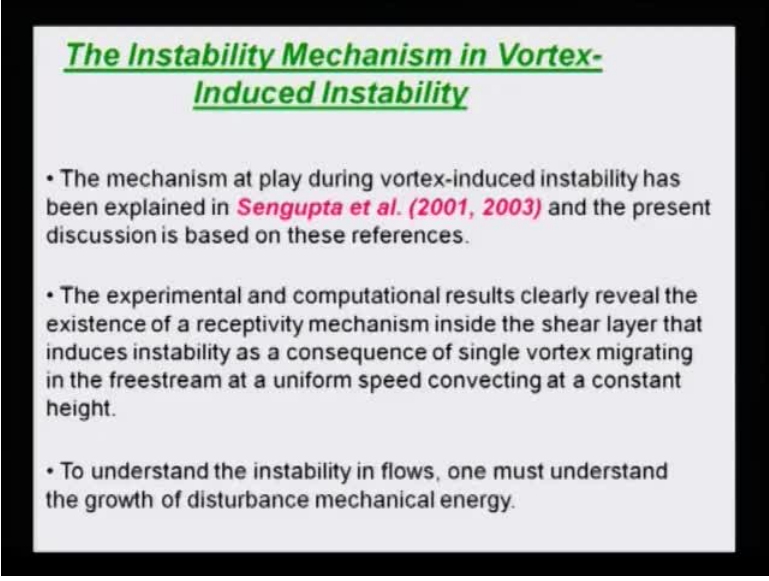


Since this is all started by a single convicting vortex, we call it a vortex- inducing instability. If you go by the definition of bypass transition by of Markovin, this can be classified as one of the bypass transition mechanism. We have been pretty much active in this area for quite many years now.

We do keep investigating it in various scenarios. In the first 2 papers, we did look at vortex inducing instability, and in the last paper we looked at, that is this paper, ((JFM)) we looked at the subcritical instability and attachment line of an infinite swept wing.

We will talk about that, but before we do that, we need to talk little more about the instability mechanism. See, all this time I have explained to you the creation of adverse pressure gradient at this covering mechanism. That is all very good qualitative description. Can we formalize it?

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The Instability Mechanism in Vortex-Induced Instability

- The mechanism at play during vortex-induced instability has been explained in **Sengupta et al. (2001, 2003)** and the present discussion is based on these references.
- The experimental and computational results clearly reveal the existence of a receptivity mechanism inside the shear layer that induces instability as a consequence of single vortex migrating in the freestream at a uniform speed convecting at a constant height.
- To understand the instability in flows, one must understand the growth of disturbance mechanical energy.

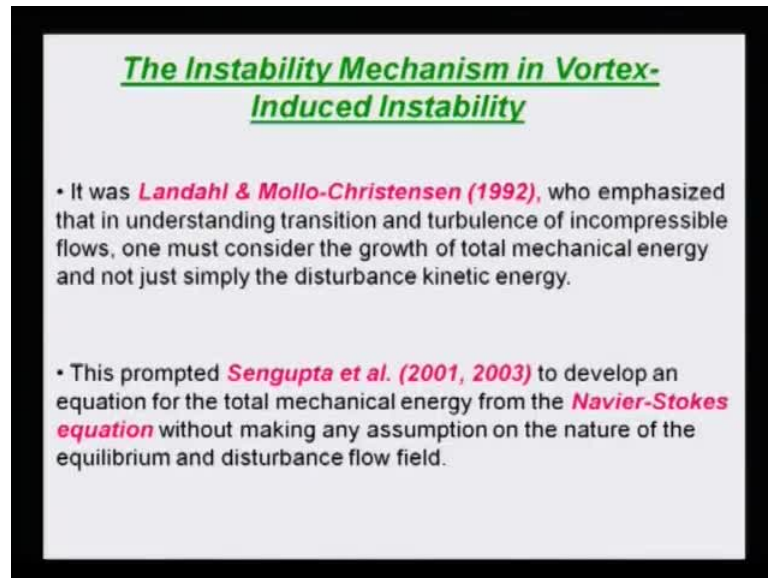
This actually we did. We will carry on the discussion, and we will see what happens. The experimental and computational results clearly reveal the existence of the receptivity mechanism. You see, the flow is receptive, that actually induces the primary and the subsequent instabilities caused by a vortex migrating in the free stream at a uniform speed.

So, to a distance the instability of the flow we must understand how the disturbance grows. When we talk about disturbance, this is where something important happens. How do you define disturbance? Those of you who may have used some package in computing turbulent flows, you would notice that, people use something called turbulence models; and one of the popular turbulence model is the k epsilon model, and the k stands for the turbulent kinetic energy, and epsilon is a dissipation.

So, what happens is the turbulence community? For say, had been looking at the turbulent kinetic energy. Both of **fresher** was actually brought in this discussion. In this book by Landahl and Mollo-Christensen, it is called Turbulence and Random process in Fluid mechanics; a very nice book, very thin one. Very extremely enjoyable, if, one is interested about chaos and turbulence. This is a very short description of the same thing. What caught our attention was, the emphasis of these two authors, that, if you really want to understand transition and turbulence of incompressible flow, then, you better look at the total mechanical energy, just do not simply **avalon** on the disturbance kinetic

energy away. This is a wrong way to go. Well, we were also aware of some of the discussions Markovin had made in a paper in 1991.

