**Instability and Transition of Fluid Flows**
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**Module No. # 01**
**Lecture No. # 20**

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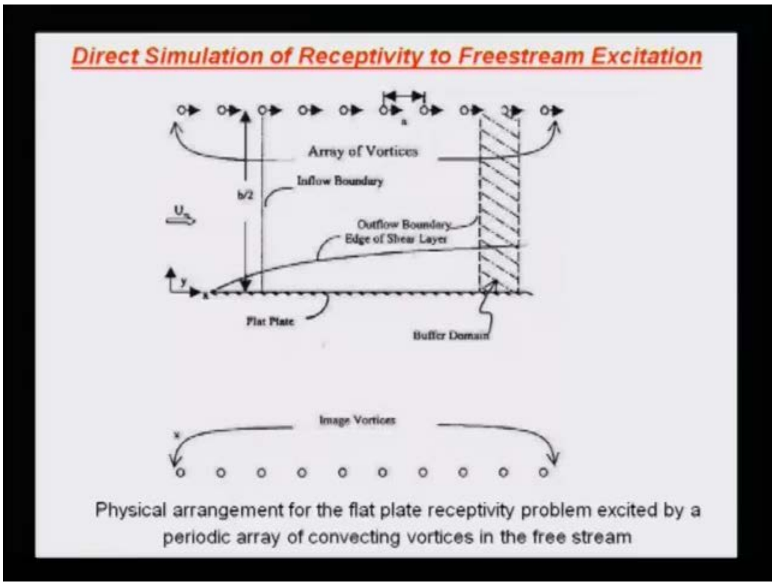


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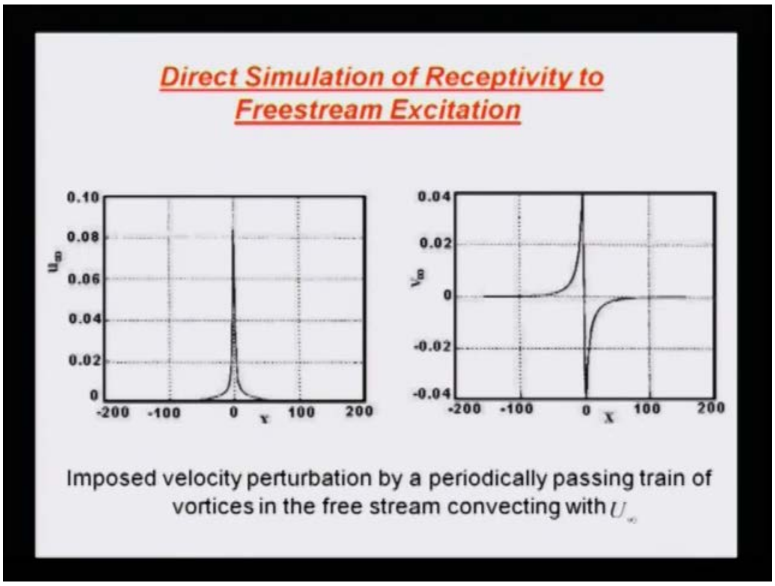


So, let us get back to our discussion on direct simulation of the receptivity problem to freestream excitation. This was the conceptual model that we were looking at. A boundary layer, developing over a flat plate is being excited by a train of vortices, moving at a constant height, with constant speed and the strength being same; since it is a train of vortex, in front of a solid wall, we need the image system; that is what we said. We went and did all kinds of things; we looked at, what kind of excitation this creates. So, these are your u infinity and v infinity, that is created at the top of the computational domain. And, you can see that, u infinity has a, kind of, a delta function like structure, around the location of individual vortex. At the location of the vortex, you have a peak and it just simply tapers off in the side; whereas, the v infinity has a feature of, like a derivative of a delta function; that is your graphical representation of a derivative of the delta function. So, that is what we see in there.

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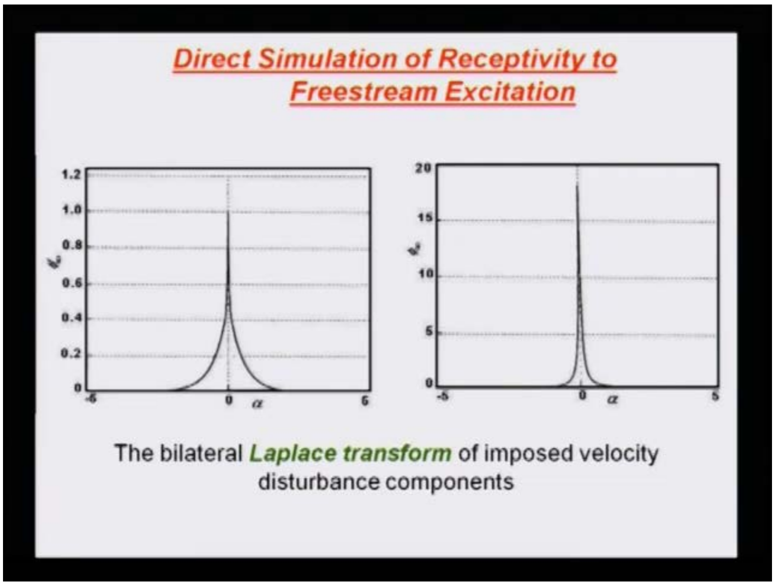


And now, we actually looked at the corresponding Fourier-Laplace transform and we find that, the Fourier-Laplace transform for the stream-wise component of velocity has an amplitude of order 1, whereas, this is the Fourier amplitude transform of v component, velocity is of the order 18 to 20.

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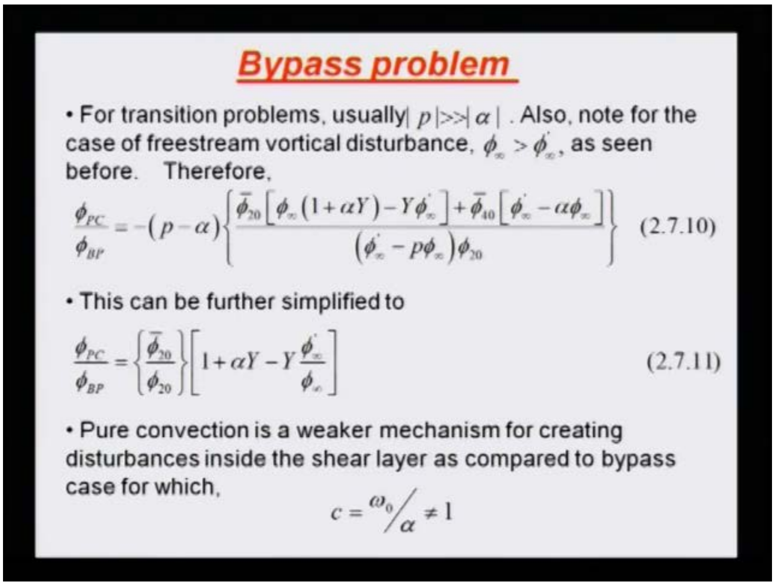


So, this disparity in the spectrum of u infinity and v infinity, helps us understanding the problems, that we have looked at. We called two classes of problem; one is the pure convection problem and there is a bypass problem. And, we figured out that, the amplitude of the equivalent wall excitation, as given by the pure convection problem divided by the bypass problem, is given by this. And, we can see that, Y is literally, large; this ratio is literally, small. So, this can be neglected, in comparison to 1 plus alpha Y. Despite the fact that, Y is large, alphas are still very small, we know. We are looking at a asymptotic solution. We also do not know, what happens to those freestream modes, at the wall; these are inviscid path.

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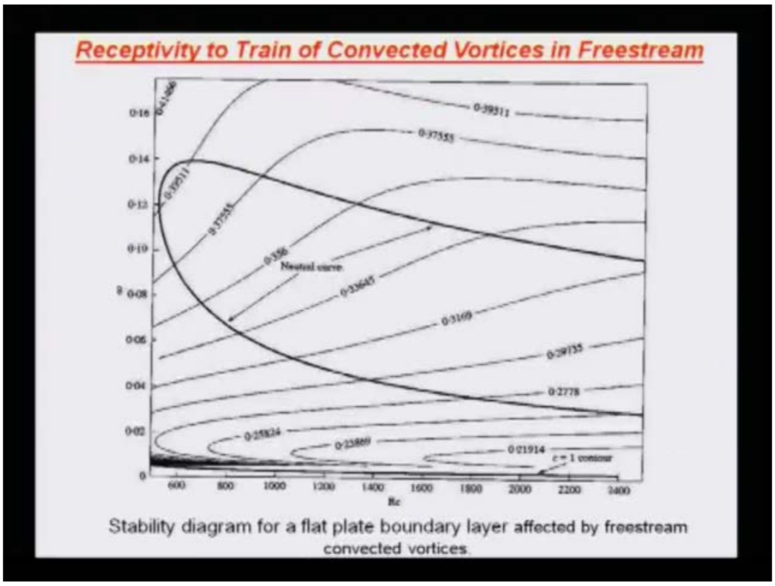


So, what you notice that, this two are essentially nothing, but events occurring due to, whether you are looking at inviscid vortex in the freestream, or viscous vortex; that is what phi P C and phi B P stands for. This is corresponding to inviscid vortex; this is corresponding to this. And one notes that, the inviscid vortex, actually plays a lesser role; and that is what we can see very clearly, because, your inviscid vortex corresponds to c equal to 1; that means, the vortex is moving at a speed of u infinity. And, that line corresponds to here, in the Re omega naught plane. And, that is so far, out of the neutral loop. So, you would know, it is a very, very stable. So, it does simply play a little or no role at all, in exciting a response field. So, that is one thing. We also noted, there are some, other distinguishing feature of freestream excitation. For example, here, the disturbances have to be tracked along c equal to constant line.

