Instability and Transition of Fluid Flows

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Lecture No.# 18

We have been talking about stability theory, and we came to receptivity theory, we focused on attention on wall excitation. We specifically made up on that stability theory the way it has been formulated, and solved refers to excitations which remains buried inside the shear layer.

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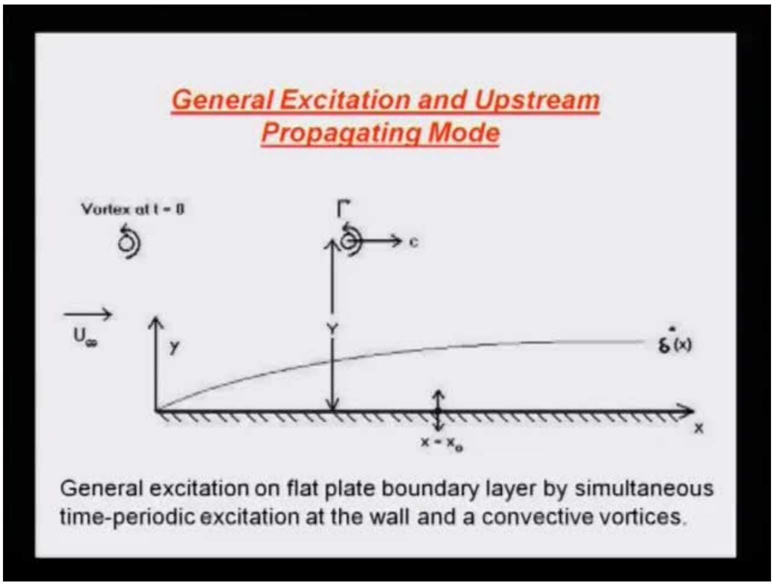

So that, basically tells you that a huge area remained blank, and that area is when you are free dynamical system is excited from outside the shear layer, and this is what happens, let say talk about a prototypical example here, and what we need to do here is consider a shear layer growing on a flat plate let say, and then, we have a wall exciter here as usual, we have done this before, but in additional, let say we have asort of a convicting vortices. We have a convicting vortices series of, then coming and may be a single vortex.

So, let us consider a case of a single vortex which is convicting at a constant speed c at a height capital Y and at t equal to 0, these vortex was let say little upstream of the leading edge, and then, once this vortex starts going, then what happens recall that in spatial stability theory what we have done, in the spatial stability theory what we have done is we imposed a time scale, and then, we saw how the disturbance grow in phase that was whole idea. So, in this case also we are imposing a time scale here through this wall vibrator that is one thing that we can see.

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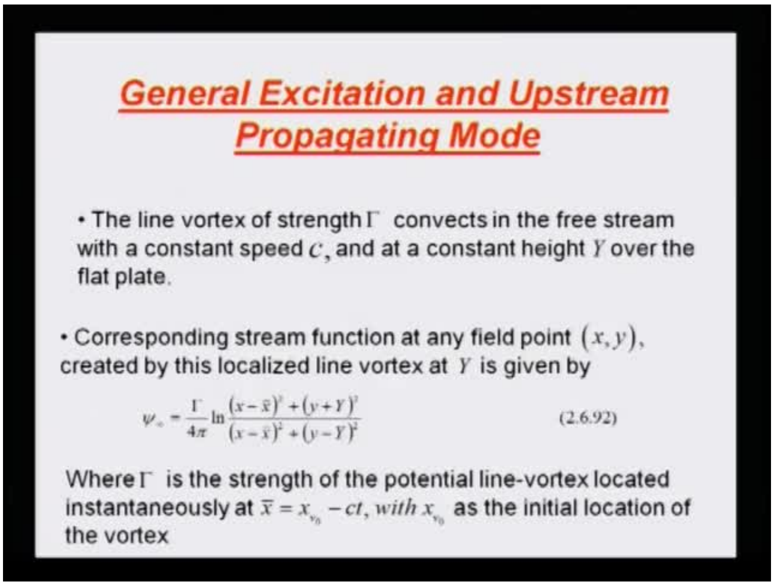

Now, what happens to this additional exciter in the free stream, and that is what we have started discussing over the last few lectures, and what we saw that the existence of that line vortex of strength gamma moving at a constant speed C at a constant height at Y gives rise to kind of a disturbance stream function, and this disturbance stream function provides a kind of a free stream in excitation.

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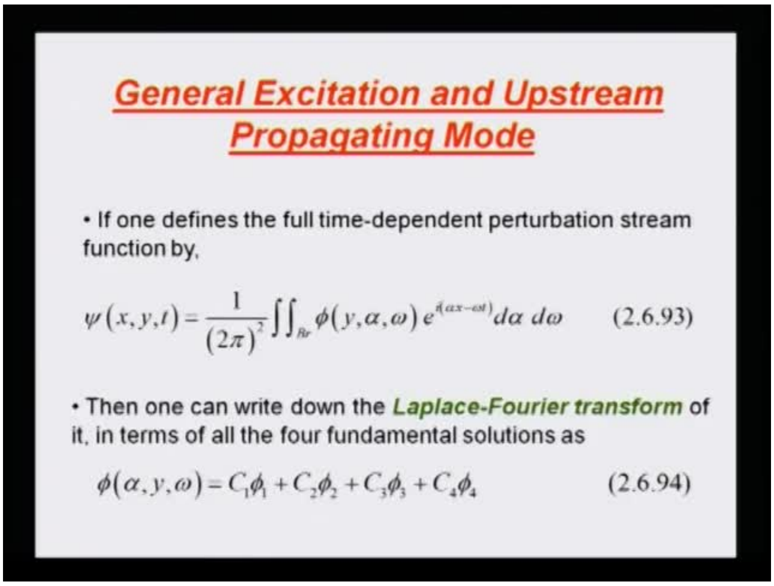

Now, if you recall what happens is that, we have these four fundamental modes of Orr-Somerfield equation, we have these four fundamental modes of Orr-Somerfield equation, and then, because now we have to satisfy the boundary conditions, not only at the wall but also in the free stream. So, you will be purpose have to keep all this 4 modes with this multiplicative constant C 1, C2, C3 and C4. This is what we are doing, where defining the disturbance stream function, in terms of it is Laplace transform and those are again split in terms of the four fundamental modes.

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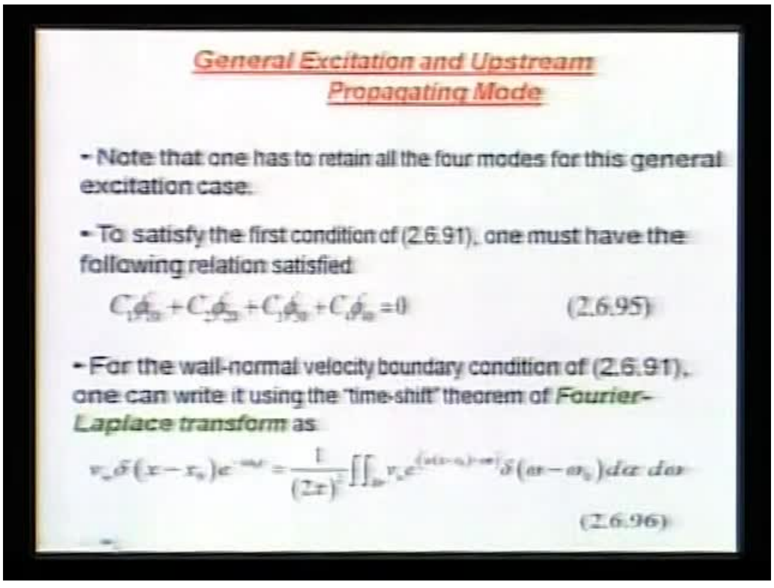

Now, once you do that, what you are going to get is to sets of boundary condition at the wall one corresponds u equal to 0, that would imply the prime quantity means derivative with respect to y that should be equal to 0, and this is the wall-normal compound of velocity which is located at x equal to x naught that gives you this delta function, and it is harmonic excitation at a frequency omega naught.

