Instability and Transition of Fluid Flows
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Lecture No. #22

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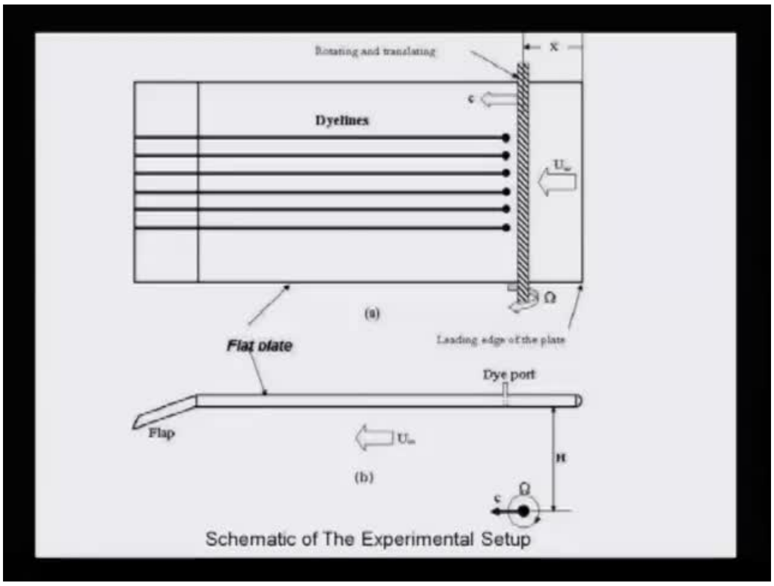

Schematic of The Experimental Setup

According to last class, we were talking about bypass transition that is triggered by a single vortex convicting in the free stream, and the experiment was conducted, whose set up is as shown in the schematic here. This is the side view of the plate, and this is the vortex, which is at height edge, this vortex is created by rotating a cylinder; the rotation rate is given by capital omega, c is the speed constant speed of convection, and there is in addition the free stream speed u infinity, so both are in the same direction. So you have basically a relative velocity between the two. And if you look at the top view of the experiment then this is your rotating cylinder, and this is the leading edge of the plate, there are 6 dye ports, which eject dye in the inside the shear layer.

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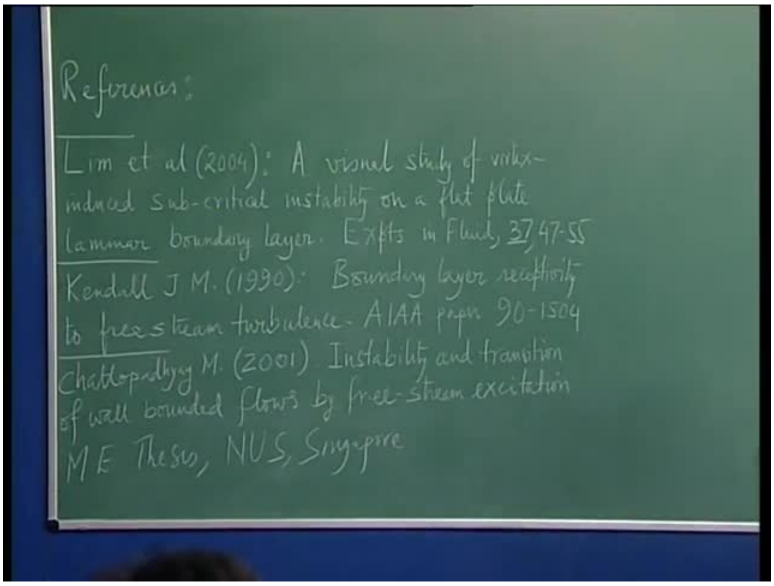

References:

Lim et al (2004): A visual study of vortex-induced sub-critical instability on a flat plate laminar boundary layer. Expts in Fluid, 37 47-55

Kendall J M. (1990): Boundary layer receptivity to free stream turbulence - AIAA paper 90-1504

Chattopadhyay M. (2001) Instability and transition of wall bounded flows by free-stream excitation ME Thesis, NUS, Singapore

And we did this visualization experiment that was reported in this paper in experiments in fluid volume 37.

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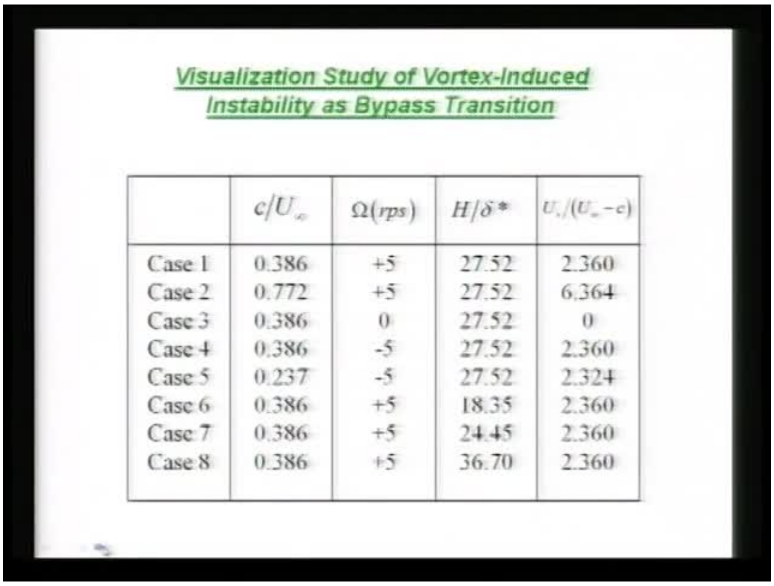

Visualization Study of Vortex-Induced Instability as Bypass Transition

	c/U_∞	$\Omega(rps)$	H/δ^*	$U_s/(U_\infty - c)$
Case 1	0.386	+5	27.52	2.360
Case 2	0.772	+5	27.52	6.364
Case 3	0.386	0	27.52	0
Case 4	0.386	-5	27.52	2.360
Case 5	0.237	-5	27.52	2.324
Case 6	0.386	+5	18.35	2.360
Case 7	0.386	+5	24.45	2.360
Case 8	0.386	+5	36.70	2.360

And the results were quite interesting namely that it showed us certain features of the flow and these are the various cases that we studied. First two cases are corresponded to with the cylinder was rotating in a counter clock wise manner at a rate of 5 rotations per second. The vortex was at a height of 27 delta star above the plate. And in the first case, the speed of convection of the vortex was 0.386 with respect to the free stream; and the second case, it was just the double of it. The third case was created just to show the importance of controlled nature of such receptivity experiment; in this case of the cylinder was just simply translated, but there was no rotation, and of course, then this last parameter which basically gives you ratio of the surface speed divided by the relative speed of the free stream with respect to the cylinder; so in that case, surface speed is 0; so that is what you are seeking. So this is the case of pure translating cylinder.

Case 4 and 5 corresponded to clockwise rotation cylinder or the same rotation rate or the speeds were kept lower, because will see that this is not very receptive while this is so. So, in this case also when the cylinder was rotated in the opposite direction, so you created a captive vortex of opposite sign. And while the other parameters remain the same, we wanted to see what happens of in front of vortex effect. For the last three cases, whereas same as the first case, because the convection speed is kept the same, the rotation rate of the cylinder is kept the same, it is only the distance that we were varied. They where three such cases; in one case, it was brought closer; the other case, it was brought some where closer to the cylinder; and this was when it was relegated further into the free stream. So, we did see the case, the case one that we shown was this.

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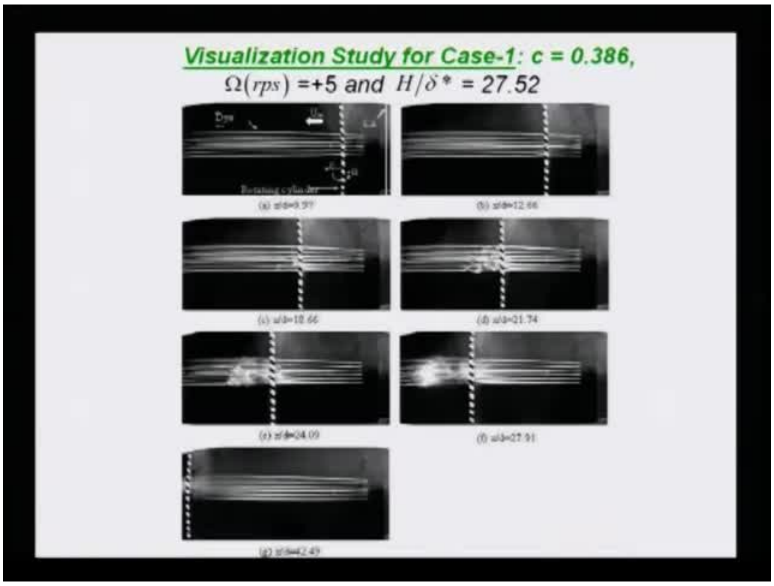

Visualization Study for Case-1: $c = 0.386$, $\Omega(rps) = +5$ and $H/\delta^* = 27.52$

(a) x/d=9.97 (b) x/d=12.66

(c) x/d=18.66 (d) x/d=21.74

(e) x/d=24.09 (f) x/d=27.91

(g) x/d=42.49

What we saw was the same plan we picture that we saw here, this is the way the picture were taken. So, this is, in this sequence, so the cylinder is moving like these, and these are the dye streaks; and this is a second frame, the third and fourth, so it has gone on that way. And as you can see when the cylinder was somewhere beyond the dye streak, you do not see appreciable disturbances however that (( )) let us starts picking up, and once that primer instability set seen that in turn leads to secondary and tertiary instabilities that is manifested here by rapid diffusion mixing; and even the aquatic nature of the flow that is what you see in subsequent frames.

And the good news about this is when the cylinder was crossed over, again you get back to your under store flow. This is what we have talked about in the last class that what you need to do is a kind of a control experiment; And the control experiment is will be such that you would have control over each and every parameter of it, and when you remove those parameters one by one, you should be able to get the undisturbed flow by itself. When and how this can happen? This can happen if by nature the flow is such that in the absence of disturbance, the flow is essentially laminar. So, this is what we did talk about.

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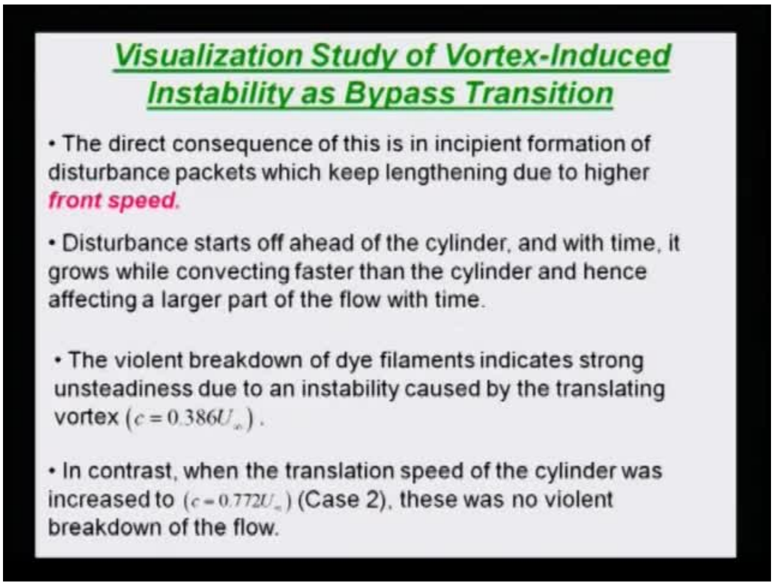

Visualization Study of Vortex-Induced Instability as Bypass Transition

- The direct consequence of this is in incipient formation of disturbance packets which keep lengthening due to higher *front speed*.
- Disturbance starts off ahead of the cylinder, and with time, it grows while convecting faster than the cylinder and hence affecting a larger part of the flow with time.
- The violent breakdown of dye filaments indicates strong unsteadiness due to an instability caused by the translating vortex $(c = 0.386U_\infty)$.
- In contrast, when the translation speed of the cylinder was increased to $(c = 0.772U_\infty)$ (Case 2), these was no violent breakdown of the flow.

