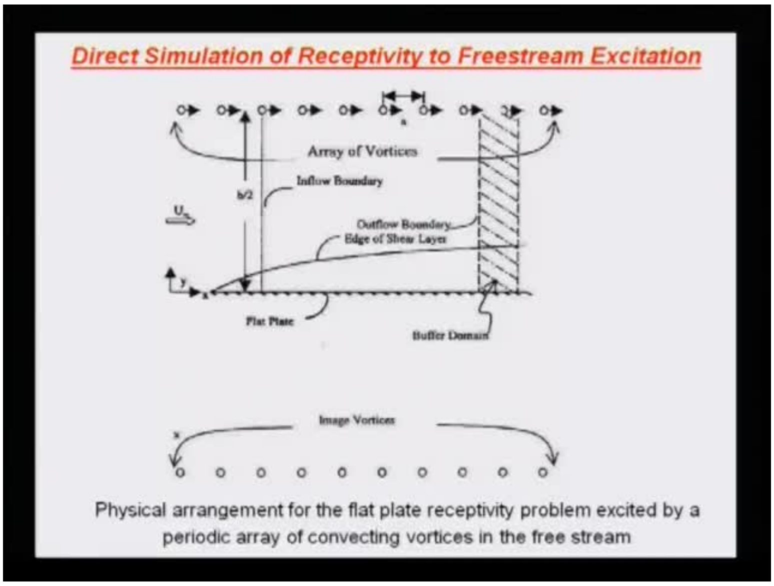
Instability and Transition of Fluid Flows
Prof. Tapan K. Sengupta
Department of Aerospace Engineering
Indian Institute of Technology, Kanpur

Module No. # 01
Lecture No. # 19

See, so far, we have looked at two different classes of excitation; one was applied from inside the shear layer and the other one was applied from outside the shear layer. And, we did, as much as we could, theoretically, using those assumptions of parallel flow, that implies that, we neglected the growth of the shear layer and we also looked at the linearized problem. Now onwards, what we would like to do is, try to get out of the shackle of this two limiting assumptions and try to incorporate, both the nonlinearity, as well as the growth of the underlying shear length; that is called the non-parallelism. So, what is the way. Well, one of the way is, of course, you do some perturbation theory; relay, relax some of this condition in a weak manner, and then, you have weakly non-linear theory and weakly nonparallel theory; those efforts are really weak. that is what people found out, for 20 years of further research; that those weak theories are really weak; we need to do something, which should have adequate strength.

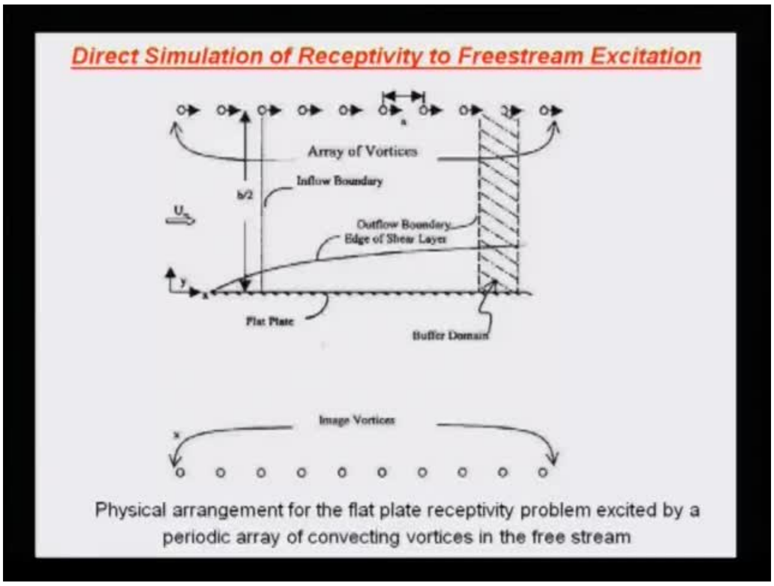
(Refer Slide Time: 01:53)



And, that is what we are now going to talk about. We are going to talk about, investigating some of this problem, directly approaching to the Navier-Stokes equation and solving them. At the same time, we will be, still be doing those control experiments and these are control numerical experiments. One of the thing that we found out in the previous set of lectures is that, if I have some kind of a free stream excitation, it does both; it creates a downstream propagating (( )) and they can also give rise to upstream propagating disturbances. So, there, what really happened, we did not talk about in a specific length, or timescale, that was introduced from a free stream excitation.

Today, we are going to look at a problem, where we are going to, kind of, specify both length and time scale. How do we do it? Well, we do it in the following way. We talk about a train of vortices, a large number of them; they all are equi-spaced, and they are all going at constant speed; and, they are also constrained to move at a constant height, so that, we can take out some of the uncertainties of the analysis. So, this is, that is why, we call it a control experiment. So, we have each of the parameters in our control. I could vary this height; I could vary this spacing; I could vary the speed of convection. All this things are our parameters of study, and that is what we need to do. Suppose, when such a thing is happening over a flat plate boundary layer with a sharp leading edge, then, what happens is, the boundary layer grows from zero thickness, all the way from downstreams, which may actually, as Reynolds number increases, saturates. It does not completely saturate, but the growth can be neglected, and that is where, we used our linear theory with parallel flow approximation.

(Refer Slide Time: 01:53).



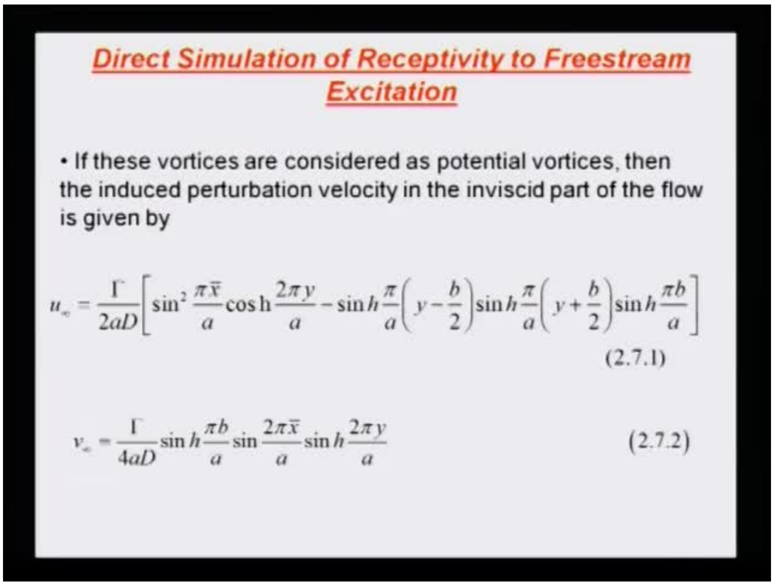
The moment that we want to simulate the flow past a flat plate like this, let us say, very uniform freestream approaching the plate at u infinity, then, because this is a no slip wall and we are putting some kind of singularities, out there, so, what happens is, this singularities will tend to create some induced velocity; say, for example, if I talk about a particular vortex here and look at a point like, here; so, what will happen, this will create a velocity normal to that radial vector. So, what happens is, that gives rise to a wall normal component and a slip component. Slip component does not create too much of a problem, because, it is going to give existing fluid particle some kind of a slip velocity; however, the normal component is a little potentially dangerous, because that implies, as if there is a, some kind of a mass source there, which is not to be there, because we are not explicitly adding any mass source.

So, what we can do is, to obviate that conceptual violation of mass conservation, we can immediately talk about a image vortex system. The image vortex system would be nothing, but it is counterpart, rotates exactly at the same height, exactly at the same location. So, what happens is, at any point, I could have a pair-wise contribution coming in the opposite direction; one is in this direction; the other is in this direction; and, you can see, the normal component will cancel out, and you will have a tangential component, add up. And, this is the whole idea of image vortex system. So, then, with this problem, then we can go ahead and solve this. What happens, we have seen that, freestream excitation is capable of creating both downstream, as well as upstream propagating disturbances. However, when we are going to compute, let us say, Navier-Stokes equation, what I could do is, I could create a box like this. I cannot do the whole thing; this would be a semi-infinite plate anyway. So, what, we would be restricting ourselves to a finite size box. So, that is the inflow of the box.