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It was a little heuristic, it was not very rigorous. Markovin suggested that, what we had seen, all that sudden unsteadiness appearing, could be due to some kind of shear noise, and that, try to look at it by looking at the Poisson equation for the static pressure. What we are intending to do here is, go away from what Markovin suggested, but instead follow the lead of Landahl and Mollo-Christensen. We developed an equation for the total mechanical energy directly from Navier-Stokes equation, without making any assumptions, what so ever. Then we talked about this disturbance field, separately from the equilibrium flow, and then, tried to follow what disturbance the mechanical energy is to.

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The Instability Mechanism in Vortex-Induced Instability

- Such restrictions are imposed in all linear viscous instability mechanism studied. For incompressible flows this disturbance equation energy equation is obtained by taking the divergence of the rotational form of the **Navier-Stokes equation**.

$$\nabla^2 E = \vec{\omega} \cdot \vec{\omega} - \vec{V} \cdot (\nabla \times \vec{\omega}) \quad (3.5.1)$$

- Where $E = \frac{p}{\rho} + \frac{\vec{V} \cdot \vec{V}}{2}$ is the total mechanical energy. The solid dots in these equations represent vector dot product. The instability is related to the rotationality of the flow and the instability is driven by the right hand side of (3.5.2.) – shown next.

Let us see what it is. Basically, what we can do is, derive this equation 5.1, that you are seeing in front of you. How does it come about? This comes about, as, I promise to you, that it will appear directly from the Navier-Stokes equation without making any assumption.

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N-S Eqⁿ:

$$\frac{\partial \vec{V}}{\partial t} + (\vec{V} \cdot \nabla) \vec{V} = -\frac{1}{\rho} \nabla p + \nu \nabla^2 \vec{V}$$

Rotational Form of NSE:

$$(\vec{V} \cdot \nabla) \vec{V} = \nabla \left(\frac{|\vec{V}|^2}{2} \right) - \vec{V} \times \nabla \times \vec{V}$$

$$\frac{\partial \vec{V}}{\partial t} - \vec{V} \times \vec{\omega} = -\nabla \left(\frac{p}{\rho} + \frac{|\vec{V}|^2}{2} \right) + \nu \nabla^2 \vec{V}$$

Take a divergence

$$-\nabla \cdot (\vec{V} \times \vec{\omega}) = -\nabla^2 E$$

Let see how we go about doing it. This is arrived at, in the following manner. Look at it this way, that, your Navier-Stokes equation is given like this. The local acceleration,

convicting acceleration, that is balanced by the pressure gradient term, and then we have the viscous if (μ) . Now, let us write down the alternative form of this Navier-Stokes equation.

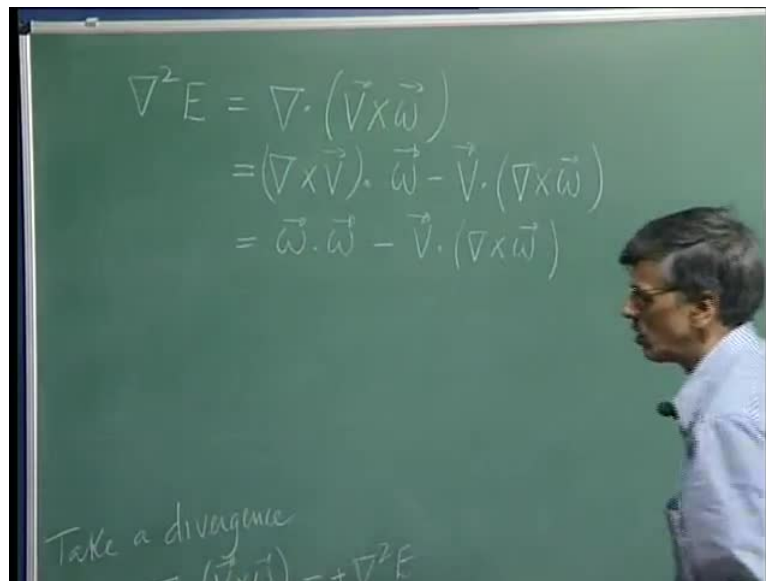
So, this is your Navier-Stokes equation, what we call as the primitive variable formulation. Let us write out the rotation in form of Navier-Stokes equation. I will just simply write NSE to indicate the Navier-Stoke. So, what we do in this is try to get this. We use this vector identity, this is the vector identity. There is the no issue with that. So, if a use this over here, what I am going to get is this $-\nabla \cdot \mathbf{V}$ $\frac{d}{dt}$. From here, what is this? $\nabla \cdot \mathbf{V}$ is the vorticity. So, I could write this as, $\mathbf{V} \times \boldsymbol{\omega}$. This term that I have here, I could put it on the right hand side. What I am going to get? There is already a minus sign; this also comes with the minus sign.