Because, that is the speed at which the vortices are moving in the freestream. And, this c equal to constant line, looks like this. And, this very clearly shows that, unlike your monochromatic wall excitation, where you would have different frequency results, corresponds to different rays originating from 0, 0, and you can see, for even, for low frequencies, it does have a significant stretch; but if you look at the corresponding c equal to constant, it enters, probably later; but, it leaves the unstable region, much, much later. So, that means, that you have a longer stretch of instability, and since it also visits in the center part of this neutral curve, that is where your maximum growth rates are. So, this particular route of creating instability by viscous vortices, moving in the freestream, is a very, very potent form. And, since this creates wave packets, and not wave, we went with the crowd and we called it a bypass mode.

But the constituents are still that Tollmien-Schlichting waves. So, we are getting a wave packet, constituting of many, many TS waves. So, that is what we should be looking at, and what we do is, we go ahead with the calculation. Now, as I promised to you that, we will have nothing to hold us back. So, we will be solving the Navier-Stokes equation, except the fact that, we are looking at the two dimensional flows.

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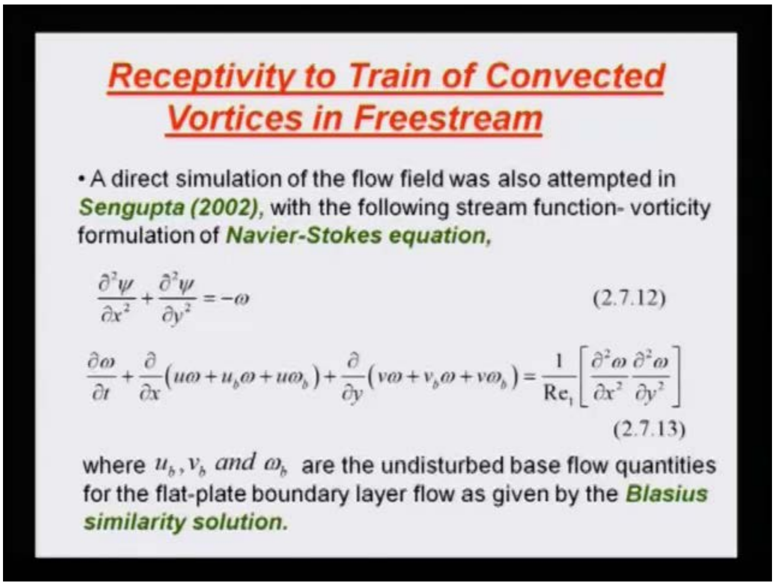


So, let us look at some two dimensional flows defined by the Navier-Stokes equation in terms of the kinematics. So, the stream function equation, is basically the equation that defines kinematically, the vorticity; that is how we get the equation 12; whereas, the following equation is the transport equation for the vorticity. So, if I am looking at the vorticity in two dimensional flow, the vorticity vector is perpendicular to the plane of the flow and that is what we are getting; unfortunately, there is a plus sign missing; please correct it there. This is the diffusion operator; these are those convection terms; this is the local acceleration term. What you notice that, this is the form of the Navier-Stokes equation, and what happens is, the way this equation is written, you can notice that, we have split the total variable into two parts; one is the basic part, which we indicate by subscript b. So, u b is the stream-wise velocity of the basic flow of the equilibrium flow; v b is the 1 normal component of the basic flow and omega b is the equilibrium vortices distribution. So, this omega, that you are seeing here, is nothing, but your disturbance vorticity, without any subscript. And, this psi is also without any subscript; so, that is also your disturbance stream function. So, we can do this calculation. We only wrote that this equilibrium flow, we can take it up as the Blasius solution; we can compute it; we can take it also like that.

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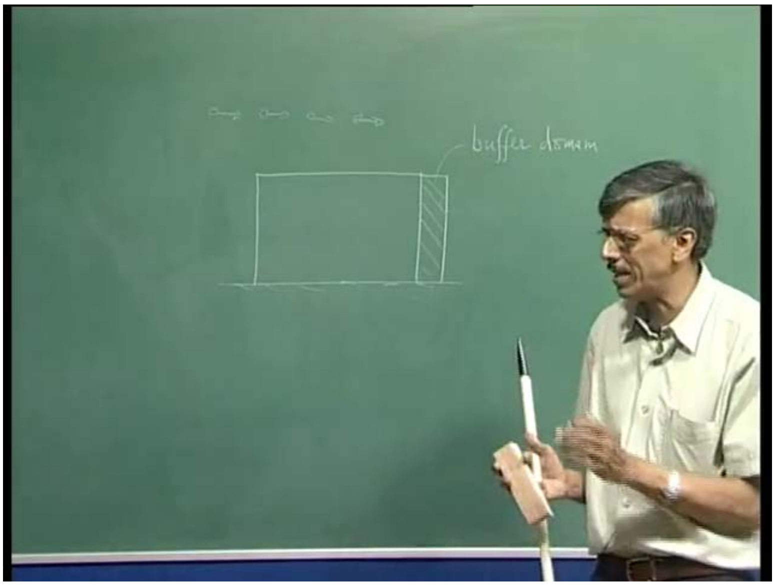


Now, what we do is the following. We have a computational domain, that we defined; like, we have a plate like this and we have a box like this. And, on this side, we have a buffer domain here; and these vortices are here, going in this direction; maintaining constant strength and constant height. And, what we do is of course, well, let me get you a better drawing; looks tilted and wrong. So, let us get you a better quality sketch, here. So, this is the buffer domain and what kind of spacing that you would require? Well, this will create some kind of a vortices, and we have seen, what kind of disturbances, this impose on the top wall. In fact, we know, that a similar foot print also, will be seen here, below.

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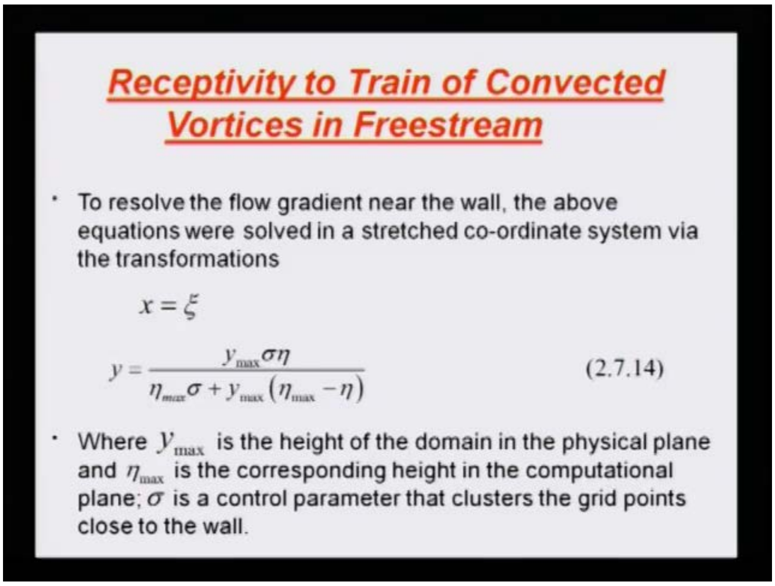


So, we will be getting some kind of a packets, immediately under each vortices; that is what we are going to see. If we are going to see that, so, in the progression of that packet, we would not like to have anything, other than uniform spacing, in this stream-wise direction; because, there is no preference in the x direction. Because, if I have a grid at that point, sometimes, the vortex will move fast; sometimes, it will be free; that we do not want to do a fancy time dependent grid. So, we will use a transform coordinate psi which is nothing, but the x itself.