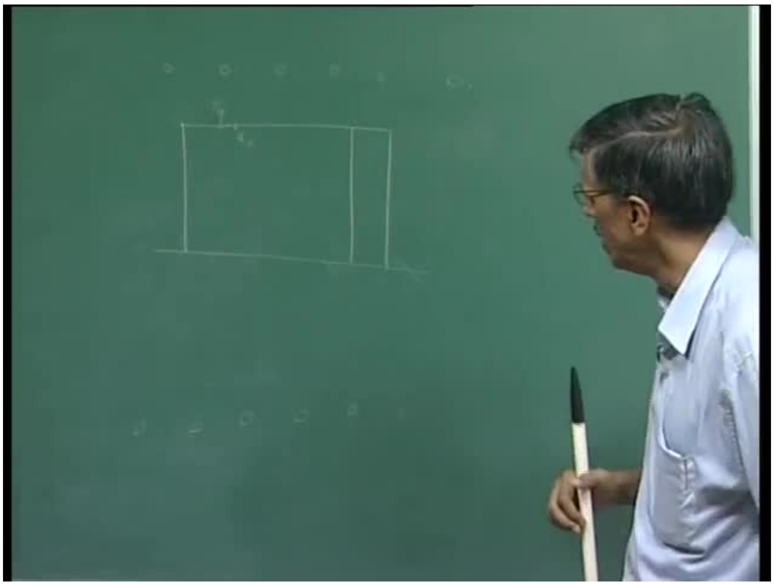
Now, the top lid I have not specified; it need not necessarily be there. I could put it somewhere down below. And, the outflow path, we noticed that, we have a very special layered structure; the hatched area indicates, it is a buffer layer. What does it do? It basically, does the following. That any disturbance, that is going to be created and that will move downstream, when they approach here, that disturbance should cleanly go out; if it cannot go out cleanly, then, there would be some kind of a reflection; and, that reflection, can build up and lead to numerical instability.

So, to discern physical instability, different from numerical instability, we have to be very cautious in avoiding such spurious reflections. So, what we could do is, we could forcibly eject it out; that is possible, doable, if we can. The other possibility is, of course that, we create a, sort of a, buffer layer, where this oncoming disturbances will be attenuated in such a way that, the reflected part will not be there; because, we are decaying the disturbance, coming in its full strength, at the beginning of the buffer domain; once it is inside the buffer domain, the amplitude will be decay. So, when it comes here, there is nothing to it. So, this is the concept of introducing a buffer domain.

(Refer Slide Time: 08:49)



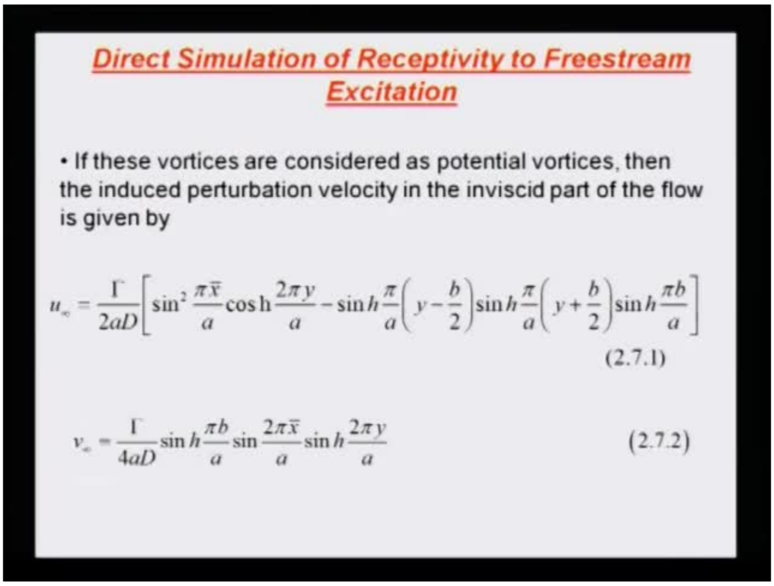
(Refer Slide Time: 08:56)



Now, let us see, what we are doing here? Well, what we are doing here, is the following that, we have the plate here. The results that I am going to show you now, today, includes a domain like this, as we seen; this is our buffer layer; there is a top lid. We have excluded the leading edge. This is the inflow and we have this vortices placed here, etcetera. And, we have seen, the corresponding image vortex system will be like this. That is it. That is what our image vortex system is going to be.

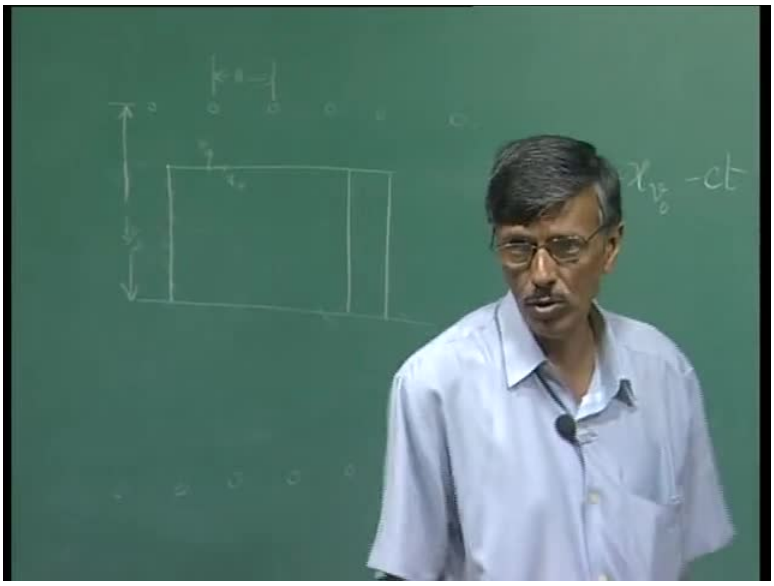
So, what is really happening in our computational domain, the presence of this two vortex systems, will induce a velocity; as they are so far outside the boundary layer, forming over the flat plate, and if we consider these vortices themselves, to be some kind of inviscid irrotational vortex, then, the effect of these vortices would be, to induce some velocity component; at each and every point, I will have a u infinity imposed and a v infinity imposed. And, that quantity is figured out here. Now, if you have followed what is done, the effect of the irrotational vortices, you know, you can use Biot-Savart Law and calculate effect of individual vortex. Then, the individual vortex effect could be integrated over the whole sequence. So, we have a infinite array of vortices, along with the image system, and that, I could do it; you can look at the book by J M Robertson. The title of the book is, I think, Hydrodynamics. You can take a look at that book, where this kind of, inviscid vortex expressions are given.

(Refer Slide Time: 08:49)



So, this is basically, the u component and v component of velocity, as it is applied on some position x and y. All this sin hyperbolic, cos hyperbolic term comes, because, we are summing over infinite such arrays. What you noticed that, the gamma, individual strength of the vortex comes into the picture; a is the spacing of the vortex.

(Refer Slide Time: 12:04)



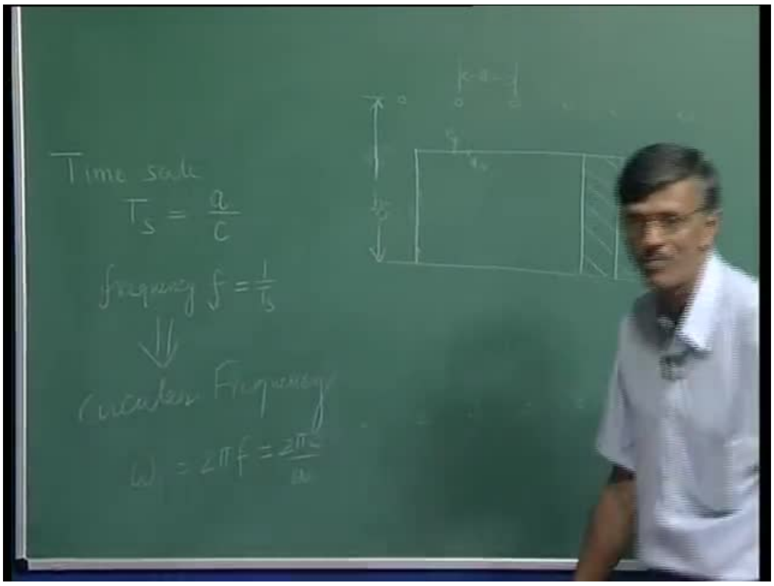
So, this is your a; a is the distance between two successive vortex; b by 2 is the distance, that we have talked about, is the distance between, well, not this; this is the distance of this; this is that. And, now, what is happening, you can see, this vortices are moving at a constant speed. So, what I could do itself, I could see that, individual location of the vortex would be given in terms of, say, the vortices initial location, minus c into t; because that is the speed with which it is translating. So, this is your x bar and those x bar, are all out to here you can see.