So, I could write it like- speed by rho plus is called by 2. That is this, and then we have this term as it is. Now, this itself is your total mechanical energy, if, you do not add any body force, etcetera.

It has two components- a pressure head and the kinetic head. This is also how you get any of Bernoulli's equation. In fact, from here itself one can derive the Bernoulli's equation. You can see how to do it; drop this term out, and then you say flow is, say, rotational or you follow along a stream line. Then if you follow along stream line this term will also contribute nothing. A steady flow, so this goes away.

If the spatial gradient of this quantity is 0, this must be constant everywhere. That is the way we come to the Bernoulli's equation. However, if I define this quantity itself as E, as we have done here, p by row plus \mathbf{V}^2 , I wrote $\mathbf{V} \cdot \mathbf{V}$ by 2. That is our total mechanical energy. Now what you do is, take a divergence of this equation. So, if I take a divergence, what will happen? $\nabla \cdot \frac{d}{dt}$ of $\nabla \cdot \mathbf{V}$ is incompressible flow, so that goes away. So, there is nothing. I am going to get there on this side, then I am going to get simply, $\nabla \cdot \mathbf{V} \times \boldsymbol{\omega}$, and on this side, on the right hand side I have, minus $\nabla \cdot \nabla$. That will be the Laplacian. What about this? I am taking again a divergence, so, that could simply just go away and we will not get this.

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So what we have is essentially this. So, del square E is equal to del dot V cross omega. Just simply use again some vector identity - del dot a cross b, and you can write it down. You get this equation, so you can see what is going to happen now. This is a very straight forward equation that we have - del square E, is nothing but- del dot V cross omega. That we can simplify again, using the incompressible flow assumption. There would be many terms, 4 terms or 2 terms will drop out, and we will have only 2 terms left behind, that will find del cross V dot omega and minus V dot del cross omega.

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This itself is again- $\omega \cdot \omega - \nabla \cdot \nabla \times \omega$. So, that is your equation 3.51. That is how we can derive it. Now, for the total field what we could do is, of course, we could simplify it little further. What you are noticing is this part. What is E ? E is something like, your E **rotational** part, it is the energy; it is got nothing to do with rotational aspect of it. On the right hand side, it is driven by the rotationality of the flow. This is how the vorticity field interacts with itself. Of course the velocity in the vorticity field interacts. That is what we are looking at, that, the instability is related to the rotationality of the flow and the instability is driven by the right hand side.

How the energy is going? Whether it is going to grow or decay depends on what this right hand side is doing. Now what you could do is you could again appeal to the giants. Sommerfeld in writing a book on partial differential equation in 1949, he talked about the properties of Poisson equation in the context of heat transfer.

He said that, if I write the Poisson equation for the temperature field, then, if the right hand side quantity is positive, it is going to be a sink of heat and if the right hand side is negative then it is a source of heat.

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The Instability Mechanism in Vortex-Induced Instability

- This is based on the observation in **Sommerfeld (1949)** that a negative right hand side indicates a source of energy, while a positive quantity represents a sink.
- If one divides E into a mean and a disturbance part via, $E = E_m + \epsilon E_d$ and substitute it in Equation (3.5.1), one gets the equation for the disturbance energy.

$$\nabla^2 E_d = 2\bar{\omega}_m \cdot \bar{\omega}_d + \bar{\omega}_d \cdot \bar{\omega}_d - \bar{V}_m \cdot (\nabla \times \bar{\omega}_d) - \bar{V}_d \cdot (\nabla \times \bar{\omega}_m) - \bar{V}_d \cdot (\nabla \times \bar{\omega}_d) \quad (3.5.2)$$

We are any way talking about energy, so we can draw the analogy, and look at the right hand side, and what the right hand side is doing; because that is what is driving. That is, what comment we make, that is what you are looking at - $\nabla^2 E$. The distribution

of E , the boundary value problem is determined by the forcing on the right hand side. So basically, we can follow the evolution of E by just simply looking at what the right hand side is doing with time.

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$$\begin{aligned} \nabla^2 E &= \nabla \cdot (\nabla \times \vec{\omega}) \\ &= (\nabla \times \nabla) \cdot \vec{\omega} - \nabla \cdot (\nabla \times \vec{\omega}) \\ &= \vec{\omega} \cdot \vec{\omega} - \nabla \cdot (\nabla \times \vec{\omega}) \end{aligned}$$

$$\begin{aligned} \nabla^2 (E_n + \epsilon E_d) &= (\vec{\omega}_n + \epsilon \vec{\omega}_d) \cdot (\vec{\omega}_n + \epsilon \vec{\omega}_d) - (\nabla_n + \epsilon \nabla_d) \cdot (\nabla_n \vec{\omega}_n + \epsilon \nabla_d \vec{\omega}_d) \\ \nabla^2 E_d &= 2 \vec{\omega}_n \cdot \vec{\omega}_d + \epsilon \vec{\omega}_d \cdot \vec{\omega}_d - \nabla_n \cdot (\nabla \times \vec{\omega}_d) + \nabla_d \cdot (\nabla \times \vec{\omega}_d) \\ &\quad + \epsilon \nabla_d \cdot (\nabla \times \vec{\omega}_d) \end{aligned}$$

We are interested not only on the total mechanical energy, but we want to find out is how the disturbance energy is growing. So, what I could do is, I could split the total energy in terms of a mean part plus a disturbance. I could do it, is it not? Once I do that, and I substitute it here, then on the left hand side I am going to get del square E_n plus epsilon E_d , that is that. On this side, what I am going to get is going to be, ω_m , write it in lower case, epsilon ω_d , that is the same thing. That is the first part, and the same way I could also split the velocity field, into mean and disturbance.