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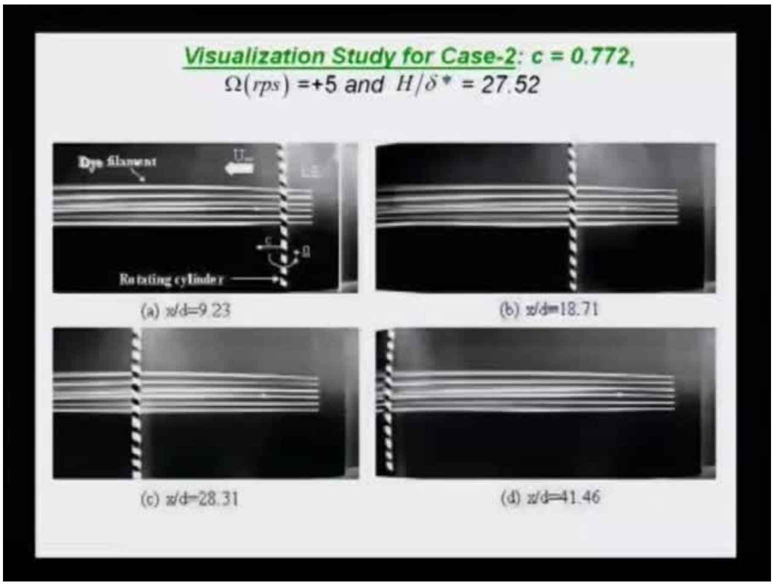

Visualization Study for Case-2: $c = 0.772$, $\Omega(rps) = +5$ and $H/\delta^* = 27.52$

(a) x/d=9.23 (b) x/d=18.71

(c) x/d=28.31 (d) x/d=41.46

Now, let us talk about the case, which we call as a case two, the speed of convection of the captive vortex has been doubled here. And up front I am telling you that there are no violent breakdowns of the flow; however what happens is, this is a scenario that you seen in the field of view here. Since the rotation rate is same, height is same, it is only the speed that is varied, and you see this is the way, this sequence is gone on. So, the captive vortex keeps moving, but the underlying shear layer does not react to it.

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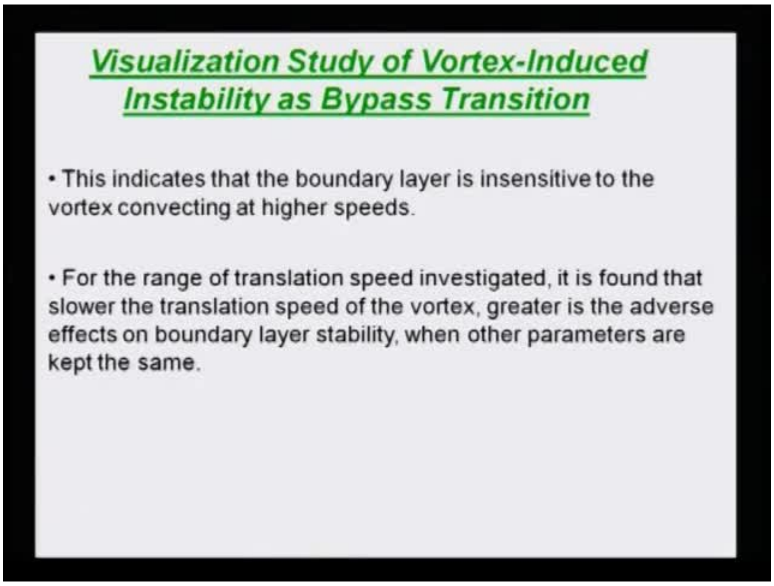

Visualization Study of Vortex-Induced Instability as Bypass Transition

- This indicates that the boundary layer is insensitive to the vortex convecting at higher speeds.
- For the range of translation speed investigated, it is found that slower the translation speed of the vortex, greater is the adverse effects on boundary layer stability, when other parameters are kept the same.

So it is kind of pretty much obvious to say that if you want to see the study the receptivity of the same boundary layer with vortex of the same strength that kept at the same height; the speed is a distinct we have important parameter. How this thing comes about so explain as we go along. This is what we are talking about; the last point please note that the range of translational speed that we investigated. We found that slower the translation speed of the vortex, greater the adverse effect of the boundary layer stability, when other parameters are kept the same. So, this is the generic observation that we can draw by looking at this two cases.

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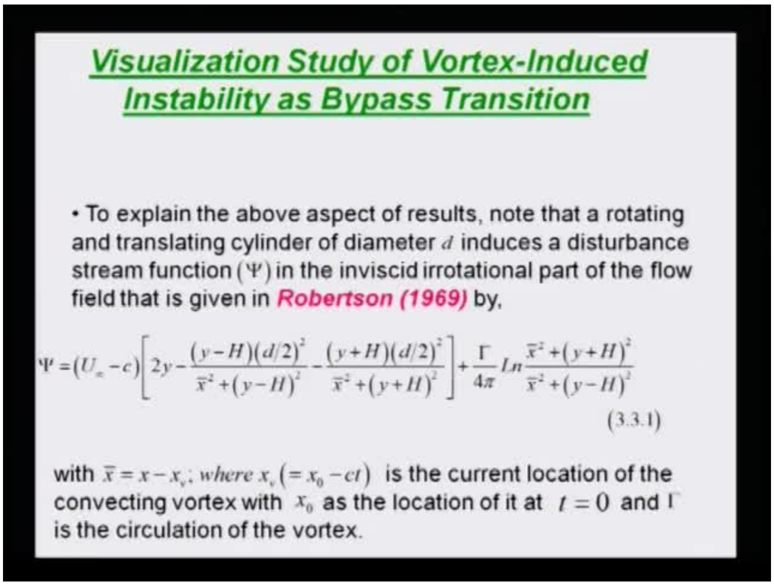

Visualization Study of Vortex-Induced Instability as Bypass Transition

- To explain the above aspect of results, note that a rotating and translating cylinder of diameter d induces a disturbance stream function (Ψ) in the inviscid irrotational part of the flow field that is given in *Robertson (1969)* by,

$$\Psi = (U_\infty - c)\left[2y - \frac{(y-H)(d/2)^2}{\bar{x}^2 + (y-H)^2} - \frac{(y+H)(d/2)^2}{\bar{x}^2 + (y+H)^2}\right] + \frac{\Gamma}{4\pi} Ln \frac{\bar{x}^2 + (y+H)^2}{\bar{x}^2 + (y-H)^2} \quad (3.3.1)$$

with $\bar{x} = x - x_v$; $where\ x_v \, (= x_0 - ct)$ is the current location of the convecting vortex with x_0 as the location of it at $t = 0$ and Γ is the circulation of the vortex.

Let us try to explain, what is happening? You see we have kept a vortex, which is a captive vortex, which is very much outside the shear layer. So, it (( )) induce a kind of a disturbance stream function. The disturbance stream function has two parts; one is due to the vortex itself, another is due to the cylinder. The cylinder is represented by a doublet in a uniform flow. And what is the uniform flow here now? u infinity minus c, because both of them are going in the same direction. So the relative speed is u infinity minus c. And this 2 y corresponds to the uniform flow part, and this corresponds to a doublet and its reflection, because it is going in front of a flat surface. So, you get a reflection the same height in one case, it is plus H; in other case, it is minus H. So, this is due to that actual vortex (( )) the doublet and this is it is reflection.

The same way you have a vortex captive with the cylinder that creates this circulatory effect, and you can see again the numerator and denominator respectively gives you the image on the original. And you note that we have indicated the x coordinate by x bar, where x bar is basically the location, where you were studying with respect to the vortex location. So x minus x v is your x bar; where x v itself is given by x naught minus c t, because it is moving at a constant speed c. So, what you are seeing is, basically these two effects. This I will call it as a circulatory effect or circulation effect, and this is what I will call it as a displacement effect right. Now you have this expression for psi. So, there is the very possibility that you could work this quantity is out yourself.

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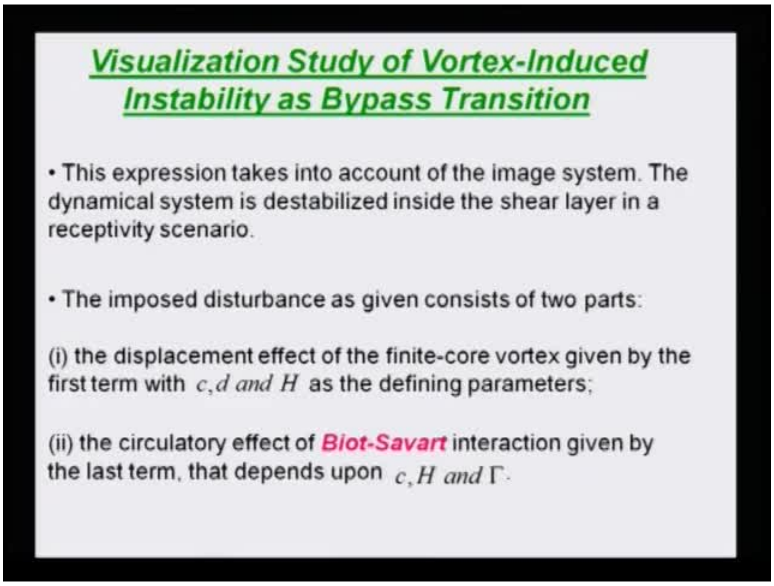

Visualization Study of Vortex-Induced Instability as Bypass Transition

- This expression takes into account of the image system. The dynamical system is destabilized inside the shear layer in a receptivity scenario.

- The imposed disturbance as given consists of two parts:

(i) the displacement effect of the finite-core vortex given by the first term with c, d and H as the defining parameters;

(ii) the circulatory effect of *Biot-Savart* interaction given by the last term, that depends upon c, H and Γ.

As I told you the expression takes account of the image system, the dynamical system is destabilized inside the shear layer in a receptivity scenario right, because we are creating a definitive disturbance and we are studying the response. I told you there are two effects; the displacement effect, and if you look at the expression, it depends on c, d - diameter of the cylinder, and H the distance. Well pretty much obvious; c is there, because we are talk about the relative speed, u infinity minus c that is how c appears there. Then d is of course, the size of the cylinder determines the dipole strength right, the doublet strength.

So that is the displacement effect; bigger the diameter, more the displacement effect you would have. An edge is coming into the picture, because of the way that the doublet is above the plate at this height. So you have another image also at minus H, so that is how in the displacement effect H comes into the picture. The circulatory effect is basically nothing but your Biot-Savart interaction that is very easy that depends upon from the expression we can see c, H and gamma.

Now, you may ask me a question that we did not see the expression for c here; we did not see the value of c appearing in this circulatory effect. How does it affect? Well, it is a certain thing, because we are creating a relative speed, and that relative speed if you recall would create at this cylinder is rotating that creates the circulation. So gamma itself that we have there is a function of c that is why it is; and dependence on h is pretty much obvious; and of course, you can also say that c comes in through the definition of x bar, because that is what you have there.

Now what happens is we can look at this expression, so this is the disturbance stream function. So if I really now consider a control volume over the flat plate; this is the disturbance that is created by the convicting vortex that we can put it on the boundary of this control volume. So what we could do is we could really estimate depending on the coordinates x and y what is it that we are getting.