So, wherever you position the computational domain n, so, you know the x and y; plug in those quantities, and you should be able to calculate the induced velocities at the box. So, that is quite easily done. So, you realize also that, there are two things. It is not only that, u infinity, v infinity is imposed on the top lid of the computation domain, you also get it happening, on the inflow too. So, you have to incorporate those two. And here, what we do is, we do not use that induced component of the velocity. Why, because, what we are saying in the following is that, this vortices impresses upon some kind of additional disturbances at the inflow and the top lid; as a consequence, some kind of disturbances will be created inside. They will amplify, depending on the stability or instability of the problem and they would like to go out.

So, here, I do not want to a priori impose the condition that, nothing has happened. We have to give some boundary conditions here, which will be independent of what the inviscid flow is doing, because, the dynamical system itself, would evolve and create additional disturbances. So, we have to get some interesting boundary condition, that we are talking about. And, we have already said that, in this buffer region, what we do is, those disturbances, plus this, all of them sum together, will be decayed, from some finite value to almost negligible value. Also, despite that, what you would also like to do, whatever may be remnant, should be allowed to pass out cleanly. So, we will talk about those boundary conditions shortly. Now, as I told you that, x bar is some x minus c t, and then, you saw that, there in the expression, there was, u infinity had some D in the denominator; please note that, D will never be equal to 0.

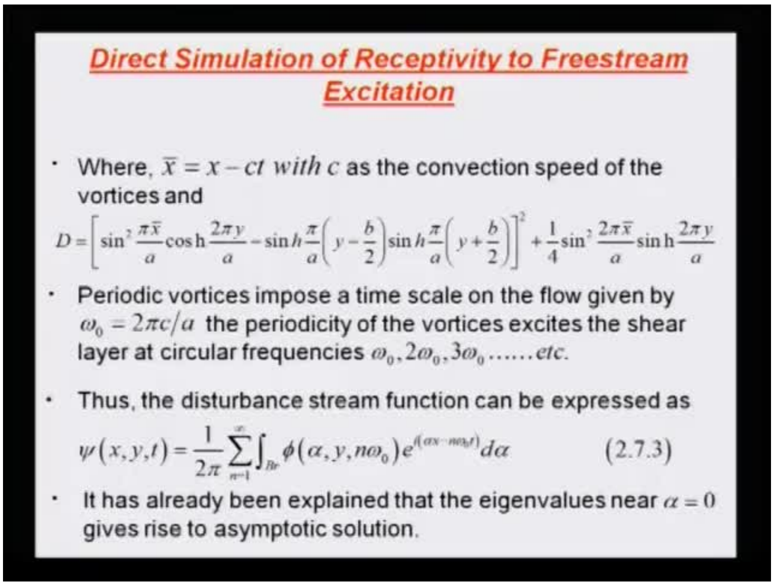
So, there is no such additional Eigen values, showing up due to inviscid excitation. So, this is the proof. It is somewhat very interesting, because, a similar such approach is taken by von Karman, when he started studing the stability of the vortex street behind the circular cylinder. The vortex street that you get, you get two rows of vortices, and in the original work of von Karman, he treated them, as some kind of inviscid vortices, and then, tried to study the stability of the street. You realize that, that may not be the correct way of doing the things, people have found out; that is a wrong way, in fact. Here also, you see, when we have this two vortices, two rows of, trains of vortices, the denominator term can never become 0, and then, you do not get any kind of instability condition due to that.

(Refer Slide Time: 17:08)



However, what we have seen that, if we create this vortices, which are at a distance a apart, and they are migrating at a speed c, I can introduce a timescale. The timescale, which I will call it as T s, would be what, will be the length scale by the velocity scale. For this convecting vortices, the length scale is a, and velocity scale is the speed of convection. So, what will be the corresponding frequency imposed, to any point we will see so many events happening per second, that is that frequency, that will be, I will call it as, 1 over this. This is your physical frequency in Hertz or cycle per second. What you could do is, from here, you could create a circular frequency, which will be nothing, but… So, that will be nothing, but 2 pi c by…You noticed that, I purposely put omega naught, because this is the fundamental unit. So, if I have an infinite sequence of vortices like this, the lowest frequency is created by 2 pi c by a. Then, of course, you are going to get this super harmonics also.

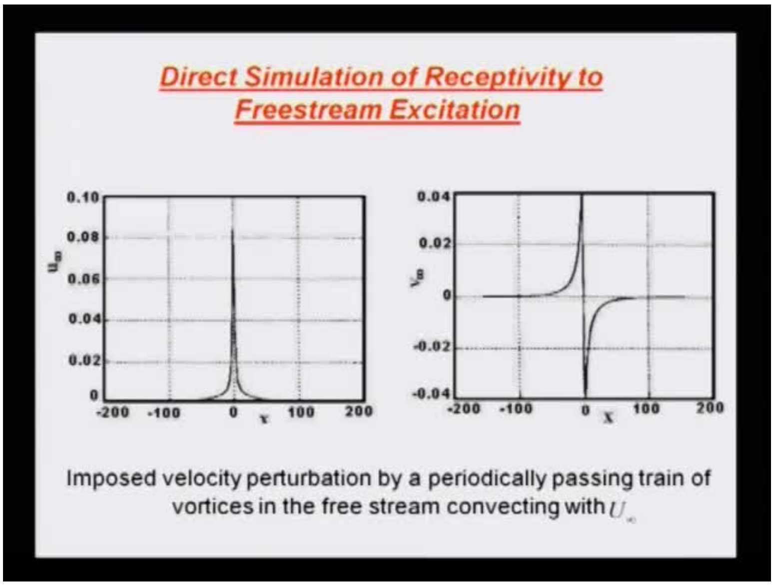
(Refer Slide Time: 19:22)



So, this is the way to ((run)) visualize the problem, that I would write down the disturbance stream function, which will be function of x, y and t; x of course, dependence can be figured out here, in terms of the phase i alpha x; alpha itself could be complex, because, we are applied it, talking about instability problem. What about the time dependence? Time dependence is going to be happening at omega naught, and its various harmonics.

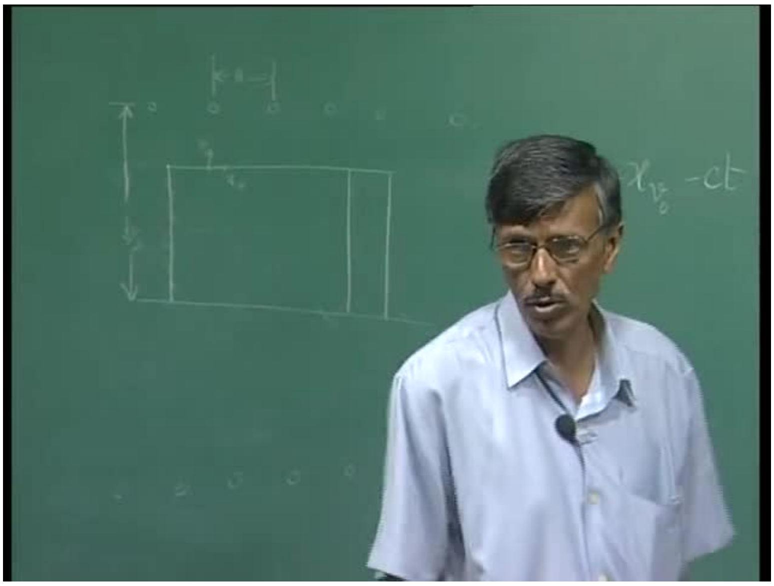
So, that is what we have done; we have put in there, e to the power minus i n omega naught t; and, omega naught, we have worked it out. And then, we should sum over, because, it is strictly periodic problem. You see, this vortices are strictly periodic. So, it will not be a integral, but it will be a sum. And, the sum will go on from n equal to 1 to infinity. And alpha, of course, will perform over the Bromwich contour. Now, we all know what a Bromwich contour is; we can do it. So, this is your expression for the disturbance stream function. Now, we also know that, if we are trying to look at the wavy solution, which is away from the exciter then, we should be looking for Eigen values close to the origin; that is what we talked about the (( )) theorem.