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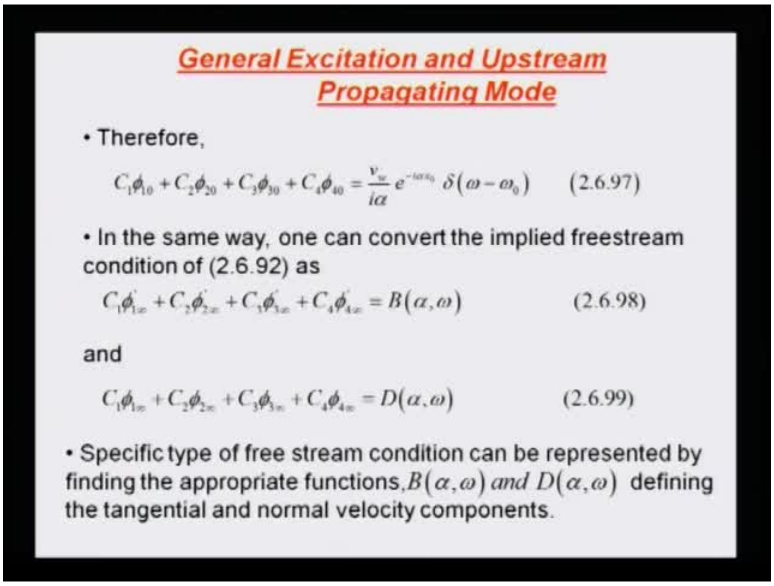

So, that is what you get e to the power minus i omega t, and this, I could write it like this; this is exactly the representation in a Fourier Laplace transform plane, and once you do that provides you the second set of condition at the wall, that is written on top, so that is your wall-normal compound of velocity, but then, we need to, now also worry about 2 sets of a boundary condition in the free stream, whatever may be there functional form, we could actually transform them in the spectral plane alpha and omega plane, and if the u velocity component in the alpha omega plane is called B, and the v velocity component in alpha omega plane is called D, then we had this two additional equations this is what we discussed.

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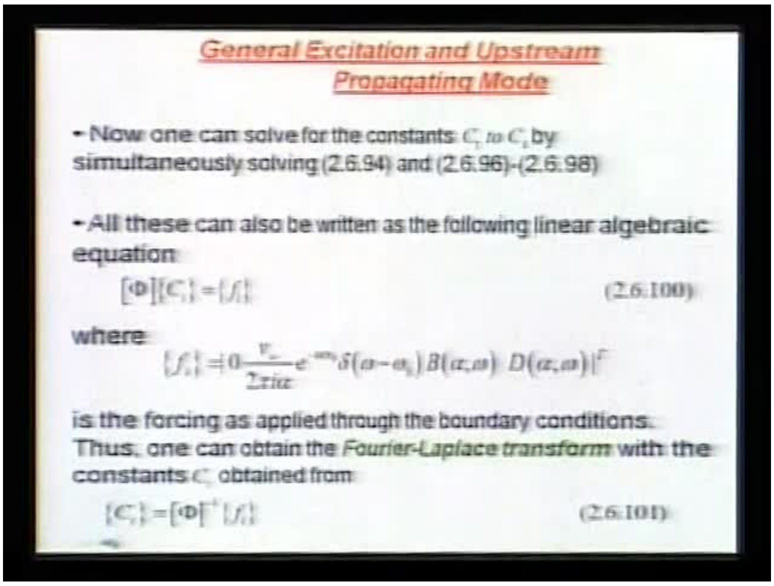

Now, then this completes that description of the boundary condition that will enable to really fix this constant C matrix, that would be given in terms of a 4 by 4 matrix, operating on the unknown constants is equal to the excitation. So, whatever may be the excitation that is put in on the right hand side. What is important, though that if this excitations are non-null, the non-zero, then what we can do is basically I can obtain this C i vector is equal to phi inverse of this.

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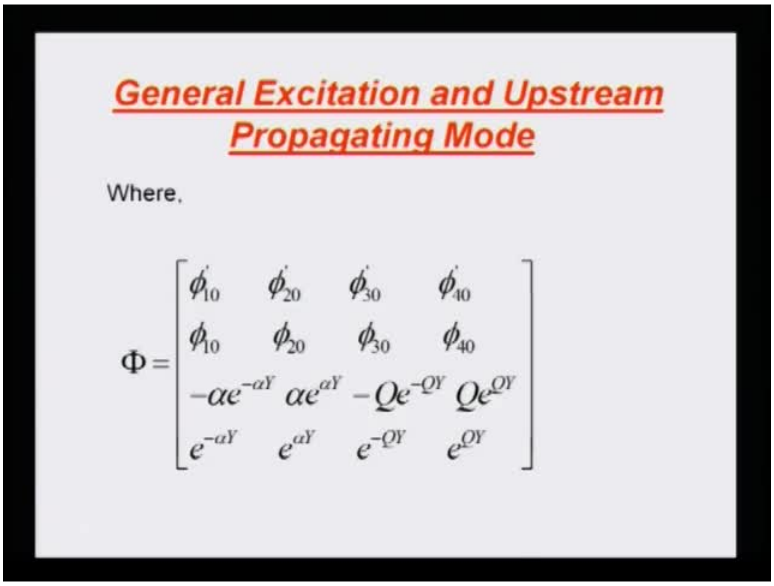

So, what we have find is, once again is very simply this, that determinant of phi some of is going to come into play, and the phi matrix is given in terms of the four fundamental solutions. Since the exact structure the fundamental solution in the free stream, we could write down these last 2 rows very explicitly. However, inside the shear layer, we have no direct way of finding them out analytically; so, at the wall we do not know that, but we just know there functional form that they will be in terms of the four components and we left that as they are.

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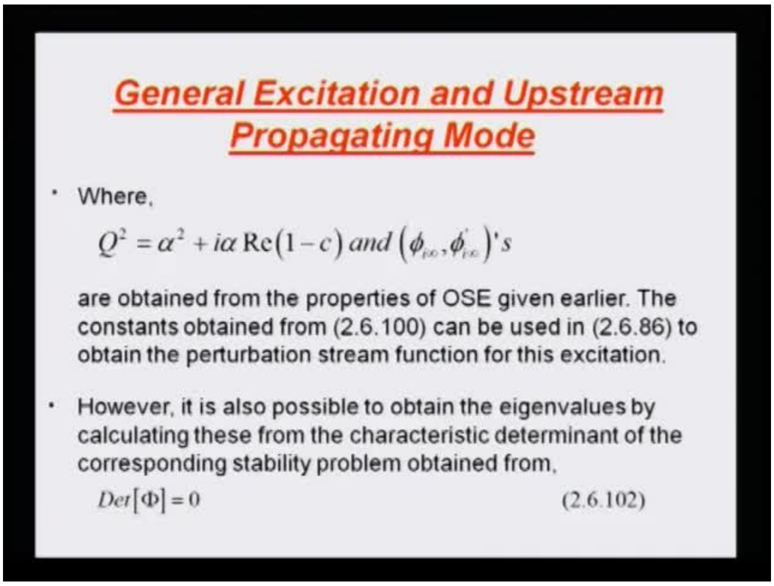

Now, since that free stream excitation would be imposed at a very large why, then what happen is we can see that, some of the parts can go to 0, and in this context, I started telling you a story, how we came to this root, we came to this root, because we were interested in what we have to call that upstream propagating mode, any upstream propagating mode here, refers to those modes for which the phase is moving upstream. So, if the phase has to move upstream for a real frequency that would imply alpha r should be equal to negative.

So, basically we are looking out for this elusive poles in the left half plane, which I told you in that nature, where ever they are marked, they are marked to the question mark, that we do not know, fortunately some sometime during midnight is here itself we figured out how to obtain them.

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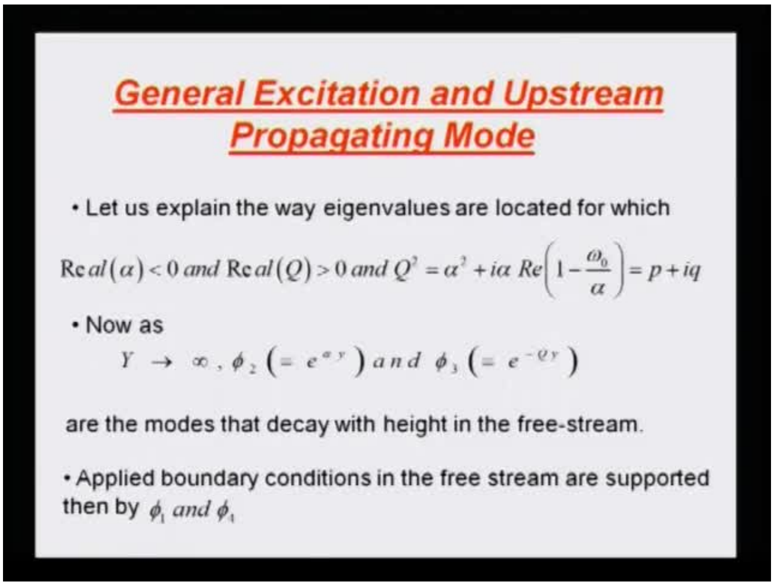

Well, they it goes a little for the back, but let us not go make it a exercise in history, but let us say that we are in the process of finding out the Eigen values, and the left half plane in the alpha the complex plane. So, basically what we have to talking about alpha r is negative, alpha i could be plus or minus. So, we are basically looking for Eigen values, those are given by real alpha less than 0, and real Q is always obtained in a mathematical sense, so that will always be positive, and then, what we could do is we could immediately see that this 2 modes, phi 2 and phi 3, in the outside the shear layer that is, when capital Y goes to infinity, they are the one's which are filled decay with y.