This is the equation now. Of course, order one quantity should balance with order one quantity. So, if I remove all the order one quantity, what am I going to get on the left hand side? Well, I have this - the order epsilon quantity on the left hand side. What about here? $\omega_m \cdot \omega_m$ that was the mean part, and that is gone. So we do not worry. ω_m will interact with this, and that will be one side of term, so that I will get $\omega_m \cdot \omega_d$.

What about this? This into this also, will give me the same thing. So I get twice of that. In addition I am simply, for the sake of writing, not leaving out anything. This is the

order epsilon square term. How the disturbance field interacts with itself? That is also there, and what about here? The mean part will go away with $\nabla \cdot \bar{\omega} \times \bar{\omega}$, so that we do not worry about. We look at this part, that we give as epsilon $\nabla \cdot \bar{\omega} \times \bar{\omega}$. This into this, and now this into this also, I will get epsilon $\nabla \cdot \bar{\omega} \times \bar{\omega}$. That is this part, and then of course, we will also have the quadratic nonlinearity, that will be this.

What you are noticing, of course, you can drop out all the epsilons, and then you are going to get this term. So, I will write here, $2 \bar{\omega} \cdot \bar{\omega}$. There would be this term, and these go away, and this. What you see as the equation 2 there, 5.2, has all the terms; only thing is we did not write, these as the order epsilon, but that is implied. This term is a lower order quantity, and so is this term.

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The Instability Mechanism in Vortex-Induced Instability

- This is based on the observation in **Sommerfeld (1949)** that a negative right hand side indicates a source of energy, while a positive quantity represents a sink.
- If one divides E into a mean and a disturbance part via, $E = E_m + \epsilon E_d$ and substitute it in Equation (3.5.1), one gets the equation for the disturbance energy.

$$\nabla^2 E_d = 2\bar{\omega}_m \cdot \bar{\omega}_d + \bar{\omega}_d \cdot \bar{\omega}_d - \bar{V}_m \cdot (\nabla \times \bar{\omega}_d) - \bar{V}_d \cdot (\nabla \times \bar{\omega}_m) - \bar{V}_d \cdot (\nabla \times \bar{\omega}_d) \quad (3.5.2)$$

If we want to be consistent, we could drop this out, and that will be your corresponding linear analysis. This is how we perform linear analysis. So basically, this is the thing. What you are noticing though is that, the disturbance energy is dictated upon how the mean vorticity interacts to the disturbance vorticity

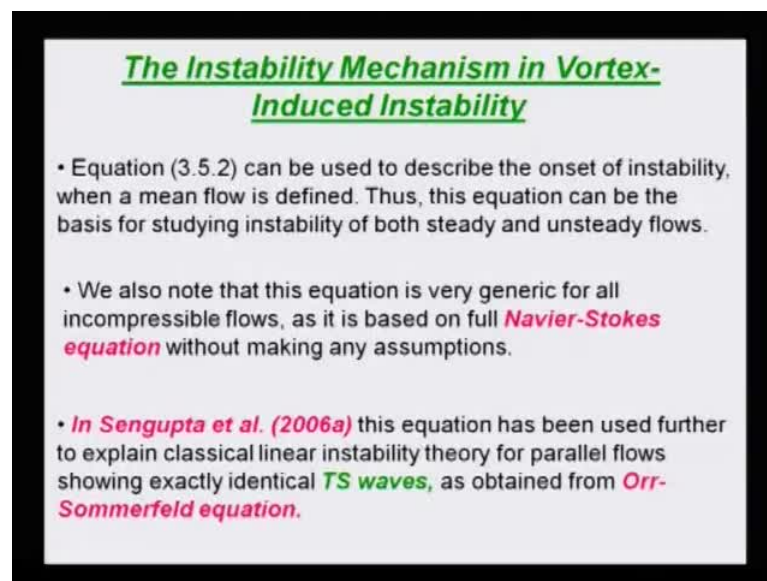
In the case of all our numerical and experimental investigation, you already have a mean flow vorticity distribution that is the boundary layer vorticity. With the boundary layer I can have a vorticity distribution. Now, over and above, this disturbance vorticity omega

d is going past it and has its foot print there that is interacting there. That is one sort of a player, in dictating what happens, and we have seen that it depends on the sign.

What we have said is that, if I have a flat plate, I have vorticity of one sign, and then if the vorticity is positive, then I get something that would give me a dominant quantity from here. If that dominant quantity happens to be a negative quantity, then we will have a source of disturbance energy.

