Instability and Transition of Fluid Flows

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Lecture No. # 17

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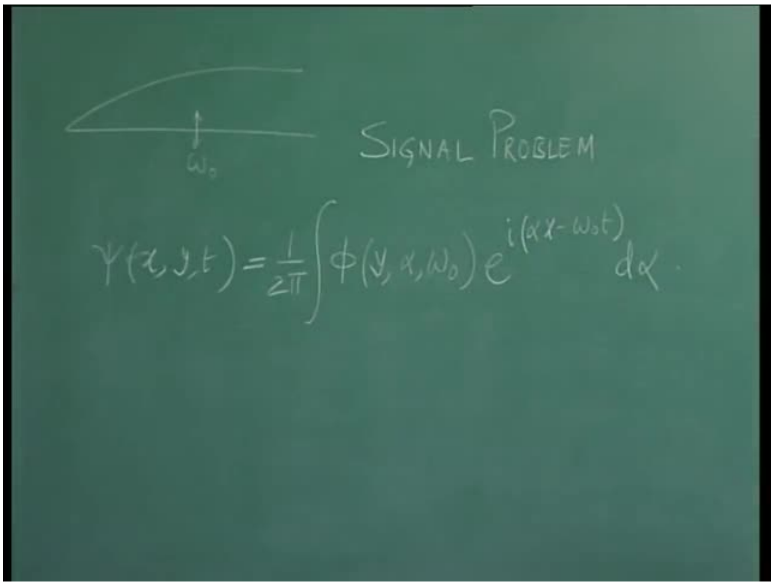

Yesterday, we concluded our discussion on the signal problem, and signal problem means, that if we have, let us say, a shear layer forming over, let say (( )) like flat plate, and we excite the system at a fixed frequency omega naught, then we wrote psi as a function of x, y, and t, in terms of 1 over 2 pi. I wrote that… are known in terms of Fourier transform - Fourier Laplace transform, we do that. This is what is assumed, that the system response at the frequency omega naught; this is what we are talking about as the signal problem, right?

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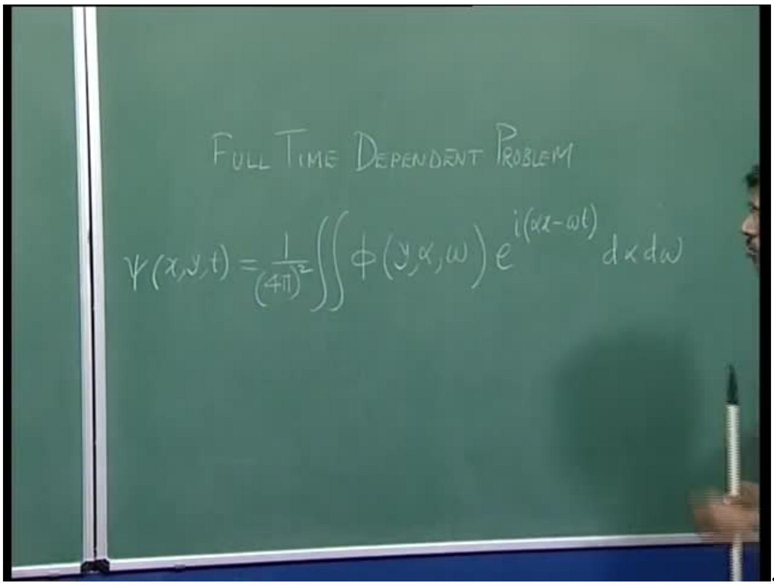

Now, in contrast to this, we could do the full problem. Why we are doing it, we will immediately explain. The full problem, what would we be doing? Well instead of writing Full Problem, let us say, a Full Time Dependent Problem.

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Now, here, we will not make this assumption. So see the signal problem assumption comes here in identifying the response, to be also at the frequency of omega naught. So this is the way the link is. If I have a vibrator vibrating at omega naught, response is also omega naught. So in this, any spatio-temporal problem, we omit that part of the temporal dependence, we fix at omega naught t frequency; whereas, in the full time dependent problem, we do not make such an assumption. So here, what we are going to write, we are going to write, in terms of the double Fourier transform - one for the space; one for the time; and then, you notice that phi is now a function of y alpha and omega, and then, we are writing the phase part is like this. I am performing the integral over d alpha and d omega.

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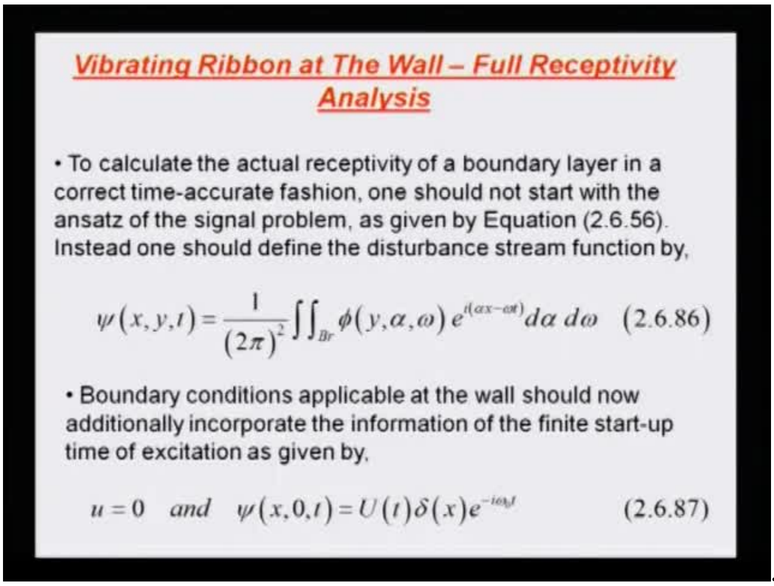

So why do we do this? That is apparent when you look at the boundary condition that we have. In the boundary condition what we have, of course, once again, you take the u velocity as 0, and psi is given like this. What does it mean? Again, we have a very localized excitation. So we are precisely starting our x-axis from the location of the exciter itself; that is why we are writing delta x. So origin is at the exciter.

However, look at this function; what is U of t? U of t is the Heaviside function. Now what does it mean? That the signal… I mean the excitation starts at t equal to 0. Before t equal to 0…. So we are also talking about a origin in time; that is given by this U of t - the Heaviside function, and note, that we are still exciting the system at frequency omega naught, and we have done this when we are discussing Tutorial on Fourier Laplace transform.

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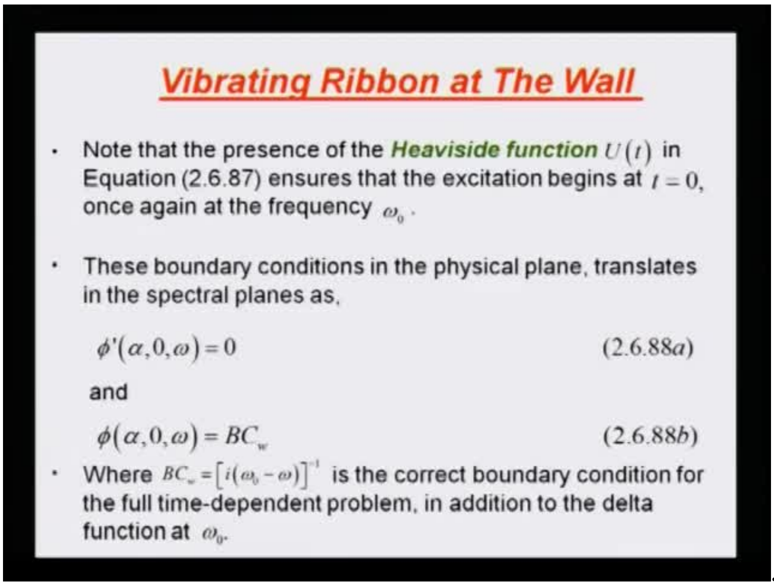

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So that is what we are going to do, is we are going to represent the unknown now in terms of two independent variables - alpha and omega, in this spectral plane. So this is essentially the idea. Now, that is what we said, that the presence of the Heaviside function U of t ensures that the excitation truly begins at t equal to 0, at the frequency omega naught

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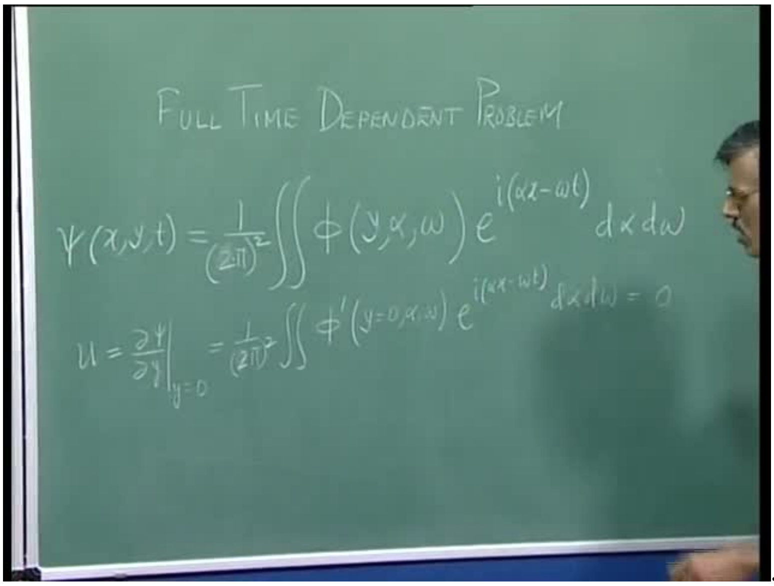

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Now what we notice that u equal to 0 will give us this; that you can see, because the u is a del psi del y and this we are doing it at the wall. So y is equal to 0 and this we are going to get from here 1 over 4 pi square; this is 2 pi whole squared, 2 pi whole squared, and one each coming from alpha 1 from and this. So this will be phi. Well, we are differentiating with respect to y. So the y dependence only comes through the definition of phi. So this will become nothing but phi prime and y, of course, is 0 and alpha omega and e to the power… again we have alpha x minus omega t and d alpha and d omega and if this u is equal to 0, then we must have this equal to 0. So that is what this 88a represents.