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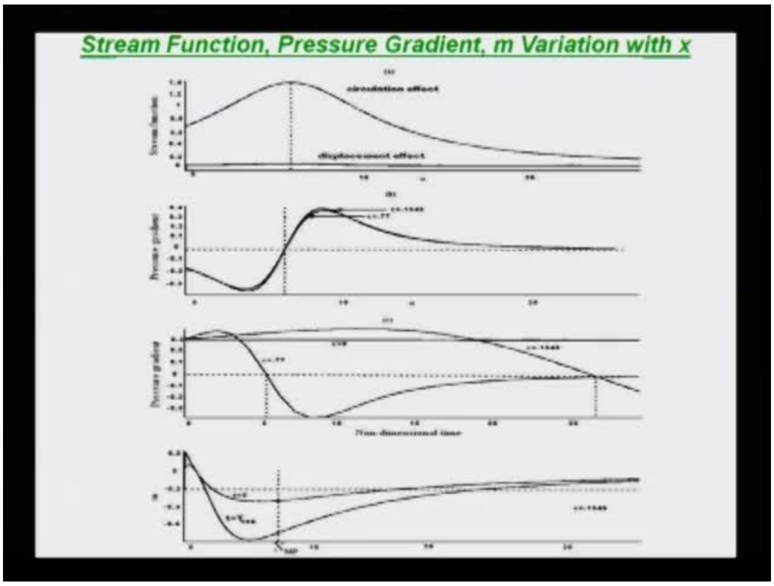

Stream Function, Pressure Gradient, m Variation with x

In this set of figure, we have shown the following quantities; the disturbance stream function itself has a function of x that is plotted and you have two paths; one path corresponds to the displacement effect; this horizontal effect quantity. And we have put in the vortex somewhere here, along that vertical dotted line. So, what happens is, of course the disturbance is maximum in the vicinity of the cylinder, and it falls of; and this is due to the circulation effect the gamma part. So, if I parameterize the value of gamma, and I can work out an expression this is how that is circulatory factor would be; how would displacement effect remains flat; it remains the same for all x(s) that is what you are seeing.

In the second frame what you are seeing basically a plot of the pressure gradient induced by this disturbance vortex as a function of x. And once again, what you notice is the around the mean position of the cylinder, you have on this side adverse effect, adverse pressure gradient, and this side you have a favorable pressure gradient. This is the case that is corresponding to case 1 and 2, where we are talking about a counter clockwise vortex. So what you notice that the movement you have a counter clockwise rotating vortex there; ahead of it, you see adverse pressure gradient; behind it you see a favorable pressure gradient.

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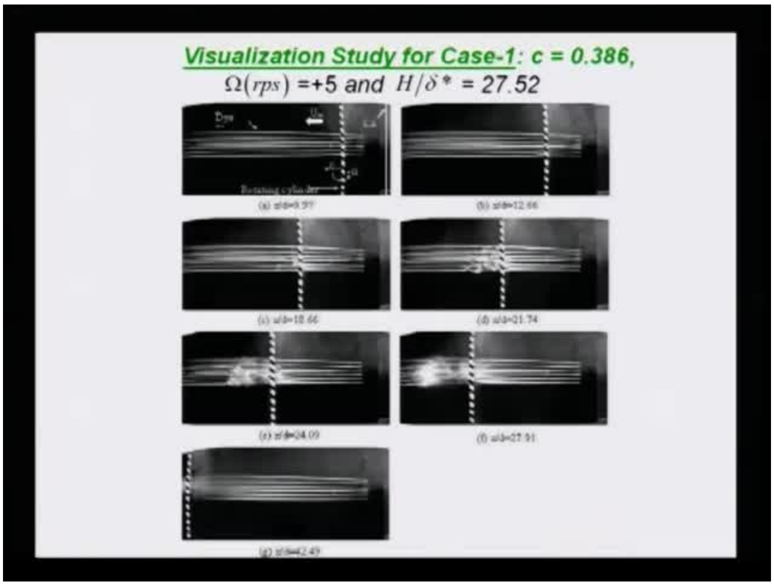

Visualization Study for Case-1: $c = 0.386$, $\Omega(rps) = +5$ and $H/\delta^* = 27.52$

Now, this is something you can convince yourself like what we seen before here. If I come back to case 1, what I notice is the flow is getting destabilize ahead of it, and that is the expression that we worked it out there. And you can also see a very interesting thing that ahead of the cylinder, the dice stray may under, but behind it is stabilizes and it becomes again smooth and nice. And that you can see that though when it is completely removed to get back your under store flow. So, it is not only it is under store flow, but because of the favorable pressure gradient again it actually stabilizes the flow, and that is precisely the expression that we noted there from that pressure gradient versus x plot.

Now, we have shown two cases of the pressure gradient for different values of c; one of the values is 0.1545 very low speed, another is 0.77. We have done this visualization cases, I did not know you 0.15, but 0.15 case had a much larger receptivity that is why we commented that more the speed, lesser will be the effect more will be the effect; and higher the speed, when the speed relative speed comes to almost 0, and then you would not see any effect at all but this figures seems to contradict that.

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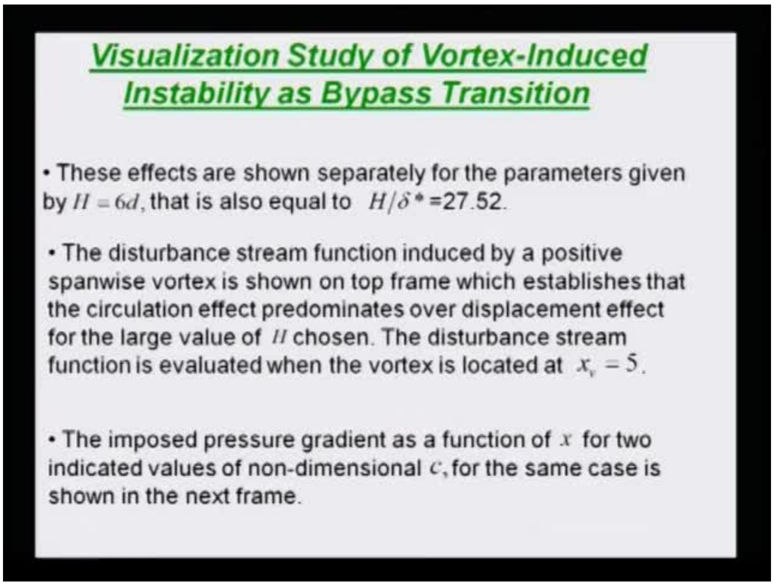

Visualization Study of Vortex-Induced Instability as Bypass Transition

- These effects are shown separately for the parameters given by $H = 6d$, that is also equal to $H/\delta^* = 27.52$.
- The disturbance stream function induced by a positive spanwise vortex is shown on top frame which establishes that the circulation effect predominates over displacement effect for the large value of H chosen. The disturbance stream function is evaluated when the vortex is located at $x_v = 5$.
- The imposed pressure gradient as a function of x for two indicated values of non-dimensional c, for the same case is shown in the next frame.

Now, why is that so? See that this is what we are talking about; that is the last point that you talk about that the impose pressure gradient as a function of x; for the two indicated values of c was shown in that frame.

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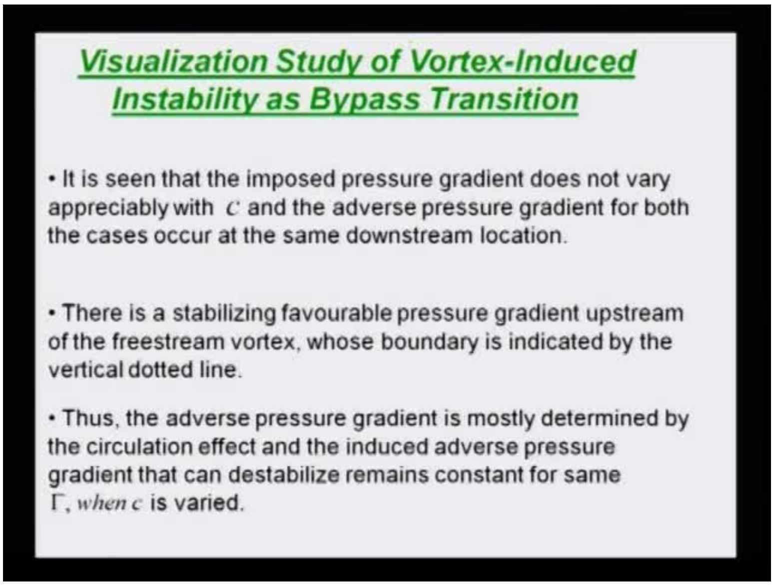

Visualization Study of Vortex-Induced Instability as Bypass Transition

- It is seen that the imposed pressure gradient does not vary appreciably with c and the adverse pressure gradient for both the cases occur at the same downstream location.
- There is a stabilizing favourable pressure gradient upstream of the freestream vortex, whose boundary is indicated by the vertical dotted line.
- Thus, the adverse pressure gradient is mostly determined by the circulation effect and the induced adverse pressure gradient that can destabilize remains constant for same Γ, $when\ c$ is varied.

That what you will notice from that figure that the imposed pressure gradient does not very appreciably with C; an adverse pressure gradient for both the cases are correct the same downstream location. However case 2 showed no receptivity; case one did show. We have talked about this that we have a stabilizing favorable pressure gradient upstream of the free stream vortex, the boundary is given by the vertical line, and the adverse pressure gradient is mostly determine by the circulation effect. And the induced adverse pressure gradient that can be stabilized remains constant for same gamma, when c is vary. Now this is what we observed.

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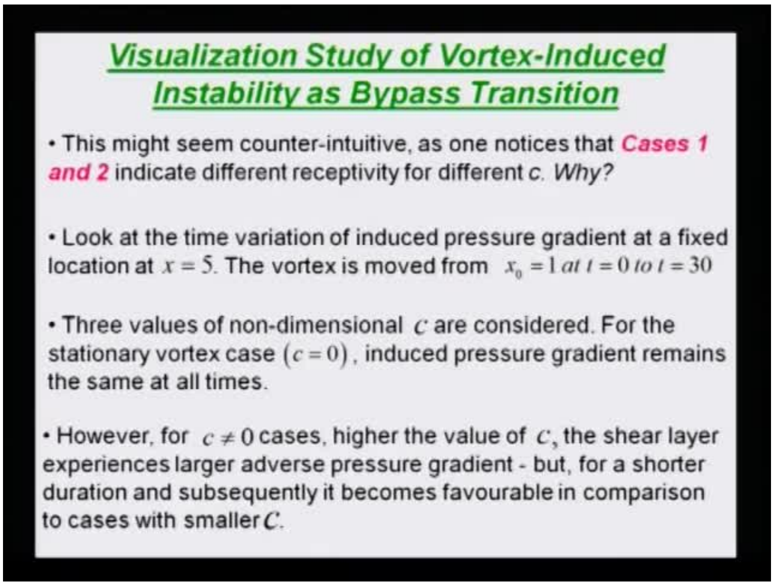

Visualization Study of Vortex-Induced Instability as Bypass Transition

- This might seem counter-intuitive, as one notices that ***Cases 1 and 2*** indicate different receptivity for different c. *Why?*
- Look at the time variation of induced pressure gradient at a fixed location at $x = 5$. The vortex is moved from $x_0 = 1\ at\ t = 0\ to\ t = 30$
- Three values of non-dimensional c are considered. For the stationary vortex case $(c = 0)$, induced pressure gradient remains the same at all times.
- However, for $c \neq 0$ cases, higher the value of c, the shear layer experiences larger adverse pressure gradient - but, for a shorter duration and subsequently it becomes favourable in comparison to cases with smaller C.

Now, we post the question that it seems counter intuitive that we have same adverse pressure gradient, but case one shows receptivity, case two does not; why? That you need to understand; if you look at the time variation of the induce pressure gradient at a fixed location, what we have done we have talked about looked at around a fix location x equal to 5, and the vortex is moved from x naught equal to 1 at t equal to 0 to t equal to 30. In this motion from 0 to 30, what happens to the pressure gradient as a function of time at x equal to 5 that is, what is plotted in the third frame. Now, that is what is plotted, if the third frame here. So this is pressure gradient verses non-dimensional time.