Now, you can see how this whole thing comes about. This is the primary term, look at this; this is a lower order term. To begin with, in the primary stage this could be suppressed, not thought about as important. It would come into play, once you are looking at secondary and tertiary instabilities.

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The Instability Mechanism in Vortex-Induced Instability

- Equation (3.5.2) can be used to describe the onset of instability, when a mean flow is defined. Thus, this equation can be the basis for studying instability of both steady and unsteady flows.
- We also note that this equation is very generic for all incompressible flows, as it is based on full **Navier-Stokes equation** without making any assumptions.
- **In Sengupta et al. (2006a)** this equation has been used further to explain classical linear instability theory for parallel flows showing exactly identical **TS waves**, as obtained from **Orr-Sommerfeld equation**.

This equation, as you are noticing here, brings out certain features that we said- how the mean and the disturbance vorticity fields interact with each other. That would probably determine your concept itself, the primary instability that we talked about. So, to really make use of this equation, what we need to know is- what is our equilibrium flow or the mean flow. Mean does not necessarily mean that we are looking at only a steady flow it could also be an unsteady flow.

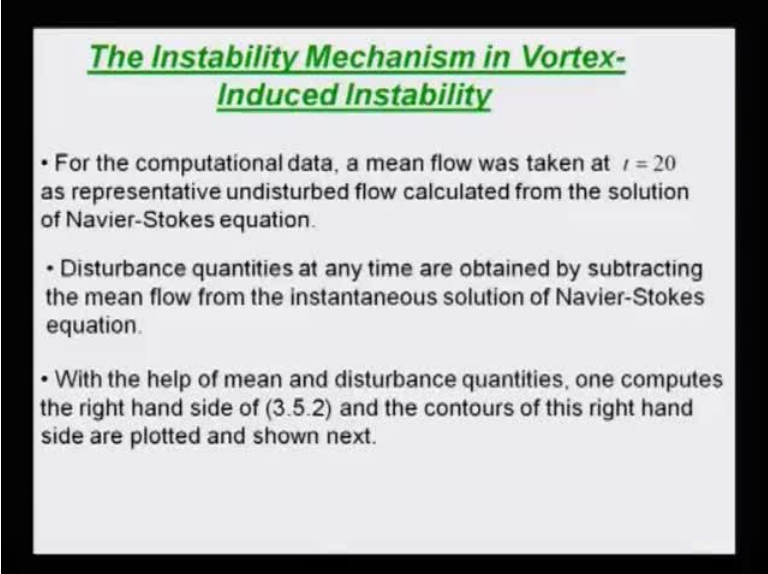
That is what we are saying, that once you have the **where wiggle** of describing what your equilibrium flow is, whether it is steady or unsteady, you can use this equation to study

its stability or instability. Do you see the quantum leap from what we have done with Orr-Sommerfeld equation? We needed a steady flow; we needed to make it parallel; and all kinds of assumptions limited our visibility. Here we are not saying that. We are ready now, to study the instability of an unsteady flow or mean flow. That is what is pointed out here. You can study both steady and unsteady flows. It does not restrict us.

This equation is very generic because we started off from Navier-Stokes equation. We did not make boundary layer assumption; we did not make parallel flow assumption; we did not talk about linearity; none of these things have come about. So, this is the most generic thing that could have happened. If it is indeed so, then I should be able to get what others have gotten with those limited approximations. How you get Tollmien-Schlichting wave? That should also be buried in this information, and indeed, we did show it in a paper in physics of fluid, and a paper in physical review letters that, you could actually make use of this equation, and explain the occurrence of Tollmien-Schlichting waves too and that has been done. Thankfully!

Now, we have really scanned the whole horizon. Then we can show that Orr-Sommerfeld equation information is also embedded in this equation set. Now, if we look at the computational data, we know the flow was evolving with time; but in the initial stage when the vorticity was very much upstream, ahead of the flat plate, then the flow was not changing with time.

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The Instability Mechanism in Vortex-Induced Instability

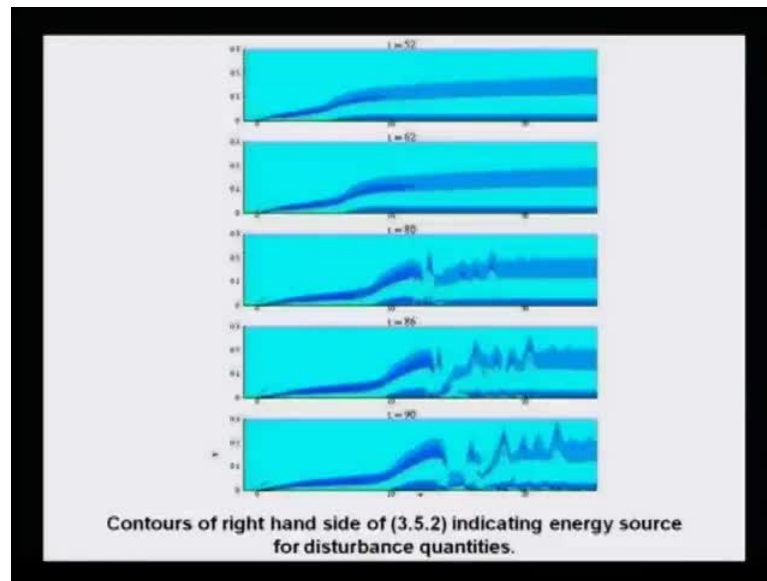
- For the computational data, a mean flow was taken at $t = 20$ as representative undisturbed flow calculated from the solution of Navier-Stokes equation.
- Disturbance quantities at any time are obtained by subtracting the mean flow from the instantaneous solution of Navier-Stokes equation.
- With the help of mean and disturbance quantities, one computes the right hand side of (3.5.2) and the contours of this right hand side are plotted and shown next.