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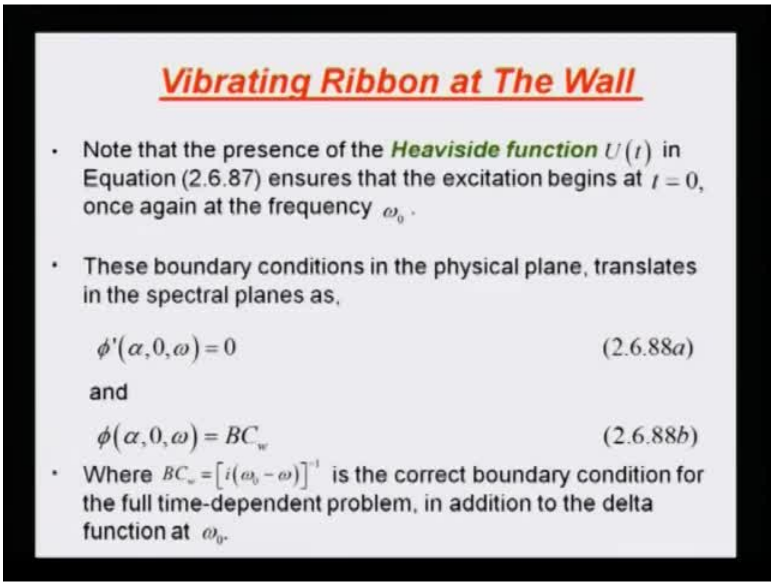

Now what about this condition? This condition, if you recall we did that, we did that when we talked about the Fourier Laplace transform of harmonic excitation, started at t equal to 0.

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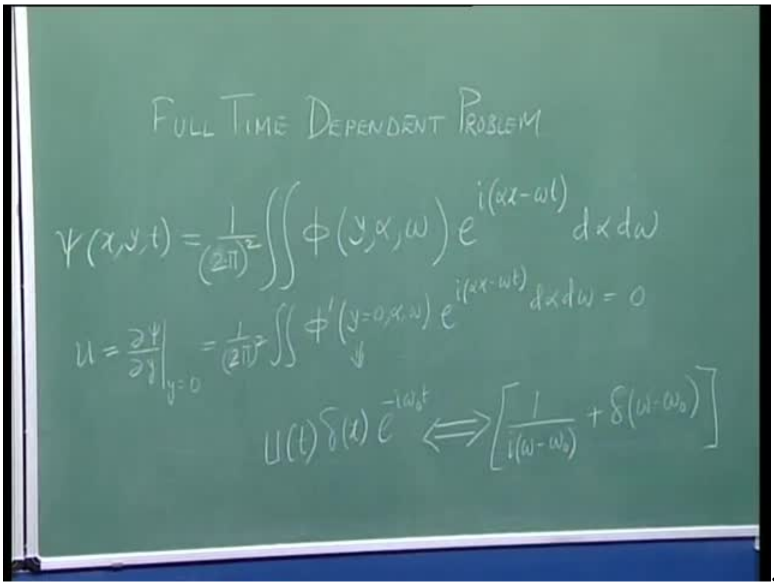

We get two parts, if you recall what we had obtained the…. If this is the original, this is our original, right? That is what we are talking about. What is its Fourier transform? Its Fourier transform, if you recall was 1 upon i omega minus omega naught plus a factor, a delta function, at omega minus omega naught. So this is what we had obtained. So what happens in the signal problem? What do you do? You do this. You go this path, but this is there, because of finite startup we can see. So you realize that how important it is for one to understand what is going on here, because the finite start of time, you are actually invoking the whole range of omega. So making that signal problem assumption, is an assumption, right? While we should have this.

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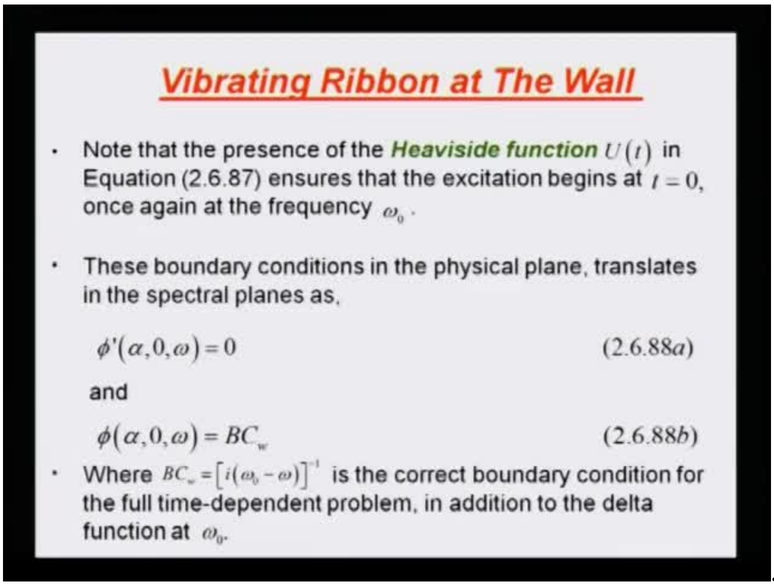

That is what we say that, that phi at y equal to 0, for any combination of alpha omega, should be some kind of a function, which I call it as a boundary condition at the wall, and which is, in this case, is 1 upon omega i minus omega naught minus. So there is a minus sign here, and then, we have to had that the delta function there. So, this is the interesting part. Why it is interesting part? Because you see, we are looking at stability and we have the familiar dispersion relation. So we know that in a dynamical system, that a priori, we do not know what is the, what are the Eigen values, right?

So, it may so happen that although we are looking at omega naught, we can also trigger the temporal dependence, which is different than the harmonic input. In the excitation, we should have the frequency dependence, which is other than omega naught, and that part is given by this. Now, for each of those omegas, we have a corresponding alpha; that is the dispersion relation, right? For very omega, I have a corresponding alpha. So what happens is, due to this transient start up, I could set up many, many Eigen value combinations other than what we have talked about in the signal problem so far, righ?

So this something that we must realize, that this adds not just simple variety to the problem, but it changes a problem qualitatively; this is something we must understand; without this, I suppose things are going to be quite different, as we will see. I will show you some results, not immediately, but later on when will revisit the problem. In fact, in anticipation of things to come in future lectures, let me tell you, when we solve this problem, we could see that what is called as a spatio-temporal wave front. See this usual signal problem gives rise to Tollmien-Schlichting waves, but when we look at the full problem, we will see that ahead of this Tollmien-Schlichting wave, there is a very localized (( )) packet that moves. In fact, to give you an analogy, this spatio-temporal growing wave front, looks almost like your tsunami. If you have noticed, you have read those accounts of the experiences people narrated during tsunami, that you get some episodes where the sea had receded, and after that a huge wave came, and everything was destroyed.

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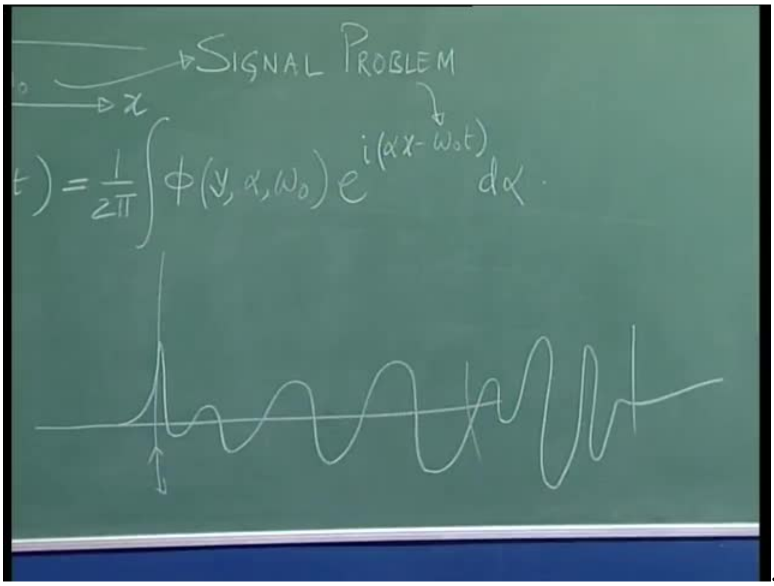

So this is something like this. Suppose this what we saw in yesterday's class, that if I create an excitation here, then it will go like this. Now this is a growing wave front. When you do the full problem, you are going see that in front there would be a massive packet here and this peaks. Ah, well, my drawing is imperfect. If this is order 1, this could be order of 1000. So you are going to get only one or two massive peaks coming in and that is precisely what people have noticed. It is one of the wave, if you are going around in the beaches, when you see, all of a sudden, sea is receding. So that is this part - the trough. So the water goes away, recedes from the beach, that is followed by this massive peak, and this massive peak is actually the wall of water that actually hits.