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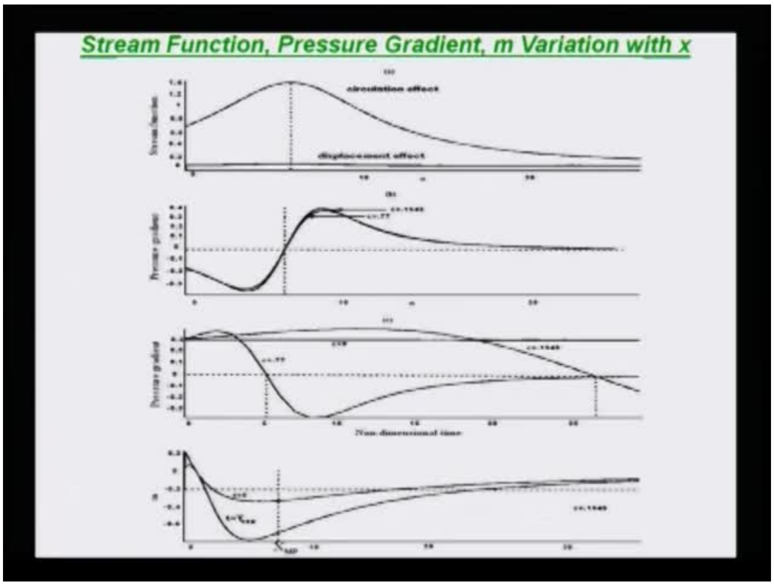

Stream Function, Pressure Gradient, m Variation with x

What you notice is the following that if you are vortex is not moving, then you going to get the case of c equal to 0; and in that case at that fix location, the pressure gradient remains the same. And what happens is that is an adverse position, so you have put in the vortex at x naught equal to 1 and at x equal 5 here, you are looking at it. So, what does it do? It basically creates a adverse pressure gradient ahead of it, and that remains in variant with time. Now you look at this case, this top figure corresponds to the case of c equal to 0.1545, whereas this curve corresponds to c equal to 0.77.

Now what you notice that adverse pressure gradient magnitude, maximum magnitude is more for 0.77 case, but it just becomes favorable after a short time; whereas for the case of 0 to low values, the adverse pressure gradient remains adverse over a long, long time for 0.145 case, you can see the flow is a experiencing adverse pressure gradient over such a long period. And c equal to 0, it is a even more interesting thing that you have the same level of adverse pressure gradient all the time.

So what happens is you can see the role, I mean it is not a static system that we have studying, we are looking at a dynamical system. So the time history is important. So what happens is 0.77 case although may produce at a fix time the same amount of adverse pressure gradient, but that maybe even little more, it is slightly more as you can see this curve goes slightly above 0.77 compared to 0.15 that is true here, but after some time it falls off. So it does not show that much of receptivity. So it is not only important that you must have a adverse pressure gradient, but it should also act over appreciable period of time; that is the important to issue to look at.

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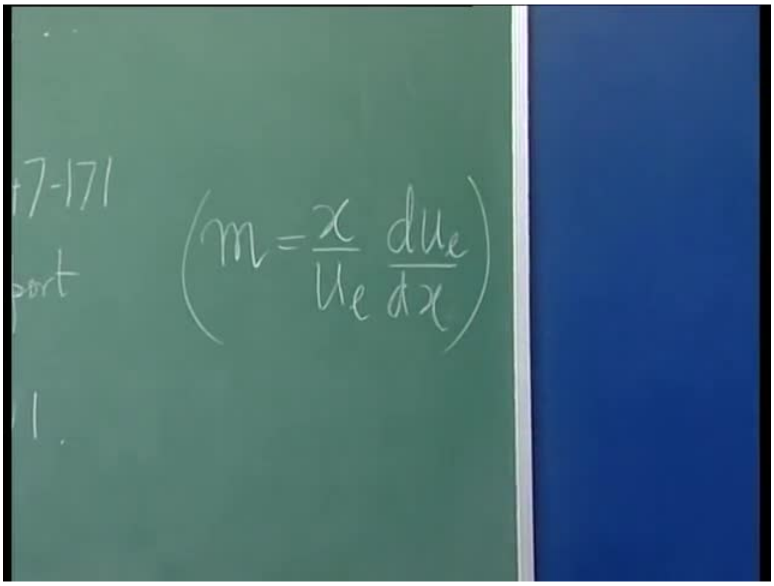

And the last quantity is that focal scan parameter that we have define as m equal to x by u e d u e d x. This is the non-dimensional pressure gradient parameter that we have plotted there; and what we show the top curve corresponds to t equal to 0; so at the initial instant what is the variation of m with respect to x, this case is shown for a low value of c, this value of c is 0.1545. So at t equal to 0, m variation is like this; what is this dotted line this dotted line corresponds to the case, where you have a steady separation, study flow separation right.

So, we know that it does so; so what happens is at even at initial time, instead then we have a range of x, where you have adverse pressure gradient that is more adverse than what you get for steady separation. So, in a dynamical science, you do create much larger adverse pressure gradient. And if you keep…now looking at it, this snapshot as a function of x that is shown here for t equal to 0. At a later time, you can see it keeps becoming more adverse; and it become adverse over a longer stretch, and even the adversity increases in quantum. You can see, this is the case, where actually you get the maximum adverse pressure gradient at this location. So this is roughly around t equal to 23, this time is around 23, maximum adverse pressure gradient is here, but you see at this location shown by this dotted line, where you see that first primary bubble that unsteady separation starts from that point.

So you can see that it is a very complex case, so you although it may appear that everything is in a static case, you know you have a study boundary layer, and you have a vortex that is going at a constant speed, but you can see so much of unsteadiness involved that is shown in these last two picture. So, basically I would assign you a home work, which I would expect you to handwritten by next Monday, where you would be doing a much more details study of all this. So I would, I will send you an email, you can follow it up, what has to be done.

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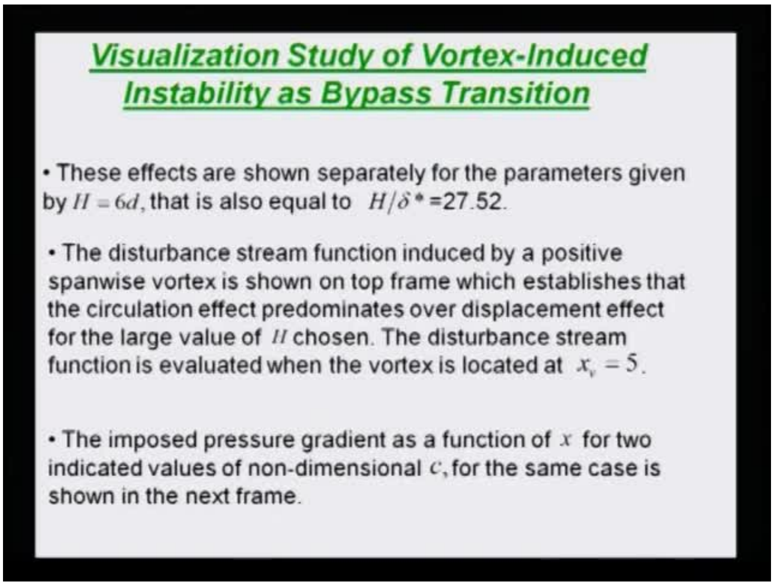

Visualization Study of Vortex-Induced Instability as Bypass Transition

- These effects are shown separately for the parameters given by $H = 6d$, that is also equal to H/δ^* =27.52.
- The disturbance stream function induced by a positive spanwise vortex is shown on top frame which establishes that the circulation effect predominates over displacement effect for the large value of H chosen. The disturbance stream function is evaluated when the vortex is located at $x_v = 5$.
- The imposed pressure gradient as a function of x for two indicated values of non-dimensional c, for the same case is shown in the next frame.

So, we have talked about all this, we have talked about we have seen all this before.

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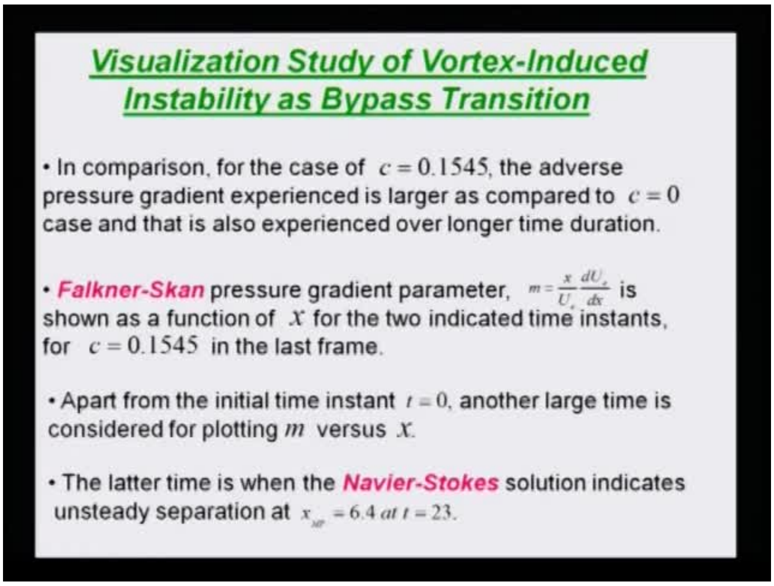

Visualization Study of Vortex-Induced Instability as Bypass Transition

- In comparison, for the case of $c = 0.1545$, the adverse pressure gradient experienced is larger as compared to $c = 0$ case and that is also experienced over longer time duration.
- *Falkner-Skan* pressure gradient parameter, $m = \frac{x}{U_e}\frac{dU_e}{dx}$ is shown as a function of x for the two indicated time instants, for $c = 0.1545$ in the last frame.
- Apart from the initial time instant $t = 0$, another large time is considered for plotting m versus x.
- The latter time is when the *Navier-Stokes* solution indicates unsteady separation at $x_{sep} = 6.4 \; at \; t = 23$.

So, what we are now going to do is, we are going to talk about the case, where when we compute the Navier-Stokes equation at t equal to 23 at this location we saw the unsteady bubble start forming, unsteady separation start forming, this is what we call as the primary instability. And in the following what we were going to do is, we are going to show some bit of computational result of it.

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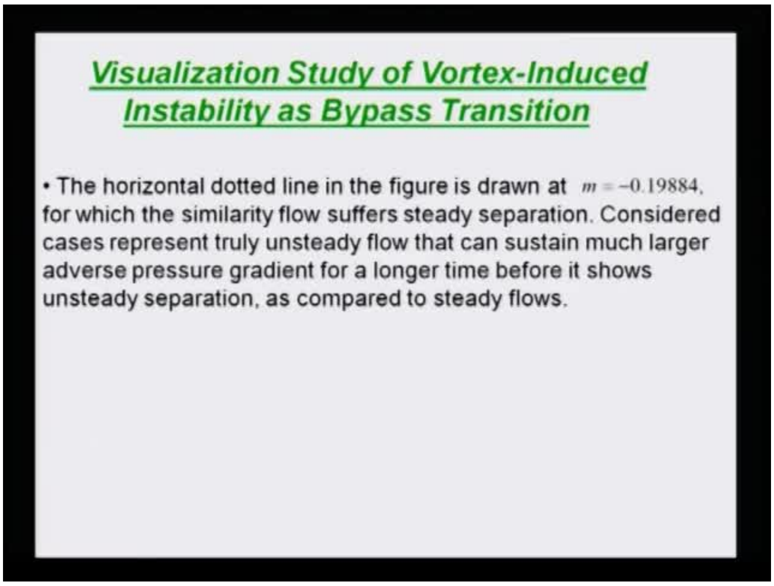

Visualization Study of Vortex-Induced Instability as Bypass Transition

- The horizontal dotted line in the figure is drawn at $m = -0.19884$, for which the similarity flow suffers steady separation. Considered cases represent truly unsteady flow that can sustain much larger adverse pressure gradient for a longer time before it shows unsteady separation, as compared to steady flows.

I explain to you those horizontal dotted line is drawn at this value of m for which the similarity flow suffers a steady separation. And when we are looking at truly unsteady flow, which consistent must higher values of m, and still do not show separation. So this is something we must understand that Prantl's criteria of steady flow separation is just a very empirical observation; it was just stated that where tau equal to 0 would lead to separation. But you see here in such cases we have much, much more adverse pressure gradient, and still we do not see it.