So, if we take one such snapshot of our representative mean flow or undisturbed flow that is subsequently perturbed by the convecting vortex, then we can make use of this. What we can do is, we can calculate the disturbance quantity because, when we are solving the Navier-Stokes equation, we are solving for the whole quantity. We are not solving for the mean separately or disturbance quantity, so what we have is the total field.

Now, if I identify a so called equilibrium flow, which is, let say the solution at t equal to 0 then, from the instantaneous solution I can subtract that. That will be my imposed disturbance, so that is this. The disturbance quantity is, at any time, could be obtained by subtracting the mean flow from the instantaneous solution of the Navier-Stokes equation. This is as simple as that. Now, with the help of the mean and disturbance quantity I could really compute the right hand side of that disturbance energy equation that we have written here. What we did and what we continue to do is, not worry about solving this Poisson equation which you can always do, but what we would rather like to do is, get a qualitative understanding of the ensuing instability, by working out the right hand side.

So, if I am solving the stream function vorticity field equation, I have- ω_m plus ω_d . From there I can split them out. I have the velocity field from the stream function equation that also, I can split into V_m and V_d , and then I can work out the right hand side. Once I have the right hand side, I know that using Somerfield's idea, looking at the sign of the right hand side; I could say where the disturbance energy sources are, or where it is working as a sink. That is interesting!

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Now, this is what we have plotted here. We calculate the right hand side and we do a contour plot. What is the legend about this plot is that, darker blue shade implies these sources of energy. So, those are the streaks along which you get negative values, and what you notice is that, this is the leading edge of the plate. So, leading edge of the plate is one location where you see disturbance sources getting created.

However, you also notice that, what is being created at the leading edge does not come inside, it kind of stays outside the shear layer. This is more or less the edge of the shear layer. So, it remains outside. The leading edge point, it is quite well known for a long time, and that has some role to play in flow instability, and that is what we are also seeing. Evaluate this have but it stays out side.

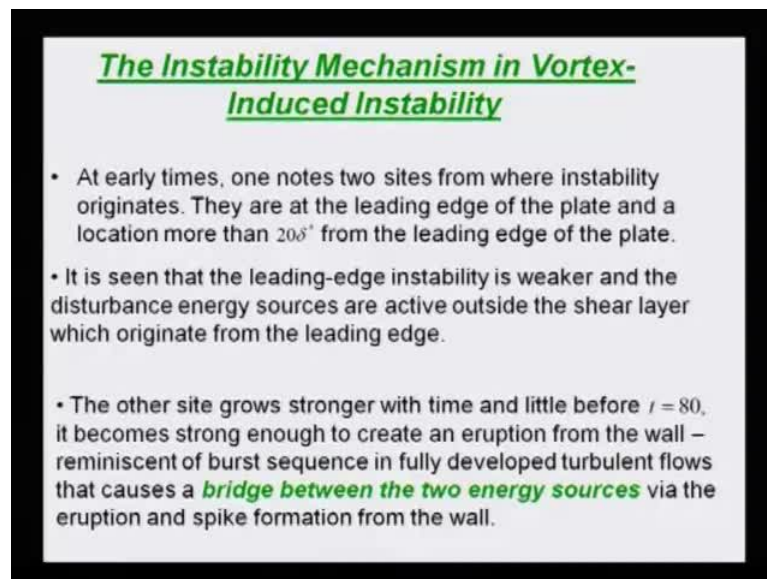
We also have a second sight, which is about 20 delta star from the leading edge. That is where I see another, more intense, source of disturbance energy. As I told you, this calculation is all for subcritical domain because, the Re_{δ^*} here is 472. So, despite that, we are seeing that, convecting vortices creates all kinds of disturbance quantities which leads to an interaction, given by the right hand side, and that shows 2 sides of instabilities. There is one vorticity field, and there is another vorticity field. They can also interact, and you will start noticing that a thing is happening; there is interaction between these 2 disturbance sources- one starting from the leading edge and another is somewhere in the middle of the domain. We did talk about those vertical eruptions; you

can see that, those are responsible for linking these 2 sources. These vertical eruptions actually build a bridge.

You notice what it does. This is a discreet event, but once this discreet event is created, it creates a link; and then, the one here actually leaves off, and this seems to be inhabitant. This is what you are going to see as a sequence of instabilities happening like this. So, as if the leading edge disturbance seems to feed energy here, and that, basically reinforces disturbances that were there on the flat plate.