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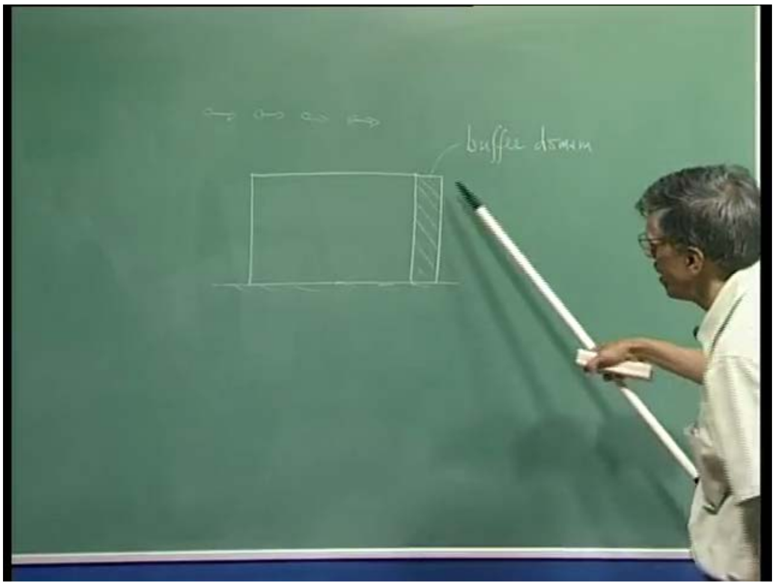

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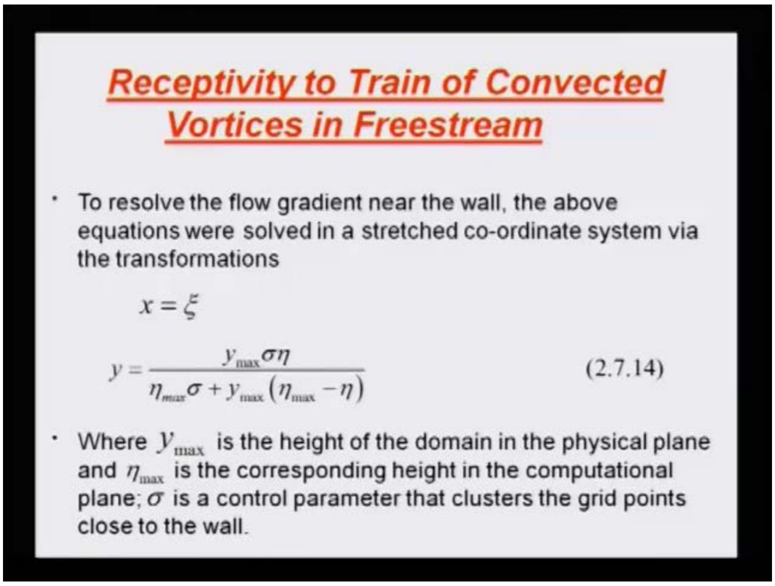


Whereas, in the 1-normal direction, we would resort to, some kind of a, stretching. So, what we are going to do is, we are going to take a very limited height, but, we are going to stretch it out significantly. So, we fixed this y max in the physical plane, and that would correspond to some computational eta max and these are the control parameters that will define our grid, in the wall normal direction. So, we will have a finer spacing near the wall and which will dilate, as we go above.

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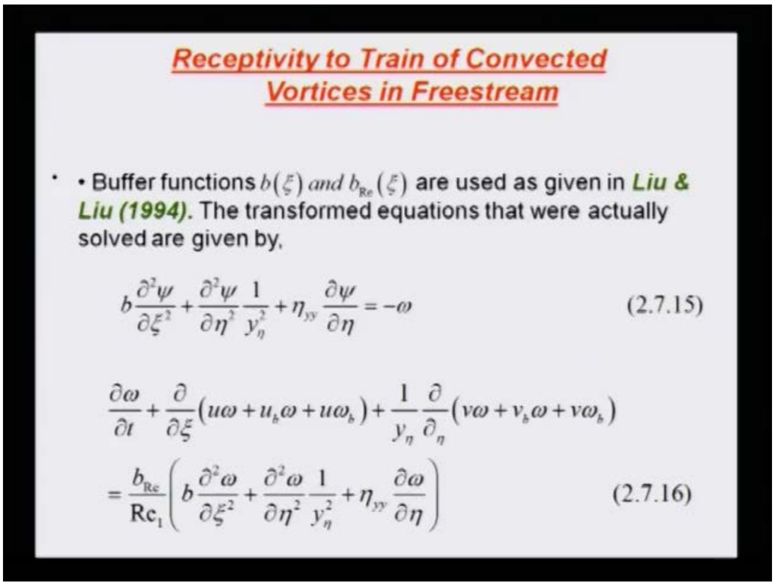

So, this is the form of equation, that we actually would be solving, for the disturbance quantity. There are certain things, that we would like to note here; for example, what we note, from the boundary layer property of viscous flows that, the stream-wise diffusion is weaker than the 1-normal diffusion; that factor is used, in defining a buffer function b of psi. So, in front of the stream-wise diffusion, what we do is, we multiply by function b.

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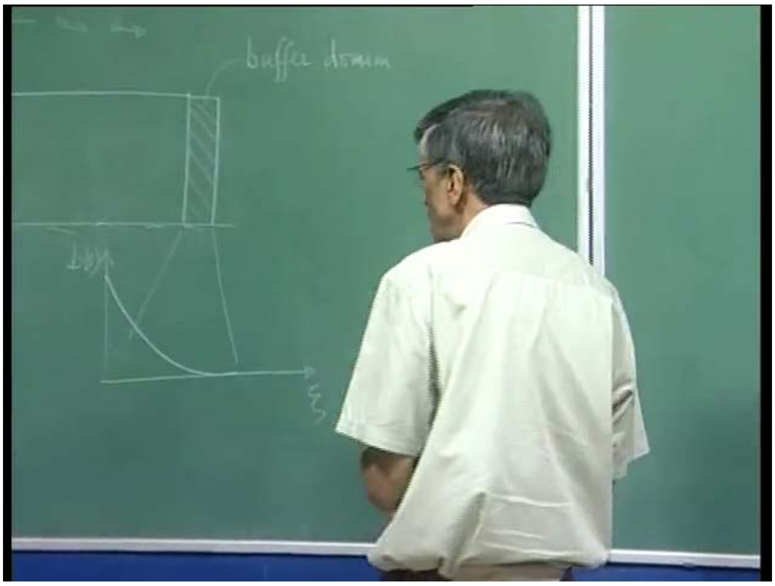

What is this b? The b is like this. So, if I stretch this part, so, I could plot b versus, say psi, here. So, if I have done it like this, so, what it does here, it takes the value b equal to 1, here, and then, it smoothly brings it down to 0, at the end of the buffer layer. So, it would be something like this. So, this is your b of psi. So, what it does, at the end of the computational domain, b is equal to 1. So, we are not tampering with anything; it is only in the buffer layer we are bringing the diffusion, further down. So, that is what we are doing.

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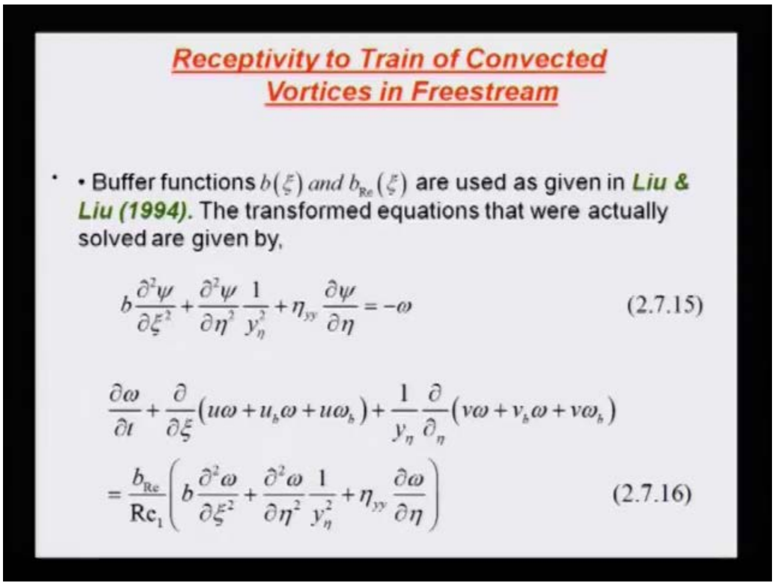


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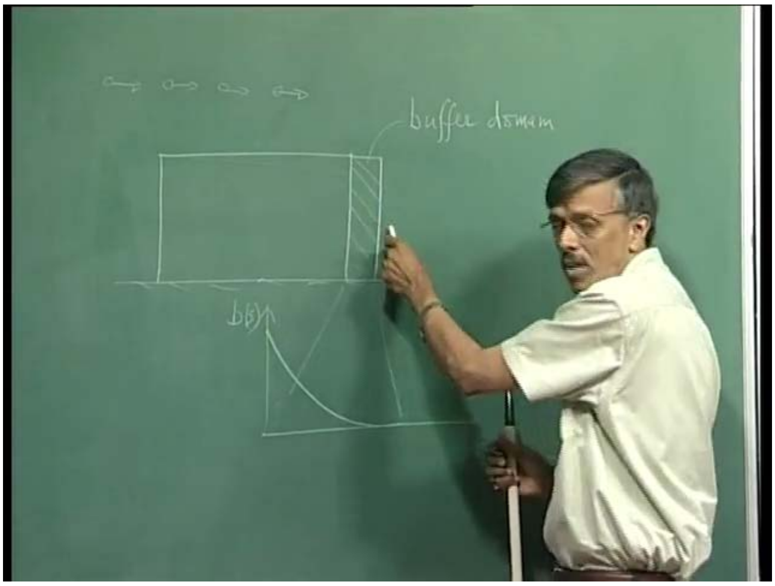


So, that is the function b. In addition, what you also notice that, we have multiplied the vorticity diffusion term by another function called b Re. b Re is also a scaled out parameter, which actually, what it does, it has a value of almost 1 here, at the end of the computational domain, and then, it effectively increases the Reynolds number, as you come to the end. So, how do you do that? So, you should have a function, that grows with psi.