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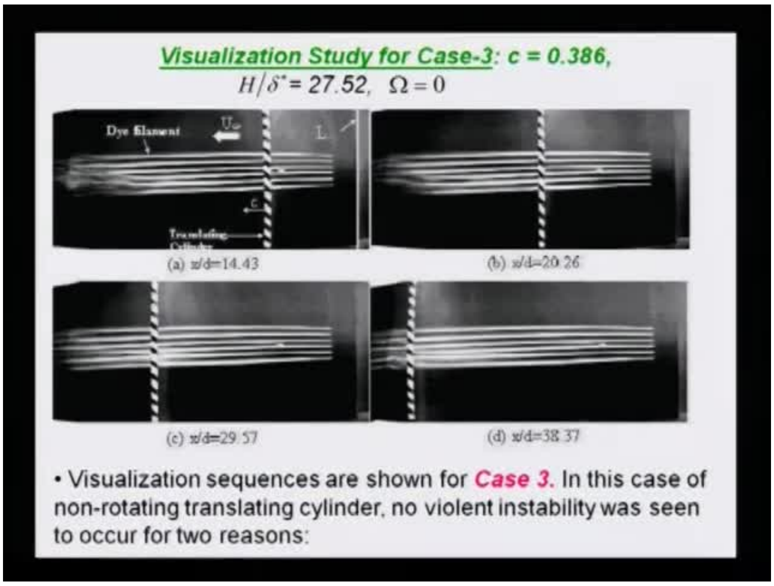

Visualization Study for Case-3: $c = 0.386$, $H/\delta^* = 27.52$, $\Omega = 0$

- Visualization sequences are shown for *Case 3*. In this case of non-rotating translating cylinder, no violent instability was seen to occur for two reasons:

Now, let us look at the case and controlled case that is what we are talking about. These cylinders just simply dragged along at this speed 0.386 of u infinity; height kept the same, but it is not rotating. When it does not rotate, we have seen the expression for psi, the circulatory part is 0.

So whatever you are getting is basically a displacement effect. And if you drag it along like this in sequence, then you see nothing really happens. So this was what we talked about also the experimental result of (( )) the JFM paper and as well as (( )) that physics of fluid paper, which had established the case that if you rotate it at a high rate, it does not. But even in this case, we do not rotate it, there might be vortex shading, but because of this large distance of separation between the vortex that is in the free stream and done below. What you are seeing essentially it is only the displacement effect that is not good enough to create the instability that is why, this whole phenomenon we have called it as vortex induced instability right.

So, as you can see this is what we are calling as vortex induced instability. Why did we add the words of critical that should be pretty much obvious to you, because in the absence of any disturbance, flow remains quiescent; it is only when I disturb the flow, I see the response; this can only happened if it is sub critical; if it was supercritical, then I would see that the flow itself has a tendency to become unstable. So that is what we are saying that in this case of non-rotating translating cylinder, we do not see any violent instability of for two reasons.

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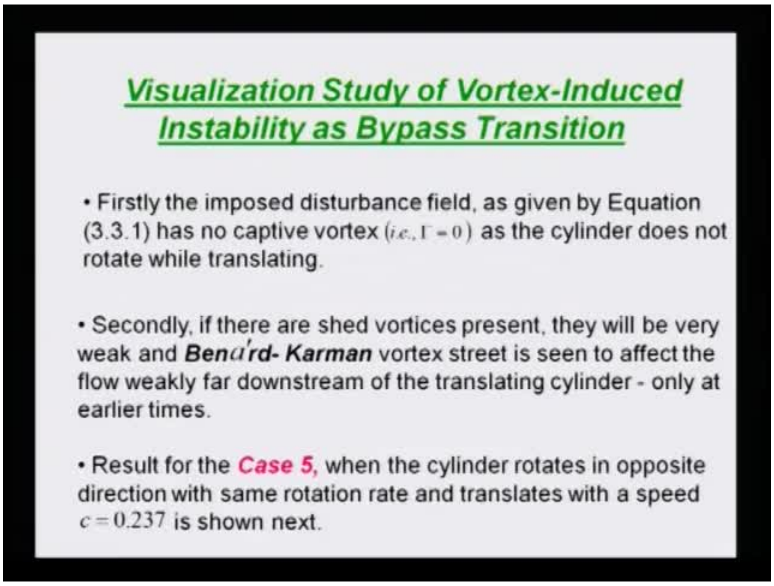

Visualization Study of Vortex-Induced Instability as Bypass Transition

- Firstly the imposed disturbance field, as given by Equation (3.3.1) has no captive vortex ($i.e., \Gamma = 0$) as the cylinder does not rotate while translating.

- Secondly, if there are shed vortices present, they will be very weak and ***Benárd- Karman*** vortex street is seen to affect the flow weakly far downstream of the translating cylinder - only at earlier times.

- Result for the **Case 5**, when the cylinder rotates in opposite direction with same rotation rate and translates with a speed $c = 0.237$ is shown next.

Firstly the imposed disturbance field as given by that equation has no captive vortex gamma equal to 0, because the cylinder does not rotate while translating. Secondly even if there are shared vortices, they will be very weak and the corresponding Bernard-Karman vortex to it is seen to affect the flow rather very weakly in for downstream location. Why does effect for the downstream location that is where a boundary layer is growing your Re delta star is growing, the transfer function is becoming more and more receptive to smaller disturbances. And that is why you may see some weak effects there that is what we are saying for downstream of the translating cylinder, only at some earlier times. So, this could be essentially due to the transient.

Now, we show a case, where the cylinder now rotates in a opposite direction. The rotation rate is kept the same, translational speed is reduced, we have seen now that if you make it dual, make it goes slower; then the adverse pressure gradient will be sustained over longer time duration. So, if the flow receptivity, it will be doing that. Now come to think of it, what the flow is doing. If this cylinder is going like this and consider this is a boundary layer in this direction; now if I rotate it like this, when a counter clockwise manner, then what does it do; what is the induced effect? It will try to cover out the boundary layer, it tries to pull it up; and that is what we are seen. I have mentioned it very clearly in the last class also that when you look at the dye that you see there are two things; one is one layer that remains parallel very much embedded, and the other one has been lifted up on its own plane, on it is own plane.

So it is basically a two-dimensional phenomenon, and this is due to this induced effects; so that is why we called it as vortex induced stability; and because it is very much dependent on the large excitation, this must be a sub critical case. Well, these are some of the other references, where experiments are conducted by J M Kendall at JPL, and we talked about boundary layer receptivity to free stream turbulence in this paper. But you see this is what we have highlighted in the last class also. You can do an experiment, but if you cannot reproduce it; time and again and again, then it becomes very, very difficult. Even though you are working in the same tunnel, it is very unlikely the two realizations give you a same thing that is because we are not doing a sub critical study.

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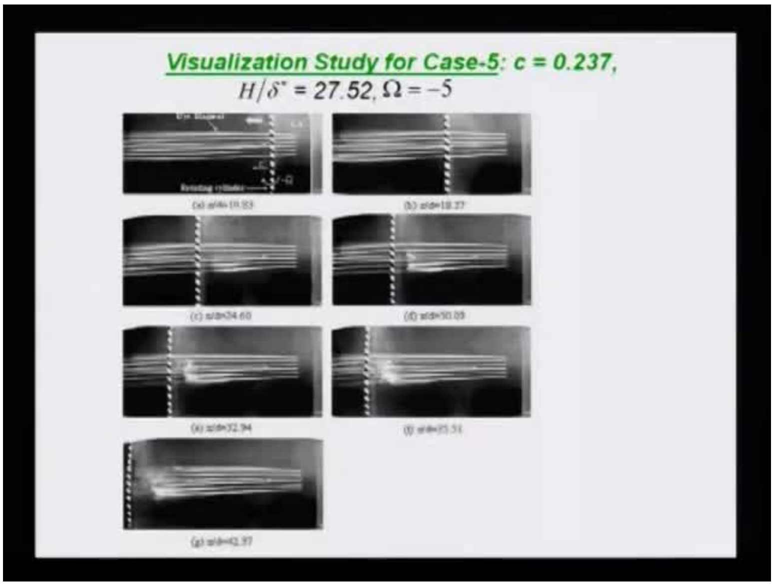

Visualization Study for Case-5: $c = 0.237$,

$H/\delta^* = 27.52, \Omega = -5$

Here, we have done a sub critical study to understand what the mechanism is, and we can reproduce it time and again, time and again. So this is something that we must understand. So now, let us take a look at the case 5, where the vortex is in a clockwise direction, and if the vortex in the clockwise direction, the vortex is moving like this like what we have shown here 1, 2, 3, 4, 5, 6, 7 in this sequence. Then you can see the, what is happening, it is going on alright, the dyes was smooth and parallel. Nothing much happens in front, but do you see there is a bit of swelling little in the behind the cylinder growing up.

Now, you can see distinct bulge here, distinct bulge here, some kind of a mixing spanner was mixing and so and so forth. So what happens is, in this case, the effect is occurring upstream of the vortex. And this is once again the same logic that we talked about, think of the boundary layer like this, and let us say it is going in the clockwise direction like this. If it does like this, then in the front, it tries to reduce the thickness of the boundary layer; it imposes a downward velocity, whereas up stream behind it, it has the other effect, it tries to cover it out. So this is the the kind of explanation that you can immediately se.

Well, you can see then that if you are going to experience some kind of a vortex inducing stability, then you can take appropriate action. And what you notice is the clockwise case was more violent as compared to… Anticlockwise case was more violent compared to the clockwise case. In the clock anticlockwise case, what happen? The disturbances were ahead of the vortex, and it was much more severe; whereas for the clockwise case, it happens behind the vortex, and it is of a much lower intensity. This is what I told you about it not much of qualitative study has been done.

But one can correlate what we have just now observe to what people experience in a flight; what is called as due to convective clouds going in, in the vicinity of the aircraft; you must have heard during the flight, pilot telling you to pass in a belt, it is start; it is basically vortex inducing stability; this convective cloud have associated vortices in them, and if the aircraft comes very close to it, it does get effected. However if you are a pilot, you know what to do, when you are approaching a vortex, how do the detect vortex in a aircraft; any idea? Where they use some kind of a Doppler radar, and from the humidity content, they try to detect.

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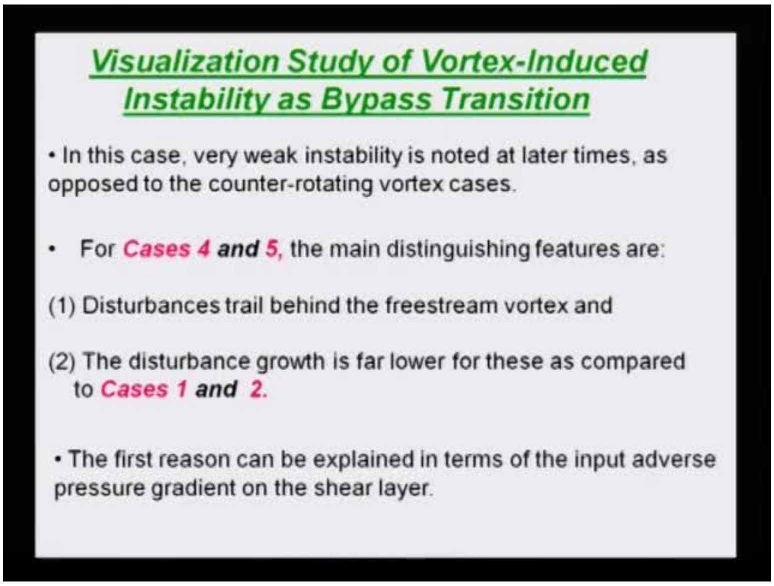

Visualization Study of Vortex-Induced Instability as Bypass Transition

- In this case, very weak instability is noted at later times, as opposed to the counter-rotating vortex cases.

- For ***Cases 4 and 5,*** the main distinguishing features are:

(1) Disturbances trail behind the freestream vortex and

(2) The disturbance growth is far lower for these as compared to ***Cases 1 and 2.***

- The first reason can be explained in terms of the input adverse pressure gradient on the shear layer.

I do not know whether it is feasible for the Doppler radar to basically tell you about those circulations, but if you does, let say then the pilot can decide to go above or below this. You understand the same thing if it is rotating like this, and I am going below it, then of course, it is going to be by counter clockwise case, and I will have wall inter stability. But if I go above it, then you can see how I have this second case, which is little more benign. So, this is something that you can understand from these experiments.