What happens phi 1 and phi 4 would complementarily will support the free stream condition, that is what we are saying that, we should be able to use that, that was a reason that we decided to keep all the four fundamental solutions which should fix (( )).

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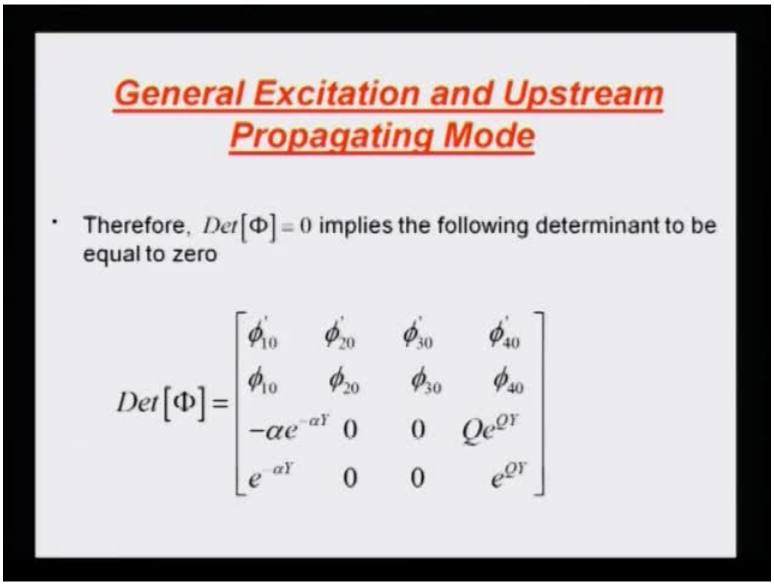

So, now, if we decide to look for those kind of modes for which real alpha is positive negative and real Q is positive then phi 2, and phi 3 are the decaying modes, and this 2 entries will go to 0, when y goes to infinity is a subdominant quantity, because we have e to the power plus alpha y so alpha r is positive.

So, that will decay with y, so that is what we are doing away with this two, and putting them they have subdominant component as 0 itself. Now, once we obtain this, we have seen the characteristic determinant is given by the determinant of this capital phi matrix, so that gives youbasically the dispersion relation.

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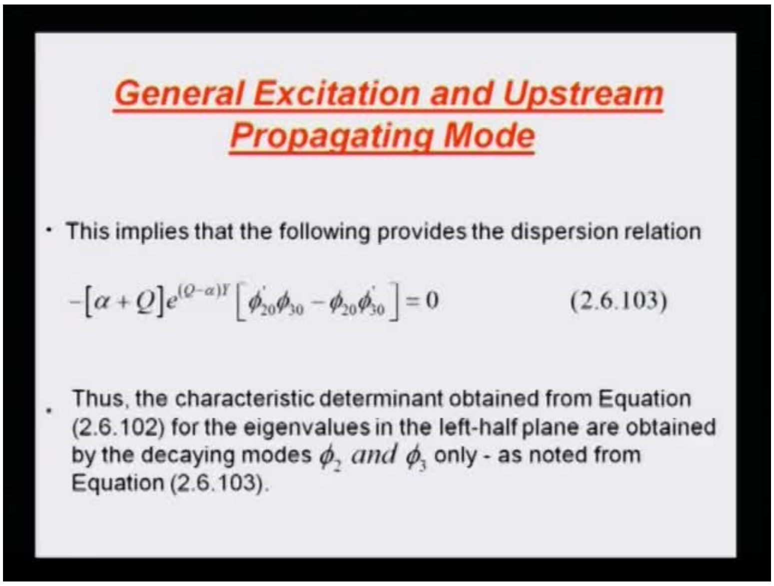

So, what is a kind of a wonderful, similarly here that when we have open it up, and simplify what we find that those growing modes with y gives rise to this part of the dispersion relation, and this part of the dispersion relation, what does it say well, and they cannot be 0. What can be 0 is this quantity and what this quantity is, this quantity is nothing but your Y1 the compound matrix variable, compound matrix variables at Y equal to 0, so this is how we get.

So, basically then we have learned a very interesting lesson, that even when we are looking for the mode in the left half plane, all we need to do is look at this same characteristic determinant, same dispersion relation composed of the decaying modes alone, and I told you a story that we figure it out in late eighty's, and we were really, I am not very sure, how to interpret it, but today now we can interpret it what we are doing.

Now, you see we have talked about the historical development of the subject per say, that well a German school was very weigh men in their pronouncement, that there is a instability problem, but experimentally it was never been obtained, and one of the experiment was done by G I Taylor, and what he did was he took a flat plat for boundary layer, and in the middle of the tunnel put in a kind of a vibrating dump by bump, and that bump was oscillating at a fixed frequency, and he could not detect any wave solution, and he pronounced that this theory would be correct.

However, people later on understood that, there was this mismatch between the theory and the experiment theory has sort of sort out Eigen values for a moderate frequencies in the orders of tens and hundreds, whereas the Taylor's experiment the bump was excited on only at 2 to 5 hertz.

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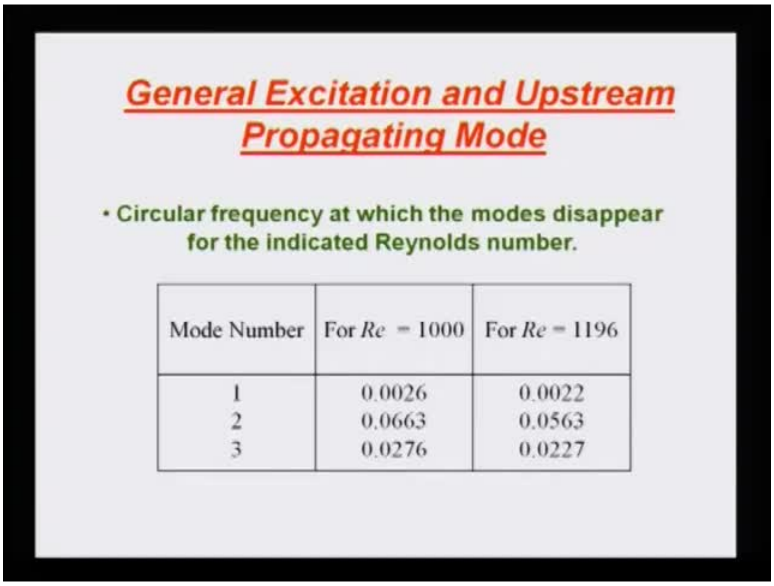

So, there is a mismatch that gave rise to this kind of acrimony between these 2 groups. However, this was re-investigated, again in early nineties by a group in came bridgemy gastro was looking at it, and they had a very nice clean tunnel noise free tunnel, and they try to replicate the experiment done by G I Taylor.

So, what they did was they put in a bump, where the local Reynolds number based on displacement thickness is11 96, and they also, they also basically vibrated the bump at a low frequency, and a very strange thing appear is the strange thing that appeared was this that you did not once again people did not see any wave per say, but what did they saw that, the whole shear layer went up and down as if the boundary layer is breathing that is why these are called breathing modes.

So, as if the whole boundary layer is heaving up and down and this breathing mode was also investigated by another group that is same famous group at NBS Washington kleban off studied this; so, kleban off noted that term if you look at a flow with noisy free stream, then you see this whole boundary layer does this.

So, he was the one who also call this is a breathing mode, but now it is named after kleban off and this is called a kleban off mode.

So, basically now what we find that 2 dispended things on one side, you have the experiment of G I Taylor repeated by mic Gaster, which was showing this boundary layer heaving up and down, and then there was this experiment done by kleban off in late fifties, where the whole boundary layer was, once again seen heaving, but then you talking about free stream turbulence, it is not a monochromatic excitation, like vibrating a bump; so, how this two desperate things are unified that is what we started looking at.

Well, you have seen that most of our earlier results, we have focused attention on Reynolds number of 1000, and the circular frequency of point one more than, so, what we did was, we seen that for that combination we have 3 Eigen values, and those 3 Eigen values are the following nature, one of them is unstable, two of them are stable.