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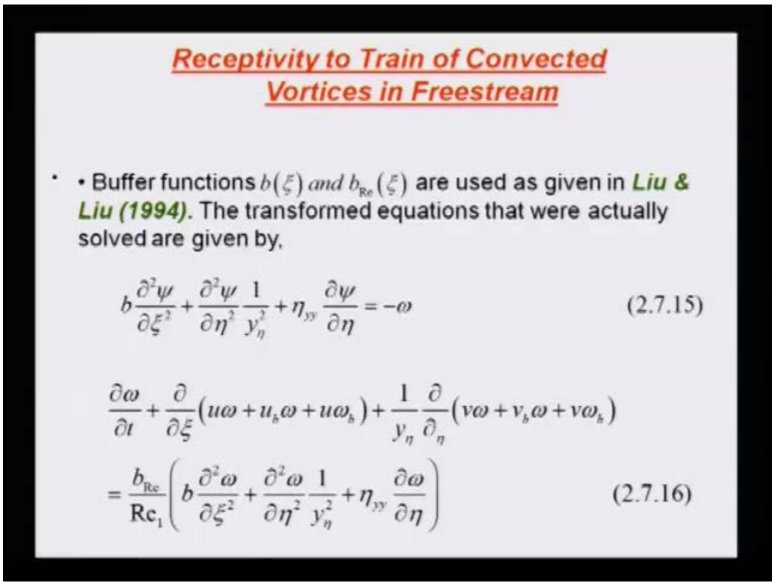


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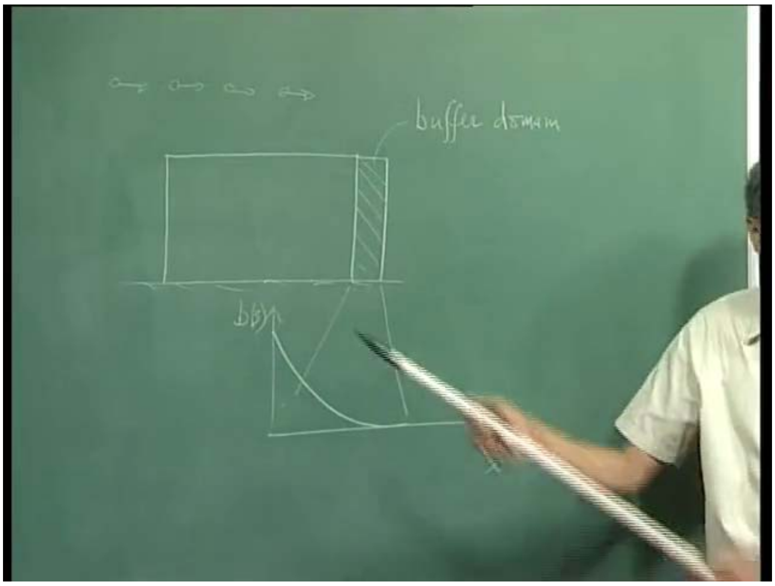


So, that is the role of this. So, you can see, these two buffer function, b of psi and b Re of psi, this were taken from this paper by Liu and Liu, and solved for this. So, why we are doing this? Basically, we are aware of the fact that, it will create some wave packets and we do not want this wave packet to go and reflect from the end of the computational domain. So, by multiplying by this two sets of buffer function, we are attenuating the disturbance so much so, that, at the end of the buffer domain, there is nothing to reflect back; that is the whole idea. In doing so, you realize, that results inside this buffer domain is unphysical. So, do not try to give any physical meaning to the solution, that you generate inside the buffer layer. You should, of course, have confidence in the result, in the main part of the computational domain, minus the buffer layer, this is what one does.

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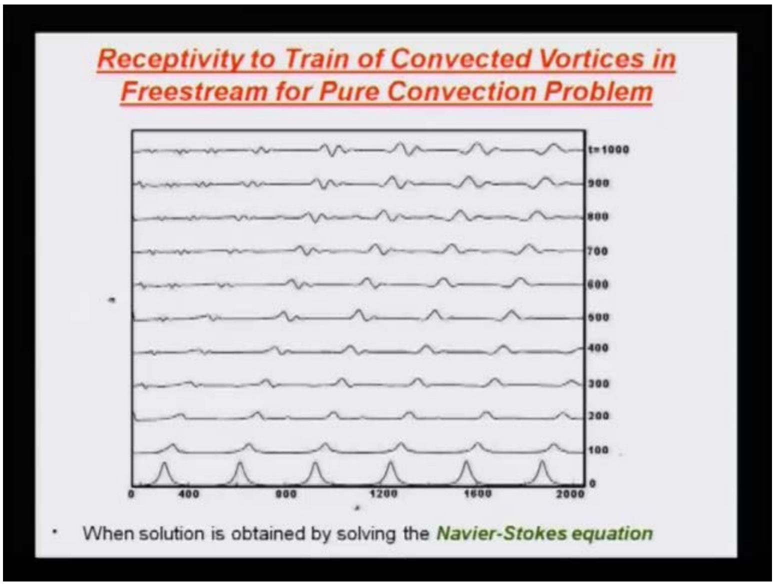


Show you some results as obtained from the simulation. Here, in the first part, I will show you some result, where the freestream vortices are moving with the freestream speed. So, that is your pure convection problem. So, this is a pure convection problem. We are putting the vortices at an interval of 100 pi, and this x axis that we have plotted here, this is nothing, but Re delta star.

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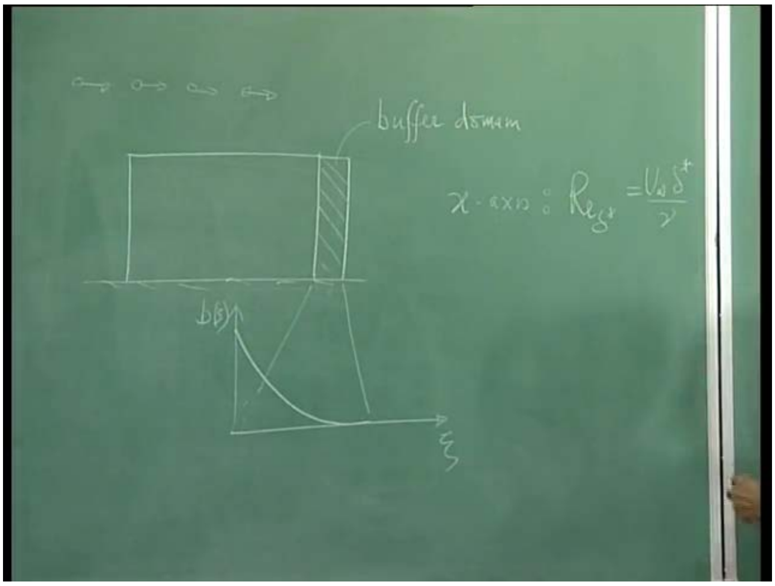


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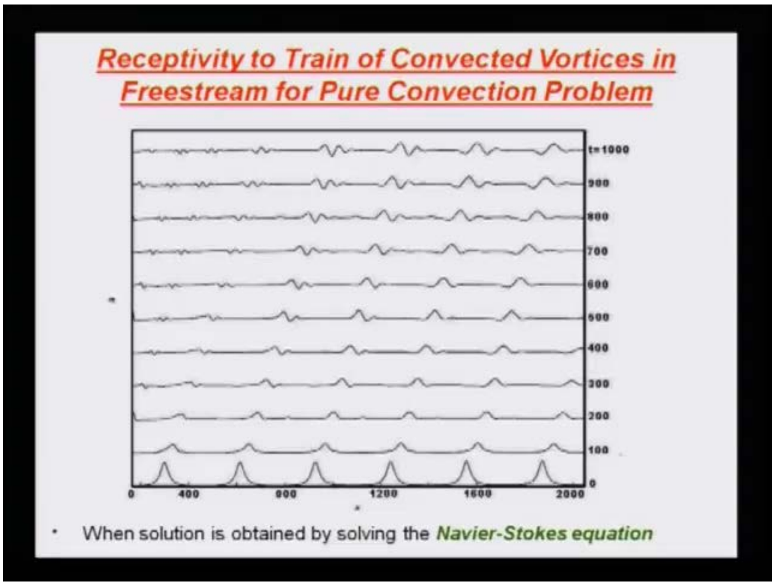


So, the x axis that you are seeing there, in this figure, that is your Re delta star; that means, u infinity delta star over nu. So, we have computed in a domain, which probably starts off around 160 or so, and goes on till 2000, Re delta star. What do we know about this axis? We know that, pure monochromatic TS, unstable waves will not be created below 520. So, here, we have taken a domain, which actually spans across from a sub-critical, to a significantly super-critical range. And, you see, in the freestream, we had vortices at an interval of 100 pi. So, at t equal to 0, this is the solution. So, you see, at the foot of each of the vortex in the freestream, we get some kind of a…And, at t equal to

100, what happens, that same disturbances, actually decay; because, we have seen, in that neutral curve, c equal to 1, corresponds to a very decayed solution and that is what you are seeing; this peak is coming down. We have also seen that, it is a periodic excitation. If it is a periodic excitation, then, I will have the excitation in terms of multiple frequencies.