Well, this is something that are comes from this part. That is what I said, in anticipation of things to come, let us keep ourselves alive to this issue. If we do not worry about this…

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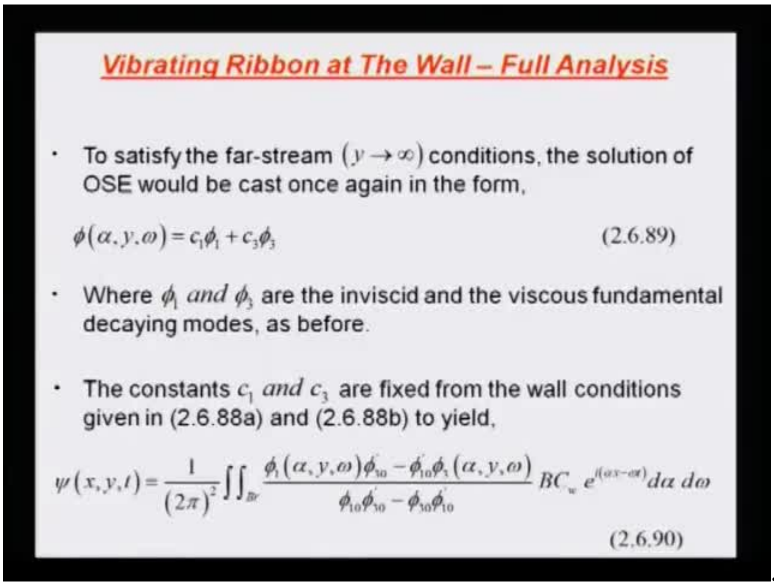

And we now would like to explain the problem in terms of those two modes, which fundamental modes, which decay with height, will be given by phi 1 and phi 3. It is the same thing. What you do, is now you fix c 1 and c 3 in terms of the new boundary conditions that we have proposed at y equal to 0, and what do you find? That it would require solving those couple of solutions for u and v condition, and this u and v condition freezes c 1 and c 3, and you can see c 1 comes from here, phi 3 prime 0 divided by this characteristic determinant.

One thing I did tell you, we did notice that this is nothing but our compound metric variable 1, y 1. So basically, what we find c 1 is nothing but phi 3 prime 0 by y 1, and c 3, we find minus phi 1 prime divided by y 1. And this is what we get. And that boundary condition, whatever we have written it down here, this path, is here. So, you can now see additionally, that omega dependence is there through this.

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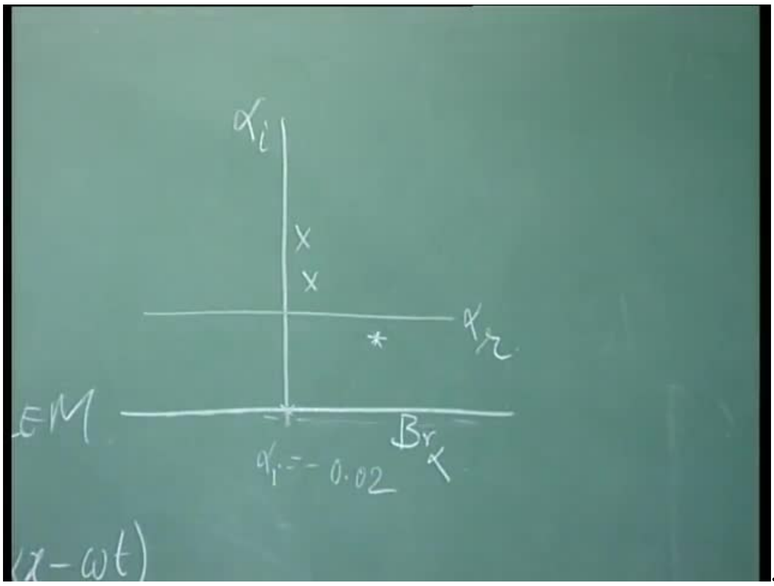

Now, if we are doing this full time dependent problem, we realize that we will have to look at the alpha plane. So give this is alpha, r alpha plane, we will have to design a Bromwich contour. And what is the way that we decided that we should have Bromwich contour? That it should be either along the real axis or parallel to real axis, because we want to use the power of Fourier transform, and use that. So what we usually do is draw a line somewhat there, why? Because we know, there are modes which go downstream and they are unstable. So if they are unstable, that means what? I am going downstream where would those Eigen values be in this plane? That will be somewhere in the fourth quadrant, right? Alpha r is positive, alpha is negative; that is what we saw. So this Eigen values if that there, those relate to your unstable t s waves.

Now, we discussed also, that if we are draw this as the Bromwich contour, so this is my say, the Bromwich contour in the alpha plane, anything above corresponds to downstream propagation, anything below corresponds to upstream propagation. That is why the choice of the Bromwich contour location has to be decided upon by yourself. They are no, some, I mean, you cannot just simply say for all problems it is like this. What you have to do, is you will have to do a little bit of grid search technique, find out all possible Eigen values, refine it by using Newton-Raphson method. Once you have got in all the Eigen values, let say, in the last case when we looked at Reynolds number of 1000 and omega naught equal 0.1 we found three such Eigen values, right? That is what we found.

Now we need to really put our Bromwich contour below all this three. So what we found, basically it is kind of our experience. The rule of thumb, that this point could be something like your alpha i equal to minus 0.02. So basic idea is what?

What ever maybe these Eigen values, while you are performing the Bromwich contour integral, at each and every point the effect of all those Eigen values are there. So this is something you understand, that how different this receptivity analysis is from Eigen value analysis, because while you are performing the Bromwich contour integral along that b r alpha that you have written, we are simultaneously getting the effect of all Eigen values, including even also the essential singularity; that is what we explained in the last two lectures - that even if I were to close this up, the contribution that comes up, supports those delta functions, right? We did see that. So this is a very elegant way of doing it, that once you perform the Bromwich contour integral, you get contributions of everything.

So you are not making the so-called normal mode analysis. What is the normal mode analysis? You look at one Eigen value at a time, but this is a much more general methodology that we are discussing.

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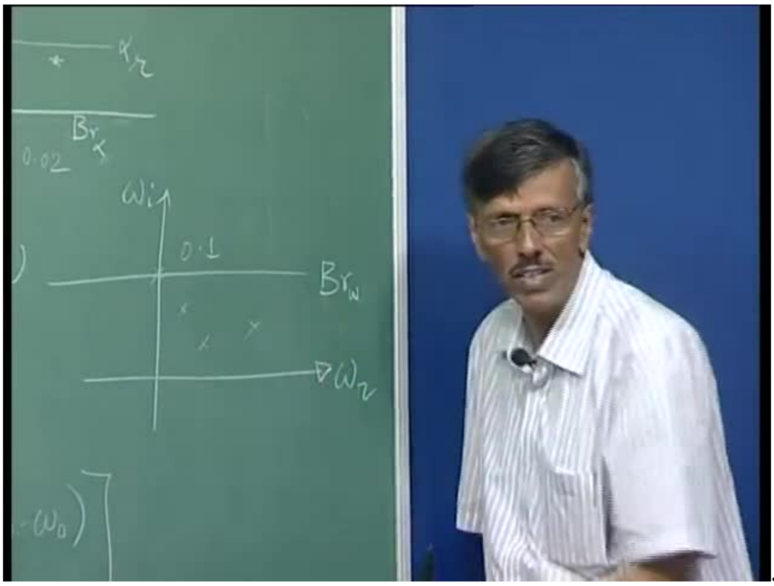

So in the signal problem, we were happy with Br alpha alone; now in the full problem, what we need to do is, we need to also worry about the omega plane also. How do you that find that omega? Now where would we position the Bromwich contour in the omega plane? Think of what we just now talked about. What we talked about, that we will have to push it in such a way that all the Eigen values remain above, because these corresponds to downstream propagation, but look at the sign here; there is already a minus sign. So here, in this case, if I put a Bromwich contour, anything above it corresponds to what happened before t minus, and if the Eigen values are below that Bromwich contour, they will correspond to t positive.

So what we would essentially do, we will choose a Bromwich contour in the omega plane, that is going to be above this; what does it mean? We are positioning in such a way that the all Eigen values would be below it. Why cannot we have any Eigen value above it? That would be disaster; physically, you understand, why? Because that would correspond to… that I am starting an experiment now, it has started effecting negative t. I have not even started, things has started happening in anticipation of what we are doing. This is what we called as a causality principle. So the causality principle demands that a Bromwich contour should be above all possible Eigen values, right? So this is something that we do. So, basically then, our experience suggests that you put it some value, have 0., well may be 0.1 or something, that should be adequate.

However, I must admit that there are no rigorous of theoretical of work that has been done in this particular field, to really show some necessary and sufficient conditions for identifying from this contour. Some mathematicians may like to adopt such a problem, and spend some time, and do something, but we have now understood the scope of the full time dependent problem vis-à-vis the signal problem, and we now also understand the power and elegance of the receptivity analysis over Eigen value analysis, right? So this is something that we must keep in mind when we go ahead.