(Refer Slide Time: 20:40)



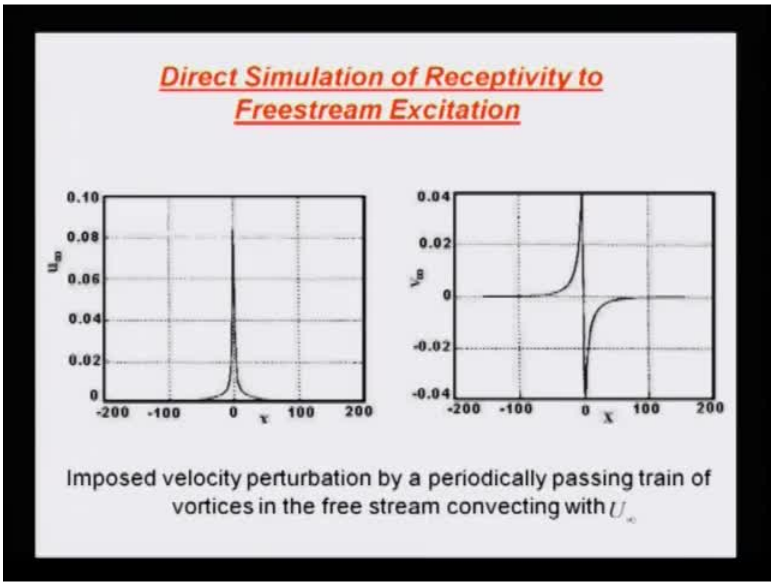
So, that is that. Then, having obtained this expression for disturbance velocity, and we have already written down the expression for u infinity and v infinity just now; now, what we could do is, we could conjure a problem here that, suppose I place this train of vortices at a non-dimensional unit of 20, or something, and I create a, some kind of a top lid, which is slightly below.

(Refer Slide Time: 12:04)



You know, my reason is that, I do not want to include singularities inside the computational domain. It is a nuisance, you know. Any discrete inviscid vortices will work as a singularities, is it not. So, to avoid that, we keep those away and our top lid is below that train. And then, we can calculate the u infinity and v infinity. And, if we calculate that, this is what we are going to see; this is shown with respect to one vortex. So, that vortex location here, has been fixed as the origin.

(Refer Slide Time: 20:40)

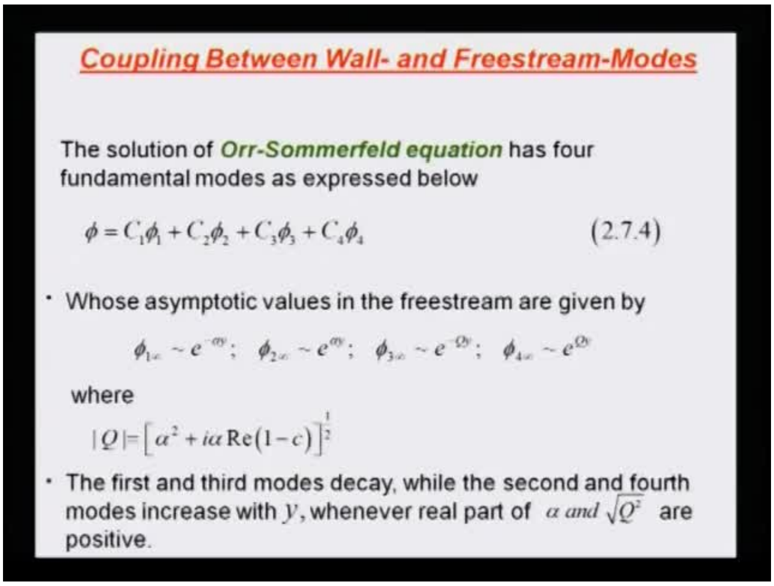


And, the example that we are seeing here, the gap between successive vortices is 100 pi. So, it is about 315, and then, you can calculate, what is happening in the intervening period. So, this is the vortex that we are focusing our attention upon; there is another vortex here, at minus 315; there is another vortex here at plus 315; in between, this is the way, the induced slip velocity is going to be.

Immediately under the vortex, I am going to get the maximum, because, that is the smallest, shortest distance. So, that is what I get this peak; and, you see this is roughly about 0.08, for the height we are talking about. What happens to the v infinity part? v infinity, actually flips sign; on the left hand side, I would have positive; on the right hand side it would be negative, because it is rotating. So, one side, it is going up, the other side it is going down. And, that is what exactly you are seeing. Please note that, u infinity and v infinity are of the same order of magnitude. It is only that, here it is flipping sign; so, it is going from plus 0.04 to minus 0.04. So, this is about 0.08 range, and that is exactly like, what you are getting; however, you note, the stream-wise extent over which the disturbance is felt, this is more localized; u infinity is more localized than v infinity. So, what will happen? Of course, when you are looking at the Fourier transform of this u infinity, you will find a wider band. So, basically, what is going to happen is that, u infinity and v infinity will excite different kinds of length scales and that, we should be able to see it, if we obtain the Fourier transform; and that is what I am showing you.

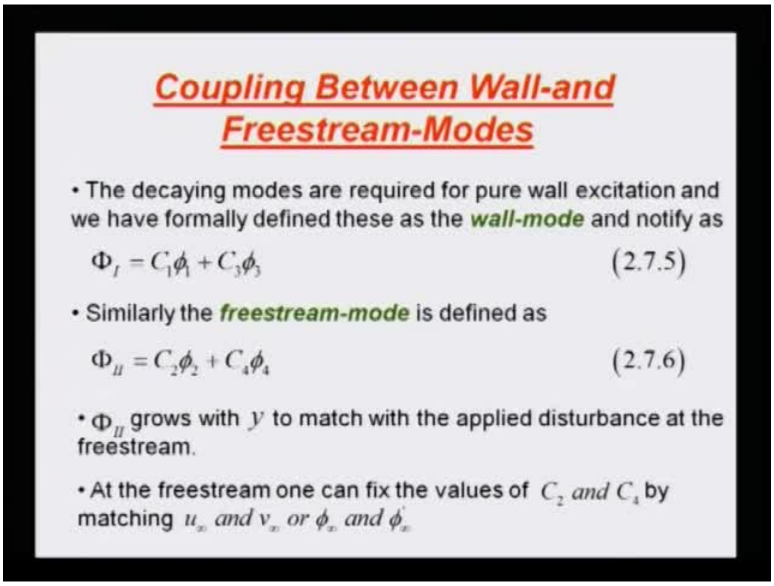
It is very interesting that, the footprint of u infinity and v infinity means a lot, when we look at in the transform plane. You know, this is one of the reasons, that we need to be looking at all fronts. If I just simply look at the physical plane, and say, they are, relative magnitude is the same, so, they are having the same effect; in the transform plane, they are not so; you can see it, very clearly, this is corresponding to u. So, this is phi infinity prime. If I look at phi infinity prime, look at the maximum; maximum is about 1. And, look at the Fourier transform of v infinity, that gives us phi infinity; and that phi infinity, look at this, about 18. So, you can see, there is a order of magnitude difference between phi infinity and phi infinity prime. We will use this information later. So, please do remember, this is what is happening; this is why is it happening is, you can also make out, because the v had a flipping of sign, and it did happen very abruptly; whereas, u infinity decayed smoothly; that is why, it has a wider band, but the 0, it does not peak up, that much.