So, what happens is, I do get, with time, I am going to see that, if I have a peak corresponding to omega naught, I will also have a secondary peak corresponding to 2 omega naught and 3 omega naught; that is what you are seeing, the secondary, partial peaks are coming up. So, this is a very interesting thing. We have created a perfectly steady excitation, and what you are seeing, is a typically unsteady solution. So, this is one way, of creating unsteadiness in a flow, by steady disturbance field; that is one thing you see. One more thing also, you notice that, if you have looked at what was the scenario at t equal to 0, that was corresponding to particular wave number; because the disturbances were like that. But if you look at the corresponding response field, it is getting richer and richer, as you go to larger and larger time.

What is happening though, here, that, you may have started off with one type of excitation band; but at later time, the band is going to get filled up. So, why is it happening? It is happening because of dispersion, because different wave number components are moving at different speed. And because of this, we are noticing one thing that, your energy spectrum can get filled up due to dispersion. This is significantly different than what we have learnt, when we talked about Kolmogorov mechanism, cascade, where, due to nonlinearity, we actually transfer energy, from low k to high k. But here, what is happening is, it is a linear mechanism; I mean, as you have seen, I have shown you those neutral curves, and all this different modes are coming, because of linear instability itself. And, that linear mechanism itself, is creating a band filling operation. So, this is something not many people appreciated before, but hopefully, now, you can see. I will come back to this case once again, somewhat later, because, this is an interesting thing; because, we talked about what, if it is a freestream excitation, then, what should we have? We should have, also both upstream and downstream.

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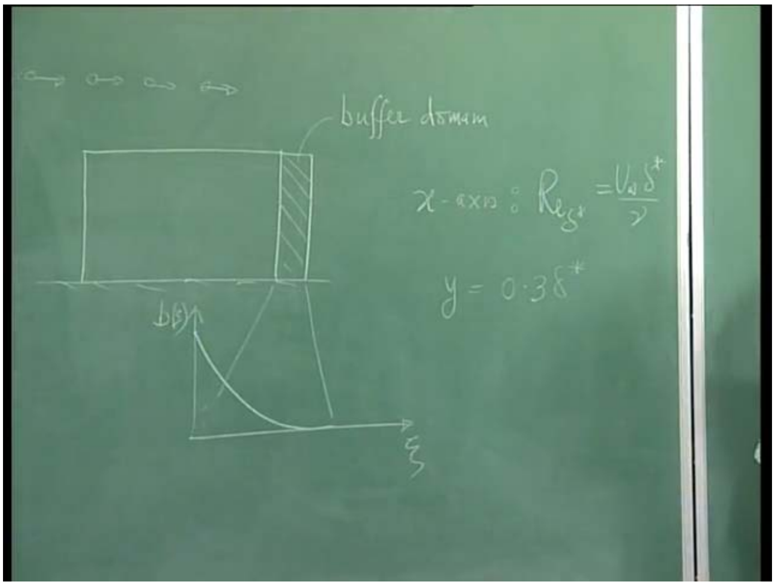


So, looking at it, can we say, whether it is going upstream or downstream? Can we say anything about it? Looks like, as if it is going downstream. So, where is the catch? We have to check the group velocity of what? See, you have to realize one thing that, what we are seeing here, is a disturbance case, very much inside the boundary layer, because this portrait that I am showing you, here, is happening at y equal to 0.3 delta star.

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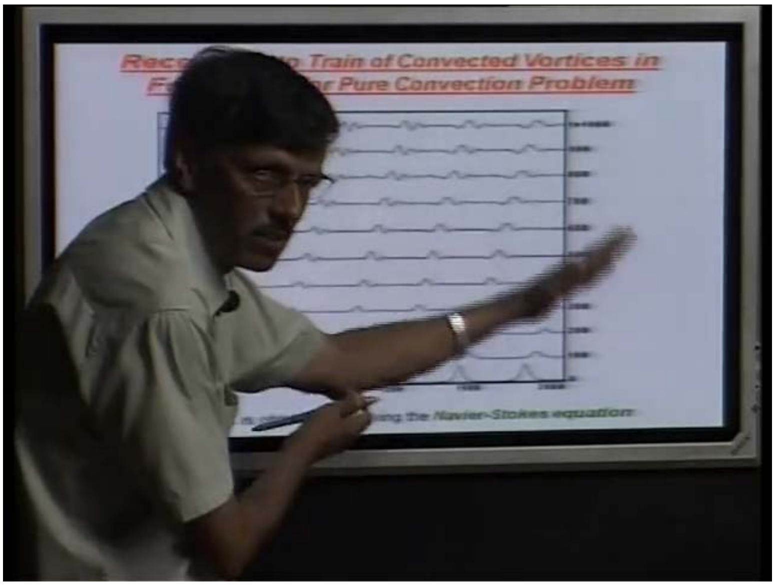


So, this is somewhere, close to your inner maximum, for your pure TS wave. So, if you look at it, the disturbances are, of course, going like this. You can, that is why, purposely, we have drawn in this manner. So, if I join the peak, then, it looks like this. So, that gives you some value of c. Is that c corresponding to the impulse c? Impulse c is going in this downstream direction, at a particular value and that is u infinity, 1. So, if c is going to be 1, in this xt plane, what that line should have been, 45 degrees. You look at these peaks. Are these 45 degrees or more than 45 degrees? More than 45 degrees. So, what is happening? The disturbances are actually, going upstream. So, you have to understand this upstream and downstream business, there is no absolute reference. You have to talk about, with respect to the input. The input is going faster; the output is going in the upstream direction, with respect to that. We will do a little bit of counting on this. So, give me a little time, we will come back to this issue again.

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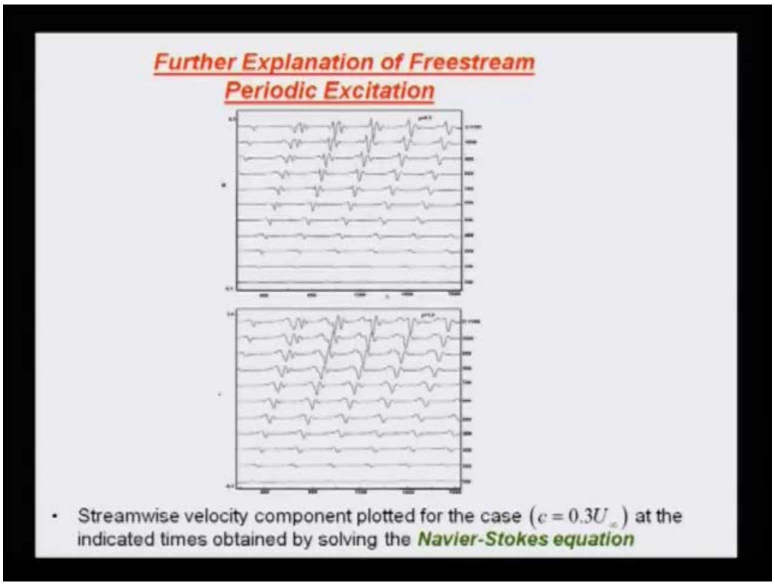


Now, this is a case, where we are looking at some freestream periodic excitation, that, this is that bypass mode, where the speed of propagation is not u infinity, but 0.3. And, if you recall, I drew that neutral curve, c equal to 0.3 would be, the case where the disturbance will enter inside the neutral curve and stay for a long, long time. So, that is the reason, that we choose this. And, we are showing you the result at two heights. I think, one of the height corresponds to…I will send you this ppt material; one of the height put up on is at 0.3 delta star and the bottom frame is at 1.5 delta star.

So, this is like corresponding to your, so called, inner and outer maximum. So, we can see that. But you also notice that, these are the traces given as a function of time. So, different curve corresponds to different time; time increasing in this particular manner. And, what do we see that, if I look at the amplitude, the amplitude is actually increasing with y. And it is had to be naturally is it not, because, we are creating a freestream excitation, so, we should have the freestream modes, that grows with y. And, that is what you are seeing. And, what you are seeing is a very, very, remarkably, interesting case, because at t equal to 0, it was virtually, like flat. Of course, those small, small bumps are there, but in the earlier case, I actually extended the y scale to show you the bump, which later on decayed. But here is the case, at t equal to 0 itself, I put it in a scale, so much, so that, I could fit in the later solutions.