One thing that we realize in studying a flow field of this kind is, if you are doing computing, you have no choice but to take the leading edge. There are many people who have tried to compute the flow by excluding the leading edge. If you exclude the leading edge, you are not going to pick this up. So, this is rather very crucial, that was the reason that we computed the flow field including the leading edge.

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We can make the following observations that, at early times we know 2 sides from where instability originates- they are at the leading edge of the plate and a location more than 20 delta star from the leading edge of plate, this we have noticed. We also noticed that, leading edge instabilities are weaker and the disturbance energy sources are active only outside due to this weak instability. What happens is the other sight that grows in the

interior, the shear layer, is stronger and that gets stronger with time. Little before this time, we saw those snapshots at 62, 82 and so on and so forth.

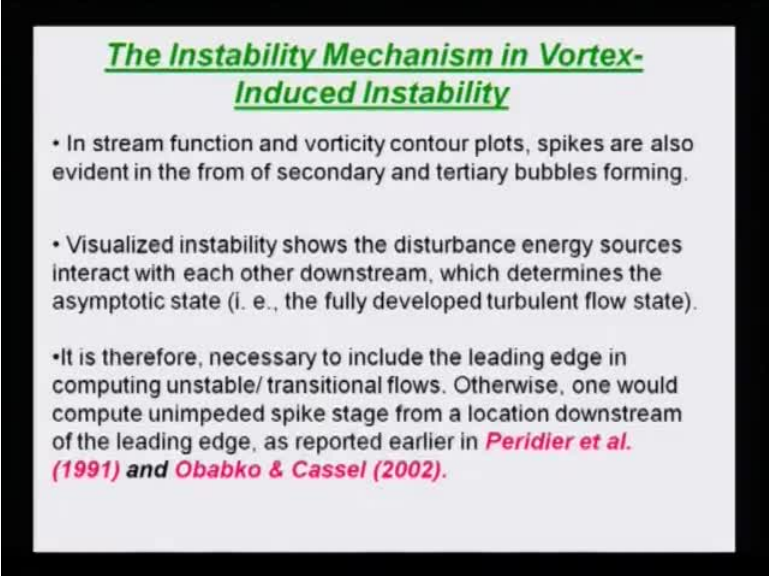
So, little before 80, we start seeing an eruption coming out of the wall, and hit those upper deck of the disturbance; that is what we have called as, the vertical eruptions. Once that happens, that creates a kind of bridge between this 2 disturbance sources. This is exactly what people have talked about in fully developed turbulent flow also. During the burst sequence of fully developed turbulent flow, people do see some kind of vertical eruptions, and that is the bridge between the 2 energy sources, via the eruption and this creates a kind of spike.

Remember, we talked about Brinkman Walkers investigation in the cross flow plane of a fully developed turbulent flow. So, what we are seeing here in the stream wise direction, we can do sort of an isolated analysis on the cross flow plane, and there also we will see some spikes forming.

You do see these kinds of things in fully developed turbulent flow. I just would like to emphasize upon all of you that, why we did not study the turbulence the way people study turbulence, because, what you see in fully developed turbulent flow is in the genesis - how it all originated.

So, that is why it is much more productive, that you try to take a composite picture of the whole thing by studying of in the primary instability, all the way down, and that is what we are also talking here, that even while we are studying instabilities, we can see some of these elements which people have found out by simply focusing up on turbulent flow itself.

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The Instability Mechanism in Vortex-Induced Instability

- In stream function and vorticity contour plots, spikes are also evident in the form of secondary and tertiary bubbles forming.
- Visualized instability shows the disturbance energy sources interact with each other downstream, which determines the asymptotic state (i. e., the fully developed turbulent flow state).
- It is therefore, necessary to include the leading edge in computing unstable/ transitional flows. Otherwise, one would compute unimpeded spike stage from a location downstream of the leading edge, as reported earlier in *Peridier et al. (1991) and Obabko & Cassel (2002)*.

This kind of a controlled experiments, controlled numerical simulations does in tremendous dividend, and we should be keeping this thing in mind. You have already seen in the stream function and vorticity contour plots, that the spikes were noted. We did note that, those spikes were nothing but your secondary and tertiary instabilities. We also notice that, our visualized instability shows that the disturbance energy sources interact with each other downstream, which actually determines the asymptotic state.

So, what we are going to see for downstream, that information is built in this initially interaction itself; that is what I say that, it is always necessary that you must include the leading edge in computing unstable of transitional flows. Otherwise, what will happen is, the top disturbance source was kind of creating a shield, it was trying to shield the inviscid flow outside, with this viscous flow inside.

If I do not take that leading edge, then I will just simply get this unimpeded spike stage. This has been done by those couple of papers that we talked about in the last class also- Peridier, Smith and Obabko. They did write couple of papers in JFM, and they did see that this spike, those having modes and they actually computation broke down.

So, they concluded from that, that, if the there are elements of vertical eruptions, those eventually leads to turbulence; but they are on the conservative side, because, they do not see the moderation due to the presence of the leading edge, and their break down is

rather spectacular. There was also another computation done by Obabko and Cassel later, at the same issue of not including the leading edge.