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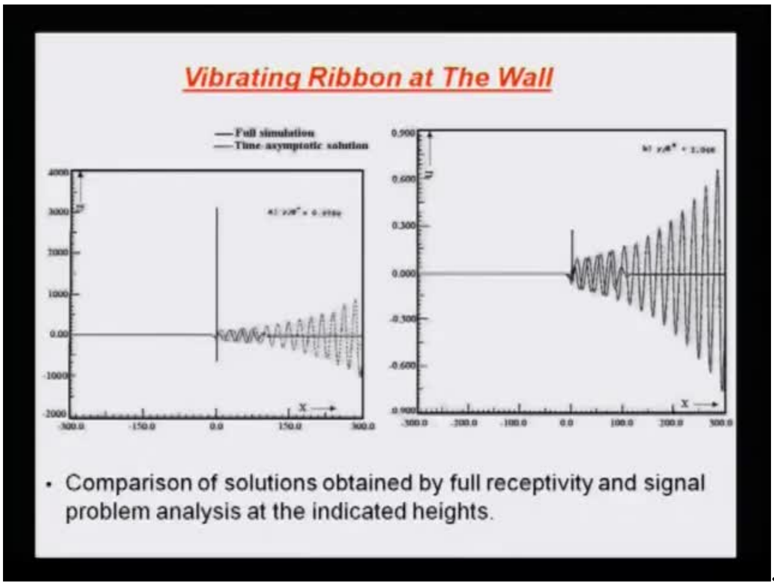

And now, if I do this analysis, what do I get? Here we got something almost 20 years ago. I show you two results - one I called it as the full simulation; the other we term it as time asymptotic solution. So this time asymptotic solution is nothing but the solution of a signal problem, and full simulation is this part, and what you notice that when you do the full simulation, these are shown at some distinct height. So this is near that inner maxim we talked about,right, where the signal is strongest for the disturbance sub quantity. So that is what we see, that it goes like this, and this is for some particular time instant.

When we look at, when we look at the signal problem, we do not have anything to worry about time, why? Because we expect that this thing will be all pervading; that is what we have to, but when it comes to the full simulation, it will depend on where exactly we are looking at, because with time, this thing will increase; this thing will increase - the stretch of the packet. So this is going to grow like a packet and go outside, and this is a solution which is obtained at the outer maximum.

So this you can see that where they exist together, they are in distribution. Only thing that you are seeing some bit of difference is efficient phase shift, and that phase shift should come from where? Well, of course, how you start off the problem; so that is that. I mean, I could have phase shifted the time asymptotic solution and made it match here, but we are purposely showing it next to each other, so that you can distinguish, but you can really see that this is what you have. And what you notice, that the signal problem indicated the combination for this case is exactly the same or equal to 1000 and omega equal to 0.1, that solution, and the full simulation solutions are identical. And that made us conclude, that indeed in some instances, the full simulation and the signal problem are just the same, if we are looking at unstable system. And this kind of thing that we talk, talked about will show - that this kind of spatio-temporal wave front that we talked about - exist for both stable as well as unstable system.

So when it comes to unstable system, we see that the full simulation and the signal problem yields the same result. And you may be thinking that, this I am contradicting myself by identifying this as the inner and this is outer maximum, but please do not… why scale. This is less than 1 and these are in thousands. So that is the kind of distinction that you get between inner and outer maximum.

Well, I also pointed out that despite this, that when you come to experiments, you are more comfortable in measuring this signal at the outer maximum, because of a larger range over which it exist. So the measurements are more reliable; they are not hyper sensitive to the location, while inner maximum is very difficult to track. Of course, today, probably with mechanized traverse mechanism, it should be probably easy to scan it and people can probably measures the whole stretch, not necessarily only at inner or outer maximum.

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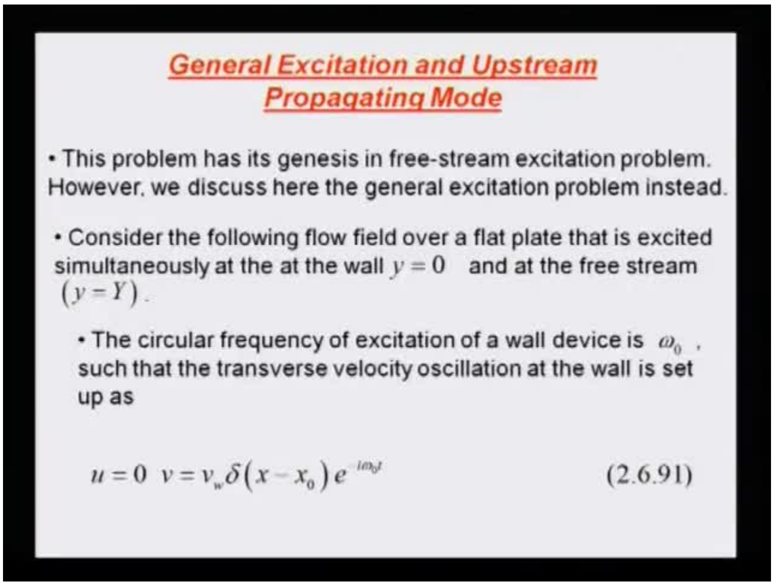

So this is something we want to talk about, next is - we are so far talking about disturbances which are generated by some excitation source being placed inside the shear layer.

What happens, what happens when we look after general excitation problem? What is a general excitation problem? General excitation problem is excitation is everywhere, like what we actually encounter; let say we fly a aircraft, it is not necessary that flow instability is caused by, let us say, the aircraft wing vibration, that would be like your wall excitation problem, but suppose I have designed an aircraft, the wing is quite rigid or the flight condition is such that the wind vibration is imperceptible, but then, still I could have disturbances coming with the oncoming flow, like what we call as the free stream turbulence problem. If we have such a situation, where we could have excitation simultaneously at the wall plus at the free stream, we want to want to study some such problem.

So, let us say, once again we are exciting the shear layer from inside by putting a vibrator at the wall itself. So we have u equal to 0, v is nonzero, and now, we are positioned exciter at x equal to x naught, because now we see, we do not have that liberty of fixing the x-axis, because now what is happening, we have the excitation here as well as excitation in the free stream (Refer Slide Time: 27:32 min). So, you know, just for the sake of convenience, we would be talking about a problem of this kind that we will probably fix the x-axis at the leading edge itself.

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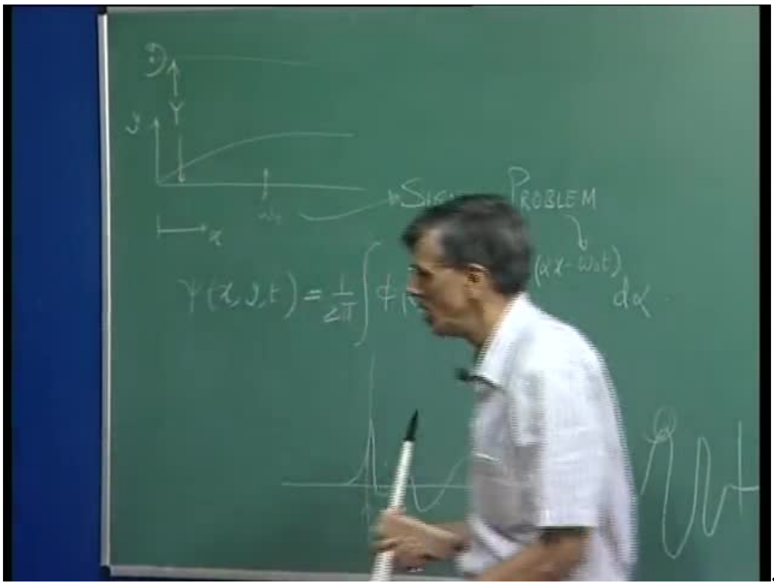

So we talking about this kind of a thing that we have excited somewhere here, and we fix the origin here. And of course, y-axis starts up here and at y equal to capital Y, let us say, we allow some kind of a vortex to convict at a fixed height and that height is what we are calling as capital Y. So that is why you notice that the wall excited is located at a location given here x naught. So that is the two wall conditions, that is what we have done.

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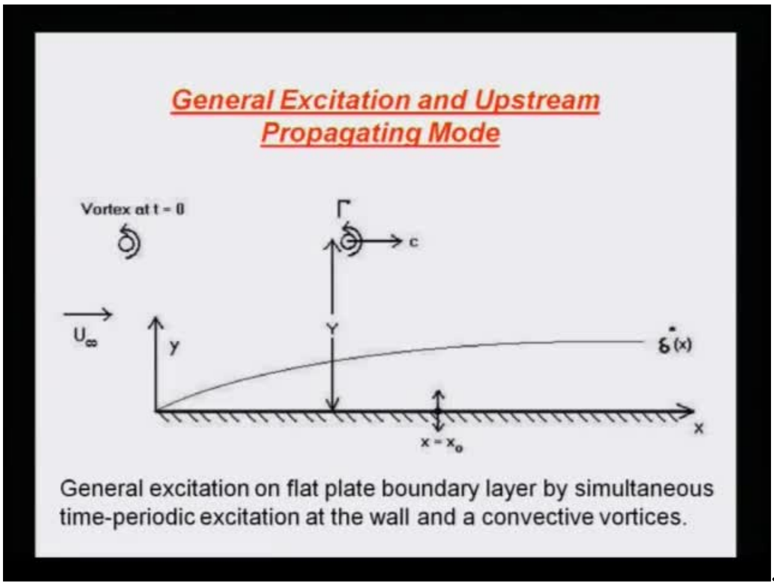

Now what we are going to do is, we are also going to provide a disturbance in the free stream. Now you see what will happen? You recall that in Orr-Sommerfeld equation we have four fundamental modes, to decay with y, to grow with y, but now, in this general excitation case, what we need to do? Can we throw away those two modes which were growing with y? Because eventually match up to this disturbance level, I cannot just stay with the wall modes, because the wall modes decay, but if I give a finite disturbance here, I must keep the other two modes also alive, but unfortunately, if you notice in our eagerness to solve this stability problem, we have all along only considered wall modes; we did not even conceive of this problem. So now what we are talking about that we need to keep those neglected modes before, and then what happens, then we still can do stability study; it is just people are not careful, they were some kind of in a rush to somehow get some results and they just simply looked at the wall mode. We can do that and that is what we are, I am explaining, that how you can set this problem up.