And, as you can see, with time increasing, the amplitude is growing. So, this is one nice example of freestream excitation case, and you also are noticing the same phenomena of dispersion. You are not going to see a single peak. Even here, you may merely imagine that, there is a significant, single peak, but there is actually a secondary peak which is running slightly ahead of it. At later time, you can see that very clearly, it is breaking up into a crest and a trough and that goes on. And, again, you can see that, they are going at a dissimilar rate, so, this, kind of, opens up. So, this is something that we did in 2 D, by solving Navier-Stokes equation and we did do this particular, well designed case, where we could see that. Now, we are talking about the propagation of this response field in the upstream direction.

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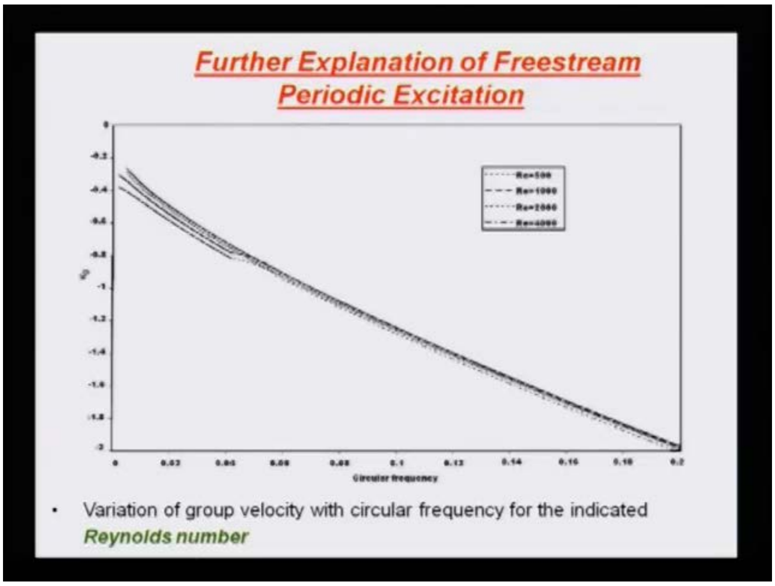

So, if I talk about c equal to 1, and if I talk about a as 100 pi, then, omega naught works out to 0.02. So, what I have done here, I have shown you, a plot of the group velocity here, versus the circular frequency, and 0.02 is there. So, if I go up, I see that, these are all negative v g; this is 0, this is minus 2; and if I look at it here, this is somewhere here, about 0.6 thereabout. So, what is going to happen that, if I now excite a system with this omega naught, this train of vortices, then, fundamental is going to go at about 0.6, with respect to the input. Do not think in terms of inertial frame. This is important for us to know. What happens to is super harmonic 0.4, that moves even faster, because, you see, this is the magnitude, increasing from 0 to 2, and 0.4 is there; 0.6 is there; 0.8 is there, so on and so forth.

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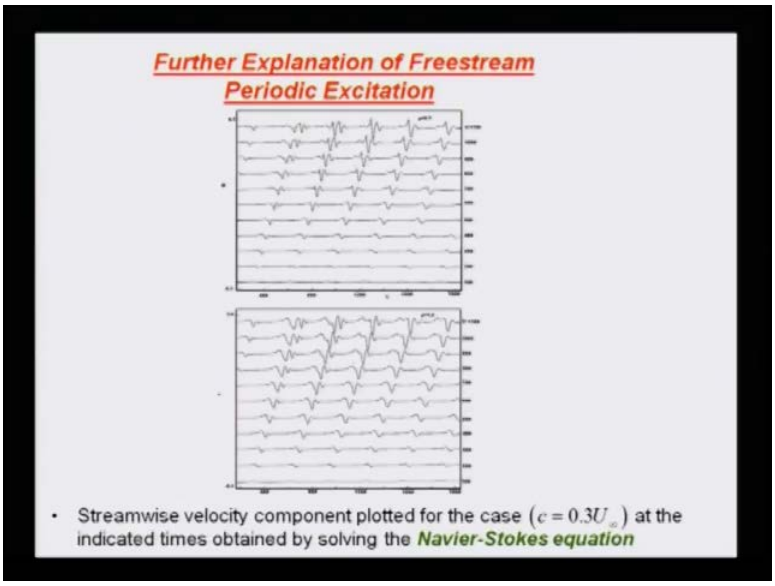


So, we can see that. But, what you notice that, in the previous solution that I showed you, there was this predominantly, two or three such modes; that is what you have seen. The one that you can see is this; this is the primary mode; the secondary mode is the one that is upstream of it, because, it is going in the upstream direction, at a higher speed. So, that is why it fills up. So, you know, I told you that, in one of the meetings, I met Doctor Kendall. He did similar, such experiment, way back in 80s.

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His experiment was like this. Say, take a tunnel. So, tunnel floor could be conceived of as supporting a shear layer; and then, what he did was, he had a support here, and in that support, you put in kind of two cylinders, and this thing was rotated. So, it was gyrating. So, what happens is, this thing gyrates, and each time, let us say, it comes there, it kind of sheds a vortex; you know, these are non-rotating cylinder. So, it is will shed vortex. And then, you basically, get vortex of…With this vortex is going this way, then, you get one side; then again, when this one is coming up, there we get…So, it is almost like what we are doing.

So, we were interested in imposing a space and time scale simultaneously, in the problem; that is what we did. And, later on, I just figured out that, this experiment was done by Doctor Kendall. And, he also noted that, there is peak in its solution, that, when this rpm was synchronized in such a way that, this goes at a speed of roughly 0.26 or something, he saw a very massive response inside the tunnel. So, we did discuss this and he was very happy to see that, we could explain it, in terms of a simple linear theory. He was not very sure, what was causing this, but this is exactly the case. You know, this may look like a very artificial problem. Butt, those of you in mechanical engineering background, you know that, this is very, very important in turbo machines. What happens in the turbo machines? You have this stator, rotor, stator, rotor multistage turbines, and if you are looking at the compressor stage itself, we will not talk about turbine stage; turbine stage, why we will not talk about, because, that is too contaminated with disturbances, due to combustion, process in the combustion chamber itself. But, if you look at the compressor part, and you look at the stator behind a rotor, then, the stator is rotating.

So, it is going to give out periodic vortices, exactly the same kind of thing. So, instead of a flat plate, you should be talking about a compressor blade and vortices going there; it is exactly, like what we…That was the reason, that we actually proposed this model, and we wanted to show the receptivity of shear layer, to some kind of a, periodically convecting vortices, and you would see identical scenario. And, it is quite relevant that, if you are looking at an internal flow inside a turbo machine, the vortices, those will be sheared from the rotor; there will be essentially, viscous vortex. They will not go at c equal u infinity; they will actually, take the bypass route. And there, this kind of things can happen. So, keeping the flow inside a turbo machine is a kind of a monumental struggle, because of this simple issue. Then, of course, you would have all kinds of other sources of disturbances, that we are not talking about. Like, as I told you, in the turbine stage will have the same issue. There, of course, what will happen? You have u infinity, also will carry lots and lots of noise, due to the combustion process.

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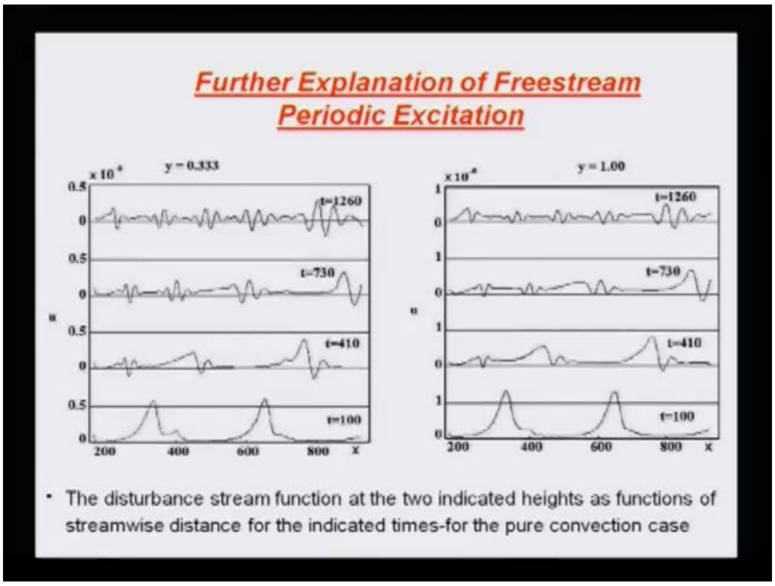


So, we did talk about the role of v g and how this affects. So, let me just show you little more, about the case. Here, we have purposely chosen, the case of c equal to 1, and shown the solution, at y equal to one third delta star, delta star by 3 and y equal to delta star. And, shown you snap shots at four instances; t equal to 100; t equal to 410, 730 and 1260. So, to begin with, we had the vortex at, I think it was at 100 pi interval. So, the first vortex was kept at around 345, and the following one was, at 350; and then, 630 and so on and so forth. If you look at t equal to 100, why would you see the freestream vortex v? That would have gone 100 to the right. But at the same time, the response field is going at 0.6. So, you can find out. So, it is basically, then going to the right, at 1 minus 0.6; somewhat 0.4. So, if you do that, you can really see, these peaks correspond to that. So, that is what you are seeing. In addition, of course, you do also get to see, some kind of a downstream propagating modes. Those downstream propagating modes go at what speed?