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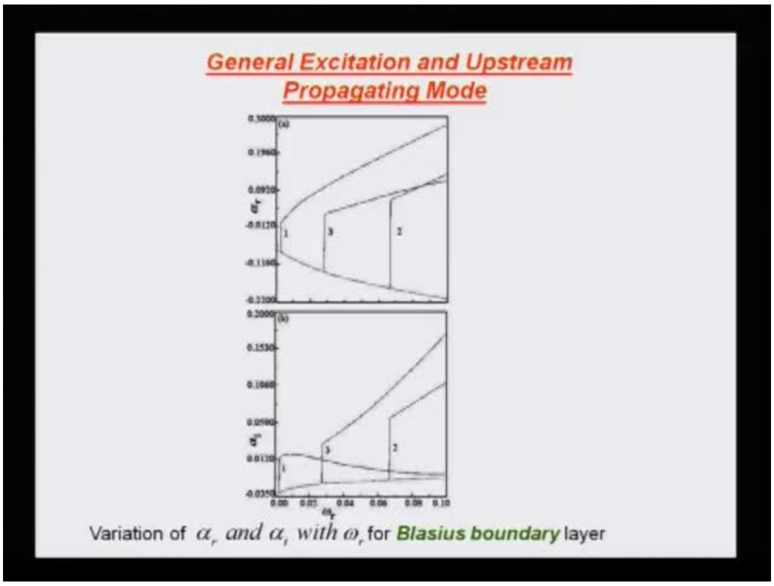

Now, what happens is, if I start from that circular frequency of omega naught equal to 0.1, and start reducing the circular frequency, then what happens is one by one the eigen values disappear, say for example, this mode 2 that we talk about, so that is what this value is so this is a alpha r positive value, then when we keep reducing omega is plotted along the x axis, it goes down, and then, all in a suddenly disappear, the second mode disappears, similar thing happens to the third mode, but this disappears at a somewhat of a lower to that number and the first one which corresponds to a t s mode.

So, this corresponds to the t s, then also eventually disappears. So, below this frequency apparently, you do not have any Eigen values, and this is a very curious thing to happen, and people did not know, why there is not a stability theory for very low frequency excitation, and we here at Kanpur decided to explain it, and that is how I am going to narrate this story to you.

Now, I will talked about the alpha i variation will tell you that this is a negative value, this one corresponding alpha is negative, but if you keep reducing the frequency, it becomes positive before if disappears. However, whenever they disappear, you do those calculations, like the way we have a just now talking about, you see that, you get a mode with alpha negative and this is what we just now studied.

So, what happens is that, along with modes whose phase propagates downstream, we always have one mode, at least we can see whose phase actually propagates upstream, and this is always there. So, when these disappear, this will still remain, so if you are at the extreme air, where there are no downstream propagating mode only upstream propagating mode, you start with that solution, and what you can do is, now you can keep increasing the frequency, you keep increasing the frequency, you come up to this point it does not go back it remains there. So, this actually shows that this quantity that this curve that you see is always there irrespective of whether those are present or not.

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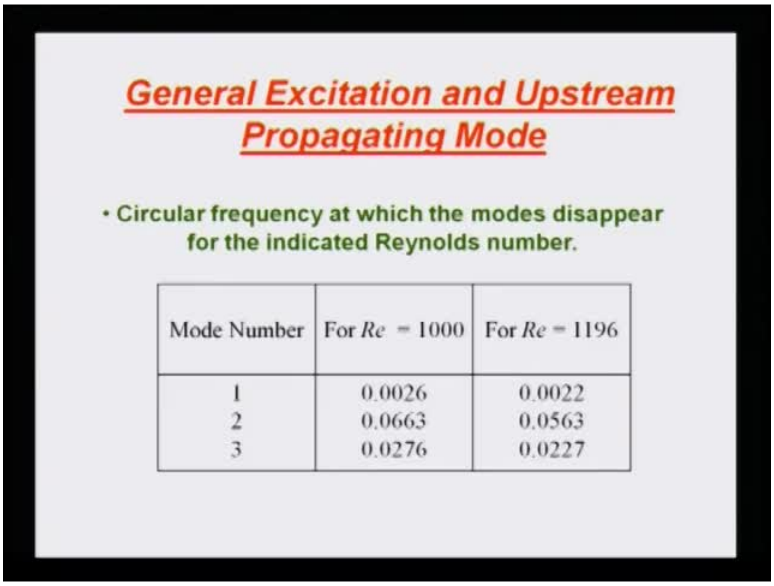

What may happen is that, these things, these modes are just abruptly disappearing, and in the previous slide, actually I indicated to you, what is really happening what are these frequencies, where this progressively one by one the mode disappears, the second mode disappear at 0.06, the third mode disappear at 0.027, and the first mode disappear at 0.0026, believe me this is the kind of value, where below which Taylor did his experiments, and when we did those calculation corresponding to Gaster result for 1196 we found those values. So, what is happening here is if your Reynolds number is increase, the frequency activate the disturbance disappear, a well reduce seven further even further, but they do all disappear, and even for this, you only have this three modes to begin with.

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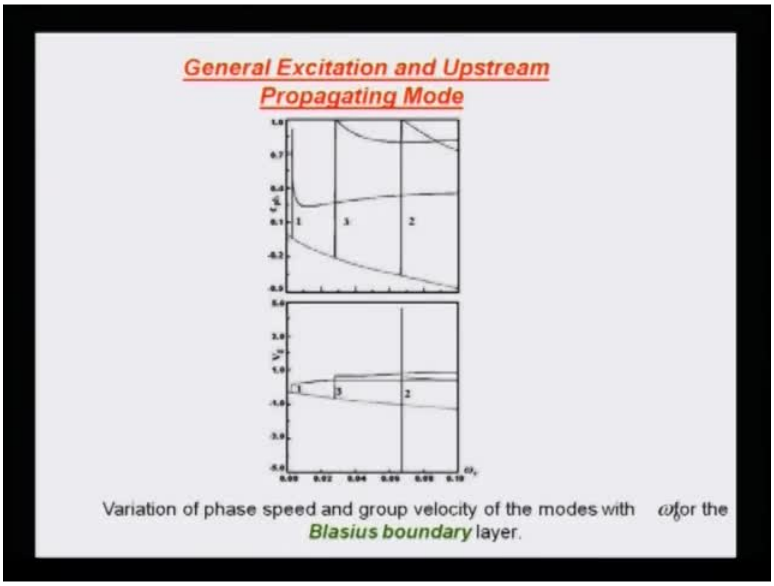

So, we have gone to this, and now he wants it to know the properties of other properties of these modes, for example, if you plot the phase speed and the group velocity versus omega, this is how it looks like. So, the second mode actually it goes there, and you see before it disappears, it takes the value of one, what it means, the phase speed is equal to u infinity.

So, this is a very interesting observation, then you notice that before disappearing, all the modes actually take a value of 1, So what happens is you can do this Eigen value analysis, and you see the movement the c approach is 1, you are virtually sure let us know later it will disappear, and it does happen, this is I slightly a sort of a failure drying it correctly, it is actually up to 1.

So, all these modes whenever C phase is equal to u infinity, those modes disappear, while these modes are always there, and you can see, they have a negative value, and you look the corresponding group velocity, and you find that group velocity is also negative for this branch, your branch. Well the upper branches they are all positive.

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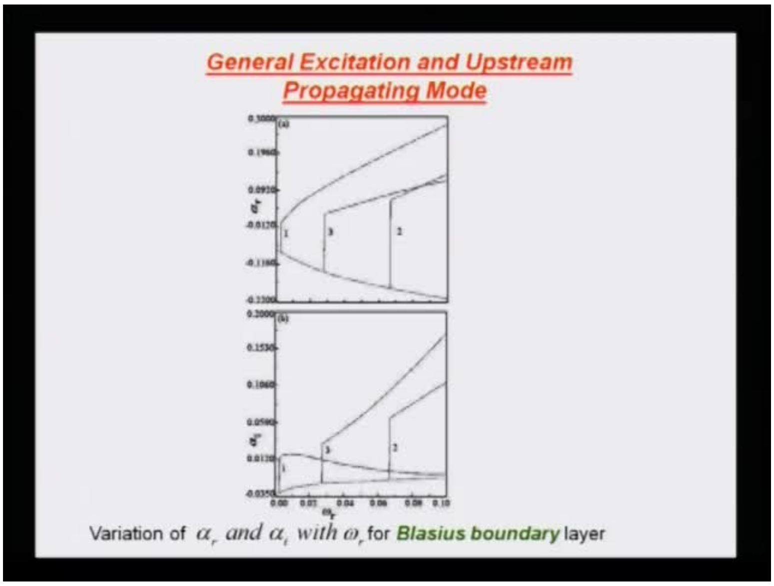

Now, if I go back and look at the alpha i plot, that should really help you understand what is happening, because the alpha i is alpha i for this upstream propagating mode is negative in a very large value compare to do.