So, we can have a kind of summarize for this clockwise rotating case that here we see a weak in stability, at later time as oppose to the counter rotating vortex cases. And for this two cases, the main distribution features disturbance trail behind, the free stream vortex, and the disturbance growth is far lower for this as compared to other two cases. Now, this first can be explained in terms of the input adverse pressure gradient, and this is what I told you about this covering effect that is exactly the adverse pressure gradient tries to do also that helps in doing that.

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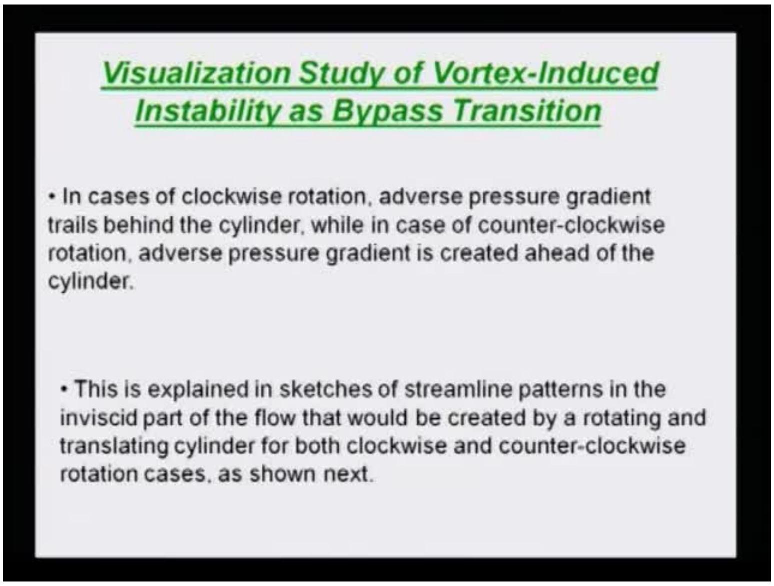

Visualization Study of Vortex-Induced Instability as Bypass Transition

- In cases of clockwise rotation, adverse pressure gradient trails behind the cylinder, while in case of counter-clockwise rotation, adverse pressure gradient is created ahead of the cylinder.

- This is explained in sketches of streamline patterns in the inviscid part of the flow that would be created by a rotating and translating cylinder for both clockwise and counter-clockwise rotation cases, as shown next.

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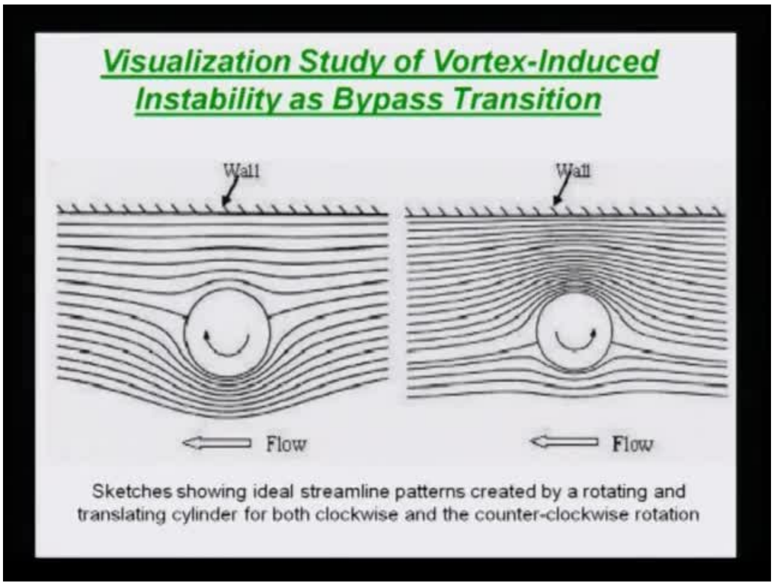

Visualization Study of Vortex-Induced Instability as Bypass Transition

Sketches showing ideal streamline patterns created by a rotating and translating cylinder for both clockwise and the counter-clockwise rotation

In case of clockwise rotation, adverse pressure gradient trails behind the cylinder, while in case of counter clockwise rotation, adverse pressure gradient is created ahead of the cylinder. That those 5, those 4 flames that I shown; you can perform redo the case; put in side by side, a clockwise vortex and anticlockwise vortex, and plot those and you can see what we are talked about. You can also see from a sketch of downstream line based on let us say in viscid flow model itself, because your captive vortex is very far away outside the shear layer, so you can draw a sketch, and this is the kind of thing that you are going to see.

So here is the case that is your wall, this is your convicting vortex, the flow is coming is like this; so if it is rotating in a clockwise manner, what do you see about the pressure gradient; how do you see? You see it from the curvature of the streamline, where is it maximum? This is here behind the cylinder; you see here it goes rapidly. Here also there is a kind of a divergence, streamline divergence, but this more like straight; whereas here you will know it was adverse in this part that is followed by a favorable pressure gradient. So, even if the flow was becoming unstable here, but when it comes very close to the cylinder itself, you actually again impose a favorable pressure gradient. So, that thing is created there, but that is again followed by a rapid (( )).

So, this is the case of clockwise case, and you can see in the case of counter clockwise case; of course, this side streamlines are converging, so you have a favorable pressure gradient, and this side it is continuously diverging, and you see the effect. And you can also now see that in this case, this kind of adverse pressure gradient is over a small region that is what if you look at the case 5, plot that I shown. The bulging happened over a very narrow range, it remain very much confined there; do not over a large distance in the upstream, because later on again you can see streamlines converts, so you have a favorable pressure gradient.

So you, that is what I was telling you about the strategy of avoiding say convicting vortices effect, whether you want to go above or below, because the adverse case is a very, very narrowly focused region; whereas in this case, adverse effect is a over a large region, I mean it keeps it all pervading right. So, this sketch is pretty much revealing that you can understand it. So again to emphasize the point, you see what happens? To the boundary layer stability or instability is dictated upon the impressed pressure gradient in the, from the outside. So, in this case, what is happening? So your shear layer may be here, embedded right next to the wall, but what is the pressure gradient that is imposed? That is given by the streamline curvature. And here you see a very interesting feature, you start off with a adverse pressure gradient, but more or less with us similar diverging that is followed by a favorable pressure gradient, again followed by a rapid adverse pressure gradient, and again followed by favorable pressure gradient.

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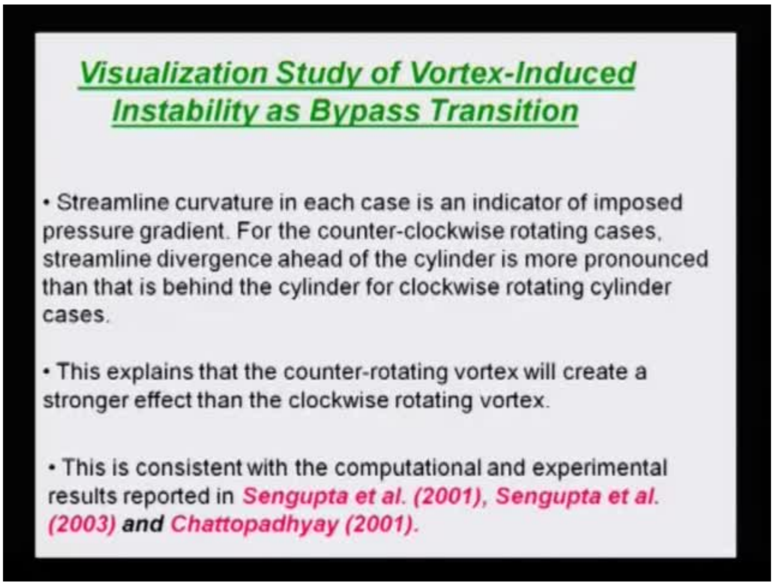

Visualization Study of Vortex-Induced Instability as Bypass Transition

- Streamline curvature in each case is an indicator of imposed pressure gradient. For the counter-clockwise rotating cases, streamline divergence ahead of the cylinder is more pronounced than that is behind the cylinder for clockwise rotating cylinder cases.
- This explains that the counter-rotating vortex will create a stronger effect than the clockwise rotating vortex.
- This is consistent with the computational and experimental results reported in *Sengupta et al. (2001), Sengupta et al. (2003)* **and** *Chattopadhyay (2001).*

So, whatever done is that can occur, can occur only here; whereas in this case, what is happening? This side is decidedly favorable pressure gradient, because a streamlines are converging, and on this side, flow is decidedly adverse, I mean unstable, because it is constantly unstable. So, this is the picture that we draw out of simple streamline consideration. That is precisely what we are also stating here with the streamline curvature in each case is the indicator; what kind of pressure gradient that you have imposed? For the counter clockwise case, the streamline divergence is ahead of the cylinder, and that is also more pronounced; then that is behind the cylinder for clockwise rotating case right, this is what we emphasized just now. This explains why a counter clockwise rotating vortex will create a stronger effect as compare to clockwise rotating.

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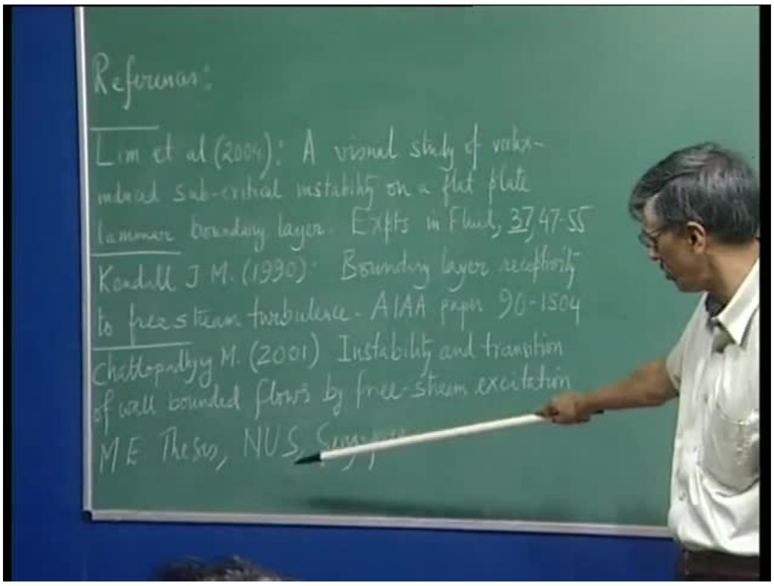

References:

Lim et al (2004): A visual study of vortex-induced sub-critical instability on a flat plate laminar boundary layer. Expts in Fluids, 37, 47-55

Kendall J M. (1990): Boundary layer receptivity to free stream turbulence. AIAA paper 90-1504

Chattopadhyay M. (2001) Instability and transition of wall bounded flows by free-stream excitation M E Thesis, NUS, Sing

This we actually did lot of computations and experiments; the experiments are reported in this paper plus (( )) thesis, it was a master's thesis done in 2001.

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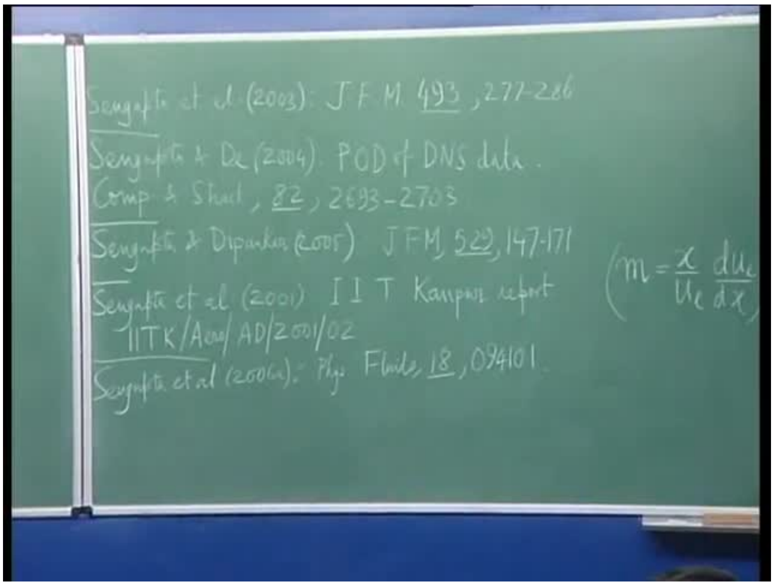

Sengupta et al (2003): J F M 493, 277-286

Sengupta & De (2004): POD of DNS data. Comp & Struct, 82, 2693-2703

Sengupta & Dipankar (2005) J F M, 529, 147-171

Sengupta et al (2001) I I T Kanpur report IITK/Aero/AD/2001/02

Sengupta et al (2006a): Phys Fluids, 18, 094101

$\left(m = \frac{x}{U_e}\frac{dU_e}{dx}\right.$

And this paper is here in this fast track paper in journal of fluid mechanics which gave a physical and theoretical explanation for these phenomena. So we did show that, we will now shortly carry on over discussion in that.