(Refer Slide Time: 25:57)



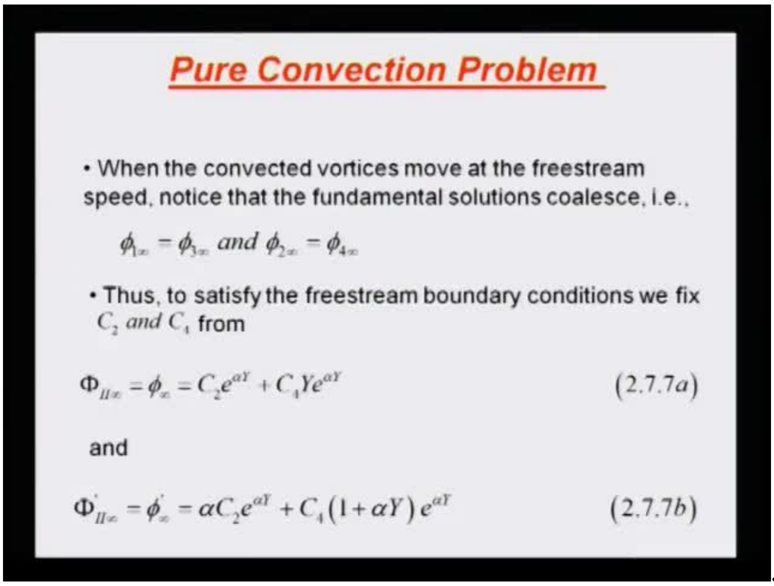
So, we have now characterized the input to the system, is it not. So, this is what we are giving it as an input, and we have talked about the Eigen values and the receptivity problem, for both the wall-modes, the modes that decay with height, and the freestream mode, that grow with height. How are these two sets of modes linked together? This is something worth investigating. Let us see what happens. If I look at the Orr-Sommerfeld equation, and the problem that we just now looked at, if we also create, some kind of a wall exciter, then, we would retain all the four modes. What is known to us is that, if once you are outside the shear layer, this four modes have, some kind of a analytical structure, that is given like this. We have seen that, if real part of alpha and q are positive, then, the first and third mode decay with height, while the second and the fourth mode grow with height.

(Refer Slide Time: 27:12)



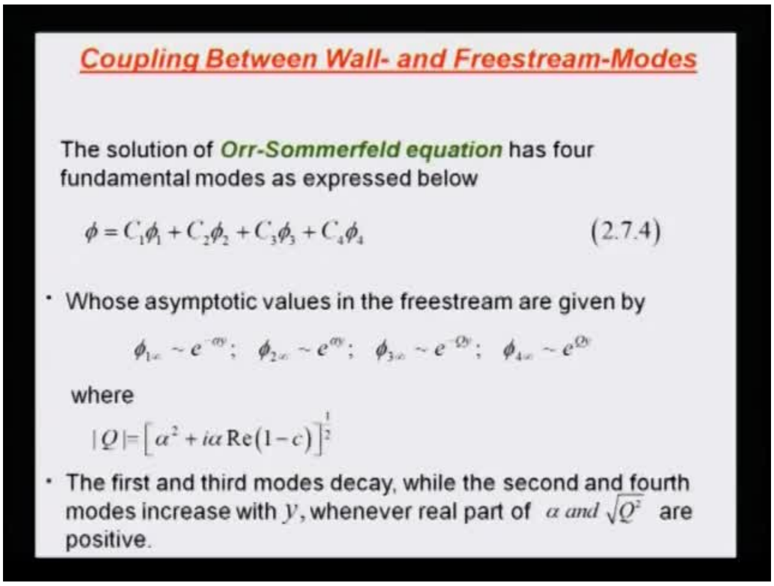
So, this is something we should remember, and if we look at the consequence, that we can basically, couple these two fundamental modes into a wall-mode C 1 phi 1 plus C 3 phi 3, which I call phi I; similarly, I could call the other two as, a freestream-mode, which is C 2 phi 2 plus C 4 phi 4, which I call phi I I. So, basically, we need this phi I I to match the imposed disturbance in the freestream. So, if I do that, if I tell you what this disturbances are, at the freestream, I have basically, phi 2 and phi 2 prime, in terms of u infinity, v infinity, we just now calculated. So, we can immediately obtain C 2 and C 4. So, we can obtain C 2 way and C 4 in terms of, either using u infinity, v infinity, or phi infinity, phi infinity prime. So, we should be able to work out. What is happening in the freestream, about this part? This part is subdominant; this does not exist; that is why.

(Refer Slide Time: 28:50)



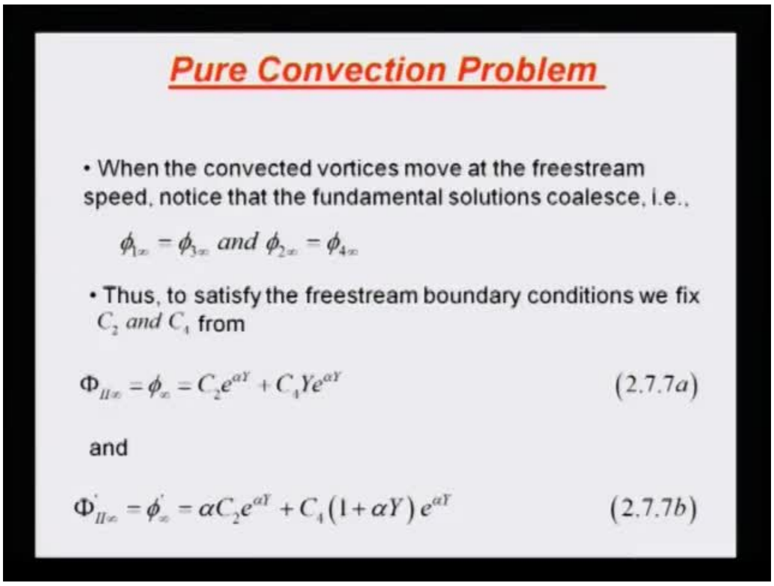
So, independently, we can fix C 2 and C 4 from the freestream condition. So, this is what we obtain, very easily. Now, having obtained C 2 and C 4, what we could do is, also talk about two distinct classes of sub-problems. You see, what has happened? In instability theory, people suffered from some misconception. One of the misconceptions was, suppose I have inviscid vortex, going in the freestream; there are no viscous effects; at what speed would it go? The vorticity transport equation is the substantial derivative of omega equal to 0; that means what, the omega is going with the local speed.

(Refer Slide Time: 25:57)



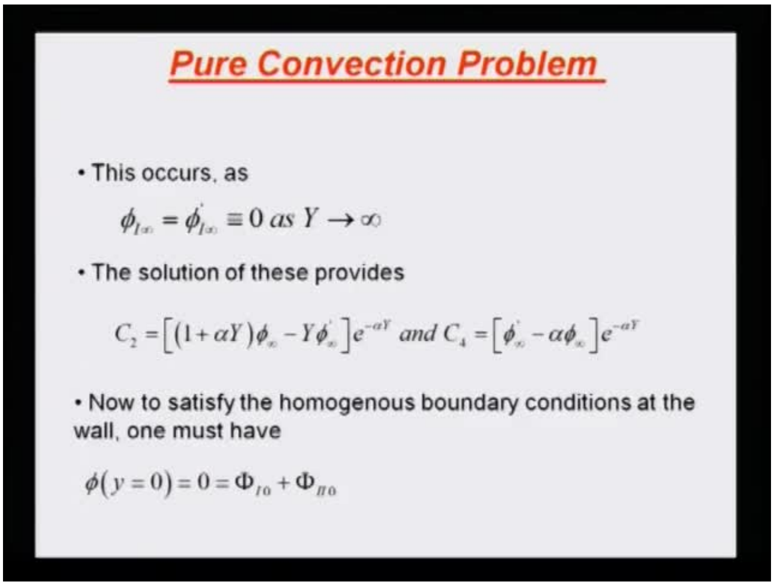
So, that is what we call as the pure convection problem. So, C will be equal to u infinity. If I do look at that, when C is equal to u infinity, then, what happens? You have seen that, this modes will coalesce. Why, because, if I look at the expression for Q, if C is equal to u infinity, then, what happens; in a non-dimensional form, C equal to 1. So, this part goes to 0. So, then, what you are getting here, Q is square root of alpha square. So, what does it mean? phi 1 and phi 3 are same; phi 2 and phi 4 are same; phi 1 is, let us say, minus alpha; phi three is also minus alpha, because, Q is equal to alpha and phi 2 equal to phi.