So, one would be really worried, if one were to just look at this result in isolation thinking, that this modes are highly unstable, if you use the same yardstick, like what you do for Toll mien – Schlichting wave, and it was for that reason I told you, someone advise me to use whitener and not pronounced the existence of this movement, because you cannot explain, but unfortunately this is a legacy of trying to follow each other's work, what you actually should do is to calculate the group velocity.

So, if you calculate the group velocity, what you find that those modes in the left half plane have a value of less than 0. So, that means what, this modes are propagating really upstream, so far we have been talk about phase propagating upstream, but now we are saying the disturbance also propagates. Now, corresponding to that, if alpha is negative, then what we have you do not have a stable mode, you do not have a unstable mode, they are actually stable mode, and there stability is put in height, because alpha I value is r minus 0.035, whereas if you recall for r equal to 1000 omega naught equal to 0.1, the value of alpha i for the t s wave was something like minus 0.0027, something minus 0.007, whereas this is minus 0.035, it is almost like a huge 20 times more but that these are damped.

So, what happened is that, we figure this out in mid-nineties, and we are today happy that we could actually explain those modes in the left half plane, and we also found that is why Blasius boundary layer, these modes are not very harmful that (( )). So, nothing greatly different happens, and that is why probably people, where not alerted if it was something like this.

This was unstable mode some, but you would have also seen it in some experiment was not see, however I must tell you one thing though that this unstable modes this upstream propagating modes, that we found this is for Blasius boundary layer 0 pressure gradient. The scenario may not remain the same, if you look at flow with pressure gradient, spatially when you look at adverse spatial gradient flow, there you may find your upstream propagating modes can become unstable.

It can become unstable, in fact one of mysterious thinking about the whole topic of separation is that, separation happens because of this phenomena, that you have upstream propagating unstable modes, this has to be systematically done, we have not looked at it seriously, but let me tell you that if you focus our attention on two flows with different pressure gradient, we may actually find out more interesting properties have upstream propagating modes.

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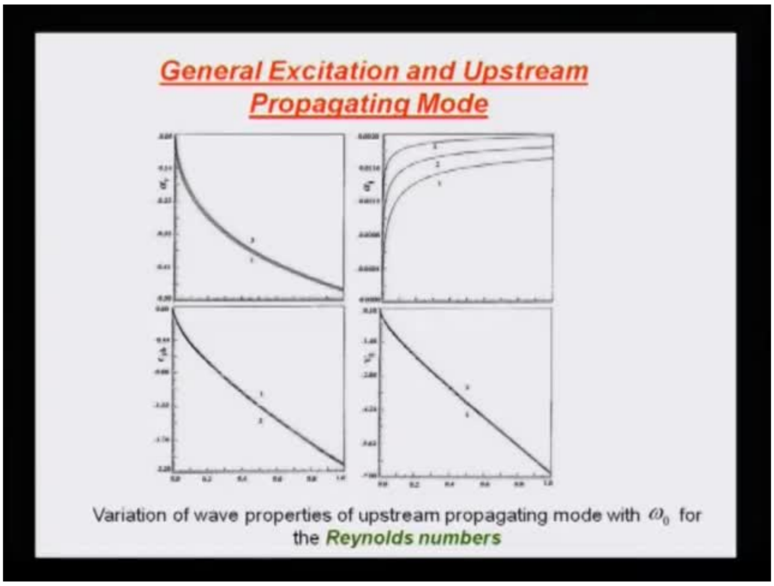

Now, hear what we are showing you basically, these four properties that we talked about alpha r, alpha i, c phase on the group velocity for three different Reynolds number that has been shown, I think one has curve corresponds to the one corresponds to Reynolds number 400, then middle one is for 1000, this is 4000, like the one that you saw the local solution also we plotted for this 3 Reynolds number, what you notice that, when you look at alpha r, when you look at alpha are there kind of banded together.

So, that is a very interesting observation, that irrespective of the spread of Reynolds number, you would always have a upstream propagating modes, and the wave number is kind of banded together. This x axis is omega naught and as omega naught increases, you can see the value of alpha i is decreased, what does decrease in alpha, I mean, alpha i means you are going to create waves of larger and larger wavelength.

So, what happens is this is the property of your upstream propagating mode, we are plotted up to omega naught equal to 1, and it kind of affects the boundary layer almost similarly, only thing is the wavelength of this disturbances keep in increasing

However, if you look at alpha I that is somewhat revealing, for the same omega naught you can see for higher Reynolds number alpha i is smaller, meaning what, these are all stable, but high Reynolds number means less stable. So, if you are looking at this upstream propagating modes, at lower Reynolds number, you will not see their effect very much, because the damping rate is very high, but as you go to the later part of the boundary layer, where Reynolds number is increasing there, this damping rate comes down, and you would be able to see this, and this might somewhat tell you of what kleban off might have been seeing, if you take a wind tunnel of a very long stretch.

So, you are going to simulate every large Re, towards the downstream end, and there what you would find that upstream propagating modes will have will be unstable; see basically what are this upstream propagating modes, what how do they come about, they come about because of thosea periodic convecting vortices that we consider in the formulation.

If we do not have that, we do not get this upstream propagating mode; you must now realize that those upstream propagating modes have their cause in this convicting vortices. So, if I do not have those convicting vortices, I would not have a upstream propagating mode.

So, if I am looking at a tunnel with a dirty flow, then those vortices discreet, vortices going over the plate would give rise to some of these modes, and the you can actually see the various values of phase, speed and group velocity will tell you there is indeed upstream propagating.

So, interestingly enough higher frequencies, when you go to the extreme right that corresponds to omega naught equal to 1, and there the group velocity actually reaches close to a value of minus 7. So, you can see, that means, what this disturbances are going at 7 times the u infinity, so they go rather rapidly, very rapidly, whereas in the lower frequency part, they do go at a smaller speed, see in the top scale I think probably, you are not being able to see it clearly this is minus 0.1 and this is about 7, minus 7.

So, they do actually go slowly, they have very lower decay rate, you can see these have a pretty much close to 0 value. So, upstream propagating modes would be there, if you are looking at thinking of the flow in terms of a first excitation, this convicting vortices would give rise to that kind of a, sort of a footprint which is what we are seeing here the low frequency part would have persistent long short wavelength which as you go along this side, well there growth rate aware the decay rate goes up, but there stretch becomes.

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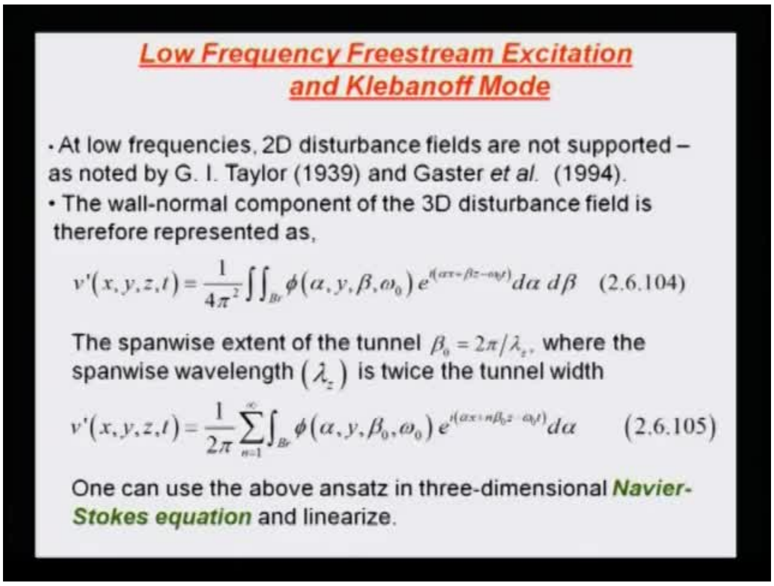

See you should be able to mentally construct what you see in an experiment or in a direct numerical simulation with the help of information like this; so, now we still have been answer that question what happens to those cases, when none of the downstream propagating modes exist, and the whole boundary layer heaps up and down, can we explain (( )), so far what we have done we have shown you results for 2-dimensional disturbance field, but whose phase that the flow field cannot be 3-dimensional.

So, you see asking some pretty innocent question some time helps, so I had a B tech student a very bright and philosophical outlook, so I told Vivek, that Vivek, let us investigate figure out, that if it is indeed the case that this two gentlemen in 1939 and 1994 come to the same observation, there must be something to it, and we cannot just simply through r power r 1, say that growth theory exists, we have to do something about it.