Depends on what we have seen before. Depending on, what is the local Re delta star and we have seen the TS waves go at, something like…In this case, of course, there would not be any TS wave going, because, c equal to u infinity; you will only have those damped modes; and if you had seen that maximum damped mode, it was going at a speed of about 0.82 or something; the unstable mode and the second stable mode, I think, it was going at around 0.4 and 0.45, in that range. But the third mode, which existed, say, for R equal to 1000, was at about 0.85. So, those actually go downstream, and those are the ones, that you are seeing, little upfront. So, this is one way of interpreting your result; that it is quite complicated, but you can see, what is happening that, you have started with a single peak, but because of dispersion, they are breaking apart; different omega naught; then, this is also breaking apart due to an upstream component and a downstream component; and you can see that, graphically goes on with time, and that kind of fills up the whole thing.

So, we started off with a very coherent excitation; but here, we end up with sort of filling up the whole space. So, this is the story of the pure…Convection case also, can provide the seed of instability in a boundary layer. So, now, but we will see, what I am trying to drive at, that we have done the eigen value analysis and we have seen that, if we create a particular length scale, and particular time scale, then, we can create TS waves. So, what, in essence, we are saying that, even though, you allow a inviscid vortex to go, to begin with, it will be pretty harmless; but with the progress in time, here we will see that, it is going to fill up all kinds of length scales. You can take this solution, and do on a 50 and you will see that, it is a, quite a wide band phenomena. Question is, about the corresponding omega naught. The omega naught also, we have seen, what can happen, that, we are not only creating omega naught, we are creating n omega naught.

So, higher frequencies are going to be excited, and those can give rise to your 2 D modes. And, the solution that you are seeing, is a, from a 2 D calculation. So, you are only seeing 2 D calculations. But, think of the actual experimental scenario. In the experimental scenario, what you are going to see? You are going to see those (( )) mode. This low frequency excitation, will make the whole boundary layer, breath up and down. So, you are going to see a very, very complex flow phenomena.

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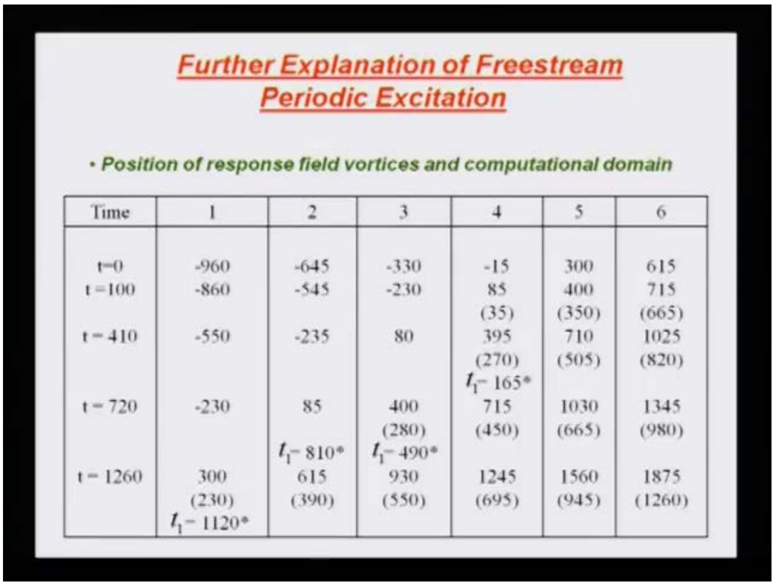

## Further Explanation of Freestream Periodic Excitation

• Position of response field vortices and computational domain

| Time | 1 | 2 | 3 | 4 | 5 | 6 |
|---|---|---|---|---|---|---|
| t=0 | -960 | -645 | -330 | -15 | 300 | 615 |
| t=100 | -860 | -545 | -230 | 85<br>(35) | 400<br>(350) | 715<br>(665) |
| t=410 | -550 | -235 | 80 | 395<br>(270)<br>$t_1$= 165* | 710<br>(505) | 1025<br>(820) |
| t=720 | -230 | 85<br>$t_1$= 810* | 400<br>(280)<br>$t_1$= 490* | 715<br>(450) | 1030<br>(665) | 1345<br>(980) |
| t=1260 | 300<br>(230)<br>$t_1$= 1120* | 615<br>(390) | 930<br>(550) | 1245<br>(695) | 1560<br>(945) | 1875<br>(1260) |

So, in essence, what we are hinting at here is that, if we do go ahead and do this sort of calculations, or you do the experiments, transition is virtually, inevitable. Say, this was supposed to be a very simple case. So, here, in this table, we have given here, proper accounting of what we just now explained, for that pure convection case. I told you the vortices are placed at 300, 615, 930, 1245, etcetera; but, there are vortices also, upstream of the computational domain. So, I have noted some of them, because, at a later time, they will move in. So, like the one which is at t equal to 0, at 15, will at a later time, will enter the computational domain. And, what happens is…So, we are writing all those locations, starting from a minus 960, minus 645, minus 330, minus 15, 300 and so on and so forth. At t equal to 100, I showed you this four frames. Where would we expect the solution to go, I mean, the input to go? At a interval of 100, it will go to 400; this will go to 715. But the response that we saw, where did you see the peak? The peak, we saw at, instead of 400, we saw at 350, and instead of seeing the peak at 715, we saw at 615.

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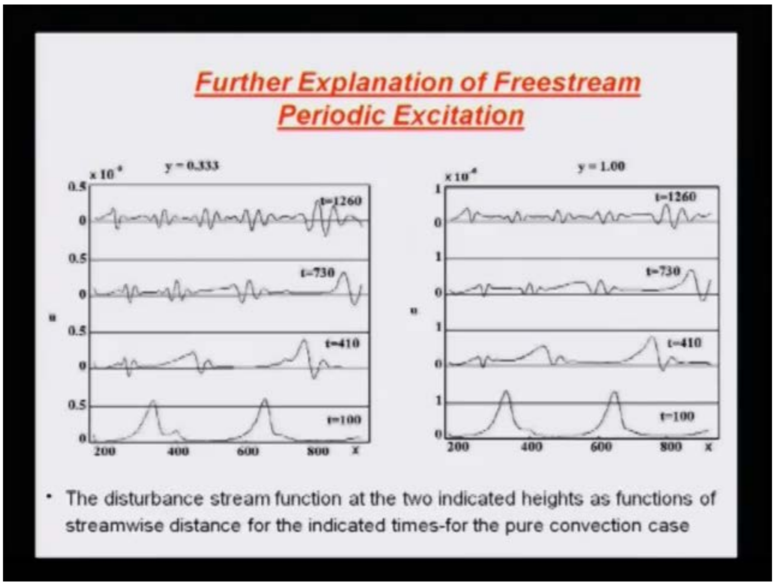


So, what does it say? That, with respect to the input, the response has moved upstream. And, what is that speed, 4.5; that is what we saw, that v g versus plot. And here, it was supposed to have gone to 85; we see a new peak coming up at 35. So, that you can basically see this that, there is a, another peak showing up here, which is not here, but that is at 30 and this is here 365. This is at 655 and this matches rather well with what we are talking about.

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## Further Explanation of Freestream Periodic Excitation

- *Position of response field vortices and computational domain*

| Time | 1 | 2 | 3 | 4 | 5 | 6 |
|---|---|---|---|---|---|---|
| t=0 | -960 | -645 | -330 | -15 | 300 | 615 |
| t=100 | -860 | -545 | -230 | 85 (35) | 400 (350) | 715 (665) |
| t=410 | -550 | -235 | 80 | 395 (270) $t_1$= 165* | 710 (505) | 1025 (820) |
| t=720 | -230 | 85 $t_1$= 810* | 400 (280) $t_1$= 490* | 715 (450) | 1030 (665) | 1345 (980) |
| t=1260 | 300 (230) $t_1$= 1120* | 615 (390) | 930 (550) | 1245 (695) | 1560 (945) | 1875 (1260) |

Now, well, anything on this side does not make sense, but the next frame that we showed, is at t equal to 410. So, now, this vortex, which was initially at 300, has gone to 710, in the freestream. The corresponding response field, would go at, is seen at 505. And, once again, you can see what this is. This is roughly around 0.5 v g. At 400, it has gone about there and this is, instead, if it was in the freestream, it is 1025. So, if I do it, 410 into 0.5; it is about 210. So, it is about 205 (( )). So, that is what it is doing and this one, which was outside, now, it has come to 270. And, this is still on the outside. Now, you see, what has actually happened, at t1 equal to 165, this entered our computational domain. So, computational domain is not starting with the leading edge. So, it is somewhat later. So, that is what, we have indicated by this t1 star, when this vortices enter the computational domain. Like this, third vortex would enter the computational domain at 490; this one would enter at 810; this one would enter at 1120, and so on and so forth.