(Refer Slide Time: 28:50)



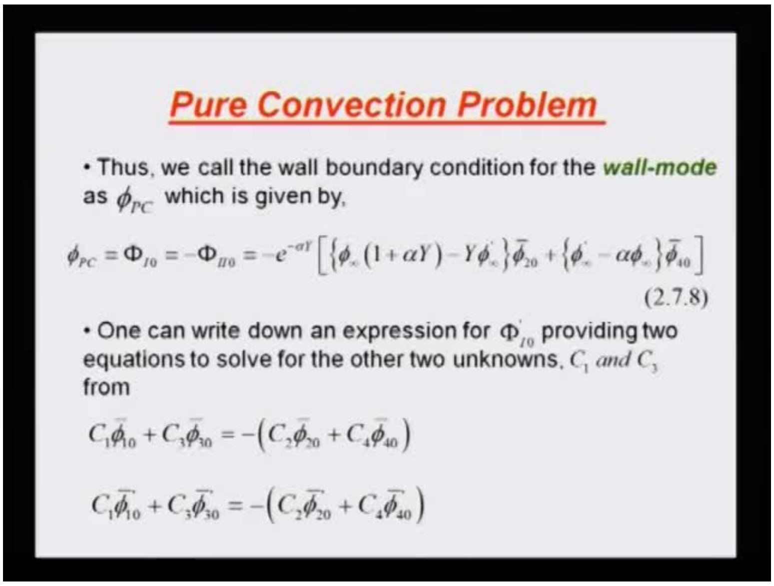
So, there is a case of mode coalescence. So, this mode coalescence have to be kept in purview. And, this is what we are talking here that, if we are looking at the pure convection problem, if the vortices move at freestream speed, then, phi 1 infinity equal to phi 3 infinity, and phi 2 infinity equal to phi 4 infinity. All of you convinced? If you are, then, we can write down the freestream mode at the freestream. So, I will write it as phi I I, evaluated at Y equal to infinity, and that is nothing, but your phi infinity itself; and that is given in terms of this. Now, please note that, because of coalescence, we have C 4 times, capital Y times, e to the power of alpha capital Y, is it not; because, this is a condition of coalescence. And then, I can also differentiate it and I get the corresponding value for phi prime I I, at Y equal to capital Y. So, now, you see, this is, these are the two equations 7 a and 7 b, will help you in solving C 2 and C 4, in terms of phi infinity and phi infinity prime. So, we know how to get this case solved.

(Refer Slide Time: 32:13)



Once you have C 2 and C 4, you can do that, and as I also mentioned to you, when you look at the other mode, the wall-mode, they actually go to 0, as Y goes to infinity. So, this do not interfere in the solution. So, we get C 2 and C 4, like this. So, here are your closed form expression for C 2 and C 4. Now, suppose, we have only the excitation in the freestream, then, what happens to the condition at the wall? Condition at the wall should be, one of no slip; so, that means, what; that means, I should have the v components 0; that means, I should have, phi at the wall should be equal to 0; and I should also have, u component 0; that means, phi prime will be 0. So, then, if phi at the wall is 0, that is equal to phi 1 0, plus phi 2 2 0. Now, what has happened? As far as this part is concerned, we at least know this multiplicative constant; but for this part, we do not know C 1 and C 3. So, we are trying to clear out a strategy, by which we can obtain those C 1 and C 3.

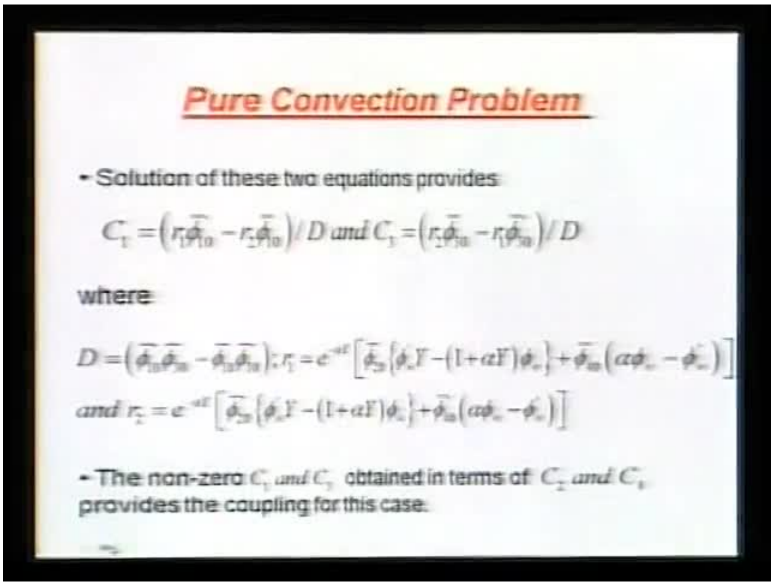
(Refer Slide Time: 33:37)



So, this is your condition for phi at the wall. You go ahead and write this as the wall boundary condition, for the wall-mode; that is what we said as capital phi of I. So, that is your boundary condition. So, that is why, I have given additional subscript 0. And this, I am writing it on purpose as phi subscript PC. PC stands for pure convection. This is the case we are considering, with C equal to u infinity. So, that is your pure convection problem. Then, what is that, phi I 0 should be equal to minus of this, because sum of this is 0. So, since I know this, I can put this. But you please note that, we do not a priori know, what this phi 2 0, phi 4 0 bar are; but supposedly, we can integrate something like Orr-Sommerfeld equation, and obtain this fundamental modes, we should be able to get these.

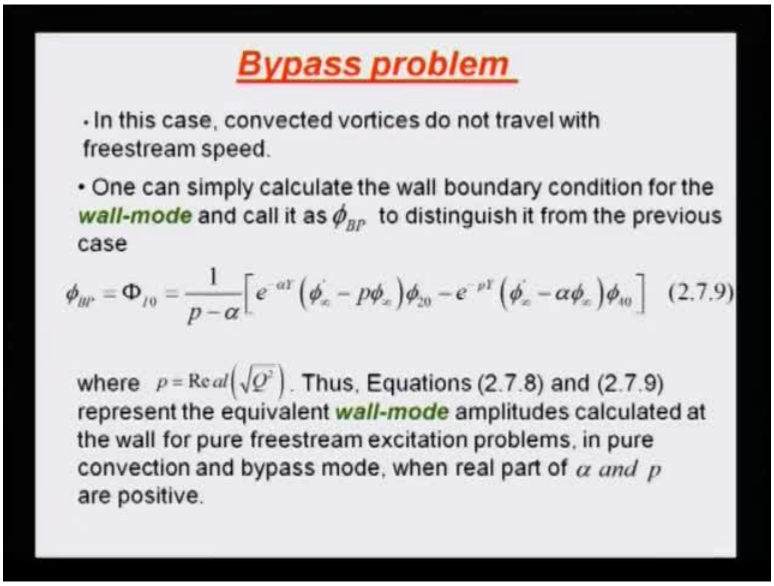
Now, what has happened now? We have now, two sets of equations to obtain for C 1 and C 3. So, this is what we are getting, by putting phi at the wall equal to 0. So, this is coming from the freestream mode, evaluated at the wall; and this is this. So, now, what you are seeing, this is the v component of velocity, and this is the u component of velocity. Whatever I did in the freestream, that in turn, gives you this nonzero C 1 and C 3, from this equation. This is what we are calling as the coupling. While I am creating a freestream mode, but that, is fixing the coefficient for the wall-mode, C 1 and C 3.

(Refer Slide Time: 36:05)



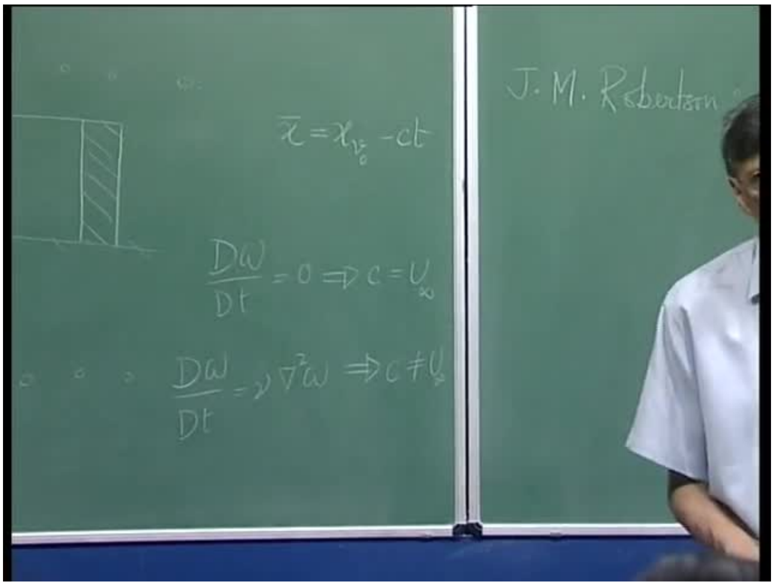
So, you see, you have not created any wall excitation, but it is happening indirectly. Because of the freestream condition, we are going to get nontrivial value of C 1 and C 3. Once you do that, you get a bit of algebra to be done and you get those C 1 and C 3 in terms of this. And, the denominator is given in terms of this. And, what is this? This is once again, your Y 1, that compound matrix variable, evaluated at the wall. But what we need to understand that, in this case, phi 1 and phi 3 are the coalescing mode. So, this is slightly special. These are special case, where Q to alpha. So, that is that; that is what we are going to get. And, you also notice, there are r 1 and r 2; those are obtained here; r 1in terms of this and r 2 in terms of this. So, basically, this nonzero values of C 1 and C 3 can be now obtained, in terms of C 2 and C 4; and that provides the coupling for this case.