So at time t equal to 0, let us say, the vortex was here; at a subsequent time, it is come there and it is flowing at a constant speed c. I particularly like this problem, because what you are noticing that we have in general an equilibrium flow which is steady, we are imposing a time scale by a vibrator here, and what about this vortex? This vortex is going at a constant y, at a constant speed c. We will see later on, that when we switch this off and we look at this problem alone, that a steady convection will excite system at impose it some additional time scale due to dispersion relation. So this is something that we would see that; that is the whole intention; that even though we look at a steady problem, we still can get unsteady solution.

Now that vortex that we saw in the previous slide, it is essentially we are starting a 2D problem. So what is the structure of this vortex? This is a line vortex perpendicular to the board; it goes from minus infinity to plus infinity in the z direction, right?

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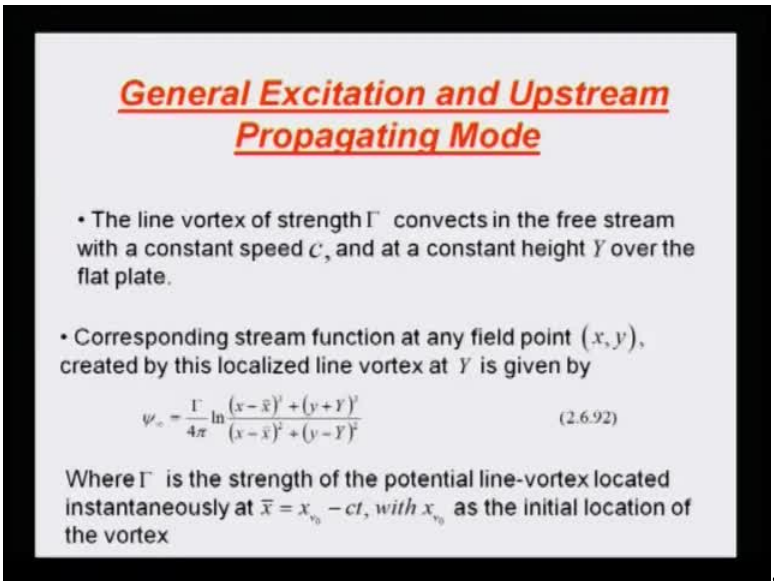

So that is what we have. We have a line vortex, at height capital Y, and which has a strength of gamma, which convects with constant speed c, at a constant height y over the (( )). Now what happens? The moment I have that vortex, I create a flow field, a disturbance field, for which a stream function can be given like this. What do we get? This the gamma is the strength. So gamma by 4 pi; it is a 2D flow; this is the l m of x minus x bar plus this y plus capital Y whole square and the denominator is this. How did you get this? I purposely did not explain. I want you to help me in understand - what is the nature of the problem that we are talking about.

So now you see what is happening. I have a vortex here, which is far outside the shear layer. So this is the viscid part of the flow. Now this vortex could be a viscous vortex; it could be an irrotational vortex, does not matter. The moment I place it in front of a solid wall to satisfy the no-slip condition, no-penetration condition, well, there will be a slip velocity, but to satisfy that there is no net wall normal velocity, I must have a image system.

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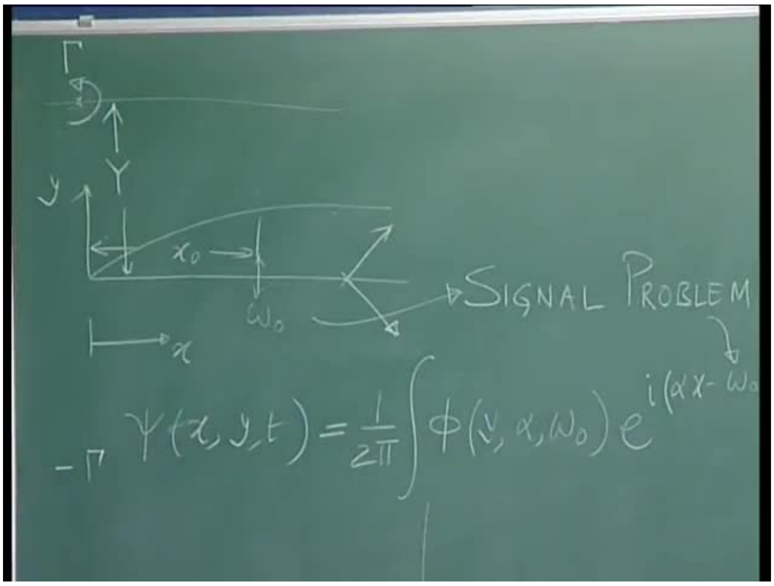

So once I have a gamma there, I should have a minus gamma here, so that if I look at any point on the wall, this will introduce, you can connect this, this will introduce a velocity in this direction, and this one will introduce a velocity in this direction, so that the wall normal component cancels and they two provide a slip velocity. You see that this is a description of the inviscid way of looking at this vortex.

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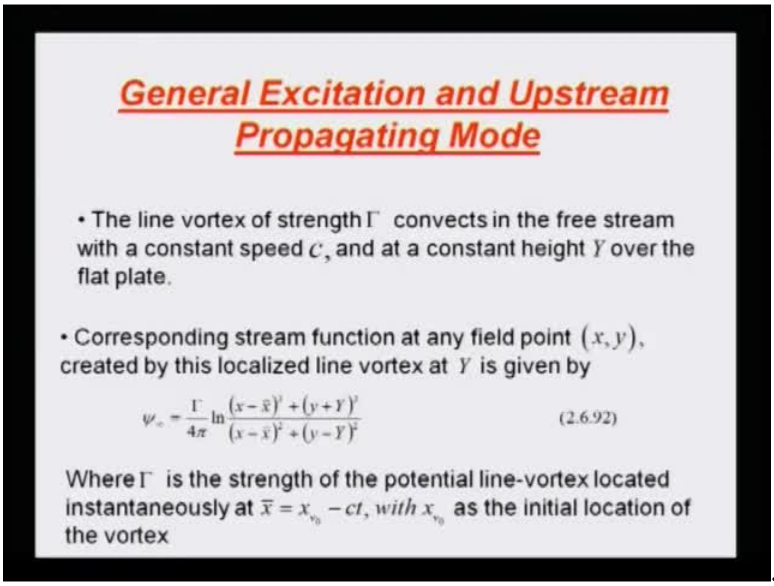

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Now what happens? This vortex itself moves at a constant speed c. So this x bar that a appears here will be a function of time. So that is what it is. If my initial location is x v naught, then I will take it minus ct, so that it moves with it. So at any instant of time, this is the imposed disturbance field by the vortex at any arbitrary x and y.

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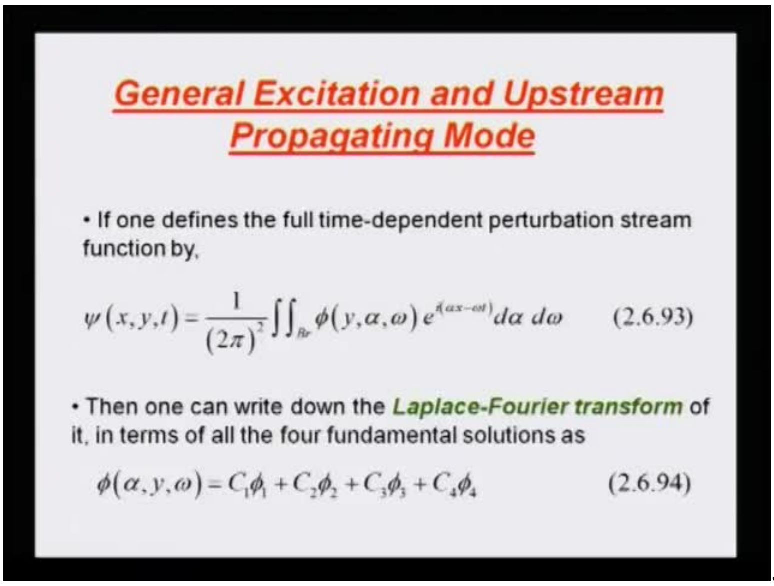