And you can see that, this kind of, very clearly, explains that, one kind of response field for pure convection problem. So, what has actually happened, we have finished this particular part. I will now show you something. So, you know, I mean, we have, as time progressed, starting from simple linear stability theory, to what I just now discussed, shows that, even under the linear theory ambit, we can see so many complex things happening. We can see that pure monochromatic excitation, as it was done in Schubauer and Skramstad experiment. We have also seen the freestream excitation by simple train of vortices going outside and that can lead to very complex things, and which we called a bypass transition. But, you know, in actual fluid flow, it so happens that, on many occasions, linear theory do not even work. Classical example is, the pipe flow we discussed. If I do linear stability theory for pipe flow, it is shown to be completely stable. The Couette flow, the shear driven flow, that if I have flow between two parallel plates, top plate moving, bottom plate stationary, that flow profile is a linear profile, and you can do the stability analysis, and you will see that, it is stable.

The other example is, look at flow inside a rectangular channel, which is called the Posi flow. The Posi flow velocity profile is parabolic; you do a linear stability analysis and what you notice is that, the critical Reynolds number is about 5700; whereas, that flow is known to become unstable, even as low as a Reynolds number of 1000. So, there are any number of examples, where people see that, you could have instability, but the linear theory is not so good. So, people have, I mean, researchers have worried about it, and one of the leading scientist in this a particular area, was professor Markovin. He coined it, this term called bypass transition, and what he meant that, they are all this nice lab experiments, where we demonstrate Tollmien-Schlichting waves, but there are many, many flow situations, where Tollmien-Schlichting waves are not at all relevant, and they are undergoing transition due to some other mechanisms.

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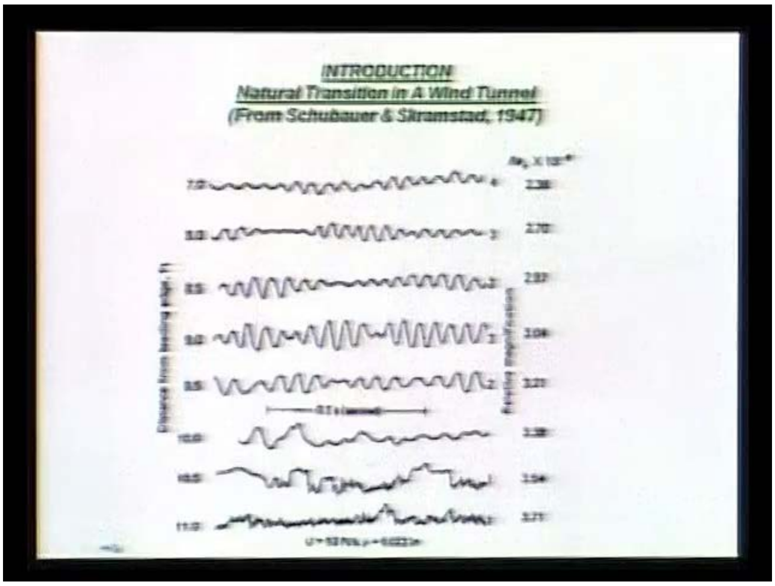

So, all this other mechanisms, taken together, he coined it as the bypass mechanism. This is something that, we have seen that, if I go to a wind tunnel, and see what is happening inside the tunnel, it is a, so called, natural transition. And, this is the result again, and showing you again, taken from that Schubauer-Skramstad paper. If I look at it, about 7 feet from the leading edge of the plate, then, what we find that, this is the kind of response field that we see. It is a major with a (( )) code and what you notice is, significant lack of monochromatic disturbances.

You see, what has happened, the initial objection to linear theory was so strong, lot of researches spent a lot of effort, to first establish the linear theory. That is what Schubauer-Skramstad's experiment was all about. But, when you look at real flow, this is what actually happens. You do not see monochromatic wave, do you? You do not see at all. You see the sequence of events going like this. But then, what you see also, progression of the growth of the disturbances with stream-wise distance. And, these are the magnifications, with which these solutions are shown. This is four times magnified; this is three times magnified. By the time we are here, we are only magnifying two times.

So, you can see that, it is growing and it is not at all a monochromatic wave. And then, all of a sudden, it has this kind of…And, when it does become turbulent, that is characterized by very rapid fluctuations, which we all know, what happens. This could be, even a transitional flow. Nobody is saying, it is fully turbulent flow. This could still be a transitional flows. And, let me add to what we already said that, this kind of scenario, that is, we are seeing here, it is because, maybe, the background disturbance was rich in harmonic content; it was not a single frequency; it has a multiple frequency.

And those responses due to different frequencies, have to be added together and this is the sum of the substance, that you are seeing. So, I will show you, (( )) a few days later that, what happens, when we excite a boundary layer with a strip, that maybe still going at a single frequency. But, because of its finite width, I would create waves of different wavelength. Why, because of the finite width, my local Re delta star is also, not a single number; it is a band. So, each point works as a source and each point, because they are a different Re delta star, for the same omega naught, will create different alpha r and different alpha i.

And, I have to sum it all up. So, this normal mode analysis does not tell you, what is the relative weight of different harmonic content. This is same omega naught, but different source point; will have different thing. If you recall that, we did all kinds of things; that Gaussian, and the strip vibration; then, we see simultaneous blowing section stream. So, we do not know, in a stability frame work, what weightage to give to each position. But, what we could do is, if, even if we can consider, that it is a kind of a white noise type of thing, that every point is weighted equally, and I add them up, I actually end up, getting pictures of this kind, of various ((hues)). I will show you some such calculations, which we have reported ourself, but what happens is, primary instability, is just one of the thing; primary instability, is just the beginning of the story. So, now, what you have, we have the basic equilibrium flow, plus a disturbance. Now, what one should do, naturally? One should naturally also study the stability of this unsteady flow. See, your basic flow could have been perfectly steady. But, because of the primary instability, now, I have the basic steady flow, plus a unsteadiness, due to this primary instability.

What prevents us from going further, and this is exactly what Landau suggested. Landau suggested that, after you have gotten the primarily unstable flow, you should study the stability of this compound flow; that is a combination of a basic flow plus a (( )). If you look at it from the point of view of Schubauer and Skramstad experiment, then, we are talking about, what? We are talking about a basic, time independent flow, plus a periodic flow, because we have created a monochromatic wave, there is some such theory, that was developed by mathematicians. This is what is called as a Floquet theory.

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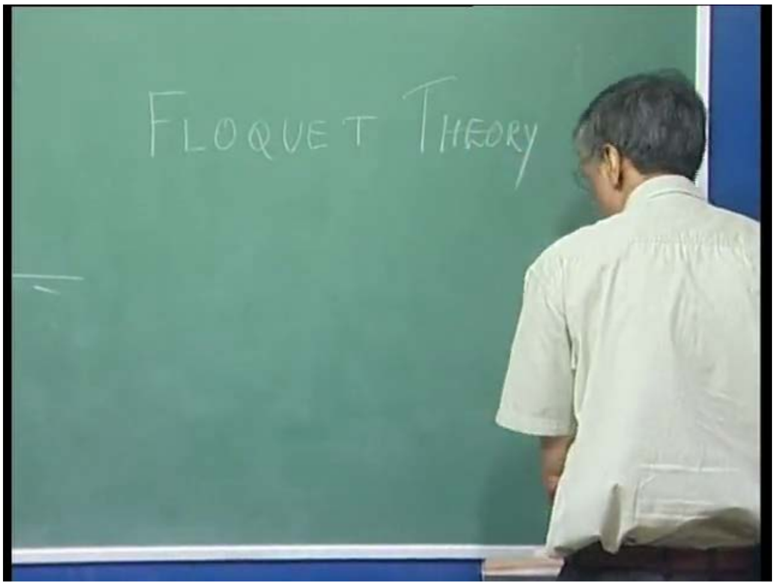


In Floquet theory, we study the stability of an equilibrium flow, which is periodic, and we can do that. So, Landau actually suggested that, Landau said, that is the root with which go to complete turbulent flow stage; that, we have the sequence of instabilities coming. Later on, in 70s and 80s, two mathematicians by name Ruelle and Takens came. They said that, you do not need to have a very large number of such instabilities. They said, maybe three or four such instabilities are good enough; that is called the Ruelle-Taken scenario. So, we are coming from the primary instability, to the Landau scenario, but then, Ruelle-Taken came. Then, they said, you did not have to go for very large number of this thing; you just simply have three or four such bifurcations, and you are done. You will get a totally, a chaotic flow. So, I think, that is where I will stop today, but in the next class, we will talk about some of this other possible routes of transition and see what happens. So, for the time being, we will keep the Tollmien-Schlichting mode in the backburner, and will look for other modes.