(Refer Slide Time: 37:25)



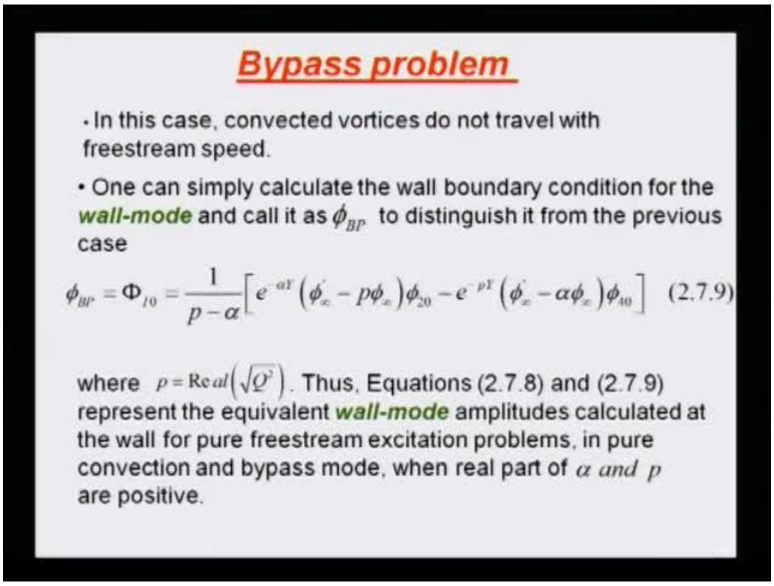
So, you can now understand, what people hoped intuitively that, wherever possible, I will excite the system. It will eventually, create those Eigen values; and here is the clue, as to the coupling between a freestream excitation and the equivalent wall excitation. Now, I told you that, there are two subclasses of problem; one case, we just now discovered, or talked about, is the pure convection problem.

(Refer Slide Time: 38:14)



However, there could be a possibility that, this vortices will not go at infinity; because, this depends on the nature of the vortex. If it is not a inviscid vortex, then, in its neighborhood, I cannot omit the diffusion term. So, my governing equation is not necessarily this. For inviscid vortex, we had the governing equation as this. And, that gave rise C equal to u infinity. But suppose, I talk about some viscous vortex, then, even in a 2 D frame work, it would be like, D omega D t should be equal to nu times this; and this, per se will give you the condition that, C will not be equal to u infinity. For a long time, people at the stability theory, are fruitlessly argued in favor of this. If you look at all the publications, till we came out into the picture, they all was used to say, C has to be equal to u infinity, and there is no receptivity. They also did not realize that, there is a case of mode coalescence, etcetera. This is something we note. But, more so, when we are looking at viscous vortices, then, this is the order; I mean, C is not going to be u infinity. And, this mode of events occurring, is what we are calling as bypass problem.

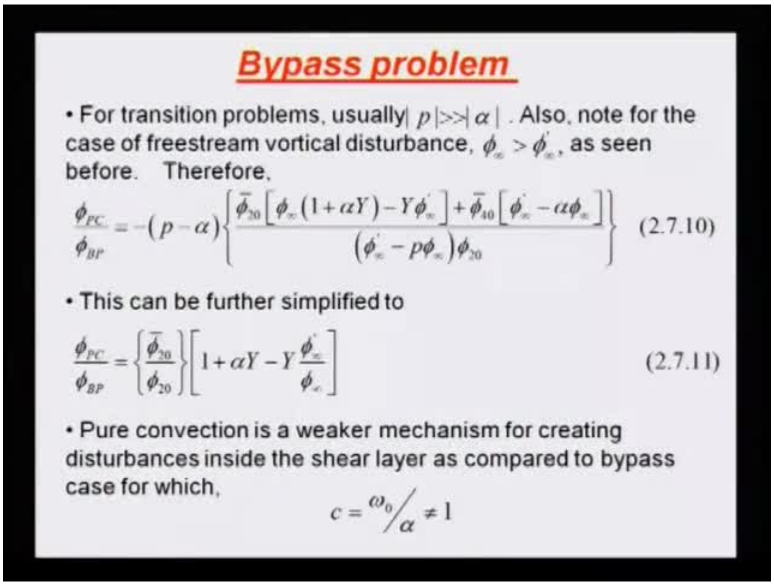
(Refer Slide Time: 37:25)



So, we are now talking about a case of convecting viscous vortices, for which C is not equal to u infinity, and that problem, we are calling it as the bypass problem, implying that, the convective vortices will not travel with the freestream speed. One can again, calculate the boundary conditions corresponding to the wall-mode, even though we are not creating any wall excitation. What we should be doing is, we should be calculating the phi, which I identify the subscript B P to indicate this bypass mode; that will be capital phi I 0. And now, what we are getting? You realize that, you have, the modes are distinct; there is no more coalescence. And, if the real part of capital Q is p, then, this is the way, that we are going to get. What we are doing here? We are again satisfying the freestream condition, to calculate C 2 and C 4. Having obtained C 2 and C 4, they are here; this is your C 2, e to the power of minus alpha Y, capital Y, into phi infinity prime, minus p phi infinity; and C 4 is this, minus of e to the power minus p Y, within parenthesis, phi infinity prime minus alpha phi infinity.

So, this is your C 4; this is your C 2. Now, this, we are evaluating at the wall. So, this is basically, phi I I, at the wall with a minus sign; and, that is equal to your, the wall-mode at the wall. So, this is what we are getting. So, this equation that we have talked about, created some kind of equivalent wall-mode amplitude in pure convection and bypass mode, when the real part of alpha and p are positive; if they are not so, we can change the condition appropriately, and work out an alternative expression. So, that is not a big deal.

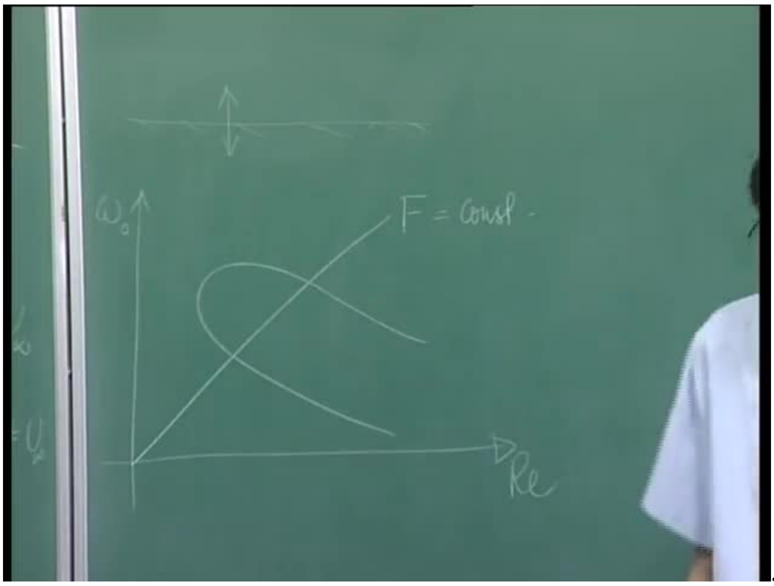
(Refer Slide Time: 42:32)



Now, what has happened here, we have calculated the effect of inviscid vortices at the wall; that is what we called as phi PC. Now, we have just now calculated, if there are viscous vortices going, what is the equivalent excitation at the wall, which we called as phi B P. We can take the ratio and it looks like this. Now, what have we used? We have used this condition. Do you recall, I have showed you, how phi infinity was of the order 1; whereas, of the order 18 to 20, whereas, phi infinity prime, was order 1. So, we have eliminated that, and we have simplified this.