So, we said look, we will work it out almost like a signal problem, but now we will consider the disturbance field as 3 d, and then work out what happens. Now, basically whenever I do some experiment in a wind tunnel, it has side walls; so, the distance between the 2 side wall, can be at most two modes, so that will be half the wavelength.

So, I can actually find out the tunnel width multiplied by 2, and I get a Span wise wave number, wavelength I will call it lambda z and the corresponding wave number I am calling it as beta naught. So, what we do is that, then I could write down, let say the v component of velocity as the combination linear combination of all this modes, because that is the maxim wavelength I can do, I can also created super harmonics, and that is what we have done here.

We have obtained beta naught, and then, we said look there should be a Span wise harmonic component, which will write as n beta naught, and n should be summed up from 1 to infinity, because we are proposing the problem as if this whole thing is repeating infinite times outside the tunnel, that this is the basis of all spectral calculations people do not spell it out, but whenever you do a spectral calculations, you are in a sense outside the computing domain, think of periodic x i extension of the thing, that you are doing inside. This is true for all computations, that is why you may be seeing very interesting aspect of flow computations, that suppose have a disturbance field which is localized, now if I compute that flow in domain like this, I get one result, if I do it over, this I get another result, but disturbance field is located in the mirror, what is the problem the problem is one of periodic extension.

So, if I take a smaller domain, then I am saying I have a p k, but outside I have a another, and I have a infinite such things and all side. Now, when increase the domain, now those periodicity has changed, and there effect of this outside once, and the interior on the flow comes down, is not it, so that is what you seen people always try to tell you a that by saying that, we have done this consisting check, we increase the number of points, you increase that domain, and we got the same resultsthey are telling half of this story, you probably should not get it like that, you should not if you get it, then the your numerical method is least with huge deception, and that so Ido not see of this, but if you are doing in a good accurate calculation to increase the domain size, believe me a result should be different, and this is something people do not say, but let us carry out with that answers that whatever we observe it actually as it is infinite periodic extension, and that is why we have this, so you see what happen, it is a 3-d problem, but we have cheated the problem, what we have done, we have not integrated over beta, we have summed over the harmonics.

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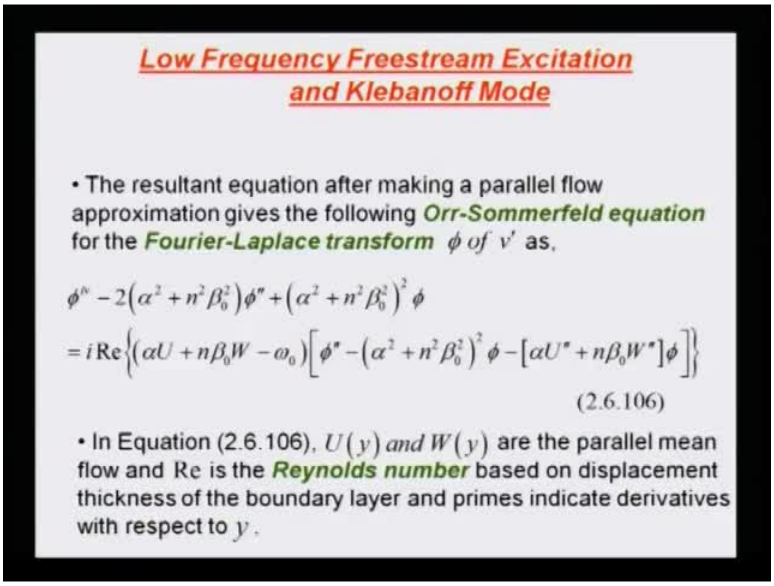

So, what happens is, we can use this, and plug it in to our 3-d disturbance equation, 3-d Orr-Somerfeld equation would be obtain, and this is how you are going to get, I would ask all of you to derive this, and give it as a submission, so that you also have practice how to get this, so what are you done, you go through a same exercise instead of saying the disturbance field is 2-dimensional, and your term what a 3-dimensional disturbance field, and then your mean flow has let say 2 components capital U and capital W, so you are talking about a formulation here, where the mean flow also good with 3-dimensional, only difference here is that, instead of the Span wise wave number beta square, we have taken those discreet, Span wise wave numbers which are given by n square beta naught square.

So, this is what we have done, so you should be able to see that, and however, I must tell you what a clear, and Gaster did both of them, both this groups still I was alone, and Gaster had it, and co authors in this both this experiments, what was run they took a two dimensional flow.

So, then, this formulation itself will work, but only we have to justsimply knock of that capital w y 1, and here, once you get Re, that is the reynolds number based on displacement thickness, and primes indicate derivative with respect to non-dimensional, y not un utilization is again done with respect to the displacement thickness.

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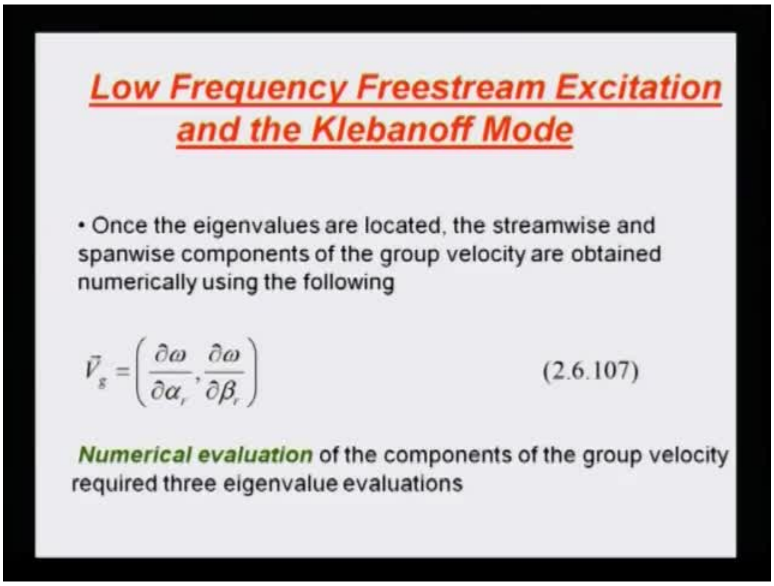

Now, what happens is, we can go through this Eigen value analysis, what we would get, if I fix a omega naught, I will get, I will also fix some beta naught, and I will get the corresponding alpha.

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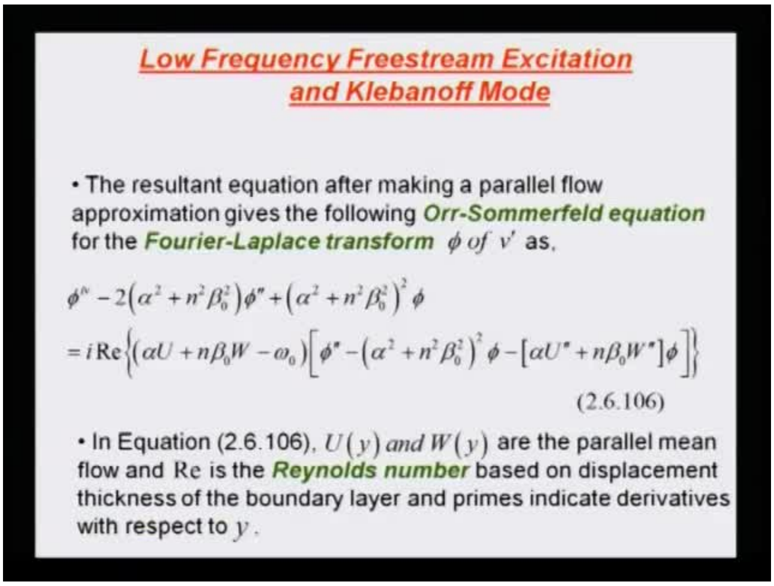

So, it is basically a, the reducing our work, by assuming the signal problem, we are just simply looking at omega naught, by assuming Span wise periodicity, we are talking about the fixing the beta naught for different values of m, we are going to get different thing.

So, we are basically talking about Orr-Somerfeld equation for the Fourier Laplace transform phi, which is the amplitude of the wall-normal components of the velocity, v prime, this prime is not derivative, this is a prime means perturbation quantity, so note that prime and the prime is here of different kinds, v prime is indicative of the wall-normal component of the velocity, and this is the generic Orr-Somerfeld equation, for 3-dimensional mean flow and 3-dimensional disturbance field, 3-dimensional mean flow is because you have U of Y and W of Y, you can split it into stream wise in a cross flow component and disturbance field is 3-dimensional means what.