We must also point out another aspect of the present energy based instability theory, that if I look at this equation, this is a like a force vibration problem. The quantity of interest is E_d that is forced by the right hand side. So, if I am trying look at the transfer function, what should look at? I should look at it without the excitation; that means I should look at it without the right hand side. Basically, I will be looking at a Laplacian of E_d , as a transfer function of this disturbance energy, and what you notice is that, nobody has really reported from a Laplacian operator that it can lead to some kind of a scene for instability.

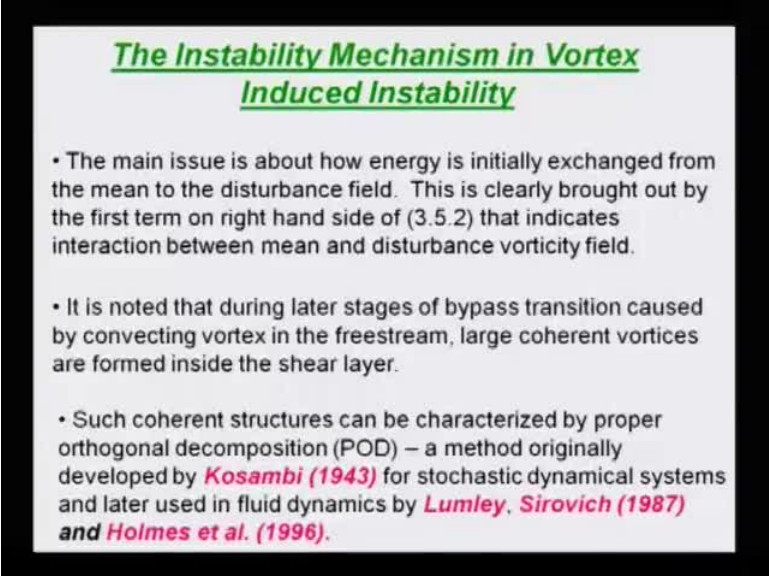
So, by itself intrinsically this is not unstable. Until and unless you have the forcing in the right hand side, nothing would happen, because, the governing equation for E_d is $\nabla^2 E_d = 0$, that is the point we are making here. The vortex induced instability is completely driven by interaction of velocity and vorticity field.

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$$\begin{aligned} \nabla^2 E &= \nabla \cdot (\vec{V} \times \vec{\omega}) \\ &= (\nabla \times \vec{V}) \cdot \vec{\omega} - \vec{V} \cdot (\nabla \times \vec{\omega}) \\ &= \vec{\omega} \cdot \vec{\omega} - \vec{V} \cdot (\nabla \times \vec{\omega}) \end{aligned}$$

$$\begin{aligned} \nabla^2 (E_m + \epsilon E_d) &= (\vec{\omega}_m + \epsilon \vec{\omega}_d) \cdot (\vec{\omega}_m + \epsilon \vec{\omega}_d) - (\vec{V}_m + \epsilon \vec{V}_d) \cdot (\nabla \times (\vec{\omega}_m + \epsilon \vec{\omega}_d)) \\ \nabla^2 E_d &= 2 \vec{\omega}_m \cdot \vec{\omega}_d + \epsilon \vec{\omega}_d \cdot \vec{\omega}_d - \vec{V}_m \cdot (\nabla \times \vec{\omega}_d) - \epsilon \vec{V}_d \cdot (\nabla \times \vec{\omega}_d) \end{aligned}$$

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The Instability Mechanism in Vortex Induced Instability

- The main issue is about how energy is initially exchanged from the mean to the disturbance field. This is clearly brought out by the first term on right hand side of (3.5.2) that indicates interaction between mean and disturbance vorticity field.
- It is noted that during later stages of bypass transition caused by convecting vortex in the freestream, large coherent vortices are formed inside the shear layer.
- Such coherent structures can be characterized by proper orthogonal decomposition (POD) – a method originally developed by *Kosambi (1943)* for stochastic dynamical systems and later used in fluid dynamics by *Lumley, Sirovich (1987)* and *Holmes et al. (1996)*.

So basically, what we are noticing is the rotationality, forcing the issue through the right hand side to determine what happens to a scalar quantity like energy. See, this is different from what Markovin conducted. Markovin did not talk about this kind of equation; he was talking about a Poisson equation for the static pressure, and he added lots of time in the right hand side, and he called those as a shear noise terms. This is a complete consistent picture. So, this is what mode? I do not know whether you have noted the issue or not, that, the main issue is really how energy is exchanged initially between the mean and disturbance field. This is clearly brought out by this term, and this term.

So, how the mean is reacting with the disturbance field? That is what we are getting here. It is also noted that, during the later stages of bypass transition large coherent vortices are found inside the shear layer. We saw those and we called them as unsteady separations or the vertical eruptions. There are coherent vortices people have been looking at it for a long time in turbulent flows. So, we basically would look at this coherence structure using proper orthogonal decomposition later in this course. We will talk about it in the next class, we will begin from here.