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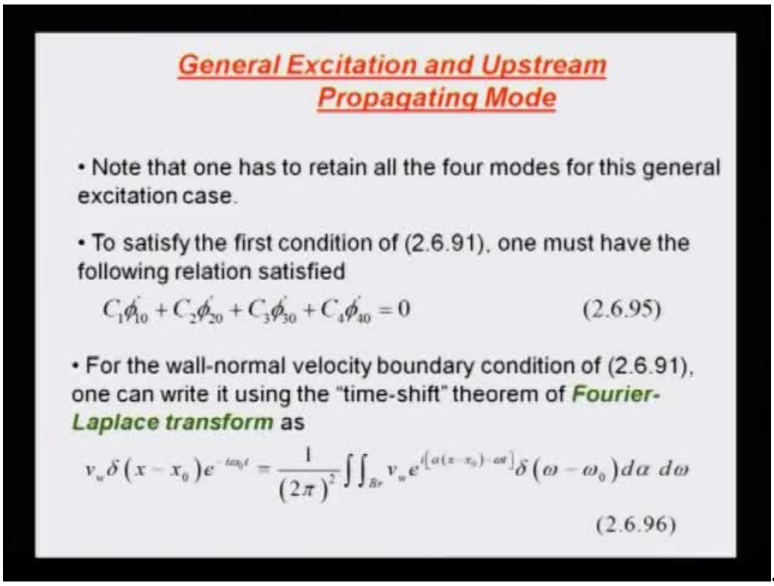

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While we are doing it, because we are actually creating a creating a disturbance field, because of this. So now what we do is we once again write down the disturbance plane is a full problem. However, we note now, this one should have all the four fundamental solution; this is the essential difference. Now, if we keep all these four conditions, well we can still do one thing. See like the inviscid solution corresponding to this gamma and its image gives rise to slip velocity, but still we are inside the boundary layer; we still should support a no slip condition; that is what we ensure by demanding this, that this c 1 c 2, c 3, c 4 should be such that a prime quantity, so that is del psi del y at the wall should be equal to 0; at the wall is indicated by addition or subscript 0, right?

Now we have seen that we have a wall exciter. The wall exciter is determined by that the magnitude v w; it is located, let us say, at x equal to x naught. It is excited at a frequency omega naught, and then, we can we can write it like this. ((Is it not?)). So once again, I am writing it like this, I alpha x minus x naught, that is because of that shift theorem, right? Because this is not at x equal to 0 but at x naught; that is why I have shifted it alpha x minus x naught.

What about omega? Well we are once again looking at the corresponding signal problem; that is why we have put it delta of omega minus omega naught. Well, we can also fill it up with the actual problem like what we have done here, we can do this exercise.

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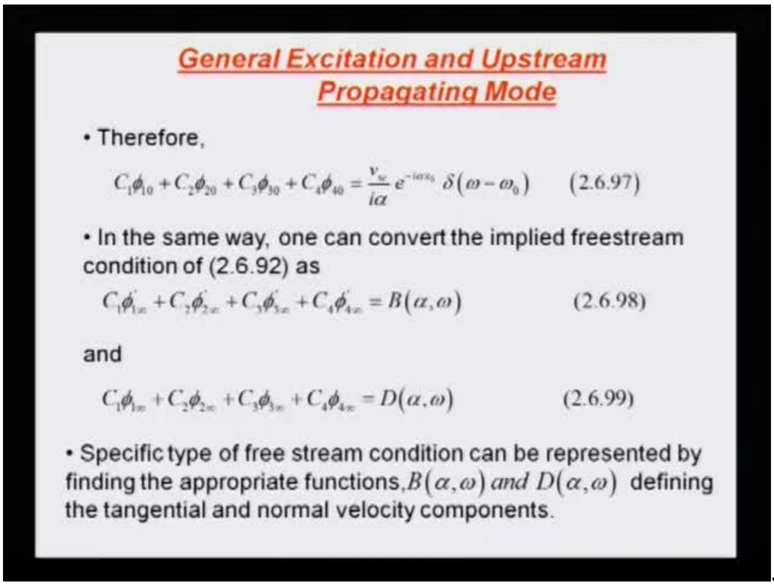

So what happens is in terms of Fourier Laplace transform that v velocity condition could yield this quantity, because we are looking at del psi del x. So del psi del x will give us I alpha times this phi. So what we have written here is in phi here, I alpha has been transported in downstream and this is what we are getting for the signal problem.

So what happens is, this is the condition at the wall. So what we can do is, we can similarly set up the other two conditions in the free stream; those free stream conditions are identified with this added subscript here - infinity; this is corresponding to the u velocity; this is corresponding to the v velocity. So, basically, you are looking at the effect of free stream excitation through this right hand side B and D. I am just leaving it very general, but for a very specific type of excitation, we can evaluate this quantity, right? We just now have seen if I have a discreet vertex moving at a constant speed c, I can immediately work out what is the expression of B and D.

Remember what we have written down? That expression psi infinity, that is for the disturbance quantity, right? And that is why we are matching those disturbance u and v with respect to this equation 98 and 99. Now, we have done this, what we have here is essentially four equations for the four unknown constants c 1 to c 4. So that is….

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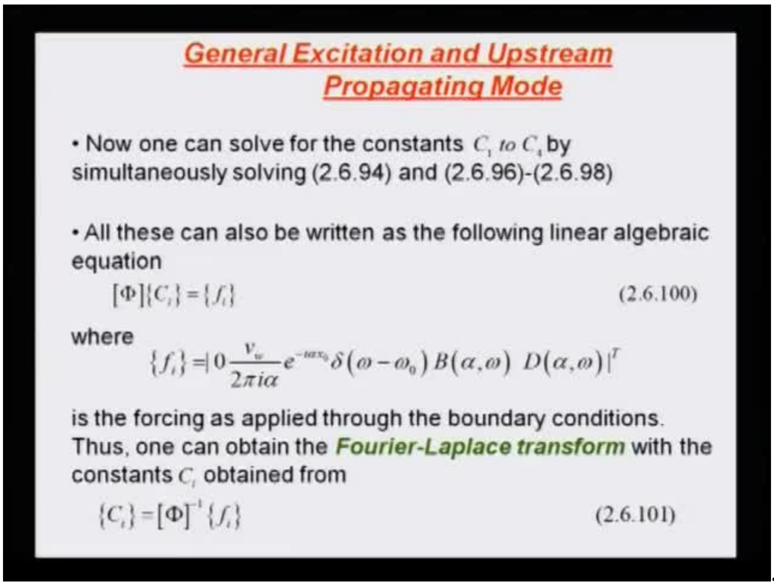

When we write it down, we can write down in a linear algebraic relation like this. Some matrix phi operating on this unknown vector c 1 to c 4 and this is our post excitation; this is our forcing; that is what we are given; the first two comes from the wall condition; the last two comes from the free stream condition. What happens? Well, I can immediately obtain c i, c 1 to c 4 as phi inverse times this, right? phi inverse times excitation in the spectral plane. So, you notice that immediately, you can smell the Eigen value problem, because of the existence of that phi inverse, because what we would it involve? It would involve in the denominator and determinant of phi matrix, right? So that is how we actually solve the wall excitation problem earlier too; there we did not do it for four, we did it for two constants - c 1 and c 3.

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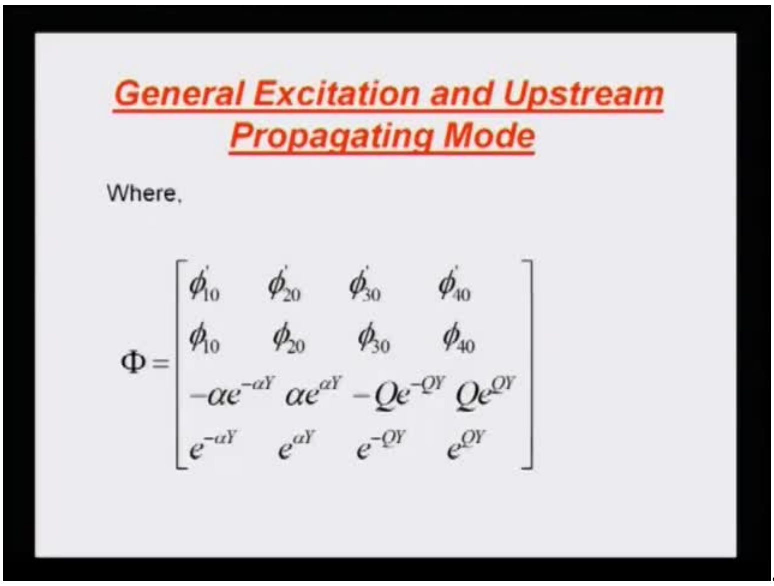

So here we are generalizing; so the moment we do that, we can immediately see that the phi matrix comes from this. You can clearly identify, the first row corresponds to the u velocity component, right? That will give you phi 1 prime, phi 2 0 prime, phi 3 0 prime, and phi 4 0 prime. The second line corresponds to your wall exciter; the weak v velocity that will be in terms of phi 1 0, phi 2 0, phi 3 0, and phi 4 0.

What about these two? Well, this is a good news, because we already know the analytical structure of the solution far away from the wall. We know the analytical solution of, for the four modes of Orr-Sommerfeld equation, but we find that they are these, right? One is Minus e to the power minus alpha y; the other one is e to the power of plus alpha y; here the third components e to the minus QY; the fourth component is e the power plus QY.