We have the amplitude as a function of y x variation is given in terms of alpha, and the z variation is given in terms of beta in this case, because we are talking about an experiment in a wind tunnel. So, we have done a periodicity in this Span wise direction and beta naught, I know the wind tunnel thickness, I can fix beta naught, and I, what I could do is, I can solve this equation for different values of n.

So, what you are basically doing, it is a 3-dimensional problem, but you are looking at each n beta naught at a time, so I can do beta naught, 2 beta naught, 3 beta naught, I can take all those harmonic and calculate the Eigen value.

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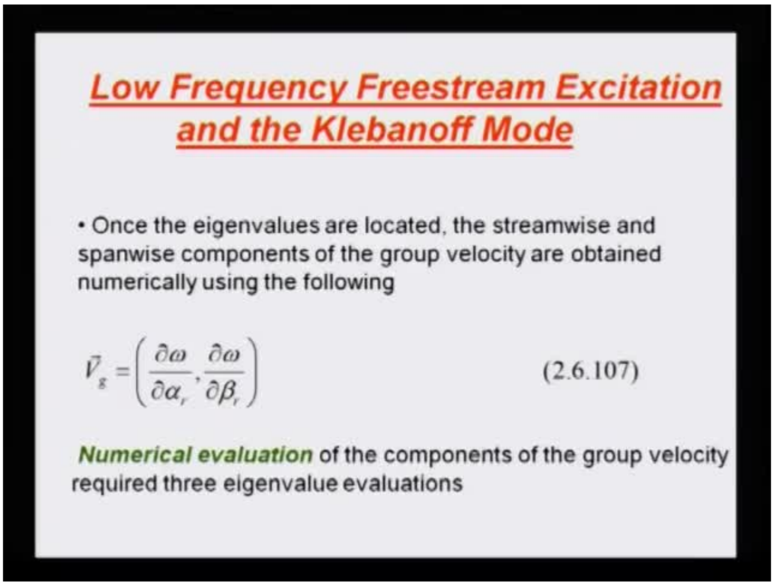

So, whatever we have done, whatever we have learnt is going to work here directly, because we opposite as a 2-d problem, now because we are looking for complex alpha. However, you can also see that, this alpha that we are going to get is for the 3-dimensional field, and your beta is also not a fix number, it has discreet values, but the omega that we are getting omega is kind of real, because we are talking about a signal problem.

So, I can calculate this quantity, what I could do is, I could change omega naught to omega naught plus d omega. So, the numerator I can get it, and I can correspondingly find out what is happening to my delta alpha r and delta beta r, delta alpha r would come about form your Eigen valuation, what about delta beta r, when we have those discreet values of beta naught to beta naught, 3 beta naught, so that is the delta b.

So, we can actually numerically evaluate is v g, and that is what is done, and this expression that you seeing their numerical evaluation of this group velocity components are in this two components in x and z direction, first one is the disturbance propagation in the stream wise direction and this is the cross flow direction.

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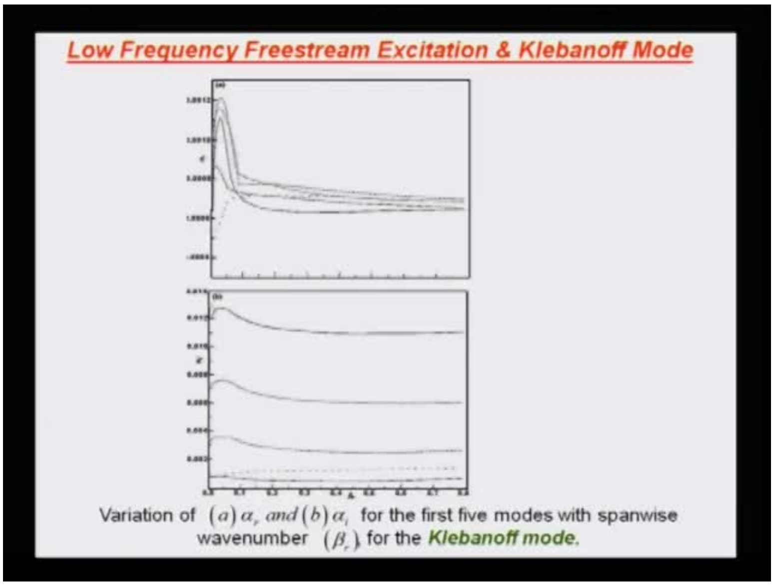

So, you can calculate this, so you basically needs 3 Eigen value evaluations, so I can do it omega naught plus d omega naught plus 2 d omega and I can calculate this. So, we can make use of that, and evaluate this group velocity, and this is what you get as the result the top figure, we have plotted alpha or versus beta r, what is alpha r, the real part of this stream wise wave number, beta r is the Span wise wave number, but you see what happened, if my beta naught is a very small value, the n beta naught will be, it is like a numerical evaluation. So, these quick points have been obtained here.

So, this is alpha versus beta r, and this is alpha I versus beta r plot, and this is now we get, and what we have, the basically done is, we have obtained say 5 such modes, that is what we are showing here 5, such curves, and how the value vary with beta r is shown here by for this 5 waves. So, now these are, what these are here 11 of mode y, because we have put in your omega naught, some values which are less than that table I showed.

So, may be less than 0.002, and then I calculated this 3-dimensional modes, first 5 modes this is the real part of this Span wise wave number, this is the imaginary part, and what you want is that, imaginary part is we are saying, they are all positive will see that, they group velocity is are also positive and then what happens.

So, these are all damp modes, but you can see those bottom 2 curves are quite small in magnitude that alpha i; so, there effect should be seen well the top three curves have a large values of alpha r.

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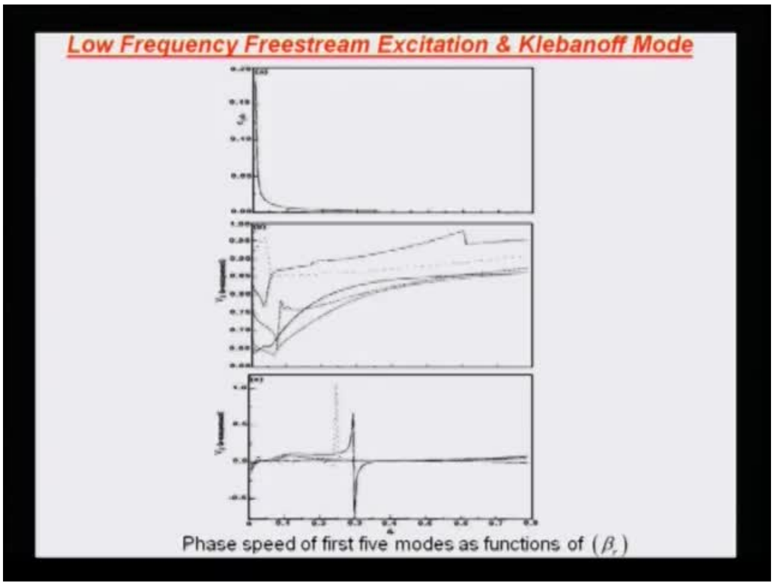

So, they may not really have much stream in them to take them in far, because they will decay, when the last two will certainly do that, and what we have here is the phase speed plotted here as a function of beta r on the top frame, and this is here v j x and v j z; v j x is nothing but del omega del alpha r and v j z is del omega del beta r.

Now, what you notice is that, burring a very small value of beta r it is virtually 0; so, the phase virtually does not move very much, so it would appear as if the whole things, if I look at that tunnel the phase is not moving. So, it would appear as a stationary, that is what I was giving us the impression of a breathing mode, we are having a 3-dimensional flow field, but c phase is virtually 0. So, we are seeing as if it is staying in a same place but which time, because it is changing with omega naught, we are fix that omega, so the whole boundary layer is heaving with that omega naught, that is what the other thing is you look at the x component for this 5 modes, they are plotted here, and they do have some kind of a numerical problem in there evaluation, and it does show that the v z x component the scale is between 0.6 and 1, so 0.6 and 1, I must confess that this was done in mid night is may be somebody should cross check them again, and probably obtain them much more higher accuracy, but this is what we obtain that, these are all downstream propagating modes; so, downstream propagating modes, but the phase is very close to 0, that is what we saw in the top frame, the velocity with which it does is roughly, two thirds to u infinity range it is bracketed in that, but look at the BGZ component is somewhat very strange, for most of the bit beta r component this lies there; so, it is like 0 component of this Span wise direction, that is what people have also complaint or observed.

What they did observe is that, you take this flow Blasius boundary layer excited at a small frequency, then you see as if the whole boundary layer is, if I had a BGZ component, it would have shown there is a oblique movement, but it did not and that is also brought out a rather clearly.