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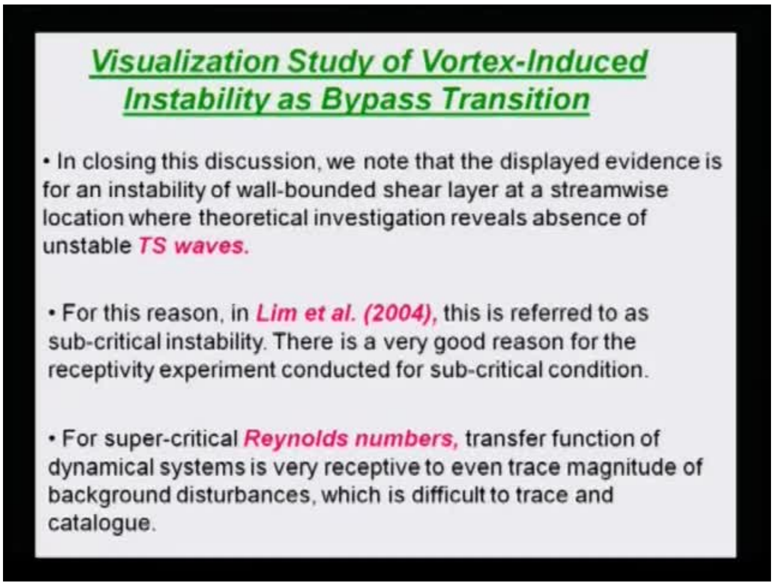

Visualization Study of Vortex-Induced Instability as Bypass Transition

- In closing this discussion, we note that the displayed evidence is for an instability of wall-bounded shear layer at a streamwise location where theoretical investigation reveals absence of unstable *TS waves.*

- For this reason, in *Lim et al. (2004),* this is referred to as sub-critical instability. There is a very good reason for the receptivity experiment conducted for sub-critical condition.

- For super-critical *Reynolds numbers,* transfer function of dynamical systems is very receptive to even trace magnitude of background disturbances, which is difficult to trace and catalogue.

Now, in trying to close a discussion on the visual signature, what we are note that the displayed evidence is for an instability of a wall bounded shear layer, at a stream wise location, where the theoretical investigation reveals absence of unstable TS waves. Later on I will show you some completed results, where the computational domain is restricted to something like where Re delta star at the outflow is less than 500. So, we are talking about a sub critical case, because TS wave only begins after 520. So that is what we are saying the theoretical investigation reveals absence of unstable TS waves. Now this is the reason we called it as a sub critical, sub critical with respect to TS wave; unfortunately, you know the initial struggle to established existence of TS wave was so much that afterwards people do not see anything much beyond TS waves; so all our vocabulary is with respect to TS wave. So this is sub critical with respect to TS wave.

So, there is a very good reason for the receptivity conducted, because why we do a sub critical test is because if we are looking at a supercritical Reynolds number, we talk about the transfer function; transfer function of the dynamical system that would be very receptive to even a trace magnitude of background disturbances. You know like what we talked about the case 3, where it was just simply translating towards the end; if there is a rapid time variation, we would see some bit of disturbances growing up; and that is precisely what happens with a super critical case. So if you are trying to establish some physical mechanism, it is in your best interest that you do the experiment on a sub critical scenario.

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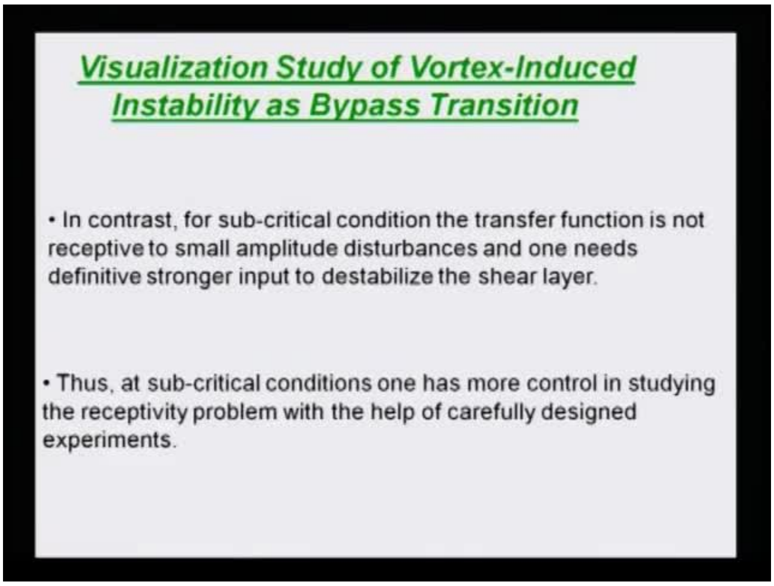

Visualization Study of Vortex-Induced Instability as Bypass Transition

- In contrast, for sub-critical condition the transfer function is not receptive to small amplitude disturbances and one needs definitive stronger input to destabilize the shear layer.

- Thus, at sub-critical conditions one has more control in studying the receptivity problem with the help of carefully designed experiments.

So that is precisely what is said that for sub critical state, transfer function is not receptive to small amplitude disturbances. And one needs definitive stronger input to destabilize (( )). So that you can reproduce it; you can do this experiment any number of time, and you would see the same result. One has more control in studying, the receptivity problem with the carefully designed experiment.

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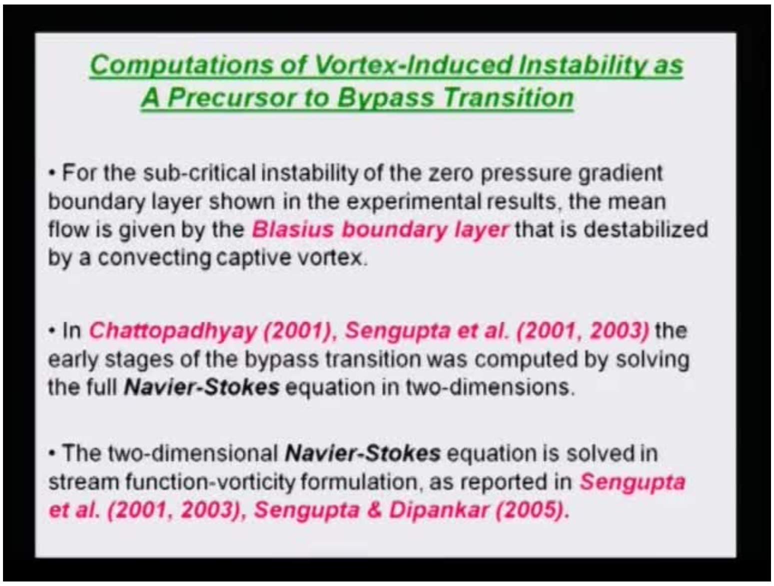

Computations of Vortex-Induced Instability as A Precursor to Bypass Transition

- For the sub-critical instability of the zero pressure gradient boundary layer shown in the experimental results, the mean flow is given by the *Blasius boundary layer* that is destabilized by a convecting captive vortex.

- In *Chattopadhyay (2001), Sengupta et al. (2001, 2003)* the early stages of the bypass transition was computed by solving the full ***Navier-Stokes*** equation in two-dimensions.

- The two-dimensional ***Navier-Stokes*** equation is solved in stream function-vorticity formulation, as reported in *Sengupta et al. (2001, 2003), Sengupta & Dipankar (2005).*

Now, we are going to talk a little bit about computations of vortex inducing stability. The instability that we see is a precursor to the overall phenomena that we talk to called it as the bypass transition. What we intend establishing here that if I have a convicting a periodic vortex that creates a kind of and primary instability; and if I have a bulge in the shear layer, then what happens, and then immediate neighborhood of the bulge, I create a secondary instability, and I can create a tertiary instability. So eventually all of this takes over. And we have seen from the experimental evidence for case one that it just happens in a very violent manner; once it appears, it just takes over and spreads over the whole domain, and you see a massive mixing.

So, this bypass transition mechanism that we talked about is a very important law. So what we can do is we can think of that we have a Blasius boundary layer that we can set it up in the lab very easily; people have done it. And then we can destabilize it by a convicting vortex. And these are the references that we have talked about we did compute the early stages of bypass transition, because we are solving 2D Navier-Stokes equation. Once span wise mixing, start occurring your results would not be a really very relevant, so that is what we do is solve 2D Navier-Strokes equation to predict the earlier stages to understand the mechanism.

And what you do is take up the Navier-Strokes equation in stream function vortices formulation, and this you do basically for the accuracy sake, because existences of psi the stream function would ensure mass conservation in a numerical sense, and basically fluid mechanics is all about vortices dynamics. So, there is nothing better than working with the vortices itself, so this formulation is always recommended these are all this references that you can see in this, in this, and in this we have define all this methodologies that is are written there.

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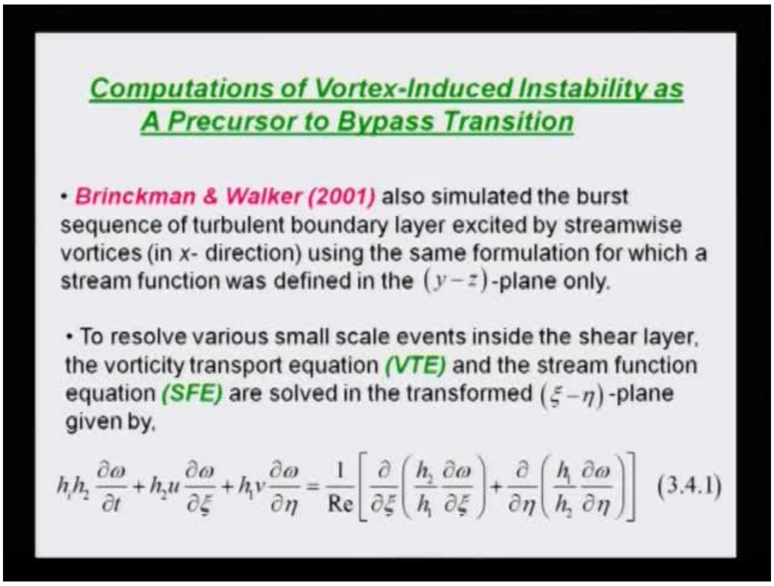

Computations of Vortex-Induced Instability as A Precursor to Bypass Transition

- ***Brinckman & Walker (2001)*** also simulated the burst sequence of turbulent boundary layer excited by streamwise vortices (in x- direction) using the same formulation for which a stream function was defined in the $(y-z)$-plane only.
- To resolve various small scale events inside the shear layer, the vorticity transport equation ***(VTE)*** and the stream function equation ***(SFE)*** are solved in the transformed $(\xi-\eta)$-plane given by,

$$h_1 h_2 \frac{\partial \omega}{\partial t} + h_2 u \frac{\partial \omega}{\partial \xi} + h_1 v \frac{\partial \omega}{\partial \eta} = \frac{1}{\mathrm{Re}}\left[\frac{\partial}{\partial \xi}\left(\frac{h_2}{h_1}\frac{\partial \omega}{\partial \xi}\right) + \frac{\partial}{\partial \eta}\left(\frac{h_1}{h_2}\frac{\partial \omega}{\partial \eta}\right)\right] \quad (3.4.1)$$

So, just we mention that even for three-dimensional flow, you can mimic this point of view that is what Brinkman and Walker did what they did, they looked at turbulent flow. And suppose the flow is in this direction and I have stream wise vortices, if I have that then I study a cross flow plane; so if this my x direction, if I study in the y z plane then I am precisely seeing what we are seeing here. So, in a 3D flow, you do not have a stream function (( )) vector potential, but if I am studying in the y z plane or the effect of a vortex in the x direction omega x is exactly like my equivalent 2D flow. So, that is what they have done. So, they looked as the vortex in the x direction, and studied the flow in y z plane.