Earlier what we did, in the presence of wall excitation alone, we just knocked these two part up and we were looking for those two solutions. Now this corresponds to your u velocity, right? I differentiate it with respect to Y and the obtain the quantity at capital Y. This is what we get; it is pretty good. You see now what happens. We are not trying to solve the problem; we are trying to understand what happens to our dispersion relation in this case.

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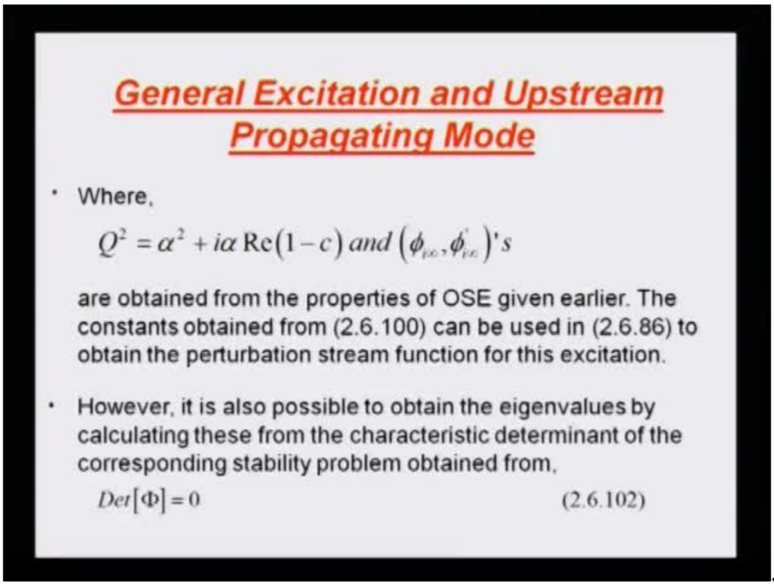

And this dispersion relation will come from where? Where the determinant of this phi matrix is equal to 0. And this is just what we said earlier that this is a Q squared is these and these are obtained from the properties of Orr-Sommerfeld equation given earlier.

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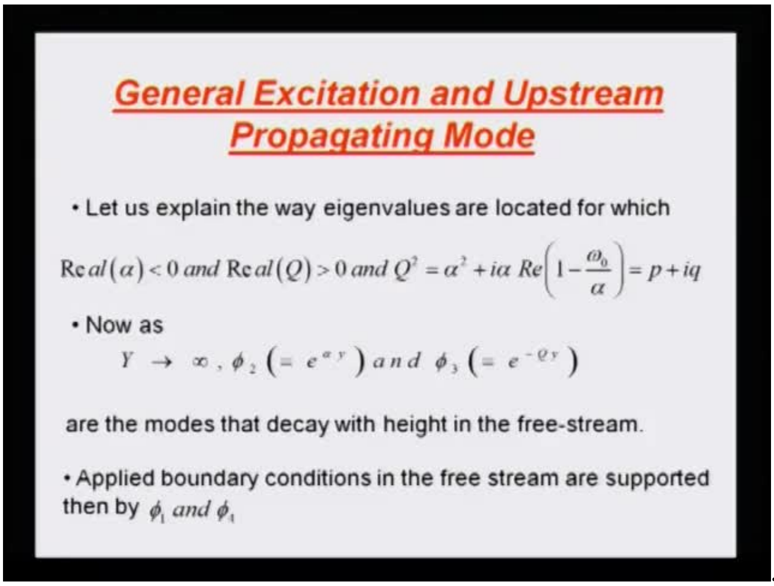

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But the characteristic determinant or the dispersion relation is obtained by putting the determinant of phi equal to 0. This is this is the actually the story, rest is all details. So, once you obtain this, then we can work it out. Now how we can obtain modes? Now what is happened, you see. When we are looking at solutions in alpha plane…

Now earlier, so far, we have somehow keep quiet about the left half plane; we have all along talked about the cases where alpha r equal to 0, the alpha r is greater than 0, why? Because we are looking at wall modes, and if we are trying to find out the waves that grow in the positive direction - downstream propagating wave, then alpha r has to be positive, because that is what we get if I scale out alpha r here, then I will get x minus ct, but now what happens? Because we are keeping all the four modes alive, we cannot just simply talk about only this part; we will also have to talk about i, minus i of alpha r i. So we are basically talking about the modes - Eigen values - which could be here; it is interesting. If you look at many research papers… well not many; one of the paper by (( )) and the book by (( )) Henningson….

(( )) Henningson, actually reproduces that figure from (( )), where they try to show this portrait, and they say this with a question mark; that means that - are they there? Not there? They are not very sure, but our discussion very clearly shows that we could also have Eigen values on the left half plane. So how do we look at those Eigen values for which real alpha is negative? That is we are looking at Eigen values in the left half plane, and if they are all Eigen values on the left half plane, where would they go? In terms of the phase, they would go upstream, right?

Please do not misunderstand me, if I say that you look at the phase and decide whether it is going upstream or downstream. To do that we have to again recompute the ((grove)) velocity, and make sure that they are indeed going upstream and downstream, and you also be very, very careful now in talking about the value of alpha i for which we will have stability or you will have instability. Why? Simply is this - that if you are looking at a upstream propagating wave, now alpha i should be what? If it is to grow, alpha i should to be positive; so this is the complimentary picture. So you look at the sign of alpha i, and you also calculate the (( )) velocity, and then you make up your mind about which Eigen value corresponds to what.

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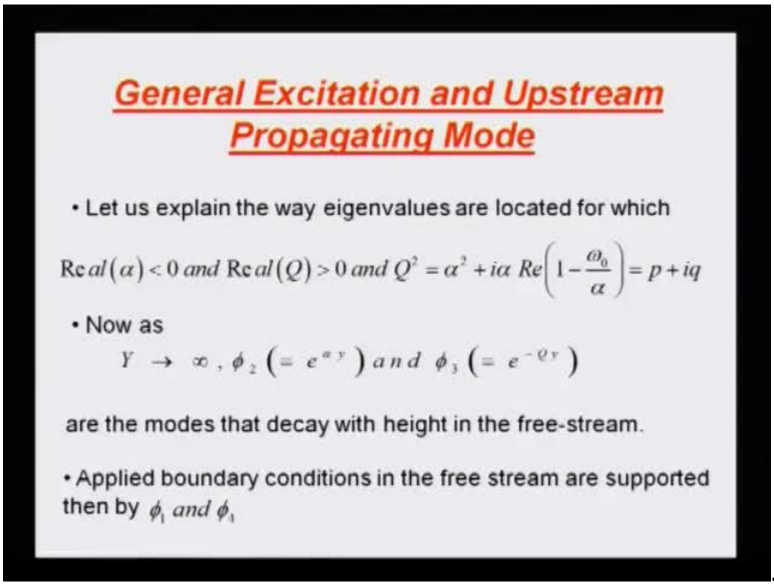

But in general we are talking about that there is a possibility that will have a upstream propagating wave mode. And we are in the lookout for that. So we are looking out for a real alpha, it is negative. Why do you do this? This part. You see Q has been always obtained from an algebraic relation, and when I compute it using a computer, I always get the plus part, and I know there is a plus part, and there is a minus par,t that is what we have. So suppose, we do this, we are now trying to find out what happens. Now as Y goes to infinity, this capital Y goes to infinity, what happens to phi 2 and phi 3?

If my alpha i is non-negative real part, then this will decay. This is also is going to decay. So these two phi 2 and phi 3 are now the decaying modes, whereas phi 1 and phi 4 will be the growing modes with Y. So, basically then, what we are in a sense saying, that phi 1 and phi 4 will support the applied free stream excitation, whereas phi 2 and phi 3 will support the wall excitation boundary condition. This is what we need to appreciate and keep the back of our mind.

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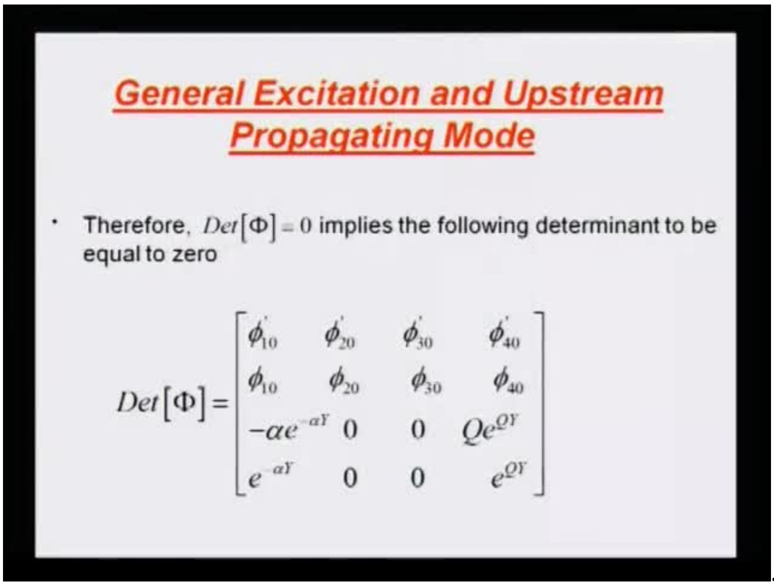

So now you see what happened? We are looking for modes in the left half plane. So what happens? Here what we had? Here we had alpha e to the power plus alpha Y. So if I talk about capital Y going to infinity, this actually goes to 0, does not continue. Same thing, this also does not continue; same thing about here. Now what happens? We are in the lookout for determinant of phi and this is the structure of phi for modes on the left half plane. So we are actually looking out for the Eigen values in the left half plane. So now we have gotten it. So you work it, you work out this expression; now it is very easy.