There is some beta of cross over in some crucial values at beta r, and this side it is slightly positive, and this side it slightly negative, we still not clearly understand what this discontinuities are may be someone will in near future, we will look at it much more carefully.

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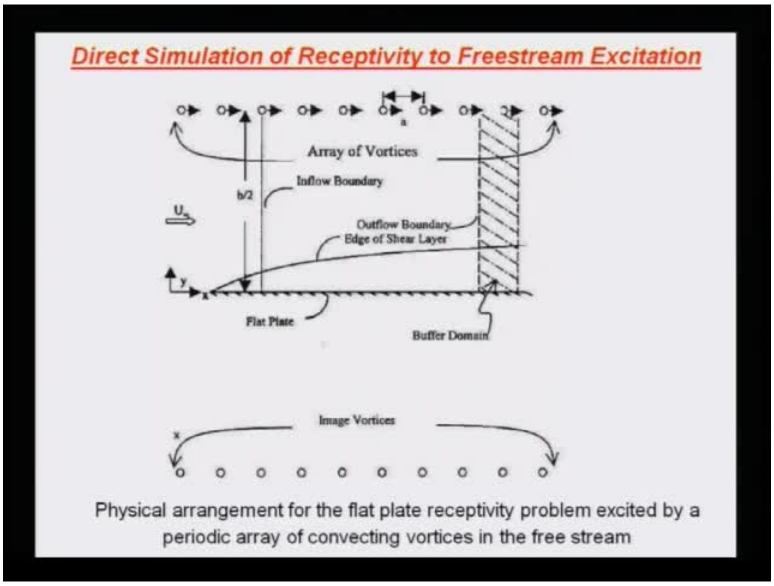

But this was a matter of great satisfaction for us that, we did achieve the following things so far in this course. We started off with what everybody does instability theory; we have now come to a stage, where you can talk about the receptivity to wall excitation, that is what you did.

We talked about all this local solutions, we sorted out the issue of any arbitrary disturbance can be explained in terms of a receptivity theory, that we have look at, we also looked at what happens, when we have a free stream excitation.

Now, we have also answer the question of, do, we have off stream propagating modes, yes, we do, we have just now talked about what happens if the boundary layer is excited at a very low frequency. A disturbance field is very spatial, it is not 2-dimensional, you get all the 3-dimensional solutions, 2 dimension of this Eigen modes disappear; however, despite the response field being 3-dimensional, it also is a very interesting attribute.

It looks like 2-dimensional, because BGZ is 0, so these are all kinds of very interesting confluence of occurrence, that tells you what people have noted, and it was very satisfying moment for anyone to be able to explain things, that one does see in experiment, and also I did not tell you another thing is about the alpha r.

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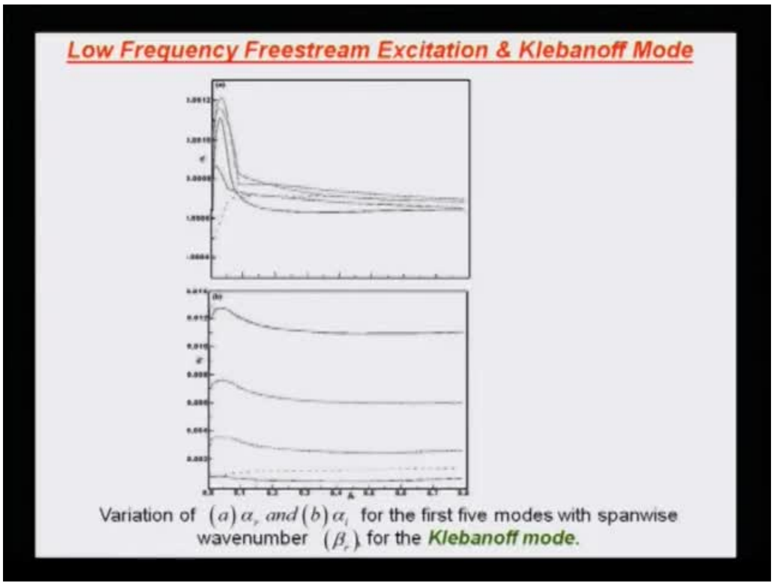

Well, let me highlight that aspect, because that is important, none of you ask me, but if you look at this top figure this alpha r values, that we are plotted, the lower stick it has 0.30 is 4.

The top of this 0.0012, so what we are saying is this is 0.304, and this is 0.0012, so if I look at a behavior somewhere here, what does it mean alpha r is 0.30 something.

What is the wavelength, what is our length scalier, length scale is in terms of displacement thickness, so if I have a alpha r, let say 0.001 also 0.001.

So, what is the corresponding wavelength 2 pi by this, so 2 pi by 2 pi into 10 to the power 3, so how much is it some 6000 delta star, how many tunnels in the world have made, where your Tess section is 600 delta 6000 delta star.

So, if I want to see this long waves, I have to have also a facility that long, unfortunately there are no such facilities, what you do is you always have a smaller stretch of (( )) tunnel and you always see a part of the wave.

Although the phase is straining, but if you are looking at a small layer stretch, it would appear the whole thing is going up and down together, but there is some variation in the elevation, because the part of your wave, but that wave is of thousands of delta star, whose experiments I have not done, so may be somebody listen to it can construct one such huge setup and look for this kleban off mode, that would be a very satisfying moment.

What is interesting, that we now cans up is also that in fluid mechanics, do not take any standard assumptions, standard assumptions always fall through the standard gaps; you will not be able to explain everything. Here, we are seeing that for very low frequency, excitation 2-dimensionality is not important, 3-dimensionality is so many a times, when you would be doing research in a life, and you would be reporting a result, you will be hitting this wall very often. They will say so and so has already written that the flow is 2-dimensional for Reynolds number up to 250, somebody will say, no, it is not 250, it is 280.

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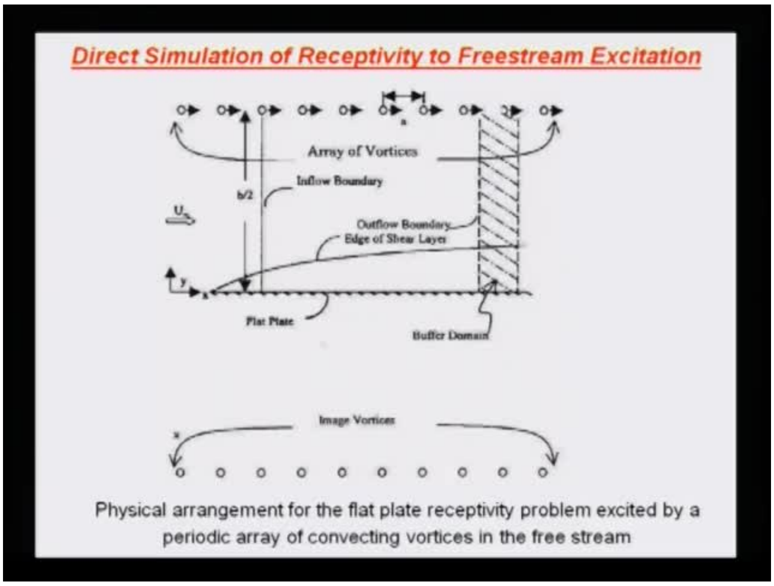

But keep your mind open, and you can see for the same flow depends on the frequency, whether you are going to get a 2D excitation or a 3D excitation so be aware, and alive to the possibilities, and you will all was come out to the very interesting results.

Now, let me talk about direct stimulation, now what does direct simulation mean, direct simulation means the following it, we are going to solve the navier stokes equation, we will solve the navier stokes equation and we aresolving a very interesting problem. See so far we have talked about as if there was a single vortex going in the free stream, but suppose you have a free stream turbulence, you do not have a single vortex, but you will have a ensemble of vortex.

So, they will go on, so here we wanted to study a prototypical case of one such thing to make our understanding complete, we look at a very standard model, the standard model is that, we will have a train of vortices, not one a large number of them and let say gap between them is fixed. The height at which they go is also fixed, so this is a basically our prototypical thought experiment, that you are conductive, and then, the boundary layer is developing, and this vortices are going very far above what will happen.

To satisfy the 0 normal velocity, I should have a image system; so, this is your physical vortices, these are the corresponding image vortices now, because we are putting those vortices at a fixed interval. There is a some kind of a Span wise scale, that is impose now on the problem, earlier we had a single vortex did not have any Span wise length scale here, we have a Span wise length scale that is determined the gap, but they are also moving at a constant speed, so if I have a Span wise wave length, and I have a constant speed, what am I doing, I am introducing a time scale, also I will stop here, we will start it from here in the next class.