So, we can actually, simplify it and we get this condition. So, what happens is, you need to basically, know what your phi 2 0 at the wall going to be, for the pure convection case and the bypass case. That, this ratio, plus this. Now, what happens here is, of course, in general case, this is a small quantity, phi infinity prime by phi infinity. So, this part is negligible. So, 1 plus alpha Y, that is what we are getting. Y is usually large; we are taking it at a very higher up, height; alpha is rather small, because we are looking at the Eigen values close to the origin. So, this is what, really a very strong thing. What really determines this ratio is the ratio of the two. And, what we find eventually that, this quantity is going to be a small quantity; that means, this equivalent wall excitation for the pure convection case, is a much weaker case than bypass mode. Well, we can see this. We do not have to be convinced immediately, because, we do not have close form estimate for this phi 2 0 bar and phi 2 0, but we can solve the Navier-Stokes equation and we can see what happens.

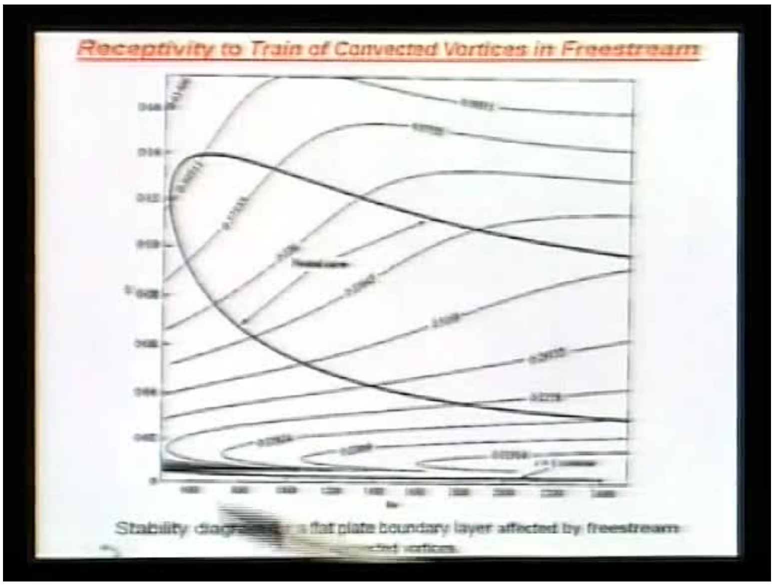
(Refer Slide Time: 45:40)



However, we are creating the wall excitation by the freestream convecting vortices. But, how would those wall disturbances go? This is something, that we must distinguish; and, this is where I think people have missed out the picture completely. And, this is, what we pointed out is that…Well, a freestream excitation is equivalent to creating a equivalent wall excitation, but what should we track. Remember, in the earlier case, when we are talking about Schubauer and Skramstad experiment, we created a wall excitation here, corresponding to a fixed frequency. And then, what we did, we did plot the neutral curve in R e omega naught case for signal problem and we found out the neutral curve like this, and the constant frequency disturbances, were tracked by rays going like this.

So, this was your, capital F equal to constant; that is what we studied; we had seen that. But now, what is happening? Here, the disturbance propagation is tricky, because it is not happening at a fixed frequency. You have already seen that, the moment I prescribed C and a naught, I created a, sort of, omega naught and it is super-harmonic. So, this is not a monochromatic disturbance. What is invariant in this problem? It is the speed at which the disturbance is propagating in the freestream. So, what I could do is, I could look at this and I could plot out the C equal to constant contours; and that is what you are seeing here. So, this is once again in R e omega naught plane. The solid line indicates the neutral curve. It is unstable inside; stable outside; and these are various C equal to constant lines.

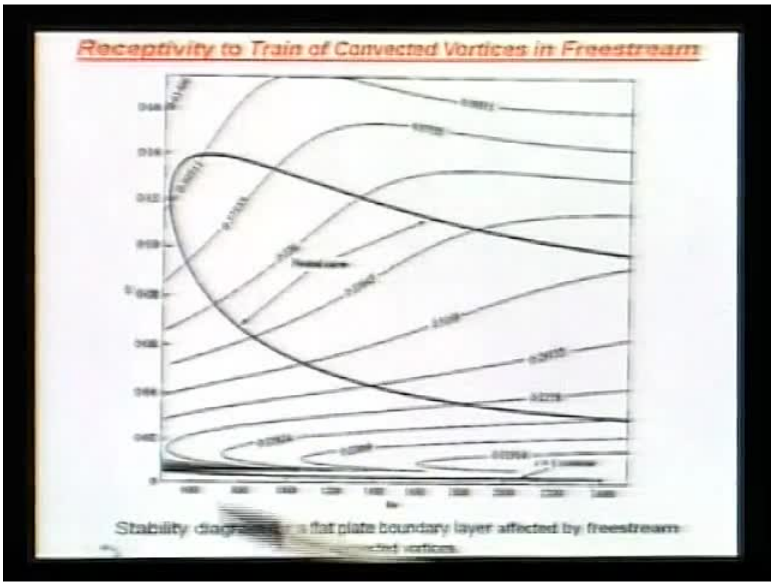
(Refer Slide Time: 47:44)



Say, for example, this line corresponds to C equal to 0.3169. So, these are all non-dimensionalized with respect to u infinity. So, if a disturbance, if those convecting vortices go at this speed, 0.3167, and if I try to trace them, I should be tracing them along this line. And, this is significantly different, than what we did for pure, monochromatic wall excitation. Here, I would probably be looking at this. And, what you notice is, basically that, these lines are very ((quire)). For example, C equal to 1; that means, when C is going at the freestream speed that would correspond to C is equal to 1 line; that is right there. And, that remains all is outside. So, that is what people have talked about that, there is no receptivity to pure convection problem.

So, if you make some inviscid vortex go in the freestream, at the freestream speed, the disturbances those are going to be created, they are going to be very stable; because you see, this is so far away from the neutral curve. So, this decay rate is rather large. So, you are not going to create that. And, this is what people have noted, theoretically; but the point remained that, experimentally, it does not happen so. I mean, you know, freestream turbulence, etcetera, they all do have, finite visible effect. So, what are we missing? We are missing what we discussed today, that, those vortices themselves could, did not necessarily be inviscid vortices; and then, C will not be equal to infinity; and, different values of C will have different instability pattern.

(Refer Slide Time: 47:44)



See, for example, this case 0.3169; it enters the neutral loop here. So, it become unstable here, and then, it remains unstable for a long stretch. This is significantly different from this monochromatic wall excitation. There, it just enters here; exits here. And now, you see, it enters here; exits here. There could be, even other cases, where it would be, actually going in a…What more, if you recall, what we have studied about the alpha i contours. This is alpha equal to 0; inside, alpha is negative. How does it increase? We have the highest value of the alpha i contour, is a line like this.

So, what happens is, the disturbances, those convect with the speed, other than u infinity, they go inside, stays over a longer region, remaining unstable, and they also visit the unstable part, and stay there for a longer time. So, because of the prevalent mindset, in late 90s, we call this as bypass transition. Why it is bypass transition? It is not really bypass, because, you have still, again, creating waves; but what you are creating, is a, basically, assembly of waves. You have excitation as omega naught, 2 omega naught, 3 omega naught. So, in the end, what you get? You get, kind of a, wave packets, consisting of TS waves. We should not have called it bypass, but if we would not have called this is bypass mode, how would we characterized it. We say, it, this is not the usual Schubauer and Skramstad type of TS wave, but it is a different kind of TS wave, and we called this as bypass mode of transition. But essentially, these are wave packets, those are created like this.

So, this is what we must understand. And, you also notice that, there is a bit of a selectivity of the value of C. We see that, this curve is something close 0.26. So, anything below 0.26 is stable; anything above 0.4, also is stable. So, there is a band; there is a band of convecting speed, in which this remains unstable. In fact, some such experiment was done by Doctor Jim Kendall, at jet propulsion lab. So, when I showed him this result, in a conference in Washington, he actually, narrated to me, couple of his papers, where he has seen this thing, but could never explain. And, this figure, very clearly says, why he saw what he saw. I think I will stop here. In the next class, I will briefly explain, what Kendall’s experiment was, what he observed; how does it all jell with this; and, you will see a very interesting story emerging there.