And in this case what did we see kind of a un-relation in the stream wise direction when you had omega z in the x y plane right, now if I do this here also I would see the same thing, I would see a what people like to call as peak value structure. In the span wise direction I will have un-relating structure that is exactly the same scenario. So, let us go head, and try to see what happens when we compute such a flow, as I told you we are using a vortices stream function formulation, so the vortices dynamics is given by this equation given there for the transform plane representation.

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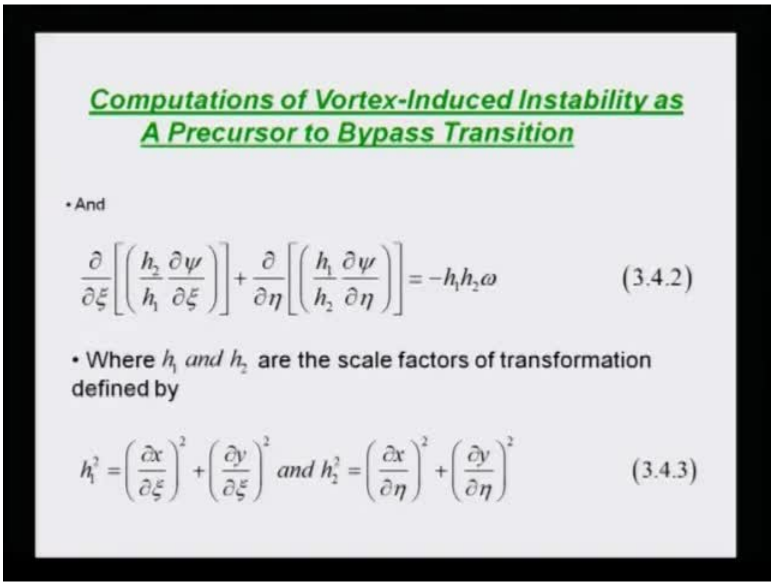

Computations of Vortex-Induced Instability as A Precursor to Bypass Transition

- And

$$\frac{\partial}{\partial \xi}\left[\left(\frac{h_2}{h_1}\frac{\partial \psi}{\partial \xi}\right)\right]+\frac{\partial}{\partial \eta}\left[\left(\frac{h_1}{h_2}\frac{\partial \psi}{\partial \eta}\right)\right]=-h_1 h_2 \omega \qquad (3.4.2)$$

- Where h_1 *and* h_2 are the scale factors of transformation defined by

$$h_1^2=\left(\frac{\partial x}{\partial \xi}\right)^2+\left(\frac{\partial y}{\partial \xi}\right)^2 \text{ and } h_2^2=\left(\frac{\partial x}{\partial \eta}\right)^2+\left(\frac{\partial y}{\partial \eta}\right)^2 \qquad (3.4.3)$$

We have gone on from the physical to transform plane, it result all the phenomena by taking a stretched grade, and if that was the vortices transport equation - the equation 2 here it is the Kinematics definition of the vortices in terms of the stream function. So, this is your (( )) equation - stream function equation, and is the h 1, and h 2 are this scale factors of transformation scale factors of transformation. So, if how x and y varied of a function of psi and eta, a priory then you can calculate h 1 and h 2. So, they are functions of psi and eta but they are still known, this is the dictated up on by the grid that you choose.

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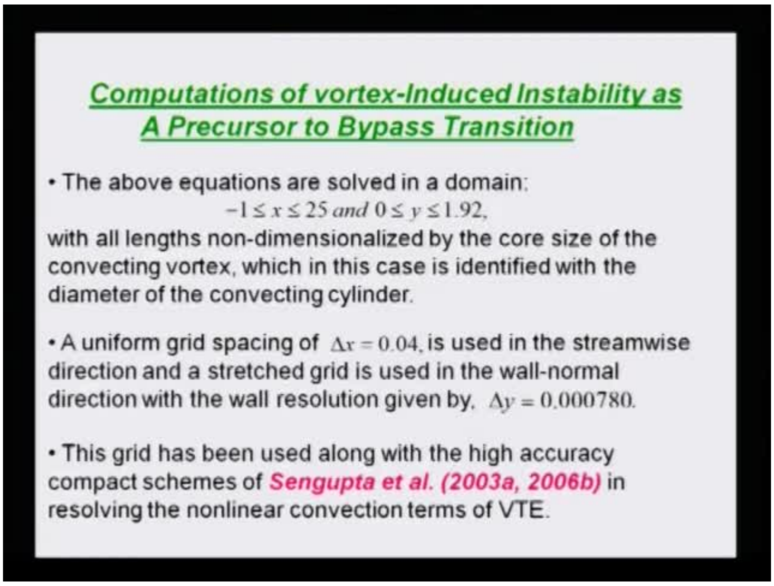

Computations of vortex-Induced Instability as A Precursor to Bypass Transition

- The above equations are solved in a domain:

$$-1 \le x \le 25 \text{ and } 0 \le y \le 1.92,$$

with all lengths non-dimensionalized by the core size of the convecting vortex, which in this case is identified with the diameter of the convecting cylinder.

- A uniform grid spacing of $\Delta x = 0.04$, is used in the streamwise direction and a stretched grid is used in the wall-normal direction with the wall resolution given by, $\Delta y = 0.000780$.
- This grid has been used along with the high accuracy compact schemes of *Sengupta et al. (2003a, 2006b)* in resolving the nonlinear convection terms of VTE.

What we did was we calculated the flow in domain in the x direction it went from minus 1 to 25. Now, what is minus 1? What is 25? Zero is a leading edge, so we have taken a domain which is little upstream on the leading (( )). So we want to include the effect of the leading edge, and we have gone significantly downstream whereas in the y direction we have gone on from this.

So these are non-dimensionally with respect to the diameter of the cylinder. So, this insignificantly more than the boundary layer thickness. So if I am talking about a 15 mm cylinder. So this is close to about 30 30 mm. So your boundary layer thickness is much lower than this. And all lengths non-dimensionally with the core size of the convicting vortex, which this is the kind of resolution that we took today we can do much better actually location as much final here. (( )) has much final calculation. Take it delta x of the order of this, and in the stream wise direction, you do not need to do any bit of stretching, u as direction you do not need to do a little bit of stretching, because this is a convicting structure. If you focus your attention at one time later on that place will have no disturbance, but while the disturbance have move there.

So it is very, very impractical to think of a grade, which will also move with the convicting structure, and we do not want know where the newer structure will come. Even if I decide to track let us see the primary instability, I do not want know, where the secondary and tertiary instabilities will stop. So all the better but is rather you take uniform grid spacing at least in the stream wise direction.

And in the wall-normal direction of course, you would like to resolve, not only the shear layer, but it instability. So, you take varies very, very small; spacing very close to the wall. So, we have used compact scheme; one of the near spectral accurate scheme that will have also developed given in some of the papers, and those are use to basically study the non-linear term, because you see most of the instabilities are basically a kind of a dynamic balance between physical dissipation and the non-linear convection term.

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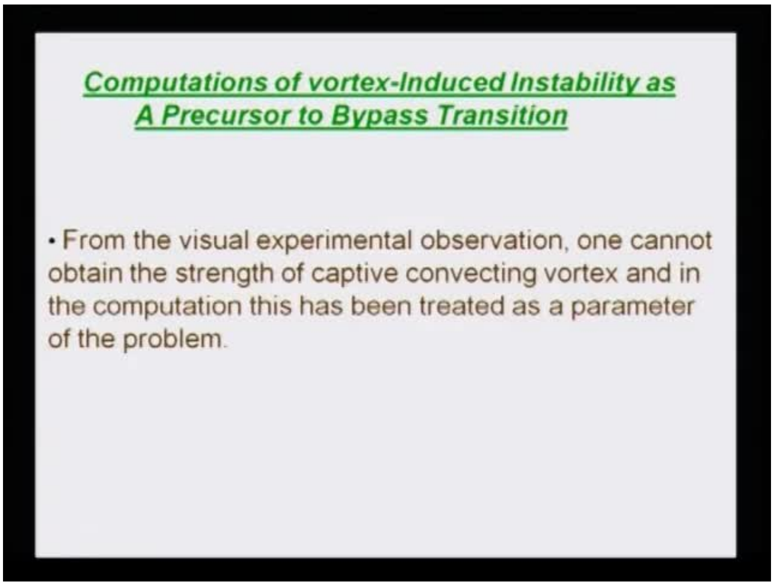

Computations of vortex-Induced Instability as A Precursor to Bypass Transition

- From the visual experimental observation, one cannot obtain the strength of captive convecting vortex and in the computation this has been treated as a parameter of the problem.

So that representing the non-linear convection term is quite challenging. I am not saying the diffusion term is irrelevant, it in fact it is equally relevant, which one can do that. Now, in computing these flows, there is only one problem that is this; we do not know a priory what the strength of the it rotating cylinder was; we can get a indirect sort of estimate. So, what we have try to do in this computations, we can treat this strength of the vortex itself as a kind of a parameter in study, what it is going to be used. You can of course, do a parallel study of (( )) of effect and work out the strength of gamma. So that is equally possible.

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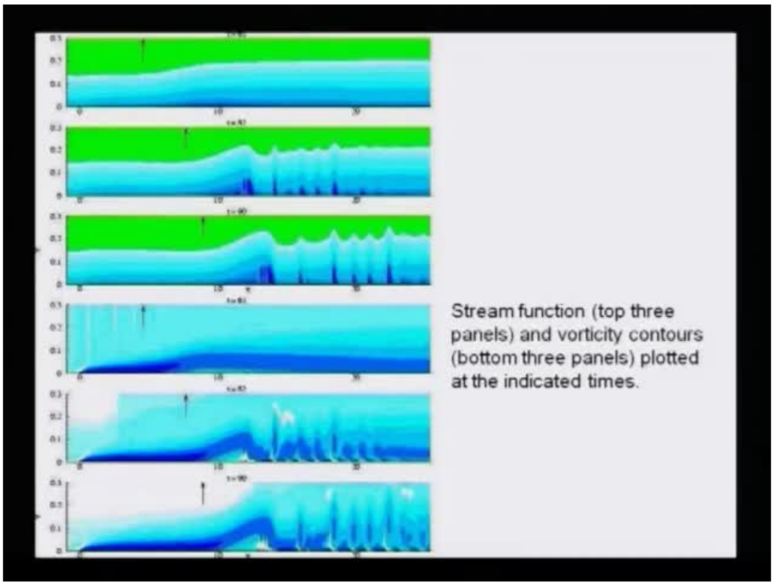

Stream function (top three panels) and vorticity contours (bottom three panels) plotted at the indicated times.

So, basically this is the kind of picture that you see. And in the top three frames, we have shown the streamline contours, and the bottom three frames are vortices contours. And this blue region kind of tells you about the region where shear layer is there. And you can see there are these faint arrows, which shows the stream wise location of the free stream vortex. So the Free stream vortex is much above, but in this frame at t equal to 62, it is here; at t equal to 82, it has come here; and t equal to 90, it is come here. And from this, you can see this, and relating streamline structure right.

So, you can see that there is some kind of a vertical eruption taking place from inside the shear layers certain structures appears through, and this is what we are calling as vortex inducing stability. And you can see the similar sort of vertical eruptions in the vorticity contours. So that I will stop here; in the next class, we will give out a theoretical model, how it creates this instability.