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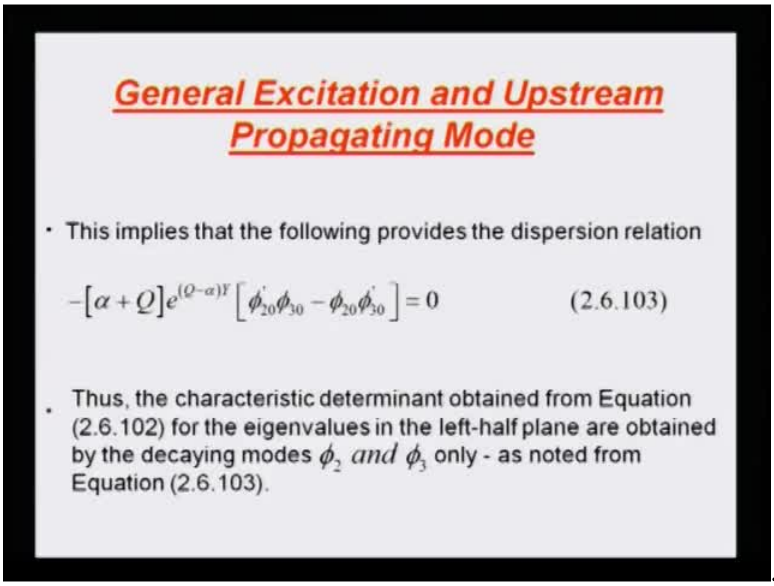

You can expanded problems using the last row, and then, you get this, you get this. Please work it out, and convince yourself that you will get the determinant is equal to minus of alpha plus Q e to the power Q minus alpha into Y, and into this. And you see the dispersion relation is uncannily resembling what we had earlier also. Earlier also we had the same dispersion relation constructed out of the decaying modes; here also, phi 2 and phi 3 are decaying modes. So it is a sort of a very fortuitous occurrence that I could use the same code and I could locate the Eigen values in both the right half as well as left half and I will get both the modes.

In fact, I will tell you a bit of a story telling, when I was in one of the leading UK university, I located some of these Eigen values, and I talked to the person with whom I was working, he said, use whitener, because you cannot explain what it is. But now, you can see, we can explain everything today to our satisfaction, that if we do identify using the same code a Eigen value also on the left half plane, they will correspond to those waves for which the phase is propagating upstream. So when we are talking about upstream propagating mode, we are basically talking about phase propagation, and then you can use the same code, and we can obtain this.

So this question mark is unwarranted. And this is what we did about 15 years ago, and we published a paper, and what we found out was completely accidental. This is what I always like to say is serendipity of research, you start looking for something, you look for worm, you dig up a snake. So this is what actually happens. What we are looking at, we were trying to looking at completely different problem.

You recall that when the German school, it was proposing all the stability theory the English school was trying to oppose it. Why? Because G. I. Taylor performed an experiment. What he did, he to a boundary layer, and then he created some wall x i term, and he did not see anywhere, and then he d that Tollmien-Schlichting waves do not exist, it is only a theoretical artifact, I did the experiment and it is not there.

But what G. I. Taylor overlooked or no one pointed out to him at that time is the frequency; he used a sort of a circular patch which was vibrated at a low frequency less than 5 hertz; whereas, Tollmien and Schlichting calculation shows waves, the stability curves, show the waves to be in the moderate range, in terms of say more than 10 hertz or so.

Now what happened is, we wanted to really see what happens, because we were also intrigued by another paper, by that same professor from that leading UK university, they also repeated G. I. Taylor's experiment, and they also reported exactly same findings, and they just came out with observation saying that for very low frequency waves instability is not there, but there is no explanation. So we were interested. Why it is not there - we wanted to explain.

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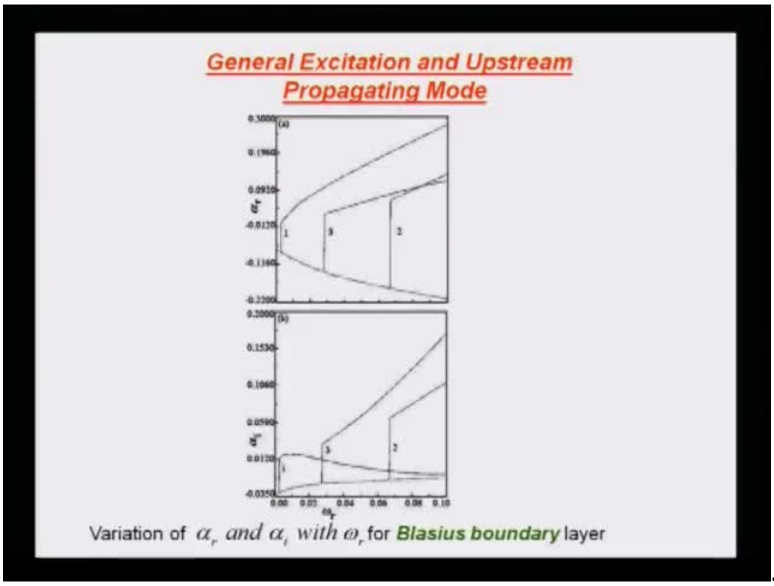

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Now let me just cleanup the blackboard and show you how we proceeded with the problem. The problem was simply this - we are looking for the Eigen values. So what we do is, let us say, we plot on this axis. May be I have some figure; I will just jump to that figure and show it to you; here it is. If we say plot say on this axis omega r and on this axis; in the top figure I am plotting alpha r; in the bottom figure I am plotting alpha i.

Now what happens is suppose I am looking at a problem of Reynolds number of 1000 and this value is let say 0.1. We know they are three Eigen values; one, two and three. Please note that there is a alpha r equal to 0 axis. So on this side you have positive; below you have negative. Now what happens is… that is the value we have obtained for alpha omega r equal to 0.1. So what we started doing is keep reducing the omega naught, and then see what happens to this three nodes. And we saw that they keep going like this.

For example, this what we called as a second mode in the previous table, when we came down to this value of omega r, it just simply disappeared. So we did that grid search, at these values we only found two Eigen values - one corresponding to this; one corresponding to that. We further went on reducing it. Then after this we found that there is only one Eigen value; the third one also has disappeared. We keep doing it, keep doing it, keep doing it, and come up to here. And beyond this we have no Eigen values and this is over exactly what that paper in physics of fluid was talking about, but they are no theories, if your frequencies are too low, because you cannot do that, but then, I recall that 20 years ago, I did find some modes, and I find the that mode is this.

So what happens is, after this, I do actually have a mode corresponding alpha i curves are like this. So these are all positive and we can show from the (( )) velocity they are also positive. So they are all decaying modes two and three, while the one is slightly negative, because your alpha i equal to 0 axis remains slightly above this line. So it remains unstable for some frequency, then it became stable and before disappearing it go in there, and this one we get it there.

Now you see what has happened. This is, this remains negative throughout. So you have when we first found out, we were worried, because alpha i is negative; alpha r is negative; so what does it mean? Well, of course, then you do is calculate the (( )) velocity and ensure what it is.

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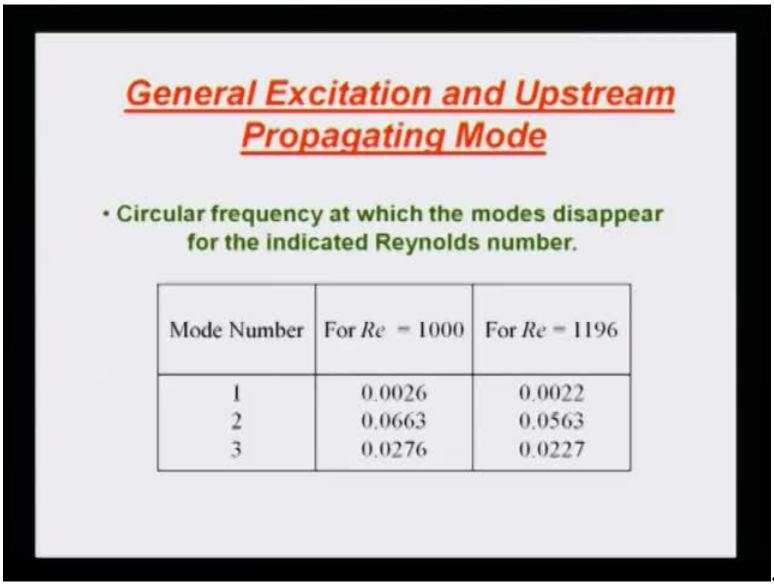

And since we are talking about here, the properties of Blasius boundary layer, so what you see is this, and this is a basically a chart, which shows those three modes one by one disappeared, right? The third mode disappears for r e equal to 1000, at this value 0.0276. The second mode disappeared earliest, right? That was around 0.0663. And the first mode also disappeared when omega naught is reduced to 0.0026. You can actually convert these values into hertz and there it will be. So and this was the physics of fluid paper that reported for a Reynolds number 1196.

We will start from here in the next class.

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